Bayesian estimation of a DSGE model for the Portuguese economy*

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This version: September 25, 2009

Abstract

In this paper, a New-Keynesian DSGE model for a small open economy integrated in a monetary union is developed and estimated for the Portuguese economy, using a Bayesian approach. Estimates for some key structural parameters are obtained and a set of exercises exploring the model's statistical and economic properties are performed. A survey on the main events and literature associated with DSGE models that motivated this study is also provided, as well as a comprehensive discussion of the Bayesian estimation and model validation techniques applied. The model features five types of agents namely households, firms, aggregators, the rest of the world and the government, and includes a number of shocks and frictions, which enable a closer matching of the short-run properties of the data and a more realistic short-term adjustment to shocks. It is assumed from the outset that monetary policy is defined by the union's central bank and that the domestic economy's size is negligible, relative to the union's one, and therefore its specific economic fluctuations have no influence on the union's macroeconomic aggregates and monetary policy. An endogenous risk-premium is considered, allowing for deviations of the domestic economy's interest rate from the union's one. Furthermore it is assumed that all trade and financial flows are performed with countries belonging to the union, which implies that the nominal exchange rate is irrevocably set to unity.

Keywords: DSGE; econometric modelling; Bayesian; estimation; small-open economy.

JEL codes: C10, C11, C13, E10, E12, E17, E27, E30, E37.

*This work has resulted from a master thesis in "Applied Econometrics and Forecasting", performed at ISEG. I would like to thank my advisor Maximiano Pinheiro for his guidance and support. I am also grateful to Jesper Lindé, Malin Adolfson and Rafael Wouters for valuable materials on their own research, and to Wouter Den Haan whose clarifications were of great use. Helpful insights were also gained at the 2008 Dynare Summer School with Michel Juillard, Stéphane Adjémian, Wouter Den Haan and Sébastien Villemot and at the 7th EABCN with Lawrence Christiano and Matthias Kehrig. Special thanks go to my fellow workers Ricardo Félix, Gabriela Castro and José Maria for their important support and contribution to the success of this research and to Banco de Portugal for providing financial support and access to a world of data and materials. A detailed Appendix is available upon request. The views expressed are mine, along with any errors or mistakes.

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1 Introduction

The modelling of macroeconomic fluctuations has changed dramatically over the last thirty years. When traditional large-scale macroeconometric models used in the 1960s and 1970s came under severe critiques, the need for a new paradigm emerged. In their seminal paper, Kydland and Prescott (1982) provided an answer by proposing a new type of model, where private agents had an optimising behaviour, benefiting from rational expectations and acting in a Dynamic Stochastic General Equilibrium (DSGE) framework. The article was in the genesis of a new way of studying macroeconomic fluctuations, the so-called Real Business Cycle (RBC) approach to macroeconomic modelling, which became one of the main toolboxes of macroeconomic research.

Although RBC models provided a fundamental methodological contribution, they soon proved to be insufficient, especially for policy analysis in central banks and other policy institutions, giving rise to a new debate in the field of macroeconomics. A new school of thought emerged from the debate, in the 1990s, the so-called New-Keynesian Macroeconomics. Although sharing the basic RBC structure, this new approach viewed the sources and mechanisms of business cycles in a very distinct way, introducing a wide set of new assumptions into DSGE models, which proved to be extremely successful among both the academia and applied economists.

In parallel with theoretical developments, major advances were also accomplished with respect to the econometric apparatus associated with DSGE models. Bayesian techniques emerged as the most promising tools to estimate and quantitatively evaluate these models, bearing important advantages when compared to the other available methods both from an economic and statistical point of view, especially for medium to large-scale models. Many studies documented the empirical possibilities and usefulness of these models, even when compared with more traditional econometric tools, making DSGE models attractive not only because of their theoretical consistency but also because of their data explanatory power.

The combination of a strong theoretical framework with a good empirical fit has turned New-Keynesian DSGE models into one of the most attractive tools for modern macroeconomic modelling and has led to their widespread use not only in the academia but also in a number of policy-making institutions. They are considered to be a privileged vehicle for economists to structure their thinking and understand the functioning of the economy, being used for a number of purposes, from policy analysis to welfare measurement, identification of shocks, scenario analysis and forecasting exercises.

New-Keynesian DSGE modelling and estimation is therefore one of the most interesting and

promising fields in modern macroeconometric research, from which no country should be left out. In the case of Portugal, although some exercises have already been performed using DSGE models, no attempt has been made to estimate a New-Keynesian DSGE model, to the best of my knowledge. In this article, I hope to contribute to fill this gap by developing and performing Bayesian estimation of a model of this type for Portugal. Estimates for some key structural parameters of the Portuguese economy are obtained and a set of exercises that explore the model's empirical properties and the results' robustness are performed. A survey on the main events and literature associated with DSGE models is also provided, as well as a comprehensive discussion on the estimation and model comparison techniques used in the study.

The model is greatly inspired in the works of Adolfson, Laséen, Lindé and Villani (2007) and Almeida, Castro and Félix (2008). The first one corresponds to a model estimated for Sweden (which like Portugal is a small-open economy) using Bayesian techniques. The second one constitutes the most recent and comprehensive DSGE model developed for the Portuguese economy and therefore, although it is calibrated, constitutes an important reference for this study. It is a New-Keynesian DSGE model for a small open economy, integrated in a monetary union, featuring five types of agents namely households, firms, aggregators, the rest of the world and the government. It includes a number of shocks and frictions, which have proven to enable a closer matching of the short-run properties of the data and a more realistic short-term adjustment to shocks. Monetary policy is totally defined by the union's central bank and the domestic economy's size is negligible, relative to the union's one, and therefore its specific economic fluctuations have no influence on the union's macroeconomic aggregates and monetary policy. A risk-premium is considered to allow for deviations of the domestic economy's interest rate from the union's one. Furthermore, for simplification, it is assumed that all trade and financial flows are performed with countries belonging to the union, which implies that the nominal exchange rate is irrevocably set to unity.

The remainder of the paper is organised as follows: in section 2 I motivate the study and provide some important references; in section 3 I explain the main methodological aspects related to the study; in section 4 I present the model; in section 5, I present the estimation results and evaluation, as well as some data and methodological considerations; and finally, in section 6, I wrap up the main conclusions and discuss some possible extensions of this work.

2 Motivation and background

In this section, a reference to the main events and literature related to the development and estimation of New-Keynesian DSGE models is made, to provide an understanding of what inspired this study and the main options concerning both the model features and the methods used when taking it to the data.

The failure of traditional macroeconometric models

During the 1960s and 1970s, large-scale macroeconometric models were the main tool available for applied macroeconomic analysis. These models were composed by a number of equations linking variables of interest to explanatory factors and while the choice of which variables to include in each equation was guided by economic theory, their coefficients were mostly determined on purely empirical grounds, based on historical data.

In the late 1970s, these models came under sharp criticism. On the empirical side, they were confronted with the appearance of stagflation, which was incompatible with the traditional Phillips curve included in these models. Furthermore, Sims (1980) questioned the usual practice of making some variables exogenous, which excluded meaningful feedback mechanisms between the models' variables. But the main critique was on the theoretical side and came from Lucas (1976), corresponding to the so-called "Lucas critique". Lucas pointed out that the empirical puzzle of stagflation was just a reflection of a more general theoretical problem. He noted that agents behave in an optimising dynamic way and form rational expectations, adapting both their current and future behaviours to anticipated changes in the economic environment. Being exclusively based on the past, traditional models simply assumed that economic relationships that were valid in a previous context would be able to explain developments in the economy even if the context changed, without considering that the anticipation of those changes by agents could alter the way they reacted, invalidating the previously estimated relationships.

Rise and fall of RBC models

As a response, economists of the 1980s departed to a new paradigm, whose genesis can be found in the seminal work by Kydland and Prescott (1982). In this article, the economy was modelled in a structural micro founded way, with economic agents making decisions and forming expectations in a DSGE framework. The model economy was perfectly competitive and frictionless, with prices and quantities immediately adjusting to their optimal levels after a shock. Fluctuations

were generated by the agents' reactions to a random technology shock, with business cycles simply resulting of an efficient response of rational optimising agents to the shock.

The model was largely adopted by macroeconomists, who introduced several sophistications over the years, exploring its theoretical and empirical possibilities. This became known as the RBC approach to macroeconomic modelling, constituting a crucial advance in modern macroeconomics, by firmly establishing DSGE models as the new paradigm of macroeconomic theory.

Despite their important methodological contribution, RBC models soon came under criticism. The main issue was that, with completely flexible prices, any change in the nominal interest rate was always matched by one-for-one changes in inflation, leaving the real interest rate unchanged. This meant that monetary policy had no impact on real variables, a result that was at odds with the widely held belief that it could influence the real side of the economy in the short run. Furthermore, since cyclical fluctuations resulted from an optimal response to shocks, stabilisation policies were not necessary and could even be counterproductive, as they would divert the economy from its optimal response. This was in sharp contrast with the Keynesian view that the troughs of the economic cycle were mainly due to an inefficient utilisation of resources, which could be brought to an end by means of economic policies. In addition, the primary role attributed to technology shocks was at odds with the traditional view of these shocks as a source of long-term growth, unrelated to business cycles, which were mostly considered to be demand-driven. Finally, the ability of RBC models to match the empirical evidence began to be disputed, as they were not able to reproduce some important stylised facts. These issues determined that although RBC models had a strong influence in the academia, they had a very limited impact on central banks and other policy-making institutions, which continued to rely on large-scale macroeconometric models despite their acknowledged shortcomings.

NKM as the new road ahead

The insufficiencies of RBC models began to be overcome in the 1990s with the birth of a new school of thought, the so-called New-Keynesian Macroeconomics (NKM). This school shared the RBC belief that macroeconomics needed rigorous microfoundations, also using DSGE models as their workhorse, but rationalised the business cycle in a substantially different way. They considered that the economy was not perfectly flexible nor perfectly competitive and that instead it was subject to imperfections and rigidities, with these being key elements to understand the dynamics of the real world. Based on this view, NKM economists introduced monopolistic

competition and various types of nominal and real rigidities, as well as a broader set of random disturbances. Some notable examples are: the introduction of sticky prices, following previous studies like Calvo (1983), which allowed for price inertia, breaking the strong RBC assumption of money neutrality; demand shocks as in Rotemberg and Woodford (1995); the extension of nominal stickiness to wages, following Erceg, Henderson and Levin (2000), which have been shown to play an important role in explaining inflation and output dynamics; the introduction of consumption habits, following previous work by Abel (1990), which helped in capturing consumption persistency; price and wage indexation and the inclusion of investment adjustment costs, as in Christiano, Eichenbaum and Evans (2005), which have enhanced the models' ability to capture the inflation persistence present in the data and investment dynamics.

These assumptions generated a meaningful role for monetary and other economic stabilisation policies and proved to be extremely successful in capturing some features of macroeconomic time series that RBC models had previously missed, determining the spreading of DSGE models from the academia to policy-making institutions.

Econometrics gathers with economic theory

In parallel with theoretical developments, major advances were accomplished concerning the econometric apparatus associated with DSGE models. Numerous procedures emerged to parameterise and evaluate DSGE models, which can be split into limited or full information methods.

In the first approach, a DSGE model mimics the world only along a particular set of dimensions. This idea is behind calibration, the method originally proposed by Kydland and Prescott (1982), which simply attributes values to the parameters, based on information from previous studies and common knowledge, such that the model is able to replicate some selected moments of the data. Another method that fits in the limited-information approach is the Generalised Method of Moments (GMM), used for e.g. in Christiano and Eichenbaum (1990), where parameters are chosen in a way that selected equilibrium equations are verified, as precisely as possible. In addition, there is the method proposed in Christiano et al. (2005), where parameters are estimated by minimising the difference between estimated impulse response functions (IRFs) to shocks in an identified VAR and those based on the DSGE model.

In contrast, the second approach takes the whole set of implications of the model into account, attempting to obtain estimates that provide a full characterisation of the observed data. There are basically two methods that fit into this category, Classical and Bayesian Maximum

Likelihood Estimation (MLE). They both rely on the specification of a probabilistic structure for the model, which enables the construction of the likelihood function that expresses the probability of observing a given dataset as a function of the model's parameters. The likelihood can be computed for different combinations of parameters, to find the combination that produces the maximum value for the function, i.e. that makes the data set (that we know to be true) "more likely". In Classical MLE, parameter estimates are directly obtained from this process. This method has been applied by Kim (2000) and Ireland (2001), for e.g.. In Bayesian MLE, an additional function is considered, the prior, representing the probability that the researcher attributes to different possible values of the parameters before observing the data, based on previous studies or on his personal beliefs. The prior is then combined with the likelihood and the resulting function can then be maximised, with respect to the parameters until the combination of parameters estimates that produces the highest value for the objective function is found, i.e. the combination that makes both the data set and the imposed a-priori beliefs to be "more likely". The application of these techniques to the estimation of a DSGE model was firstly done by DeJong, Ingram and Whiteman (2000) and has since then been adopted by several researchers as is the case of Smets and Wouters (2003), Rabanal and Rubio-Ramírez (2005), Adolfson, Laséen, Lindé and Villani (2007), and Christoffel, Coenen and Warne (2008).

Bayesian techniques stand out

When comparing the two approaches, although both have advantages and disadvantages, the superiority of the full-information one is clear. On the economic side, it is appealing since estimates are obtained employing all the restrictions implied by the model, allowing for a more consistent characterisation of the data generation process. On the statistical side, efficiency is enhanced by the use of all available information.

Within full-information methods, Bayesian MLE is the one that has gathered more supporters, with Classical MLE being feasible only for relatively small systems and not for the large-scale models that have been used recently. The main issue has to do with identification problems. In models with a large amount of parameters to estimate it is hard to obtain correct information about all of them from the data, which implies that quite often two problems arise: the likelihood is flat in large subsets of the parameters space, i.e. different values of the parameters lead to the same joint distribution for the observed data, making the maximisation of the likelihood a very hard task; or the likelihood peaks in "strange areas", producing estimates that

are at odds with additional information that the researcher may have, the so-called "dilemma of absurd parameter estimates". In Bayesian inference, the likelihood function is "reweighted" by the prior which produces a function with more curvature that is able to minimise, both the "flatness" and "dilemma of absurd parameter estimates" problems[1]. Furthermore, in Classical MLE, the outcome of the overall estimation process is highly sensitive to the estimated value of each parameter meaning that if one or more parameters are poorly estimated, this may strongly affect the results for all the other parameters. In Bayesian MLE this problem is minimised, since the estimation of each parameter does not embody a particular value for the other parameters being estimated, but instead, it takes a whole distribution into account for each parameter, which encompasses a wide range of possible estimates. In addition, the Bayesian approach allows one to formally incorporate the use of a-priori information, coming either from previous studies or simply reflecting the subjective opinions of the researcher, while Classical estimation cannot incorporate even the most non-controversial prior information. Also, the Bayesian approach provides a natural framework for model evaluation and comparison, through the computation of posterior odds ratios[2]. These have Classical analogues in likelihood ratios, but go beyond the comparison of two parameter vectors of greatest likelihood to involve information about other possible values by comparing weighted-average values of likelihoods, with weights given by the priors. Finally, another important advantage of Bayesian methods is that they produce probability distributions for the model's parameters, IRFs, forecasts, etc., and thereby directly provide a measure of the uncertainty associated with model-based analysis and forecasting.

A large scope of application

The development of a deeper econometric framework surrounding DSGE models has considerably enlarged their scope of application. They are now attractive not only because of their theoretical consistency but also because the most recent vintages, estimated with Bayesian techniques, have made big a progress on the empirical front. Indeed, they have proven to fit economic data quite reasonably and to compare well and in many cases outperform more traditional tools such as Vector Autoregressions (VAR), Vector Error Correction Models (VECM), Bayesian Vector Autoregressions (BVAR), among others. These findings can be found in many reference studies such as Smets and Wouters (2003), Fernandez-Villaverde and Rubio-Ramirez (2004), Del Negro, Schorfheide, Smets and Wouters (2005), Adolfson, Laséen, Lindé and Villani (2005), Juillard,

[1]For more on identification problems in DSGE models see Canova (2007) and Iskrev (2009).
[2]This concept will be defined further on in Section 3.

Kamenik, Kumhof and Laxton (2006) and Adolfson, Lindé and Villani (2007).

An important milestone in this evolution was the development of Dynare, a pre-processor and a collection of publically available Matlab routines for the solution, simulation and estimation of DSGE models[3]. It has enabled an easier access to quantitative DSGE modelling and is by now the software adopted by a large fraction of macroeconomists working with DSGE models.

The combination of a strong theoretical framework with a good explanatory power of empirical evidence has turned New-Keynesian DSGE models into the state of the art instrument for macroeconomists, used for a number of purposes from policy analysis to welfare measurement, identification of shocks, scenario analysis and forecasting. They are the object of attention not only in the academia but also in a number of policy-making institutions, such as central banks, which has brought the later closer to academic research and knowledge. Some prominent examples of organisms using these models are: the International Monetary Fund (IMF) whose model, presented in Kumhof and Laxton (2007), has been used for e.g. to analyse fiscal policy issues; the Bank of Sweden who has used its model, described in Adolfson, Laséen, Lindé and Villani (2007), both for policy analysis and forecasting; the Bank of Finland with the AINO model, that can be seen in Kilponen and Ripatti (2006), which besides being applied to the study of specific issues, like ageing and demographics, is already the official forecasting model of Finland's central bank; and the European Central Bank (ECB) with the New Area Wide Model, exposed in Christoffel et al. (2008), which is being used for a wide range of purposes.

For what has been exposed, New-Keynesian DSGE models and their estimation is certainly one of the most interesting and promising fields in modern macroeconometric research, from which no country should be left out. In the case of Portugal, very few exercises have been performed using DSGE models, namely: Pereira and Rodrigues (2002) (DGE model) and Fagan, Gaspar and Pereira (2004) where calibrated New-Keynesian models are used to analyse the impact of a tax reform package and the macroeconomic effects of structural changes, respectively; Silvano (2006) where an RBC model estimated with GMM is used to study business cycle movements; and Almeida et al. (2008) and Adão (2009) where the effects of increasing competition in the nontradable goods and labour markets and a monetary policy shock are examined, respectively, using calibrated large-scale New-Keynesian models. However, no attempt has yet been made to estimate a New-Keynesian DSGE model using Portuguese data, which has led to the conduct of this study.

[3]For more on Dynare please go to http://www.cepremap.cnrs.fr/dynare/. A reference manual is Griffoli (2007).

3 Methodology

A general overview of the main methodological aspects inherent to this work is now provided[4]. A more detailed description of some issues is provided in the Appendix.

3.1 Model construction and solution

The construction of the model starts with the specification of the characteristics of the economy, its environment, agents, preferences, technologies and constraints, and the set of structural shocks to which the economy is subject. The objectives of each agent are then presented, involving dynamic optimisation problems for private agents and policy rules for the fiscal and monetary authorities. Solving the problems of the optimising agents with the Lagrange multipliers method a set of first order conditions is obtained, defining those agents' decision rules. The model equations are then subject to aggregation procedures, to obtain the behaviour of economic agents as a whole. Furthermore, a set of market clearing conditions is specified, which are necessary to "close" the model.

As a result of this process, a set of equations is obtained expressing the endogenous variables at each period as a function of its past, present and expected future path, a set of parameters and a set of stochastic disturbances. These equations are the starting point of a set of operations which in the end allow for the computation of a stable, unique, solution to the model.

Stationarising the model

The first step is rendering all variables stationary, with a well defined steady-state to which they return after the economy has been hit by temporary shocks. For a model like the one in this work, which comprises a unit-root technology shock and inflation in the steady-state, the following transformations are needed: all non-stationary real variables have to be scaled by the level of technology; all nominal variables have to be scaled by the numeraire of the economy; the consumption lagrange multiplier has to be multiplied by the technology level.

Some issues should be referred at this point. First, variables are scaled with the technology level and numeraire price level of the period in which they are decided and therefore, all predetermined variables such as the capital stock are scaled with the lagged values of technology and/or price. Second, being assumed to grow in line with productivity (which grows with

[4]These aspects mostly correspond to those applied by Dynare, since this was the adopted software.

technology), the nominal wage rate needs to be scaled by both the price and technology levels. Finally, foreign real variables are scaled by the foreign technology level, not the domestic one.

Writing the model in a general form

The set of stationary model equations are then put into the following matrix general form:

$$E_t \left\{ f\left(y_{t+1}, y_t, y_{t-1}, e_{t+1}, e_t\right) \right\} = 1 \tag{1}$$

where y_t is the vector of (stationary) endogenous variables, 1 is a vector of 1's and e_t is the vector of innovations, assumed to be Gaussian white noise processes, satisfying:

$$e_t \sim N(0, \Sigma_e) \quad E(e_t e_s') = 0 \quad t \neq s$$

We want to find the solution of the model, i.e. obtain the path for the model's endogenous variables. For this, we need to categorise them into predetermined (or state) and non-predetermined (or forward-looking) variables. The former are variables that at t are already determined, i.e. totally known at the end of $t-1$, while the later are variables that are only known at t. We want to express the forward-looking variables at t as a function of the available information set composed by the predetermined variables and the innovations occurring in that period. Formally, we wish to obtain a function g(.), called the policy function, such that:

$$y_t = g\left(y_{t-1}, e_t\right) \tag{2}$$

Now note that using (2) we can rewrite (1) as:

$$E_t \left\{ f\left(g\left(g\left(y_{t-1}, e_t\right), e_{t+1}\right), g\left(y_{t-1}, e_t\right), y_{t-1}, e_{t+1}, e_t\right) \right\} = 1 \tag{3}$$

Ideally, we would obtain the model solution directly from analytical manipulation of this system of equations.However, these are highly non-linear, which makes the task of finding the exact expression for $g(.)$ a complex one. Instead, a tractable linear approximation is performed.

Computing the steady-state of the model

To perform the approximation, it is necessary to compute the steady-state of the model, which corresponds to the situation in which there are no innovations and variables assume a constant

value in every period. Denoting the steady-state value of y_t by $\bar{y}$, and using (1) and (2), the steady-state of the model is represented by:

$$f(\bar{y}, \bar{y}, \bar{y}, 0, 0) = 1 \tag{4}$$

$$\bar{y} = g(\bar{y}, 0) \tag{5}$$

Approximating the model with a log-linearisation

Having computed the steady-state, each equation of (1) is approximated by the expected value of a first order Taylor expansion of its logarithm around the steady-state. As a result, a "new" set of equations is obtained defining an approximate linear model whose endogenous variables correspond to the percentage deviations of the original variables from their steady-state. Using these equations, (1) and (2) can then be approximated by the following system:

$$E_t \{f_{y+1}\hat{y}_{t+1} + f_y\hat{y}_t + f_{y-1}\hat{y}_{t-1} + f_{e+1}e_{t+1} + f_e e_t\} = 0 \tag{6}$$

$$\hat{y}_t = g_{y-1}\hat{y}_{t-1} + g_e e_t \tag{7}$$

where f_{y+1}, f_y and f_{y-1} are the matrix derivatives of $f(.)$ with respect to y_{t+1}, y_t and y_{t-1}, respectively, evaluated at the steady-state; f_{e+1} and f_e are the matrix derivatives of $f(.)$ with respect to e_{t+1} and e_t, respectively, evaluated at the steady-state; g_{y-1} and g_e are the matrix derivatives of $g(.)$ with respect to y_{t-1} and e_t, evaluated at the steady-state; $\hat{y}_{t+1}$, $\hat{y}_t$ and $\hat{y}_{t-1}$ are vectors containing the percentage deviations of the original endogenous variables from their steady-state at times $t+1$, t and $t-1$, respectively.

Equation (7) is the solution of the approximate linear model (6), dependent on g_{y-1} and g_e, which are a function of the parameters of the model but whose exact form is yet unknown.

Solving the approximate linear model

To find g_{y-1} and g_e some techniques for solving Linear Rational Expectations (LRE) models are applied, which I present here in only rough terms, since a detailed explanation is considered to be beyond the scope of this study[5]. Start by noting that using (7) we can rewrite (6) as:

$$E_t \{f_{y+1}(g_{y-1}g_{y-1}\hat{y}_{t-1} + g_{y-1}g_e e_t + g_e e_{t+1}) + f_y(g_{y-1}\hat{y}_{t-1} + g_e e_t) + f_{y-1}\hat{y}_{t-1} + f_{e+1}e_{t+1} +$$
$$+ f_e e_t\} = 0 \Leftrightarrow (f_{y+1}g_{y-1}g_{y-1} + f_y g_{y-1} + f_{y-1})\hat{y}_{t-1} + (f_{y+1}g_{y-1}g_e + f_y g_e + f_e)e_t = 0 \tag{8}$$

[5]To obtain a detailed exposition of this subject, please refer to Klein (2000) and Sims (2002).

This equation has to hold for any $\hat{y}_{t-1}$ and any e_t and therefore each parentheses must be null. Thus the matrices g_{y-1} and g_e that we seek must satisfy:

$$f_{y+1}g_{y-1}g_{y-1} + f_y g_{y-1} + f_{y-1} = 0 \tag{9}$$

$$f_{y+1}g_{y-1}g_e + f_y g_e + f_e = 0 \tag{10}$$

Now note that equation (8) can be rearranged in the following way:

$$f_{y+1}g_{y-1}\left(g_{y-1}\hat{y}_{t-1} + g_e e_t\right) + \left(f_y g_{y-1} + f_{y-1}\right)\hat{y}_{t-1} + \left(f_y g_e + f_e\right)e_t = 0 \Leftrightarrow$$

$$\Leftrightarrow f_{y+1}g_{y-1}\hat{y}_t + \left(f_y g_{y-1} + f_{y-1}\right)\hat{y}_{t-1} + \left(f_y g_e + f_e\right)e_t = 0 \tag{11}$$

We can then rewrite our approximate linear model, characterised by (7) and (11) as:

$$Ax_{t+1} = Bx_t + Ce_t \tag{12}$$

$$A = \begin{bmatrix} 0 & f_{y+1} \\ I & 0 \end{bmatrix} \quad B = \begin{bmatrix} -f_{y-1} & -f_y \\ 0 & I \end{bmatrix} \quad x_t = \begin{bmatrix} I \\ g_{y-1} \end{bmatrix}\hat{y}_{t-1} \quad C = \begin{bmatrix} -\left(f_y g_e + f_e\right) \\ g_e \end{bmatrix}$$

This represents the approximate linear model as a first order linear stochastic difference equation, from which, by applying techniques used for solving LRE models, we can obtain g_{y-1}. This includes applying a generalised Schur decomposition to matrices A and B, solving the generalised eigenvalue problem, $\lambda Ax = Bx$, verifying the well-known Blanchard-Kahn condition (the number of generalised eigenvalues that are larger than one in absolute value must be equal to the number of endogenous forward-looking variables) and a rank condition on one of the matrices resulting from the decomposition. Recovering g_e is then straightforward from (10).

Having found the form of matrices g_{y-1} and g_e we simply have to define some initial conditions for the endogenous variables, $\hat{y}_0$, and obtain values for the parameters, to be able to use the model for all types of "computational experiments". I impose the assumption that $\hat{y}_0$ are drawn from a Normal distribution, which will become clear in the next subsection.

Note that since the endogenous variables are expressed as percentage deviations of the original variables from the steady-state, the results of the simulations have to be interpreted accordingly. Furthermore, it is important to be aware that the performed approximation is only a local one, around the steady-state, and therefore experiments should not deviate the economy considerably from the steady-state, since in this case the approximation will no longer be valid.

3.2 Estimation

Calibration

As commonly done in the DSGE literature, some parameters were calibrated from the outset, instead of estimated. This helps to cope with the already discussed identification problems from which DSGE models commonly suffer, which in small scale models may be solved by carefully analysing each equation, but in medium/large-scale models is hardly solvable. Furthermore, incorporating fixed parameters in the estimation process can be viewed as imposing a very strict prior, being therefore consistent with the Bayesian approach to estimation.

All parameters exclusively affecting the dynamic behaviour of the model were estimated, with only parameters affecting the steady-state being calibrated. Among these, the calibrated parameters were those belonging to at least one of the following categories: those crucial to obtain a model's steady-state able to replicate the main steady-state key ratios of the Portuguese economy; those for which reliable estimates already exist; and those for which, although estimation was attempted, a satisfactory identification was not achieved.

Likelihood function

To perform the estimation of the remaining parameters, using Bayesian MLE, the first step is obtaining the likelihood function, which corresponds to the joint density of all variables in the data sample, conditional on the structure and parameters of our model. For this, we first need to establish a relation between the data and the model, which is done by considering that the measured variables can be explained partly by the model's variables and partly by some factors that the model is unable to measure, i.e.:

$$y_t^* = F\hat{y}_t + Gu_t \tag{13}$$

where y_t^* represents the vector of observed variables, F is a matrix establishing the link between the model's endogenous variables and the data, u_t stands for the vector of measurement errors and G consists of a matrix defining the role of measurement errors in each observed variable. The measurement errors are assumed to be Gaussian white noise processes, such that:

$$u_t \sim N(0, \Sigma_u) \quad E(u_t u_s') = 0 \quad t \neq s$$

Denoting g_{y-1} and g_e in (7) by D and E, respectively, and combining with (13) we obtain the model's state-space representation:

$$\hat{y}_t = D\hat{y}_{t-1} + Ee_t \tag{14}$$

$$y_t^* = F\hat{y}_t + Gu_t \tag{15}$$

Since e_t, $\hat{y}_0$, and u_t are all normally distributed, it must be that $\hat{y}_t$ and y_t^* are also normal. Denoting the whole data sample by y^*, and using a predictor error decomposition, we can then write the log-likelihood function as:

$$\mathcal{L}(y^*|\theta) = -\frac{Tn}{2}\log 2\pi - \frac{1}{2}\sum_{t=1}^{T}\log|\Sigma_{y^*\,t|t-1}| - \frac{1}{2}\sum_{t=1}^{T}\left(y_t^* - y_{t|t-1}^*\right)'\Sigma_{y^*\,t|t-1}{}^{-1}\left(y_t^* - y_{t|t-1}^*\right) \tag{16}$$

where $y_{t|t-1}^*$ is a predictor of y_t^* using information up to $t-1$, $\Sigma_{y^*\,t|t-1}$ is a predictor of the variance-covariance matrix of y_t^* using information up to $t-1$, and θ is the vector of parameters being estimated, on which $y_{t|t-1}^*$ and $\Sigma_{y^*\,t|t-1}$ depend. These are computed in a recursive way, using the Kalman filter.

Prior distributions

The next step is the specification of the prior distributions, $p(\theta)$. Each prior is a probability density function of a parameter, constituting a formal way of specifying probabilities to the values that parameters can assume, based on past studies or/and occurrences or simply reflecting subjective views of the researcher. It is a representation of belief in the context of the model, set without any reference to the data, constituting an additional, independent, source of information.

The prior's functional form is specified on the basis of each parameter's characteristics, namely: inverse gamma distribution for parameters bounded to be positive; beta distribution for parameters bounded between zero and one; normal distribution for non bounded parameters.

To set the parameters defining each distribution (mean and standard deviation) parameters are grouped into those for which there are relatively strong a priori convictions, which comprises the model's core structural parameters, and those for which there is great uncertainty, which includes the parameters characterising the shocks. Priors for the first type of parameters are based on the existing empirical evidence and their implications for macroeconomic dynamics. For parameters of the second type, although existing studies are also used, the strategy is mainly to set priors with reasonable means and a large support so that the distribution can cover a

considerable range of parameter values, i.e., priors that are only weakly informative.

Since priors are generated from well-known densities, its computation is straightforward.

Posterior distributions

Having derived the likelihood and specified the priors the posterior distribution, $p(\theta|y^*)$, which represents the probabilities assigned to different values of the parameters after observing the data, is then estimated. It basically constitutes an update of the probabilities given by the prior, based on the additional information provided by the variables in our sample. Applying the Bayes theorem to the two random events θ and y^*, the posterior distribution is given by:

$$p(\theta|y^*) = \frac{p(\theta, y^*)}{p(y^*)} = \frac{p(y^*|\theta)\, p(\theta)}{p(y^*)} \tag{17}$$

where $p(\theta, y^*)$ is the joint density of the parameters and the data, $p(y^*|\theta)$ is the density of the data conditional on the parameters (the likelihood), $p(\theta)$ is the unconditional density of the parameters (the prior) and $p(y^*)$ is the marginal density of the data. Note that $p(y^*)$ does not depend on θ and therefore can be treated as a constant for the estimation, producing:

$$p(\theta|y^*) \propto p(y^*|\theta)\, p(\theta) = \mathcal{K}(\theta|y^*) \tag{18}$$

where $\mathcal{K}(\theta|y^*)$ is the posterior kernel, proportional to the posterior by $p(y^*)$. Taking logs:

$$\ln \mathcal{K}(\theta|y^*) = \ln p(y^*|\theta) + \ln p(\theta) = \mathcal{L}(y^*|\theta) + \ln p(\theta) = \mathcal{L}(y^*|\theta) + \sum_{x=1}^{\gimel} \ln p(\theta_x) \tag{19}$$

where $\gimel$ is the number of parameters being estimated and priors are assumed to be independent.

This function is analytically intractable, implying the use of numerical methods. Specifically, Christopher Sims's routine, csmiwell, is used to maximise (19) with respect to θ to obtain estimates for the mode of the posterior, θ^m, and for the Hessian matrix evaluated at the mode, $H(\theta^m)$ (note that the maximum of $p(\theta|y^*)$ will be the same as the maximum of $\mathcal{K}(\theta|y^*)$) and then the Metropolis-Hastings (MH) algorithm is used to simulate the posterior distributions.

3.3 Evaluation

To assess the quality of the estimated model, two classes of aspects are considered: validation of the estimation procedures and results; ability of the model to fit the data's characteristics.

Checking the estimation diagnosis and results

The first check is the quality of the posterior kernel maximisation, which is done by plotting the minus of the function for values around the estimated mode, for each parameter in turn. If the mode is not at the trough of the function, the numerical procedure is having a hard time finding the optimum, which can be due to poor priors or identification problems.

Secondly, the parameters mode and standard deviation estimates are inspectioned, to ensure that they are plausible both from a statistical and an economic point of view. For this, comparison with previous studies and evidence from micro data can be of extreme usefulness.

Thirdly, the convergence properties of the MH algorithm are inspectioned. For this, several runs of MH simulations are conducted, each one with a different initial value, and for each run a large number of draws is performed. If convergence is achieved, and the optimiser did not get stuck in an odd area of the parameter subspace: results within each run's iterations should be similar; results between different runs should be close.

Fourthly, the simulated posteriors are inspectioned to check if: they are approximately normal; are not too different nor too similar to the priors[6]; the modes are not too far from the ones obtained from the maximisation of the posterior kernel.

Fifthly, the estimates of the innovations are inspectioned to eye-ball their plausibility, particularly if they exhibit a stationary, i.i.d, behaviour and are centered around zero.

Finally, sensitivity analysis are performed by changing some assumptions concerning the priors and comparing the obtained results with the benchmark model.

Assessing the fit of the model

Having checked the estimates robustness and reasonability, the ability of the estimated model to match the properties of the data is inspectioned.

This is done by, firstly, comparing a set of relevant statistics (mean, variance, etc.) computed for the actual data to the same set of statistics computed for data simulated with the model. Secondly, the Kalman filter estimates of the observed variables can be given the same interpretation as the fitted values of a regression, and are therefore loosely interpreted as the

[6]If they are very distinct, priors may be imposing erroneous restrictions on the data. If they are very close, results are probably being mostly led by the prior and only marginally by the data. While the prior should exclude regions of the parameter space that are unreasonable, it should also be reasonably uninformative on the interesting portions of the parameters space to let the data speak and avoid misleading conclusions. The shift from prior to posterior is an indicator of the tension between the two sources of information, prior and data. If, for a given parameter, the two distributions are virtually the same, one can conclude that the estimates are being determined by the prior and that either the data is silent on that parameter or we are not allowing it to speak.

in-sample fit of the model. Thirdly, the relative fit of the model is explored, through comparison with alternative specifications of the model.

The crucial tool to perform model comparison in a Bayesian framework is the marginal likelihood function, $p(y^*|i)$. It corresponds to the density of the data, conditional on the model but unconditional on the parameters, constituting a measure of the likelihood attributed by the model to the observed data independently of the parameters, being therefore a measure of the model's unconditional overall fit. It is obtained by integrating out the parameters from the joint density of the data and the parameters which, recalling (18), is equivalent to:

$$p(y^*|i) = \int_{\theta_i} p(y^*, \theta_i|i)\, d\theta_i = \int_{\theta_i} p(y^*|\theta_i, i)\, p(\theta_i|i)\, d\theta_i = \int_{\theta_i} \mathcal{K}(\theta_i|y^*, i)\, d\theta_i \tag{20}$$

where $p(\theta_i|i)$, $p(y^*|\theta_i, i)$ and $\mathcal{K}(\theta_i|y^*, i)$ are simply the prior, likelihood and posterior kernel, stated as a function of model i[7]. This function is analytically intractable, being necessary to recur to numerical approximations, the Laplace approximation and the Harmonic mean estimator.

Using the marginal likelihood, one can compute the posterior odds ratio between any two models, i and j, given by the ratio of each model's posterior probability, i.e. the probability of the model being true, after observing the data:

$$PO_{i,j} = \frac{p(i|y^*)}{p(j|y^*)} = \frac{\frac{p(y^*|i)p(i)}{p(y^*)}}{\frac{p(y^*|j)p(j)}{p(y^*)}} = \frac{p(i)}{p(j)}\frac{p(y^*|i)}{p(y^*|j)} = \frac{p(i)}{p(j)} BF_{i,j} \tag{21}$$

where $BF_{i,j} = \frac{p(y^*|i)}{p(y^*|j)}$ is the Bayes factor, summarising the evidence in the data in favour of model i over model j and $\frac{p(i)}{p(j)}$ is the prior odds ratio, summarising the relative probability one attributes a priori to the two different models.

This way, the posterior odds ratio provides a measure of the relative adequacy of each model based not only on their fit, given by the Bayes factor, but also on the beliefs one has a-priori on the models' quality, given by the prior odds ratio. The optimal decision is naturally to select the model with the highest posterior support, i.e., if $PO_{i,j} > 1$ choose model i and vice-versa.

Finally, some applications of the estimated model are explored namely the IRFs, which provide estimates of the effect of each shock on the endogenous variables, and the variance decomposition, which decomposes the variance of the endogenous variables into the contributions of the variance of each shock. For the IRFs, we obtain not only point estimates but also confidence intervals, by using the draws produced by the MH algorithm.

[7] This was not made explicit before for notational simplicity.

4 The model

A general overview of the model is now provided[8]. A detailed description is given in the Appendix including all steps needed to obtain the model equations at their stationary, aggregate, levels as well as the steady-state and log-linear versions of the model.

The model is a New-Keynesian DSGE model for a small open economy (Portugal) integrated in a monetary union (Euro area). The economy is composed by five types of agents namely households, firms, aggregators, the rest of the world (RW) and the government. It is assumed from the outset that monetary policy is totally defined by the union's central bank (ECB) and that the domestic economy's size is negligible, relative to the union's one, and therefore its specific economic fluctuations have no influence on the union's macroeconomic aggregates and monetary policy. Furthermore, it is assumed that the RW is strictly composed by the members of the monetary union (excluding the domestic economy) and that therefore the nominal exchange rate is irrevocably set to unity. The model includes several shocks, which enable a closer matching of the short-run properties of the data, and a number of real and nominal frictions, which allow for a more realistic short-term adjustment to shocks.

4.1 Households

There is a continuum of households, indexed by $i \in (0,1)$. A representative household derives utility at time t from consumption of a private consumption good, $C_t(i)$, relative to a consumption habit, $H_t(i)$, and from leisure, $1 - L_t(i)$, where $L_t(i)$ is the amount of labour supplied by the household. The household's lifetime utility function, describing the expected value of the discounted sum of future utilities, is given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t U_t(i) = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \varepsilon_t^c \ln \left[C_t(i) - H_t(i) \right] - \frac{\varepsilon_t^l}{1+\sigma_l} L_t(i)^{1+\sigma_l} \right\} \tag{22}$$

where $U_t(i)$ is the instantaneous utility function, representing each period contribution to lifetime utility [9], $0 \leq \beta \leq 1$ is the time discount factor, which captures the fact that households typically value future utility less than present utility, $0 < \sigma_l < \infty$ is the inverse of the elasticity of work effort with respect to (w.r.t.) the marginal disutility of labour and ε_t^c and ε_t^l are preference shocks to consumption and labour, respectively.

[8] The expectations operator has been purposely suppressed from the equations, for the sake of notation simplicity.

[9] Following Woodford (2003) and most of the recent literature a cashless limit economy is considered.

Habits are endogenous, defined as a proportion of the household's consumption in $t-1$:

$$H_t(i) = hC_{t-1}(i) \tag{23}$$

where $0 \leq h \leq 1$ determines the degree of habit persistence.

Labour is differentiated over households, with each one being a monopoly supplier of a particular variety of labour. Households sell these varieties to a labour aggregator who bundles them to produce an homogeneous labour input, which is then supplied to domestic good firms. Being a monopoly supplier of a particular variety of labour, each household has some decision power over the wage it charges, $W_t(i)$. However, it cannot set its wage optimally in every period, being subject to an indexating variant of a Calvo type mechanism. The household can only set its price freely if it receives a "wage-change signal" which occurs randomly at a constant (exogenous) probability, $1-\xi_w$, in which case it sets a new, optimal, wage, $\tilde{W}_t^0(i)$. $1-\xi_w$ is also the proportion of households that get to reoptimise their price in each period and defines the duration of wage contracts, given by $\frac{1}{1-\xi_w}$. Households that do not receive the "signal" update their previous period wage by indexating it to the current inflation rate target, $\bar{\pi}_t = \overline{\left(\frac{P_t}{P_{t-1}}\right)}$, modelled as a shock, the previous numeraire inflation rate, $\pi_{t-1} = \frac{P_{t-1}}{P_{t-2}}$ and the current growth rate of the technology level, ζ_t. More formally, a household who does not reoptimise in period t sets its wage as:

$$W_t(i) = \pi_{t-1}^{\kappa_w} \bar{\pi}_t^{1-\kappa_w} \zeta_t W_{t-1}(i) \tag{24}$$

where $0 \leq \kappa_w \leq 1$ is the degree of wage indexation to π_{t-1}.

Besides consuming and working, households invest in capital stock and rent it to domestic good firms at the rental rate R_t^k. Capital follows an accumulation equation, which states that the capital stock available at the beginning of period $t+1$, $K_{t+1}(i)$, (or equivalently, the capital stock available at the end of period t) is equal to the capital stock available in the beginning of period t, $K_t(i)$, net of period t capital stock depreciation, $\delta K_t(i)$, with $0 \leq \delta \leq 1$ being the depreciation rate, plus the amount of capital accumulated during period t, which is determined by the investment made during that period, $I_t(i)$. Investment is subject to adjustment costs, which are a positive function of changes in the household's investment from period $t-1$ to period t, $S\left(\frac{I_t(i)}{I_{t-1}(i)}\right)$, which satisfies the properties $S(\zeta) = 0$, $S'(\zeta) = 0$, $S''(\zeta) > 0$. Furthermore,

investment is subject to a shock, ε_t^i. The capital accumulation equation is then given by:

$$K_{t+1}(i) = (1-\delta)K_t(i) + \varepsilon_t^i \left[1 - S\left(\frac{I_t(i)}{I_{t-1}(i)}\right)\right] I_t(i) \tag{25}$$

Furthermore, each household saves both in domestic and foreign bonds. Domestic bonds, $B_t(i)$, are bought from the government and yield the domestic nominal interest rate prevailing at the time the decision is taken, R_{t-1}. The stock of bonds is assumed to be non-negative, meaning that households are not allowed to borrow from the government. Foreign bonds, $B_t^*(i)$, are bought from the RW and can be either positive or negative, with the household being a net borrower when $B_t^*(i) < 0$ and a net lender when $B_t^*(i) > 0$. As it is common in the literature on small-open economy models, foreign bonds yield the foreign nominal interest rate, R_{t-1}^*, adjusted by a risk-premium, $\Phi\left(\ddot{b}_t^*, \varepsilon_{t-1}^\phi\right)$, assumed to be a decreasing function of the real stationary holdings of foreign assets of the entire domestic economy, $\ddot{b}_t^* = \frac{B_t^*}{z_{t+1}P_{t+1}}$ and an increasing function of a risk-premium shock, ε_t^ϕ[10]. The premium is assumed to be neutral (equal to one) when $\ddot{b}_t^*$ and ε_t^ϕ are zero, larger than one when $\ddot{b}_t^*$ is negative or/and ε_t^ϕ is positive, and smaller than one when $\ddot{b}_t^*$ is positive or/and ε_t^ϕ is negative. This implies that when the economy is a net debtor, households will have to pay a remuneration higher than R_{t-1}^* to contract foreign debt whereas when the economy is a net lender they will receive a remuneration lower than R_{t-1}^* if they wish to sell their foreign assets. This way, the risk-premium works as a disincentive to buy/sell foreign bonds, acting as a stabiliser of $B_t^*(i)$, being crucial to pin down a well-defined steady-state for consumption and assets. It is assumed that $\Phi\left(\ddot{b}_t^*, \varepsilon_{t-1}^\phi\right) = \exp\left(-\ddot{b}_t^* + \varepsilon_{t-1}^\phi\right)$. In the steady-state, it is assumed that $R = R^*$, implying that $\ddot{b}^* = \varepsilon^\phi = 0$.

Households pay taxes on consumption, τ_t^c, and labour-income, τ_t^l, to the government who provides them with lump-sum transfers, TR_t. The after tax price of consumption good, $P_t = (1+\tau_t^c)\, P_t^c$, is assumed to be the numeraire with $\pi_t = \frac{P_t}{P_{t-1}}$ being the numeraire inflation rate.

Each household participates in a market of state-contingent securities, with the net cash inflow from participating in it being given by $P_t A_t(i)$. Having state-contingent securities, each household is insured against its labour-income uncertainty (arising from the fact that it does not know which wage it will be able to charge in each period) so that $W_t(i)L_t(i) + P_t A_t(i)$ is equal for all households, eliminating the labour-income uncertainty, making expenditure decisions perfectly symmetric for all households. The aggregate value of the state-contingent assets is assumed to be zero, i.e. $P_t \int_0^1 A_t(i)di = 0$.

[10] See for example Benigno (2001), Schmitt-Grohé and Uribe (2001) and Schmitt-Grohé and Uribe (2003).

Finally, households own all the firms in the economy receiving their profits in the form of dividends, $DIV_t(i)$. It is assumed that dividends are distributed equally among households and therefore $DIV_t(i) = DIV_t$.

Thus, in every period, households have at their disposal: the domestic and foreign bonds they accumulated from the previous period plus the interest they earned on them, $R_{t-1}B_t(i) + R^*_{t-1}\Phi\left(\ddot{b}^*_t, \varepsilon^{\phi}_{t-1}\right)B^*_t(i)$; the wage they receive for their labour supply, subtracted by the labour-income tax, $\left(1-\tau^l_t\right)W_t(i)L_t(i)$; the net income from their state-contingent securities, $P_tA_t(i)$; the income received from renting the capital stock they accumulated in the previous period, $R^K_tK_t(i)$; and the lump-sum transfers they receive from the government, TR_t. These resources can be used for: accumulation of domestic and foreign bonds, $B_{t+1}(i) + B^*_{t+1}(i)$; consumption, together with the consumption tax payment, $(1+\tau^c_t)P^c_tC_t(i)$; and investment, $P^i_tI_t(i)$. The household's flow budget constraint, defining that in each period its expenditure must equal its resources, is then given by:

$$B_{t+1}(i) + B^*_{t+1}(i) + (1+\tau^c_t)P^c_tC_t(i) + P^i_tI_t(i) = R_{t-1}B_t(i) + R^*_{t-1}\Phi\left(\ddot{b}^*_t, \varepsilon^{\phi}_{t-1}\right)B^*_t(i) + \\ + \left(1-\tau^l_t\right)W_t(i)L_t(i) + P_tA_t(i) + R^k_tK_t(i) + TR_t + DIV_t \tag{26}$$

A complete description of the household's problem requires the specification of a limit on borrowing to prevent Ponzi-type schemes. This in turn requires the introduction of the stochastic discount factor concept, used by households to obtain the present value of its future financial income streams, at any period $t+s$, which is given by:

$$\rho_{t+s} = \prod_{l=1}^{s}\frac{1}{R_{t+l-1}} \quad \text{for} \quad s>0 \quad (=1 \quad \text{for} \quad s=0) \tag{27}$$

The no-Ponzi game condition is then given by:

$$\lim_{s\to\infty} E_t\left\{\rho_{t+s}\left(B_{t+s} + B^*_{t+s}\right)\right\} = 0 \tag{28}$$

which states that the present discounted value of debt at infinity must be zero.

The representative household's optimisation problem is to choose the levels of consumption, domestic and foreign bonds, investment and capital stock that maximise (22) subject to the constraints imposed by (23), (25) and (26). Furthermore, households also have to set the utility maximising wage for their labour services, subject to the constraints imposed by (24), (26)

and the demand from the labour aggregator given in (43). A household who does not get to reoptimise, simply sets its wage according to (24). A household who gets to reoptimise, considers all future possible states of nature, noting that its entire utility flow will possibly depend on the wage it sets in t, according to whether it gets to reoptimise again or not. Solving this problem for every period, we see that it can be summed down to the maximisation of expected utility considering only the scenario where the household never reoptimises again, weighting each period utility by the probability that the household does not reoptimise its wage in that period.

Solving the optimisation problem, we obtain six first order conditions (FOC), which can be summarised in the following five equations. The consumption Euler equation:

$$\frac{U_{t+1}^{c,life}(i)}{U_t^{c,life}(i)} = \frac{1}{\beta R_t^r} \tag{29}$$

where $R_t^r = \frac{R_t}{\pi_{t+1}}$ is the domestic real interest rate and $U_t^{c,life}(i) = \frac{\varepsilon_t^c}{C_t(i)-hC_{t-1}(i)} - \beta h \frac{\varepsilon_{t+1}^c}{C_{t+1}(i)-hC_t(i)}$ is the extra utility a household obtains from increasing consumption by one unit in t. Note that, using (29), (27) can be written as:

$$\rho_{t+s} = \prod_{l=1}^{s} \beta \frac{P_{t+l-1}}{P_{t+l}} \frac{U_{t+l}^{c,life}(i)}{U_{t+l-1}^{c,life}(i)} = \beta^s \frac{P_t}{P_{t+s}} \frac{U_{t+s}^{c,life}(i)}{U_t^{c,life}(i)} \tag{30}$$

The modified uncovered interest rate parity (UIP) condition:

$$R_t = R_t^* \Phi\left(\ddot{b}_{t+1}^*, \varepsilon_t^{\phi}\right) \tag{31}$$

The FOC w.r.t. investment:

$$\begin{aligned} &Q_t(i)\varepsilon_t^i \left[1 - S\left(\frac{I_t(i)}{I_{t-1}(i)}\right) - S'\left(\frac{I_t(i)}{I_{t-1}(i)}\right)\frac{I_t(i)}{I_{t-1}(i)}\right] + \\ &+ Q_{t+1}(i) R_t^r \varepsilon_{t+1}^i S'\left(\frac{I_{t+1}(i)}{I_t(i)}\right)\left(\frac{I_{t+1}(i)}{I_t(i)}\right)^2 = \frac{P_t^i}{P_t} \end{aligned} \tag{32}$$

where $Q_t(i)$ is the marginal utility of capital in terms of the marginal utility of consumption, which gives the relative value of investing in more capital stock, in terms of consumption.

The FOC w.r.t. the capital stock:

$$Q_t(i) = R_t^r \left(\frac{R_{t+1}^k}{P_t} + Q_{t+1}(i)\,(1-\delta)\right) \tag{33}$$

For a household that gets to reoptimise its wage in t, the FOC w.r.t. wage:

$$E_t \sum_{s=0}^{\infty} (\beta\xi_w)^s L_{t+s}(i) \left\{ U^l_{t+s}(i)\mu_w + \frac{\tilde{W}^0_t(i)}{P_t z_t} \frac{X^w_{t+s}}{\frac{P_{t+s}}{P_t}} \left(1 - \tau^l_{t+s}\right) z_{t+s} U^{c,life}_{t+s}(i) \right\} = 0 \tag{34}$$

$$X^w_{t+s} = \begin{cases} 1 \quad \text{if} \quad s = 0 \\ \\ (\pi_{t+s-1} ... \pi_t)^{\kappa_w} (\bar{\pi}_{t+s} ... \bar{\pi}_{t+1})^{1-\kappa_w} \quad \text{if} \quad s > 0 \end{cases}$$

and where $U^l_{t+s}(i) = -\varepsilon^l_t L_t(i)^{\sigma_l}$ the marginal (dis)utility of wage optimising households from supplying an additional unit of labour.

Furthermore, note that all optimising households face the same conditions, behaving symmetrically, and therefore $K_t(i) = K_t$, $I_t(i) = I_t$, $C_t(i) = C_t$, $B_t(i) = B_t$, $B^*_t(i) = B^*_t$, $U^{c,life}_t(i) = U^{c,life}_t$, $Q_t(i) = Q_t$[11]. We can then easily define households' aggregate demand for consumption, investment and domestic and foreign bonds and supply for capital as $C_t = \int_0^1 C_t(i)di$, $I_t = \int_0^1 I_t(i)di$, $B_t = \int_0^1 B_t(i)di$, $B^*_t = \int_0^1 B^*_t(i)di$ and $K_t = \int_0^1 K_t(i)di$, respectively.

4.2 Firms

Focusing now on the supply side, there are two categories of firms operating in the economy: intermediate and final good firms.

Intermediate good firms

Intermediate good firms are of three types: domestic, import and composite good firms.

Domestic good firms

There is a continuum of domestic good firms, indexed by $j \in (0, 1)$. A representative firm produces a specific variety of domestic good, $Y^d_t(j)$, by combining capital, $K_t(j)$, and labour, $L_t(j)$, selling its product to the domestic good aggregator. The good is produced using the following Cobb-Douglas technology:

$$Y^d_t(j) = z_t^{1-\alpha_d} \varepsilon^a_t K_t(j)^{\alpha_d} L_t(j)^{1-\alpha_d} - z_t \phi_d \tag{35}$$

[11] Note however that this is not valid for $W_t(i)$ and $L_t(i)$, since these include households who are not optimising their wages, and therefore have an asymmetric behaviour from households that are optimising their wages.

where $0 \leq \alpha_d \leq 1$ is the capital income share, ϕ_d is a fixed cost of production, z_t is a unit root technology level, common to the domestic and foreign economies, with growth rate $\zeta_t = \frac{z_t}{z_{t-1}}$, modelled as a shock, and ε_t^a is a domestic, stationary, technology shock. The fixed cost is introduced to ensure zero profits in the steady-state and is assumed to grow with z_t since otherwise it would vanish and profits would systematically be positive.

Each firm rents capital and hires labour from households in perfectly competitive markets taking the wage, W_t, and the rental rate of capital, R_t^k, as given. In their output market, however, firms work in a monopolistically competitive environment, exploiting the power they have over their price, $P_t^d(j)$, arising from the fact that their product is differentiated. Like in the wages case, the price setting decision is modelled as an indexation variant of the Calvo mechanism. Firms only update their prices if they receive a "price-change signal", which occurs with a constant (exogenous) probability, $1 - \xi_d$, in which case it sets a new, optimal, price, $\tilde{P}_t^d(j)$. $1 - \xi_d$ is also the proportion of firms that get to reoptimise and defines the duration of domestic good price contracts, given by $\frac{1}{1-\xi_d}$. Firms that do not receive the "signal" update their previous period price by partially indexating it to the current inflation rate target and to the previous period domestic good inflation rate, $\pi_{t-1}^d = \frac{P_{t-1}^d}{P_{t-2}^d}$. More formally, a domestic good firm which does reoptimise its price sets it according to:

$$P_t^d(j) = {\pi_{t-1}^d}^{\kappa_d} \bar{\pi}_t^{1-\kappa_d} P_{t-1}^d(j) \tag{36}$$

where $0 \leq \kappa_d \leq 1$ is the degree of domestic good price indexation to π_{t-1}^d.

All firms choose labour and capital to minimise the cost of producing a certain amount of domestic good, subject to (35), which produces two FOC that can be summarised into an equation for the capital-labour demand ratio:

$$\frac{K_t(j)}{L_t(j)} = \frac{\alpha_d}{1-\alpha_d} \frac{W_t}{R_t^k} \tag{37}$$

In addition, we obtain the firm's marginal cost, given by:

$$MC_t^d(j) = \frac{1}{{z_t}^{1-\alpha_d} \varepsilon_t^a} \left(\frac{1}{\alpha_d}\right)^{\alpha_d} \left(\frac{1}{1-\alpha_d}\right)^{1-\alpha_d} W_t^{1-\alpha_d} {R_t^k}^{\alpha_d} \tag{38}$$

Besides solving the cost minimisation problem, firms have to decide on the profit maximising price to charge for their output. A firm who does not get to reoptimise, will simply set its price

according to (36). A firm who receives the "Calvo signal", and therefore gets to reoptimise, will choose a price that maximises its expected profits in all future possible states of nature, taking into account that the entire flow of profits will possibly depend on the price set in t, according to whether the firm reoptimises its price again or not. Just as in the wages case, the problem can be summed down to the maximisation of expected profits only considering the scenario where the firm never gets to reoptimise again, weighting each period's profit by the probability that the firm does not reoptimise in that period and incorporating the constraints imposed by (36) and the demand from the domestic good aggregator, given in (45). Since firms are owned by households, they will maximise expected profits using the stochastic discount factor applied by households to discount their future financial income streams, ρ_{t+s}. Solving this problem, the following FOC is obtained:

$$E_t \sum_{s=0}^{\infty} (\beta\xi_d)^s \frac{U_{t+s}^{c,life}}{\mu_{t+s}^d - 1} Y_{t+s}^d(j) \frac{P_{t+s}^d}{P_{t+s}} \left\{ \frac{X_{t+s}^d}{\frac{P_{t+s}^d}{P_t^d}} \frac{P_t^{0,d}(j)}{P_t^d} - \frac{MC_{t+s}^d}{P_{t+s}^d} \mu_{t+s}^d \right\} = 0 \tag{39}$$

$$X_{t+s}^d = \begin{cases} 1 \quad \text{if} \quad s = 0 \\ \\ \left(\pi_{t+s-1}^d ... \pi_t^d\right)^{\kappa_d} \left(\bar{\pi}_{t+s} ... \bar{\pi}_{t+1}\right)^{1-\kappa_d} \quad \text{if} \quad s > 0 \end{cases}$$

Import good firms

There is a continuum of import good firms, indexed by $m \in (0, 1)$. A representative firm buys a certain amount of homogeneous foreign good, M_t, and turns it into a differentiated import good, $Y_t^m(m)$, by brand naming, selling it to the import good aggregator. Like domestic good firms, import good firms are subject to fixed production costs, $z_t \phi_m$.

These firms operate in perfect competition in their input market, taking the price of the foreign good, P_t^*, as given, which constitutes the firm's marginal cost, being equal for all firms. In their output markets, however, they work in monopolistic competition, deciding on the price to charge for their product $P_t^m(m)$, in a perfectly analogous way to domestic good firms.

Composite good firm

There is one composite good firm that buys the homogeneous domestic good, Y_t^d, and the homogeneous import good, Y_t^m, from their respective aggregators and combines them to produce a homogeneous composite good, Y_t^h,which is then sold to final good firms. The good is produced

via the following CES technology:

$$Y_t^h = \left[(1-\omega_h)^{\frac{1}{\vartheta_h}} Y_t^{d\,\frac{\vartheta_h - 1}{\vartheta_h}} + \omega_h^{\frac{1}{\vartheta_h}} Y_t^{m\,\frac{\vartheta_h - 1}{\vartheta_h}} \right]^{\frac{\vartheta_h}{\vartheta_h - 1}} \tag{40}$$

where $0 \leq \omega_h \leq 1$ is the quasi-share of import good in the production of composite good and $1 < \vartheta_h < \infty$ is the elasticity of substitution between domestic and import goods.

The firm operates in a perfectly competitive environment, taking the prices of the domestic and import goods, P_t^d and P_t^m, and the price of its output, P_t^h, as given.

The firm's problem is to decide on the combination of domestic and import good that minimises the cost of producing a certain quantity of composite good subject to (40), which produces two FOC that can be summarised into an equation for the import-domestic good demand ratio:

$$\frac{Y_t^m}{Y_t^d} = \frac{\omega_h}{1-\omega_h} \left(\frac{P_t^d}{P_t^m} \right)^{\vartheta_h} \tag{41}$$

Note that the composite firm's demand for import good must correspond to total imports, i.e. $Y_t^m = M_t$.

Final good firms

Final good firms are of four types: private consumption, investment, government consumption and export, indexed by $f \in \{C, I, G, X\}$. For each type there is a continuum of firms, indexed by $n \in (0, 1)$, who buy an amount of composite good, Y_t^f, and differentiate it, by brand naming, producing different varieties of type f final good, $Y_t^f(n)$, which are then sold to their respective aggregators. Like domestic and import good firms, each final good firm is subject to fixed production costs, $z_t \phi_f$.

These firms operate in perfect competition in their input markets, taking the price of the composite good, P_t^h, as given, which constitutes each firm's marginal cost, being equal for all firms. In their output markets, however, they work in monopolistic competition, deciding on the price to charge for their differentiated products $P_t^f(n)$, in a perfectly analogous way to domestic and import good firms.

4.3 Aggregators

Aggregators solve the mismatch between the supply of differentiated products and the demand for homogeneous products. For each type of differentiated product being supplied, there is an

aggregator that buys the different varieties and combines them to produce an homogeneous product that can satisfy the economy's demand, using a CES technology. All the aggregators operate in a perfectly competitive environment both in their input and output markets, and therefore take the price of both their inputs and output as given.

The labour aggregator buys the different labour varieties from households and combines them to produce an homogeneous labour input, L_t, which it then sells to the domestic good firms at wage W_t. The homogeneous labour input is given by:

$$L_t = \left(\int_0^1 L_t(i)^{\frac{1}{\mu_w}} di \right)^{\mu_w} \tag{42}$$

where $1 < \mu_w < \infty$ is the wage markup, which is dependent on the elasticity of substitution between varieties of labour, $1 < \vartheta^w < \infty$, such that $\mu_w = \frac{\vartheta^w}{\vartheta^w - 1}$[12].

The problem of the labour aggregator is to decide on the combination of different labour varieties that minimises the cost of producing L_t, subject to (42). This produces the following FOC w.r.t. each variety of labour:

$$L_t(i) = \left(\frac{W_t}{W_t(i)} \right)^{\frac{\mu_w}{\mu_w - 1}} L_t \tag{43}$$

The domestic/import good aggregators buy the different varieties of domestic/import good from the domestic/import good firms and combine them to produce an homogeneous domestic/import good, Y_t^d / Y_t^m, which they then sell to the composite good firm at price P_t^d / P_t^m. For each type of final good there is an aggregator, which takes the different varieties of type f final good and combines them to produce an homogeneous type f final good, Y_t^f, which is then sold to households, the government and the RW at price P_t^f.

Analogously to the labour case, the homogeneous goods are given by:

$$Y_t^s = \left(\int_0^1 Y_t^s(r)^{\frac{1}{\mu_t^s}} dr \right)^{\mu_t^s} \tag{44}$$

and the FOC w.r.t. each variety of each good is equal to:

$$Y_t^s(r) = \left(\frac{P_t^s}{P_t^s(r)} \right)^{\frac{\mu_t^s}{\mu_t^s - 1}} Y_t^s \tag{45}$$

[12]The wage markup is considered to be time-invariant, contrary to the other markups, to prevent identification problems generated by the coexistence of two shocks, the wage markup shock and the labour supply shock, in the log-linearised wage equation. For a thorough discussion of this, please refer to Adolfson et al. (2005).

for $s = d, m, f$ and $r = j, m, n$, and where $1 < \mu_t^s < \infty$ is the good's price markup, modelled as a shock, dependent on the elasticity of substitution between varieties of the good, $1 < \vartheta_t^s < \infty$, such that $\mu_t^s = \frac{\vartheta_t^s}{\vartheta_t^s - 1}$.

4.4 Rest of the world

The RW is strictly composed by the members of the monetary union (excluding the domestic economy) implying that the nominal exchange rate is irrevocably set to unity. It interacts with the home economy by selling an homogeneous foreign good, buying the final export good and selling foreign bonds. It combines the domestic economy's export good with its own domestic good to produce its good, Y_t^*, using a CES technology. The demand for the domestic export good, which corresponds to domestic exports, X_t, is exogenously given as:

$$X_t = \omega_* \epsilon_t^{\vartheta_*} Y_t^* \tag{46}$$

where $\epsilon_t = \frac{P_t^*}{P_t^x}$ is the real exchange rate, $0 \leq \omega_* \leq 1$ is the quasi-share of domestic export good in the production of the foreign good and $1 < \vartheta_* < \infty$ is the foreign economy's elasticity of substitution between the domestic export good and the foreign good.

RW's variables (inflation, output and interest rate) are assumed to be exogenous, given as shocks. Furthermore, a stationary asymmetric (or foreign) technology shock, $\zeta_t^* = \frac{z_t^*}{z_t}$, is assumed where z_t^* is the permanent technology level abroad (used to render Y_t^* stationary), to allow for temporary differences between domestic and foreign permanent technological progresses.

4.5 Government

The government spends resources on the acquisition of the government consumption good, $P_t^g G_t$, payment of debt services, $(R_{t-1} - 1) B_t$, and transfers to households, TR_t, and obtains resources from taxes, $\tau_t^c P_t^c C_t + \tau_t^l \int_0^1 W_t(i) L_t(i) di$, and debt issuance, B_t.

The government's primary deficit, i.e. the difference between the government's current spending and revenue, excluding debt related resources and expenditures, is then given by:

$$SG_t^{prim} = P_t^g G_t + TR_t - \tau_t^c P_t^c C_t - \tau_t^l \int_0^1 W_t(i) L_t(i) di \tag{47}$$

The government's total deficit also includes interest outlays, being given by:

$$SG_t^{tot} = SG_t^{prim} + (R_{t-1} - 1)\, B_t \tag{48}$$

The government's budget constraint, defining that its total resources must equal its total expenditures, can then be written as:

$$B_{t+1} + \tau_t^c P_t^c C_t + \tau_t^l \int_0^1 W_t(i) L_t(i) di = R_{t-1} B_t + TR_t + P_t^g G_t \Leftrightarrow B_{t+1} = B_t + SG_t^{tot} \tag{49}$$

To prevent an explosive debt path a fiscal rule is imposed, which restricts SG_t^{prim} such that TR_t adjusts endogenously to ensure that the debt to Gross Domestic Product (GDP) ratio converges to a long-term, pre-specified value. The rule is given in its stationary form by:

$$\frac{\ddot{sg}_t^{prim}}{\ddot{gdp}} = -d_g \left(\frac{\ddot{b}_{t+1}}{\ddot{gdp}} - \left(\frac{\ddot{b}}{\ddot{gdp}} \right)^{tar} \right) \tag{50}$$

where $\left(\frac{\ddot{b}}{\ddot{gdp}}\right)^{tar}$ is the target value for the stationary debt to GDP ratio. Whenever the debt to GDP ratio is above its target value, transfers automatically decrease in order to reduce the government's expenditures and consequently minimise its deficit and future period debt.

The fiscal policy variables (taxes and expenditures) are exogenously given as shocks. Steady-state expenditures are determined by a government expenditure to output ratio parameter, g_y.

4.6 Market clearing conditions

Finally, consider the market clearing conditions needed to close the model.

In the labour market, supply by the aggregator must equal demand by domestic good firms:

$$L_t = \int_0^1 L_t(j) dj \tag{51}$$

In the capital market, supply by households must equal demand by domestic good firms:

$$K_t = \int_0^1 K_t(j) dj \tag{52}$$

In the composite good market, supply by the composite good firm must equal demand by

the final good aggregators:

$$Y_t^h = Y_t^c + Y_t^i + Y_t^g + Y_t^x \tag{53}$$

In the final good market, supply by the aggregators must equal demand by households, the government and the RW:

$$Y_t^c = C_t \tag{54}$$

$$Y_t^i = I_t \tag{55}$$

$$Y_t^g = G_t \tag{56}$$

$$Y_t^x = X_t \tag{57}$$

In the foreign bond market, households' net bond holdings must equal the economy's trade net position:

$$B_{t+1}^* - R_{t-1}^* \Phi\left(\ddot{b}_t^*, \varepsilon_{t-1}^{\phi}\right) B_t^* = P_t^x X_t - P_t^* M_t \tag{58}$$

Finally, consider the measure of GDP, which following the National Accounts definition corresponds to the sum of demand expenditures, including consumption taxes and excluding expenditures with imports:

$$GDP_t = P_t C_t + P_t^i I_t + P_t^g G_t + P_t^x X_t - P_t^* M_t \tag{59}$$

4.7 Shocks

The stochastic behaviour of the model is driven by twenty structural shocks which are all given by the following univariate representation:

$$\xi_t^i = \left(1 - \rho_{\xi^i}\right) \bar{\xi}^i + \rho_{\xi^i} \xi^i{}_{t-1} + \eta_{\xi^i,t} \qquad \eta_{\xi^i,t} \sim N(0, \sigma_{\xi^i}^2) \tag{60}$$

where all innovations are assumed to be independent and identically distributed (i.i.d.), $i = \left\{\varepsilon^i, \varepsilon^c, \varepsilon^\phi, \varepsilon^l, \varepsilon^a, \mu^d, \mu^m, \mu^c, \mu^i, \mu^g, \mu^x, \bar{\pi}, \zeta, \zeta^*, \pi^*, \ddot{Y}^*, R^*, \tau^l, \tau^c, \ddot{G}\right\}$ and $E(\xi_t^i) = \bar{\xi}^i$. In the cases of $\left\{\varepsilon^i, \varepsilon^c, \varepsilon^l, \varepsilon^a, \zeta^*\right\}$ it is assumed that $\bar{\xi}^i = 1$.

5 Estimation for the Portuguese economy

This section provides all empirical aspects concerning the estimation of the model with Portuguese data.

5.1 Data

To compute the key steady-state ratios, used in the calibration, I used annual data, between 1988-2007, taken from the annual National Accounts dataset, available from the Statistics Portugal Institute (INE). Some of the chosen series have an extremely erroneous behaviour prior to 1988, which motivated the exclusion of this period from the sample.

For the estimation I used quarterly data, over the same period. Portuguese data was taken from the "Quarterly Series for the Portuguese Economy: 1977-2007" in the Summer 2008 issue of Banco de Portugal's Economic Bulletin. Euro area data was taken from the Area Wide Model (AWM) database[13], originally presented in Fagan, Henry and Mestre (2001), that has become a standard reference for empirical studies using euro area data. To my knowledge however, the most recent update is only until 2005Q4. To obtain data for 2006 and 2007, I recurred to the Eurostat database and used the implied growth rates in its series to extend the AWM ones.

I chose to match thirteen variables: GDP inflation, private consumption good inflation (including taxes), investment good inflation, real wages, real private consumption, real investment, real GDP, employment, real exports, real imports, nominal interest rate, foreign real GDP and foreign nominal interest rate. All inflation rates were obtained as the fourth order difference of the log of their respective deflator. Real wages were obtained by scaling nominal wages by the private consumption good deflator.

To render the data stationary an HP-filter was used with $\lambda = 7680$, as in Almeida and Félix (2006). At this point, it is important to refer some problems I encountered when dealing with this issue. I applied different strategies in the attempt of rendering the data stationary, in particular first-order differentiation and linear detrending. However, due to the highly non-stationary behaviour of Portuguese data, marked by the economic instability that the country suffered during a considerable time span, none of these strategies was able to produce "well-behaved" series, to use in the estimation. The HP filter was indeed the only methodology (among the ones I tried) that provided a reasonable treatment of the data. I am however, conscious that this method suffers from some problems, in particular the well-known end-of-sample bias.

[13]The data is available at http://www.eabcn.org/data/awm/index.htm.

The vector of variables used in the estimation can then be described by:

$$\Upsilon_t' = \begin{bmatrix} \pi_t^d & \pi_t & \pi_t^i & w_t & C_t & I_t & gdp_t & L_t & X_t & M_t & R_t & gdp_t^* & R_t^* \end{bmatrix}^{HP}$$

The correspondent time series are shown in Figure 1.

5.2 Calibration

Being a small open economy in the euro area, Portugal's steady-state real growth and inflation were set according to those of the euro area. ζ was set at 1.005, which as referred in Almeida et al. (2008) corresponds to the euro area's potential output growth and also seems a plausible estimate for Portugal in view of the results of Almeida and Félix (2006). $\pi = \bar{\pi} = \pi^*$ was set at 1.005, in line with the ECB goal of 2% for the euro area annual inflation rate. Conditional on these values, and according to the consumption Euler equation in the steady-state, the discount rate β was then set to 0.999, to produce a steady-state long-run nominal interest rate in line with that of the euro area, i.e. 4.5% following Christoffel et al. (2008).

Conditional on the value assumed for ζ, and according to the capital accumulation equation in the steady-state, the depreciation rate was calibrated to match the empirical long-run annual investment to capital ratio of 8%, implying $\delta = 0.014$, i.e. an annual depreciation rate of 6%. The steady-state government to domestic output ratio, was set to match its empirical counterpart implying $g_y = 0.14$. The steady-state tax rates were taken from Almeida et al. (2008), where $\tau^c = 0.304$ and $\tau^l = 0.287$, and the target debt to GDP ratio, $\left(\frac{\ddot{b}}{\dot{gdp}}\right)^{tar}$, was set at 60%, the upper bound of the limit imposed the Maastricht Treaty criteria. The capital income share, the quasi-share of import good in composite good, the quasi-share of domestic export good in foreign good production and the elasticity of substitution between domestic and import goods were then calibrated to closely follow Almeida et al. (2008) and ensure that the model key steady-state ratios closely follow the correspondent ratios in the data. This amounted to $\alpha_d = 0.323$, $\omega_h = 0.32$, $\omega_* = 0.02$ and $\vartheta_h = 1.000001$.

Besides these, two more parameters were calibrated that ideally would have been estimated, but for which satisfactory estimates were not obtained. These comprise the domestic good price markup, for which an unreasonably high markup was obtained, and the elasticity of substitution between final export good and foreign good, for which a value below one was obtained, which violates the assumption that this elasticity must be larger than one. μ_d was set at 1.15 and ϑ_* at 1.5 following Almeida et al. (2008).

5.3 Priors

In Table 1 the assumptions concerning the priors are presented.

For the inverse of the elasticity of labour effort, the steady-state markups and the standard deviations of shocks the inverse gamma distribution was used. For the first one, the prior mean was set at 2, according to the calibration in Almeida et al. (2008), and the standard deviation at 0.5, producing a reasonably loose prior. The prior means for the steady-state markups were mostly based on the calibration in Almeida et al. (2008) being set at 25% in the wages case and 5% in the final goods case. As for import prices, a markup of 20% was considered, as in Adolfson, Laséen, Lindé and Villani (2007). The standard deviations for these priors were set at 0.2 for the wages and domestic good price markups, producing reasonably loose priors, and 0.05 for the final goods markups, as a markup lower than one would not make sense. For the shocks standard deviations, I did not have strong a-priori convictions and therefore priors were set as harmonised and loose as possible. For most of them, the mean was set at 0.15, a value that fits into what is usually used in the literature, while for the remaining it was set at a considerably lower level, 0.02, to ensure the success of the numerical optimisation of the posterior kernel. The dispersions were all set equal to the means, which produced rather uninformative priors.

The habit persistence parameter, the shocks' autoregressive parameters, the Calvo and the indexation parameters were assumed to follow a beta distribution. For the habit parameter, the prior mean was set at 0.7, according to the calibration in Almeida et al. (2008), and the standard deviation was set at 0.2. Concerning the autoregressive parameters, for which I had no strong a-priori information, priors were completely harmonised, with their mean set at 0.6. The means for the priors of the non-reoptimisation Calvo probability were mostly set according to the calibration in Almeida et al. (2008), with wages assumed to be readjusted every 6 quarters, domestic good prices every 4 quarters and final good prices every 2.5 quarters. Import prices were assumed to adjust every 2 quarters, according to the prior in Adolfson, Laséen, Lindé and Villani (2007). As for the indexation parameters, all prior means were set at 0.5, in line with Adolfson, Laséen, Lindé and Villani (2007). The standard deviation of all these parameters was set at 0.1, which is in line with what is usually assumed in the DSGE literature.

Finally, a normal distribution was considered for the investment adjustment cost parameter, with prior mean and standard deviation set at 7.69 and 1.5, respectively, as in Adolfson, Laséen, Lindé and Villani (2007).

5.4 Results and evaluation

The results for the 63 estimated parameters are presented in Table 1 and the corresponding posteriors in Figures 3-4. In Table 1, the posterior maximisation columns report the parameters mode and standard deviation obtained by maximising the posterior kernel. The MH sampling columns contain the mean and 5th and 95th percentiles of the posterior distributions computed with the MH algorithm, based on 500000 draws with 5 distinct chains. In the bottom of the table, the value of the marginal likelihood is presented, computed with both the Laplace approximation and the Harmonic mean estimator, together with the average acceptance rate in each chain.

Starting with the parameters influencing the steady-state, the estimation for the inverse of the elasticity of labour confirms a value close to 2, although the posterior seems to be very much driven by the prior. The habit parameter seems to be slightly higher than the assumed prior mean, close to 0.8, with the obtained estimate appearing to be highly data driven, indicating a quite high persistence of Portuguese households' consumption between periods. In the case of the markups' steady-state values, the prior conviction of low markups in the final goods sector seems to be confirmed by the data, with values between 3% and 6%, indicating a reasonable degree of competition in these markets. These results seem however to be highly influenced by the respective priors. The wage and import price markups, on the other hand, are estimated to be high, 38% and 23% respectively, which is in line with prior conviction of higher markups in these markets and with the estimates obtained in Adolfson, Laséen, Lindé and Villani (2007). The low degree of competition in the Portuguese labour market, in particular, is well documented in Almeida et al. (2008). For both these parameters, results seem to be reasonably data driven, especially in the case of the wage markup.

Turning to the model's frictions, the investment adjustment cost parameter is estimated to be around 10, a value slightly above the ones commonly found in the literature, with the corresponding posterior being reasonably data driven. Considering the Calvo stickiness parameters, prices in the labour and consumption good markets are estimated to be quite sticky, with average durations of price contracts close to 1.5 years. The high stickiness of wages is a fairly intuitive result in the case of the Portuguese economy, where wage negotiations are centralised and performed on a yearly basis. Domestic, investment, government consumption and export goods prices are estimated to be more flexible, being renegotiated once a year, on average. Import prices estimates, in turn, point to a high degree of flexibility with renegotiations occurring every 1.5 quarters. The posteriors for all these parameters are quite distinct from the assumed priors,

indicating that the estimates draw important information from the data. As for the indexation parameters, results suggest that inflation persistence is the highest for wages, around 59%, and the lowest for import prices, around 43%. These results compare quite favourably with what is usually obtained, particularly in the wages case, as it is very frequent that the price of labour is the most related to past inflation among prices considered in DSGE models. This makes particular sense for the Portuguese economy, since wage negotiations are very much based on past inflation figures (though no formal indexation mechanism exists). The comparison between priors and posteriors for these parameters points to some influence from the data in the results, but an apparent predominance of the information imposed by the priors.

Finally, consider the persistence and volatility parameters of structural shocks. The autoregressive parameters are estimated to lie between 0.35, for the import goods price markup, and 0.89, for the foreign output shock. None of the shocks is excessively persistent, in the sense that the posterior distribution's 95th percentile does not exceed 0.93, which gives an indication on the absence of unit roots in these processes. It is interesting to note that when compared to Smets and Wouters (2003) (closed economy), the obtained estimates are considerably lower, while closer to the estimates of Adolfson, Laséen, Lindé and Villani (2007) (open economy). This seems to be a reasonable feature, since for an open economy there is an extra possibility of propagation of shocks hitting the economy, and therefore these are likely to be less persistent. Turning to the estimated standard deviations, it is possible to conclude that the most volatile shock considered in the model is the investment specific one (0.21), while the least volatile are the consumption tax rate and the foreign interest rate (0.00). For most of these parameters results seem to be driven to a reasonable extent by the data.

Looking at the diagnosis concerning the numerical maximisation of the posterior kernel, overall they indicate that the optimisation procedure was able to obtain a robust maximum for the posterior kernel. The corresponding graphs can be seen in the Appendix. As for the MH sampling algorithm, a diagnosis of the overall convergence is provided in Figure 2. The information is summarised in three graphs, with each graph representing specific convergence measures and having two distinct lines that represent the results within and between chains. Those measures are related to the analysis of the parameters mean (interval), variance (m2) and third moment (m3). Convergence requires that both lines, for each of the three measures, become relatively constant and converge to each other. Diagnosis for each individual parameter were also obtained, following the same structure as the overall, being available in the Appendix.

Most of the parameters do not seem to exhibit convergence problems, notwithstanding the fact that for some of them this evidence is stronger than for others.

Figure 5 displays the estimates of the innovation component of each structural shock. These appear to respect the i.i.d. properties assumed from the beginning and are centered around zero, which gives some positive indication on the statistical validity of the estimated model.

To assess the sensitivity of the results to the priors, these were changed in several ways and the model re-estimated for each of them[14]. Results concerning three particular cases are presented in Table 3, for illustrative purposes. In Case I, all prior means and standard deviations were increased by 10%. Comparing the obtained results with those of the benchmark model, in Table 1, we can see that for the majority of the parameters estimates changed but not substantially. In Case II, the prior means were kept unchanged and the prior standard deviations were increased substantially, by 50%. Although results exhibit a more substantial change than in Case I, the overall conclusions remained broadly the same as the ones of the benchmark model. We can therefore conclude that for reasonable changes in the values of the priors mean and standard deviation, quantitative results are somewhat sensible but the overall qualitative results are quite robust. In Case III, a uniform distribution is assumed for the price indexation parameters, which amounts to considering an uninformative prior. This produces unreasonable values for at least two parameters, the import and export goods ones, illustrating that, as expected, the shape provided by the priors in the benchmark model is in fact crucial for the results.

Turning now to the assessment of the fit of the model, the one step ahead Kalman filter estimates are presented in Figure 1, for each of the thirteen variables used in the estimation, together with the actual data. As can be seen, the obtained estimates are quite close to the true data, indicating that the overall absolute fit of the model is satisfactory.

In addition, a set of unconditional moments were considered computed for the actual data and for artificial time series generated from the estimated model, namely the: standard deviation, autocorrelation coefficients and cross-(contemporary) correlation coefficients. Results are presented in Table 3. In general, the model seems to produce time series with higher volatility than the actual data, which may be related to the fact that, contrary to the observed data, the estimated data has not been hp-filtered. Concerning the autocorrelation coefficients, the model seems to do a good job in capturing the data features for the first 3 lags and to a lesser extent

[14]For this purpose, I did not use the MH sampling mechanism since the focus was mainly the comparison between point estimates and not the entire posterior distributions. Therefore, estimates only concern the results of the posterior kernel maximisation and the corresponding Laplace approximation to the marginal likelihood.

for the 4th and 5th lags. As for the cross correlation coefficients, the quality of the results is mixed, with the model having some difficulties in capturing all the data features correctly.

Furthermore, the fit of the benchmark model relative to alternative specifications was explored. Results concerning two particular cases are presented in Table 4, for illustrative purposes. In Case IV, the persistence coefficients of the markup shocks were calibrated to be zero, an assumption relatively common in the DSGE literature. In Case V, the relevance of indexation to past inflation in pricing decisions was tested, by calibrating all the indexation parameters to be zero. In both cases, the diagnosis concerning the posterior kernel maximisation deteriorated, indicating that the imposed restrictions spurred identification problems. Furthermore, in Case IV, the standard deviations of the markup shocks increased substantially reaching values that appear to be less reasonable in light of what is usual in the literature. Computing the posterior odds ratio for both models relative to the benchmark model, smaller than one figures were obtained, indicating that the latest is more likely than the formers[15].

Additionally, a variance decomposition was performed to make a formal assessment of the contribution of each structural shock to fluctuations in the endogenous variables, which is presented in Table 5 for the main endogenous variables of the model. The most crucial shock, that motivates important fluctuations in most variables, is the unit root technology shock, accounting for more than 70% of developments in capital, consumption, investment, wages and the rental rate of capital. Crucial in explaining inflation developments are the markup shocks, the inflation target shock, and the foreign inflation shock, which together account for more than 70% of the variation in all inflation rates. Finally, the asymmetric technology shock seems to be also relevant, particularly in explaining trade developments.

Finally, the IRFs to the model's shocks were inspectioned, considering shocks of one standard deviation. For all the 20 shocks variables returned to their steady-state value, reinforcing the conclusion given by the Blanchard-Kahn and rank conditions that the model is stable. Furthermore, in general, results seem to make sense from an economic point of view. The IRFs for all the shocks are given in the Appendix. For illustration purposes, the IRFs for a positive stationary productivity shock are presented in Figure 6. Higher productivity leads to lower marginal costs in the production of domestic goods, which implies smaller domestic good inflation and consequently a decrease in the relative price of domestic good. This lowers the price of composite good, which translates into smaller final good inflation and consequently lower

[15]For the computation of all posterior odds ratios, the prior odds ratio was considered to be one.

relative prices of final goods. This induces an increase in the relative price of foreign and import goods and an exchange rate depreciation, which generates higher demand for export goods. The decrease in the relative price of domestic good and increase in the relative price of import good decreases imports. Higher exports and lower imports has a positive impact on the economy's net foreign asset position, which decreases the domestic interest rate. This increases the value of the capital stock, stimulating capital accumulation. Furthermore, the decrease in investment good price stimulates investment, also contributing for a more intensive capital accumulation. Higher capital supply decreases its rental rate. The decrease in the domestic interest rate also stimulates private consumption, decreasing the marginal utility of consumption, and lowers savings in domestic bonds, turning them less attractive relative to foreign bonds. The higher demand for consumption, investment and export goods is matched by an increase in production, and consequent increase in capital and labour demand. However, the decrease in inflation leads to a higher real wage, which has a negative effect over labour demand that more than offsets the increase via stronger production. After the initial impact, the increase in wages puts pressure on marginal costs and consequently on inflation, which leads to a reversion of some of the previous effects (like a decrease in investment) and intensification of others (like a further decrease in the real interest rate). Overall, the shock has an expansionary effect in the economy, producing a rise in real GDP. As the temporary shock begins to vanish its effects over the economy decrease, with all variables returning to the steady-state after twenty years.

6 Conclusions and directions for further work

In this study, a New-Keynesian DSGE model for a small open economy integrated in a monetary union was developed and estimated for the Portuguese economy using a Bayesian approach. The results were analysed using several techniques, which overall indicated that the estimated model has both statistical and economic plausible properties. In addition, a survey on the main events and literature associated with DSGE models was provided, as well as a comprehensive discussion of the Bayesian estimation and model comparison techniques applied.

The obtained estimates for the parameters of interest are generally in line with the DSGE literature and previous studies for Portugal. Among them, some are particularly noteworthy. Firstly, the finding of high markups in the import goods and labour markets, indicating a low degree of competition in these markets, which is in line with the estimates obtained in Adolfson, Laséen, Lindé and Villani (2007) (for the import good price markup) and the calibration

in Almeida et al. (2008) (for the wage markup), where the low degree of competition in the Portuguese labour market is well documented. Secondly, wages are estimated to be the stickier prices and import good prices the more flexible, a result commonly found in the literature. The high stickiness of wages, in particular, is a fairly intuitive result for the Portuguese economy, where wage negotiations are centralised and performed on a yearly basis. Thirdly, all prices exhibit a considerable degree of indexation to past inflation, particulary wages, a result that seems to make sense for the Portuguese labour market, since wage negotiations usually rely heavily on past inflation developments.

Some problems were however encountered, which should be mentioned. In particular, the treatment of the data was a far from trivial task, leading to the use of hp-filtered data, which is recognised to be subject to caveats. Also, despite the overall quality of the diagnostic measures, this was not an homogeneous result, indicating that some identification problems may exist, which is in fact a common problem in the estimation of medium-large scale DSGE models. Furthermore, although the data was informative in the majority of the cases, some estimates seemed to be quite influenced by the chosen priors, an influence that ideally should be as small as possible. Finally, the estimation produced unreasonable results for two parameters, which consequently had to be calibrated.

The existence of these caveats indicates that this work can be improved in some dimensions. In particular, a more profound data analysis should be performed and the estimation should be carried out with unfiltered data. Furthermore, the fiscal and foreign variables of the model could be modelled outside the model, like in Adolfson, Laséen, Lindé and Villani (2007), which would allow for a more realistic treatment of these variables and a considerably reduction of the "estimation burden" imposed on the current model. Finally, a more comprehensive analysis of the model's properties could be performed by considering other benchmark models such as VARs or further exploring the model's applications, in particular forecasting exercises.

Tables

Table 1: Estimation results

Parameters	Prior			Posterior maximisation		Metropolis-Hastings sampling			
	Type	Mean	Stdev	Mode	Stdev	Mean	Mode	5%	95%
σ_l	invg	2.00	0.50	1.81	0.39	2.05	1.81	1.29	2.78
h	beta	0.70	0.20	0.79	0.04	0.78	0.79	0.72	0.84
μ^w	invg	1.25	0.20	1.38	0.11	1.38	1.38	1.21	1.56
μ^m	invg	1.20	0.20	1.23	0.20	1.34	1.23	0.96	1.71
μ^c	invg	1.05	0.05	1.06	0.10	1.08	1.06	0.91	1.24
μ^i	invg	1.05	0.05	1.02	0.09	1.05	1.02	0.89	1.20
μ^g	invg	1.05	0.05	1.01	0.09	1.04	1.01	0.89	1.19
μ^x	invg	1.05	0.05	1.03	0.09	1.06	1.03	0.90	1.22
S	norm	7.69	1.50	9.73	1.27	9.89	9.73	7.83	11.98
ξ_w	beta	0.83	0.10	0.86	0.03	0.83	0.86	0.78	0.89
ξ_d	beta	0.75	0.10	0.74	0.04	0.75	0.74	0.69	0.82
ξ_m	beta	0.50	0.10	0.30	0.06	0.30	0.30	0.21	0.40
ξ_c	beta	0.60	0.10	0.84	0.02	0.84	0.84	0.80	0.88
ξ_i	beta	0.60	0.10	0.73	0.04	0.74	0.73	0.68	0.80
ξ_g	beta	0.60	0.10	0.74	0.06	0.73	0.74	0.64	0.83
ξ_x	beta	0.60	0.10	0.73	0.05	0.73	0.73	0.66	0.81
κ_w	beta	0.50	0.10	0.59	0.06	0.61	0.59	0.51	0.71
κ_d	beta	0.50	0.10	0.47	0.08	0.47	0.47	0.33	0.61
κ_m	beta	0.50	0.10	0.43	0.10	0.44	0.43	0.28	0.60
κ_c	beta	0.50	0.10	0.47	0.07	0.48	0.47	0.36	0.60
κ_i	beta	0.50	0.10	0.48	0.09	0.48	0.48	0.34	0.63
κ_g	beta	0.50	0.10	0.52	0.10	0.53	0.52	0.37	0.68
κ_x	beta	0.50	0.10	0.54	0.11	0.53	0.54	0.36	0.69
ρ_{ε^i}	beta	0.60	0.10	0.39	0.08	0.39	0.39	0.27	0.52
ρ_{ε^c}	beta	0.60	0.10	0.61	0.10	0.59	0.61	0.43	0.74
ρ_{ε^ϕ}	beta	0.60	0.10	0.75	0.07	0.73	0.75	0.61	0.85
ρ_{ε^a}	beta	0.60	0.10	0.59	0.10	0.60	0.59	0.44	0.76
ρ_{ε^l}	beta	0.60	0.10	0.62	0.11	0.62	0.62	0.47	0.78
ρ_{μ^d}	beta	0.60	0.10	0.66	0.07	0.62	0.66	0.50	0.75
ρ_{μ^c}	beta	0.60	0.10	0.59	0.08	0.59	0.59	0.46	0.72
ρ_{μ^i}	beta	0.60	0.10	0.60	0.07	0.58	0.60	0.46	0.71
ρ_{μ^g}	beta	0.60	0.10	0.78	0.06	0.75	0.78	0.64	0.86
ρ_{μ^x}	beta	0.60	0.10	0.62	0.08	0.60	0.62	0.48	0.73
ρ_{μ^m}	beta	0.60	0.10	0.35	0.07	0.35	0.35	0.23	0.46
ρ_ζ	beta	0.60	0.10	0.53	0.08	0.52	0.53	0.38	0.65
ρ_{ζ^*}	beta	0.60	0.10	0.87	0.03	0.86	0.87	0.81	0.91
ρ_{τ^l}	beta	0.60	0.10	0.67	0.08	0.63	0.67	0.48	0.77
ρ_{τ^c}	beta	0.60	0.10	0.74	0.08	0.72	0.74	0.59	0.85
ρ^g	beta	0.60	0.10	0.68	0.12	0.69	0.68	0.52	0.87
ρ_{π^*}	beta	0.60	0.10	0.69	0.09	0.69	0.69	0.55	0.82
ρ_{y^*}	beta	0.60	0.10	0.89	0.03	0.88	0.89	0.84	0.93
ρ_{r^*}	beta	0.60	0.10	0.83	0.04	0.82	0.83	0.76	0.89
$\rho_{\bar{\pi}}$	beta	0.60	0.10	0.66	0.08	0.64	0.66	0.52	0.77
$\sigma_{\eta^{\varepsilon^i}}$	invg	0.15	0.15	0.21	0.03	0.21	0.21	0.15	0.27
$\sigma_{\eta^{\varepsilon^c}}$	invg	0.15	0.15	0.04	0.01	0.05	0.04	0.03	0.06
$\sigma_{\eta^{\varepsilon^\phi}}$	invg	0.02	0.02	0.01	0.00	0.01	0.01	0.01	0.01
$\sigma_{\eta^{\varepsilon^a}}$	invg	0.02	0.02	0.01	0.00	0.01	0.01	0.01	0.01
$\sigma_{\eta^{\varepsilon^l}}$	invg	0.15	0.15	0.09	0.04	0.16	0.09	0.05	0.28
$\sigma_{\eta^{\mu^d}}$	invg	0.15	0.15	0.08	0.02	0.09	0.08	0.05	0.13
$\sigma_{\eta^{\mu^c}}$	invg	0.15	0.15	0.06	0.01	0.07	0.06	0.04	0.10
$\sigma_{\eta^{\mu^i}}$	invg	0.15	0.15	0.09	0.02	0.11	0.09	0.06	0.15
$\sigma_{\eta^{\mu^g}}$	invg	0.15	0.15	0.12	0.04	0.15	0.12	0.07	0.22
$\sigma_{\eta^{\mu^x}}$	invg	0.15	0.15	0.07	0.02	0.09	0.07	0.05	0.13
$\sigma_{\eta^{\mu^m}}$	invg	0.15	0.15	0.06	0.01	0.07	0.06	0.05	0.09
σ_{η^ζ}	invg	0.15	0.15	0.03	0.00	0.03	0.03	0.02	0.03
$\sigma_{\eta^{\zeta^*}}$	invg	0.15	0.15	0.03	0.00	0.03	0.03	0.02	0.03
$\sigma_{\eta^{\tau^l}}$	invg	0.15	0.15	0.13	0.06	0.15	0.13	0.05	0.24
$\sigma_{\eta^{\tau^c}}$	invg	0.02	0.02	0.00	0.00	0.00	0.00	0.00	0.00
σ_{η^g}	invg	0.02	0.02	0.01	0.00	0.01	0.01	0.00	0.01
$\sigma_{\eta^{\pi^*}}$	invg	0.02	0.02	0.01	0.00	0.01	0.01	0.01	0.01
$\sigma_{\eta^{y^*}}$	invg	0.02	0.02	0.08	0.01	0.09	0.08	0.07	0.10
$\sigma_{\eta^{r^*}}$	invg	0.02	0.02	0.01	0.00	0.01	0.00	0.00	0.01
$\sigma_{\eta^{\bar{\pi}}}$	invg	0.02	0.02	0.01	0.00	0.01	0.01	0.01	0.01
Marginal likelihood (Laplace)				3266					
Marginal likelihood (Harmonic mean)				3267					
Average acceptance rate per chain				0.31	0.31	0.31	0.31	0.31	

Table 2: Sensitivity analysis

Parameters	Case I					Case II					Case III				
	Prior			Posterior maximisation		Prior			Posterior maximisation		Prior			Posterior maximisation	
	Type	Mean	Stdev	Mode	Stdev	Type	Mean	Stdev	Mode	Stdev	Type	Mean	Stdev	Mode	Stdev
σ_l	invg	2.20	0.55	1.92	0.40	invg	2.00	0.75	1.71	0.52	invg	2.00	0.50	1.75	0.36
h	beta	0.77	0.22	0.79	0.04	beta	0.70	0.30	0.80	0.04	beta	0.70	0.20	0.79	0.04
μ^w	invg	1.38	0.22	1.37	0.10	invg	1.25	0.30	1.42	0.14	invg	1.25	0.20	1.33	0.10
μ^m	invg	1.32	0.22	1.35	0.22	invg	1.20	0.30	1.22	0.25	invg	1.20	0.20	1.20	0.19
μ^c	invg	1.16	0.06	1.16	0.11	invg	1.05	0.08	1.04	0.12	invg	1.05	0.05	1.09	0.20
μ^i	invg	1.16	0.06	1.12	0.10	invg	1.05	0.08	1.00	0.11	invg	1.05	0.05	1.02	0.09
μ^g	invg	1.16	0.06	1.11	0.10	invg	1.05	0.08	1.00	0.11	invg	1.05	0.05	1.01	0.09
μ^x	invg	1.16	0.06	1.14	0.10	invg	1.05	0.08	1.02	0.11	invg	1.05	0.05	1.03	0.09
S	norm	8.46	1.65	10.72	1.39	norm	7.69	2.25	10.47	1.75	norm	7.69	1.50	9.73	1.28
ξ_w	beta	0.91	0.11	0.86	0.03	beta	0.83	0.15	0.85	0.05	beta	0.83	0.10	0.88	0.02
ξ_d	beta	0.83	0.11	0.75	0.04	beta	0.75	0.15	0.74	0.05	beta	0.75	0.10	0.74	0.04
ξ_m	beta	0.55	0.11	0.30	0.06	beta	0.50	0.15	0.28	0.08	beta	0.50	0.10	0.39	0.07
ξ_c	beta	0.66	0.11	0.85	0.02	beta	0.60	0.15	0.86	0.03	beta	0.60	0.10	0.84	0.03
ξ_i	beta	0.66	0.11	0.74	0.04	beta	0.60	0.15	0.75	0.04	beta	0.60	0.10	0.74	0.04
ξ_g	beta	0.66	0.11	0.76	0.06	beta	0.60	0.15	0.75	0.08	beta	0.60	0.10	0.73	0.06
ξ_x	beta	0.66	0.11	0.75	0.05	beta	0.60	0.15	0.73	0.05	beta	0.60	0.10	0.74	0.05
κ_w	beta	0.55	0.11	0.62	0.06	beta	0.50	0.15	0.62	0.07	unif			0.61	0.08
κ_d	beta	0.55	0.11	0.49	0.09	beta	0.50	0.15	0.41	0.11	unif			0.44	0.15
κ_m	beta	0.55	0.11	0.47	0.12	beta	0.50	0.15	0.42	0.17	unif			-0.14	0.20
κ_c	beta	0.55	0.11	0.50	0.07	beta	0.50	0.15	0.46	0.09	unif			0.45	0.12
κ_i	beta	0.55	0.11	0.50	0.10	beta	0.50	0.15	0.44	0.14	unif			0.40	0.19
κ_g	beta	0.55	0.11	0.57	0.11	beta	0.50	0.15	0.54	0.15	unif			0.51	0.29
κ_x	beta	0.55	0.11	0.58	0.12	beta	0.50	0.15	0.52	0.18	unif			0.98	0.27
ρ_{ε^i}	beta	0.66	0.11	0.38	0.08	beta	0.60	0.15	0.27	0.09	beta	0.60	0.10	0.39	0.08
ρ_{ε^c}	beta	0.66	0.11	0.64	0.11	beta	0.60	0.15	0.59	0.15	beta	0.60	0.10	0.62	0.09
ρ_{ε^ϕ}	beta	0.66	0.11	0.81	0.07	beta	0.60	0.15	0.83	0.07	beta	0.60	0.10	0.74	0.08
ρ_{ε^a}	beta	0.66	0.11	0.65	0.11	beta	0.60	0.15	0.56	0.16	beta	0.60	0.10	0.59	0.10
ρ_{ε^l}	beta	0.66	0.11	0.69	0.11	beta	0.60	0.15	0.65	0.17	beta	0.60	0.10	0.62	0.11
ρ_{μ^d}	beta	0.66	0.11	0.65	0.08	beta	0.60	0.15	0.68	0.09	beta	0.60	0.10	0.66	0.07
ρ_{μ^c}	beta	0.66	0.11	0.61	0.08	beta	0.60	0.15	0.61	0.11	beta	0.60	0.10	0.58	0.08
ρ_{μ^i}	beta	0.66	0.11	0.61	0.08	beta	0.60	0.15	0.60	0.10	beta	0.60	0.10	0.62	0.08
ρ_{μ^g}	beta	0.66	0.11	0.81	0.06	beta	0.60	0.15	0.80	0.07	beta	0.60	0.10	0.77	0.06
ρ_{μ^x}	beta	0.66	0.11	0.65	0.08	beta	0.60	0.15	0.62	0.10	beta	0.60	0.10	0.58	0.09
ρ_{μ^m}	beta	0.66	0.11	0.34	0.08	beta	0.60	0.15	0.27	0.10	beta	0.60	0.10	0.40	0.07
ρ_ζ	beta	0.66	0.11	0.56	0.09	beta	0.60	0.15	0.49	0.11	beta	0.60	0.10	0.53	0.08
ρ_{ζ^*}	beta	0.66	0.11	0.89	0.03	beta	0.60	0.15	0.91	0.03	beta	0.60	0.10	0.86	0.03
ρ_{τ^l}	beta	0.66	0.11	0.71	0.08	beta	0.60	0.15	0.70	0.09	beta	0.60	0.10	0.68	0.08
ρ_{τ^c}	beta	0.66	0.11	0.81	0.07	beta	0.60	0.15	0.83	0.09	beta	0.60	0.10	0.74	0.08
ρ^g	beta	0.66	0.11	0.80	0.13	beta	0.60	0.15	0.90	0.06	beta	0.60	0.10	0.68	0.11
ρ_{π^*}	beta	0.66	0.11	0.74	0.09	beta	0.60	0.15	0.79	0.10	beta	0.60	0.10	0.68	0.09
ρ_{y^*}	beta	0.66	0.11	0.90	0.03	beta	0.60	0.15	0.92	0.03	beta	0.60	0.10	0.89	0.03
ρ_{r^*}	beta	0.66	0.11	0.86	0.04	beta	0.60	0.15	0.87	0.04	beta	0.60	0.10	0.83	0.04
$\rho_{\bar{\pi}}$	beta	0.66	0.11	0.67	0.08	beta	0.60	0.15	0.66	0.10	beta	0.60	0.10	0.67	0.07
$\sigma_{\eta^{\varepsilon^i}}$	invg	0.17	0.17	0.23	0.04	invg	0.15	0.23	0.24	0.04	invg	0.15	0.15	0.21	0.03
$\sigma_{\eta^{\varepsilon^c}}$	invg	0.17	0.17	0.05	0.01	invg	0.15	0.23	0.04	0.01	invg	0.15	0.15	0.04	0.01
$\sigma_{\eta^{\varepsilon^\phi}}$	invg	0.02	0.02	0.01	0.00	invg	0.02	0.03	0.01	0.00	invg	0.02	0.02	0.01	0.00
$\sigma_{\eta^{\varepsilon^a}}$	invg	0.02	0.02	0.01	0.00	invg	0.02	0.03	0.01	0.00	invg	0.02	0.02	0.01	0.00
$\sigma_{\eta^{\varepsilon^l}}$	invg	0.17	0.17	0.10	0.04	invg	0.15	0.23	0.08	0.04	invg	0.15	0.15	0.09	0.04
$\sigma_{\eta^{\mu^d}}$	invg	0.17	0.17	0.08	0.02	invg	0.15	0.23	0.07	0.02	invg	0.15	0.15	0.08	0.02
$\sigma_{\eta^{\mu^c}}$	invg	0.17	0.17	0.07	0.02	invg	0.15	0.23	0.06	0.02	invg	0.15	0.15	0.06	0.01
$\sigma_{\eta^{\mu^i}}$	invg	0.17	0.17	0.10	0.03	invg	0.15	0.23	0.10	0.03	invg	0.15	0.15	0.09	0.02
$\sigma_{\eta^{\mu^g}}$	invg	0.17	0.17	0.12	0.04	invg	0.15	0.23	0.10	0.04	invg	0.15	0.15	0.12	0.04
$\sigma_{\eta^{\mu^x}}$	invg	0.17	0.17	0.08	0.02	invg	0.15	0.23	0.07	0.02	invg	0.15	0.15	0.08	0.02
$\sigma_{\eta^{\mu^m}}$	invg	0.17	0.17	0.07	0.01	invg	0.15	0.23	0.06	0.01	invg	0.15	0.15	0.07	0.01
σ_{η^ζ}	invg	0.17	0.17	0.03	0.00	invg	0.15	0.23	0.02	0.00	invg	0.15	0.15	0.03	0.00
$\sigma_{\eta^{\zeta^*}}$	invg	0.17	0.17	0.03	0.00	invg	0.15	0.23	0.03	0.00	invg	0.15	0.15	0.03	0.00
$\sigma_{\eta^{\tau^l}}$	invg	0.17	0.17	0.12	0.05	invg	0.15	0.23	0.14	0.06	invg	0.15	0.15	0.15	0.08
$\sigma_{\eta^{\tau^c}}$	invg	0.02	0.02	0.00	0.00	invg	0.02	0.03	0.00	0.00	invg	0.02	0.02	0.00	0.00
σ_{η^g}	invg	0.02	0.02	0.01	0.00	invg	0.02	0.03	0.01	0.00	invg	0.02	0.02	0.01	0.00
$\sigma_{\eta^{\pi^*}}$	invg	0.02	0.02	0.01	0.00	invg	0.02	0.03	0.01	0.00	invg	0.02	0.02	0.01	0.00
$\sigma_{\eta^{y^*}}$	invg	0.02	0.02	0.08	0.01	invg	0.02	0.03	0.08	0.01	invg	0.02	0.02	0.08	0.01
$\sigma_{\eta^{r^*}}$	invg	0.02	0.02	0.01	0.00	invg	0.02	0.03	0.00	0.00	invg	0.02	0.02	0.01	0.00
$\sigma_{\eta^{\bar{\pi}}}$	invg	0.02	0.02	0.01	0.00	invg	0.02	0.03	0.01	0.00	invg	0.02	0.02	0.01	0.00
Marginal likelihood			3264					3300					3151		

Table 3: Unconditional moments

Variable	Stdev	Autocorrelation					Contemporaneous cross-correlation												
		Lag1	Lag2	Lag3	Lag4	Lag5	$\hat{\pi}_t^d$	$\hat{\pi}_t$	$\hat{\pi}_t^i$	$\hat{w}_t$	$\hat{\hat{C}}_t$	$\hat{\hat{I}}_t$	$\hat{R}_t$	$\hat{L}_t$	$G\hat{\hat{D}}P_t$	$\hat{\hat{X}}_t$	$\hat{\hat{M}}_t$	$\hat{\hat{Y}}_t^*$	$\hat{R}_t^*$
Data																			
$\hat{\pi}_t^d$	0.010	0.699	0.449	0.197	-0.203	-0.248	1.000	0.598	-0.102	0.103	0.325	0.253	0.394	0.313	0.358	0.172	0.277	0.429	0.284
$\hat{\pi}_t$	0.009	0.796	0.472	0.124	-0.175	-0.226		1.000	0.233	-0.097	0.284	0.099	0.222	0.296	0.311	0.161	0.145	0.463	0.293
$\hat{\pi}_t^i$	0.012	0.689	0.357	0.041	-0.238	-0.166			1.000	-0.242	-0.124	0.092	-0.234	0.175	0.080	0.302	0.052	0.243	0.019
$\hat{\hat{w}}_t$	0.021	0.951	0.838	0.682	0.505	0.328				1.000	0.759	0.270	0.702	0.345	0.479	-0.130	0.183	0.262	0.631
$\hat{\hat{C}}_t$	0.022	0.953	0.880	0.780	0.658	0.529					1.000	0.601	0.511	0.608	0.751	0.048	0.596	0.586	0.582
$\hat{\hat{I}}_t$	0.065	0.923	0.838	0.721	0.607	0.506						1.000	0.021	0.818	0.804	0.467	0.836	0.713	0.341
$\hat{R}_t$	0.015	0.948	0.841	0.696	0.537	0.373							1.000	0.252	0.434	0.159	-0.026	0.402	0.796
$\hat{L}_t$	0.014	0.956	0.904	0.812	0.712	0.615								1.000	0.898	0.531	0.572	0.819	0.553
$G\hat{\hat{D}}P_t$	0.021	0.942	0.877	0.800	0.702	0.597									1.000	0.586	0.634	0.884	0.654
$\hat{\hat{X}}_t$	0.034	0.856	0.750	0.598	0.402	0.240										1.000	0.439	0.713	0.389
$\hat{\hat{M}}_t$	0.047	0.781	0.686	0.534	0.426	0.330											1.000	0.616	0.261
$\hat{\hat{Y}}_t^*$	0.012	0.948	0.858	0.747	0.609	0.470												1.000	0.670
$\hat{R}_t^*$	0.011	0.923	0.762	0.572	0.376	0.203													1.000
Model																			
$\hat{\pi}_t^d$	0.015	0.772	0.474	0.234	0.077	-0.012	1.000	0.693	0.513	0.151	0.133	-0.005	-0.305	0.128	0.167	0.258	0.210	0.024	-0.001
$\hat{\pi}_t$	0.011	0.757	0.557	0.376	0.237	0.137		1.000	0.486	0.077	0.117	0.000	-0.149	0.104	0.156	0.166	0.137	0.016	0.001
$\hat{\pi}_t^i$	0.018	0.712	0.361	0.105	-0.042	-0.107			1.000	0.072	0.070	0.025	-0.121	0.104	0.152	0.160	0.111	0.012	0.000
$\hat{\hat{w}}_t$	0.107	0.996	0.989	0.978	0.964	0.948				1.000	0.795	-0.753	-0.053	-0.734	0.120	-0.059	0.022	0.016	-0.001
$\hat{\hat{C}}_t$	0.102	0.980	0.941	0.896	0.849	0.804					1.000	-0.509	-0.054	-0.421	0.553	0.242	0.234	0.028	-0.012
$\hat{\hat{I}}_t$	0.223	0.985	0.954	0.915	0.874	0.834						1.000	-0.213	0.894	0.367	0.402	0.393	0.034	-0.012
$\hat{R}_t$	0.034	0.892	0.709	0.509	0.327	0.180							1.000	-0.306	-0.283	-0.171	-0.359	-0.070	0.159
$\hat{L}_t$	0.143	0.967	0.917	0.864	0.813	0.766								1.000	0.427	0.536	0.391	0.031	-0.010
$G\hat{\hat{D}}P_t$	0.062	0.947	0.855	0.750	0.644	0.545									1.000	0.641	0.577	0.063	-0.016
$\hat{\hat{X}}_t$	0.091	0.915	0.792	0.660	0.534	0.426										1.000	0.715	0.061	0.000
$\hat{\hat{M}}_t$	0.058	0.911	0.779	0.650	0.535	0.436											1.000	0.119	-0.021
$\hat{\hat{Y}}_t^*$	0.009	0.885	0.784	0.694	0.614	0.544												1.000	0.000
$\hat{R}_t^*$	0.009	0.829	0.687	0.570	0.472	0.391													1.000

Table 4: Model comparison

Parameters	Case IV					Case V				
	Prior			Posterior kernel maximisation		Prior			Posterior kernel maximisation	
	Type	Mean	Stdev	Mode	Stdev	Type	Mean	Stdev	Mode	Stdev
σ_l	invg	2.00	0.50	1.81	0.39	invg	2.00	0.50	1.81	0.39
h	beta	0.70	0.20	0.76	0.04	beta	0.70	0.20	0.79	0.04
μ^w	invg	1.25	0.20	1.43	0.13	invg	1.25	0.20	1.36	0.13
μ^m	invg	1.20	0.20	1.15	0.18	invg	1.20	0.20	1.22	0.20
μ^c	invg	1.05	0.05	1.04	0.09	invg	1.05	0.05	1.05	0.10
μ^i	invg	1.05	0.05	1.00	0.09	invg	1.05	0.05	1.02	0.09
μ^g	invg	1.05	0.05	1.00	0.09	invg	1.05	0.05	1.01	0.09
μ^x	invg	1.05	0.05	1.00	0.09	invg	1.05	0.05	1.04	0.09
S	norm	7.69	1.50	9.24	1.28	norm	7.69	1.50	9.73	1.29
ξ_w	beta	0.83	0.10	0.86	0.03	beta	0.83	0.10	0.87	0.03
ξ_d	beta	0.75	0.10	0.84	0.03	beta	0.75	0.10	0.78	0.03
ξ_m	beta	0.50	0.10	0.37	0.06	beta	0.50	0.10	0.37	0.07
ξ_c	beta	0.60	0.10	0.87	0.02	beta	0.60	0.10	0.86	0.02
ξ_i	beta	0.60	0.10	0.84	0.02	beta	0.60	0.10	0.76	0.03
ξ_g	beta	0.60	0.10	0.80	0.05	beta	0.60	0.10	0.73	0.06
ξ_x	beta	0.60	0.10	0.73	0.04	beta	0.60	0.10	0.72	0.04
κ_w	beta	0.50	0.10	0.65	0.05	-	-	-	-	-
κ_d	beta	0.50	0.10	0.68	0.06	-	-	-	-	-
κ_m	beta	0.50	0.10	0.48	0.10	-	-	-	-	-
κ_c	beta	0.50	0.10	0.46	0.07	-	-	-	-	-
κ_i	beta	0.50	0.10	0.68	0.07	-	-	-	-	-
κ_g	beta	0.50	0.10	0.62	0.09	-	-	-	-	-
κ_x	beta	0.50	0.10	0.65	0.09	-	-	-	-	-
ρ_{ε^i}	beta	0.60	0.10	0.37	0.08	beta	0.60	0.10	0.40	0.08
ρ_{ε^c}	beta	0.60	0.10	0.58	0.10	beta	0.50	0.10	0.64	0.09
ρ_{ε^ϕ}	beta	0.60	0.10	0.77	0.06	beta	0.50	0.10	0.75	0.07
ρ_{ε^a}	beta	0.60	0.10	0.60	0.10	beta	0.50	0.10	0.60	0.10
ρ_{ε^l}	beta	0.60	0.10	0.62	0.11	beta	0.50	0.10	0.63	0.11
ρ_{μ^d}	-	-	-	-	-	beta	0.50	0.10	0.72	0.05
ρ_{μ^c}	-	-	-	-	-	beta	0.50	0.10	0.59	0.08
ρ_{μ^i}	-	-	-	-	-	beta	0.50	0.10	0.70	0.05
ρ_{μ^g}	-	-	-	-	-	beta	0.60	0.10	0.80	0.05
ρ_{μ^x}	-	-	-	-	-	beta	0.60	0.10	0.67	0.07
ρ_{μ^m}	-	-	-	-	-	beta	0.60	0.10	0.37	0.07
ρ_ζ	beta	0.60	0.10	0.56	0.08	beta	0.60	0.10	0.51	0.08
ρ_{ζ^*}	beta	0.60	0.10	0.89	0.03	beta	0.60	0.10	0.88	0.03
ρ_{τ^l}	beta	0.60	0.10	0.69	0.07	beta	0.60	0.10	0.68	0.09
ρ_{τ^c}	beta	0.60	0.10	0.75	0.08	beta	0.60	0.10	0.73	0.08
ρ^g	beta	0.60	0.10	0.88	0.04	beta	0.60	0.10	0.68	0.11
ρ_{π^*}	beta	0.60	0.10	0.72	0.07	beta	0.60	0.10	0.67	0.09
ρ_{y^*}	beta	0.60	0.10	0.89	0.03	beta	0.60	0.10	0.89	0.03
ρ_{r^*}	beta	0.60	0.10	0.83	0.04	beta	0.60	0.10	0.83	0.04
$\rho_{\bar{\pi}}$	beta	0.60	0.10	0.63	0.07	beta	0.60	0.10	0.76	0.06
$\sigma_{\eta^{\varepsilon^i}}$	invg	0.15	0.15	0.20	0.03	beta	0.60	0.10	0.21	0.03
$\sigma_{\eta^{\varepsilon^c}}$	invg	0.15	0.15	0.04	0.01	beta	0.60	0.10	0.04	0.01
$\sigma_{\eta^{\varepsilon^\phi}}$	invg	0.02	0.02	0.01	0.00	beta	0.60	0.10	0.01	0.00
$\sigma_{\eta^{\varepsilon^a}}$	invg	0.02	0.02	0.01	0.00	beta	0.60	0.10	0.01	0.00
$\sigma_{\eta^{\varepsilon^l}}$	invg	0.15	0.15	0.09	0.04	beta	0.60	0.10	0.09	0.04
$\sigma_{\eta^{\mu^d}}$	invg	0.15	0.15	0.33	0.10	beta	0.60	0.10	0.08	0.02
$\sigma_{\eta^{\mu^c}}$	invg	0.15	0.15	0.07	0.02	beta	0.60	0.10	0.06	0.01
$\sigma_{\eta^{\mu^i}}$	invg	0.15	0.15	0.41	0.12	beta	0.60	0.10	0.09	0.02
$\sigma_{\eta^{\mu^g}}$	invg	0.15	0.15	0.57	0.28	invg	0.15	0.15	0.12	0.04
$\sigma_{\eta^{\mu^x}}$	invg	0.15	0.15	0.17	0.05	invg	0.15	0.15	0.07	0.02
$\sigma_{\eta^{\mu^m}}$	invg	0.15	0.15	0.08	0.02	invg	0.02	0.02	0.06	0.01
σ_{η^ζ}	invg	0.15	0.15	0.03	0.00	invg	0.02	0.02	0.03	0.00
$\sigma_{\eta^{\zeta^*}}$	invg	0.15	0.15	0.03	0.00	invg	0.15	0.15	0.03	0.00
$\sigma_{\eta^{\tau^l}}$	invg	0.15	0.15	0.14	0.05	invg	0.15	0.15	0.12	0.06
$\sigma_{\eta^{\tau^c}}$	invg	0.02	0.02	0.00	0.00	invg	0.15	0.15	0.00	0.00
σ_{η^g}	invg	0.02	0.02	0.01	0.00	invg	0.15	0.15	0.01	0.00
$\sigma_{\eta^{\pi^*}}$	invg	0.02	0.02	0.01	0.00	invg	0.15	0.15	0.01	0.00
$\sigma_{\eta^{y^*}}$	invg	0.02	0.02	0.08	0.01	invg	0.15	0.15	0.08	0.01
$\sigma_{\eta^{r^*}}$	invg	0.02	0.02	0.01	0.00	invg	0.15	0.15	0.01	0.00
$\sigma_{\eta^{\bar{\pi}}}$	invg	0.02	0.02	0.01	0.00	invg	0.15	0.15	0.01	0.00
Marginal likelihood	3244					3245				

Table 5: Variance decomposition

Variable	Shock																			
	η^{ε^i}	η^{ε^c}	η^{ε^ϕ}	η^{ε^a}	η^{ε^l}	η^{μ^d}	η^{μ^m}	η^{μ^c}	η^{μ^i}	η^{μ^g}	η^{μ^x}	$\eta^{\bar{\pi}}$	η^{ζ}	η^{ζ^*}	η^{π^*}	η^{y^*}	η^{r^*}	η^{τ^l}	η^{τ^c}	η^{g}
$\hat{\hat{K}}_t$	3.0	0.2	0.0	0.0	0.0	0.2	0.0	0.1	0.2	0.0	0.0	0.1	95.6	0.4	0.1	0.0	0.0	0.2	0.0	0.0
$\hat{\hat{C}}_t$	2.1	4.5	0.0	0.1	0.1	1.9	0.8	0.2	1.2	0.0	0.0	1.7	81.2	4.2	1.3	0.1	0.0	0.4	0.0	0.1
$\hat{R}_t$	6.8	2.9	6.9	0.5	0.0	10.8	19.8	0.3	0.6	0.0	0.0	0.3	14.1	31.8	0.2	0.6	3.4	0.1	0.1	0.7
$\hat{\hat{I}}_t$	5.6	2.4	0.0	0.2	0.2	5.8	1.0	0.9	1.9	0.0	0.1	1.6	70.2	7.2	1.3	0.2	0.0	1.4	0.0	0.1
$\hat{Q}$	10.8	3.3	1.2	0.4	0.0	11.2	9.5	0.3	2.5	0.0	0.0	2.4	26.3	26.2	4.1	0.5	0.7	0.1	0.0	0.5
$\hat{\hat{w}}_t$	2.4	0.3	0.0	0.1	0.2	3.7	0.1	0.8	0.1	0.0	0.0	0.1	88.9	2.0	0.1	0.1	0.0	1.2	0.1	0.0
$\hat{r}^k$	2.4	0.2	0.0	0.3	0.0	3.5	1.0	0.3	0.1	0.0	0.5	2.5	83.8	3.3	1.8	0.1	0.0	0.1	0.0	0.0
$\hat{\hat{Y}}_t^d$	1.6	0.2	0.0	0.4	0.5	10.7	5.0	0.2	0.2	0.0	2.3	14.2	37.0	13.4	10.8	0.3	0.0	3.0	0.0	0.1
$\hat{L}_t$	1.8	0.3	0.0	1.0	0.2	6.2	2.9	0.1	0.1	0.0	1.4	7.7	62.6	8.0	5.9	0.2	0.0	1.4	0.0	0.1
$\hat{\hat{M}}_t$	0.9	0.3	0.1	0.1	0.1	2.4	6.4	0.1	0.1	0.0	8.8	2.6	5.8	67.9	2.2	1.5	0.1	0.4	0.0	0.2
$\hat{\hat{X}}_t$	0.8	0.1	0.0	0.2	0.2	4.1	1.5	0.3	0.1	0.0	32.3	9.4	18.6	22.6	8.0	0.5	0.0	1.5	0.0	0.0
$\hat{\hat{B}}_t$	3.9	1.6	2.5	0.2	0.0	5.6	9.7	0.3	0.4	0.0	0.0	0.9	53.2	18.0	1.7	0.4	1.3	0.0	0.0	0.4
$\hat{\hat{B}}_t^*$	7.3	3.1	1.7	0.5	0.0	11.6	21.3	0.3	0.7	0.0	0.0	0.3	15.1	34.1	0.2	0.7	2.0	0.1	0.1	0.8
$G\hat{\hat{D}}P_t$	0.8	0.7	0.1	0.4	0.4	10.6	4.7	0.4	2.6	6.0	0.2	9.0	33.5	21.3	6.5	0.5	0.1	2.4	0.0	0.1
$\hat{\pi}^d$	0.7	0.0	0.0	1.6	0.2	39.5	1.2	0.3	0.1	0.0	0.7	12.8	15.7	6.6	19.7	0.2	0.0	0.9	0.0	0.0
$\hat{\pi}^m$	0.0	0.0	0.0	0.0	0.0	0.0	89.9	0.0	0.0	0.0	0.0	0.1	0.0	0.0	10.1	0.0	0.0	0.0	0.0	0.0
$\hat{\pi}^c$	0.3	0.0	0.0	0.1	0.1	3.1	1.2	13.8	0.0	0.0	0.2	37.7	5.7	2.3	35.1	0.1	0.0	0.4	0.0	0.0
$\hat{\pi}^i$	0.2	0.0	0.0	0.2	0.0	3.9	1.8	0.1	64.9	0.0	0.1	7.7	3.9	1.6	15.5	0.0	0.0	0.3	0.0	0.0
$\hat{\pi}^g$	0.1	0.0	0.0	0.1	0.0	1.5	0.7	0.0	0.0	86.6	0.1	2.8	1.5	0.6	6.0	0.0	0.0	0.1	0.0	0.0
$\hat{\pi}^x$	0.2	0.0	0.0	0.2	0.1	5.1	2.4	0.1	0.0	0.0	57.5	8.1	4.9	2.0	19.1	0.1	0.0	0.3	0.0	0.0
$\hat{\pi}$	0.2	0.0	0.0	0.1	0.1	2.8	1.1	12.6	0.0	0.0	0.2	34.4	5.2	2.1	32.0	0.1	0.0	0.4	8.8	0.0
$\hat{\epsilon}$	0.9	0.1	0.0	0.2	0.3	4.9	1.8	0.3	0.2	0.0	38.3	11.1	22.0	8.5	9.5	0.2	0.0	1.8	0.0	0.0
$\hat{R}^r$	6.4	2.6	5.9	0.4	0.0	9.5	18.0	0.7	0.6	0.0	0.1	2.2	15.4	31.3	2.9	0.6	2.9	0.1	0.0	0.6

Figures

Figure 1: Observed and filtered data

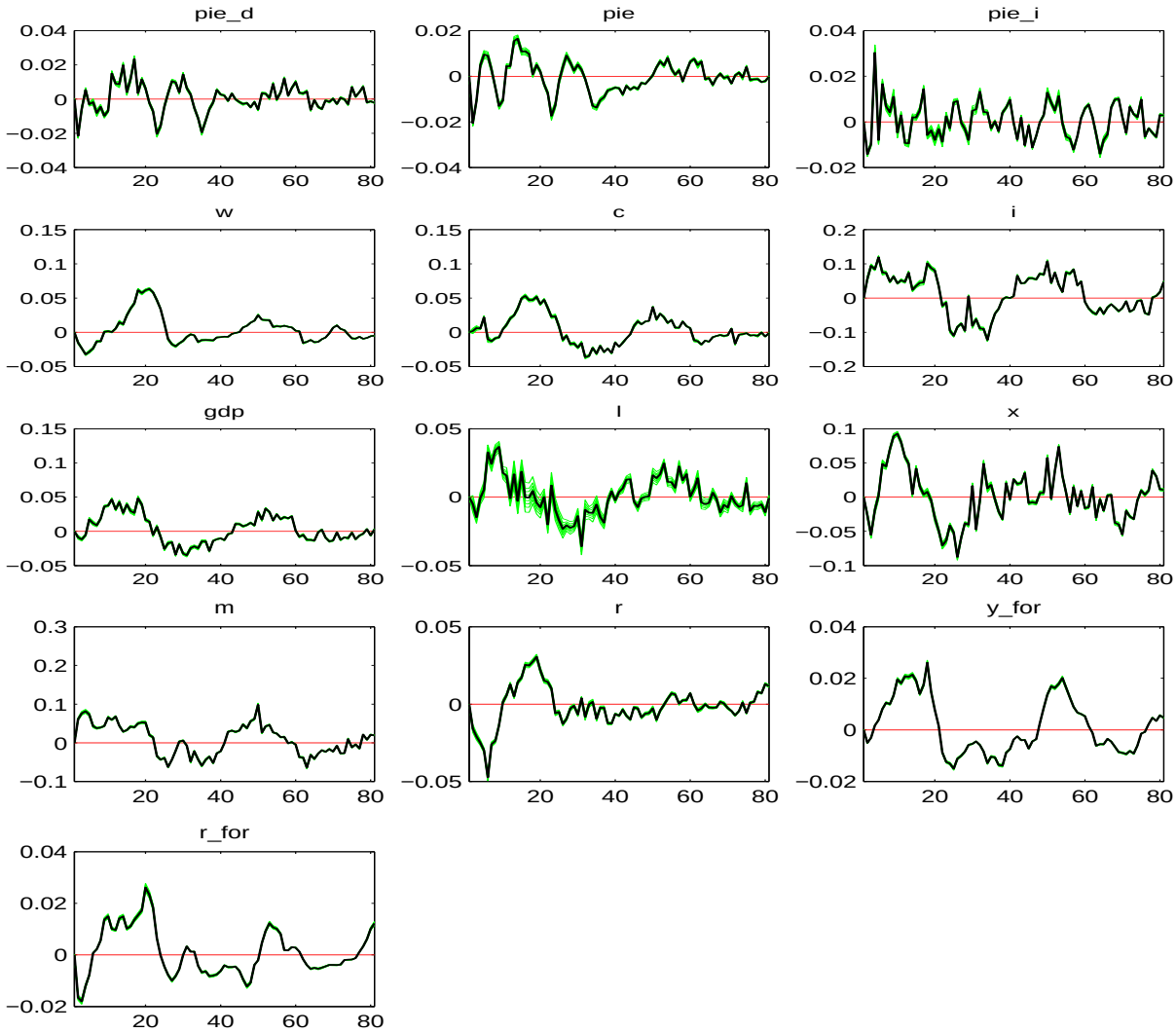


Figure 2: Multivariate MH convergence diagnosis

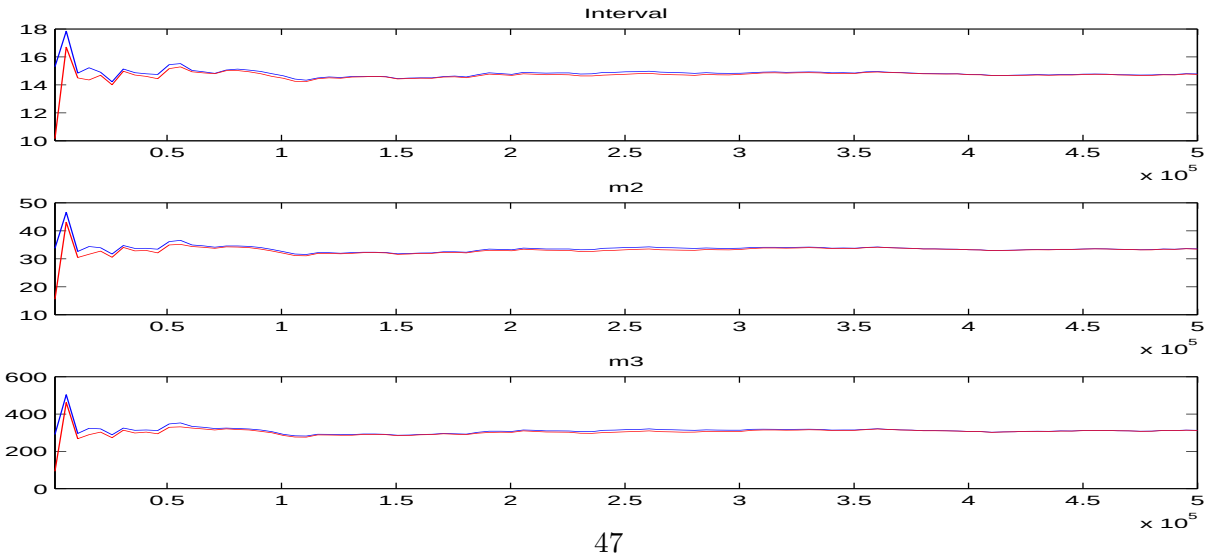


Figure 3: Priors and posteriors

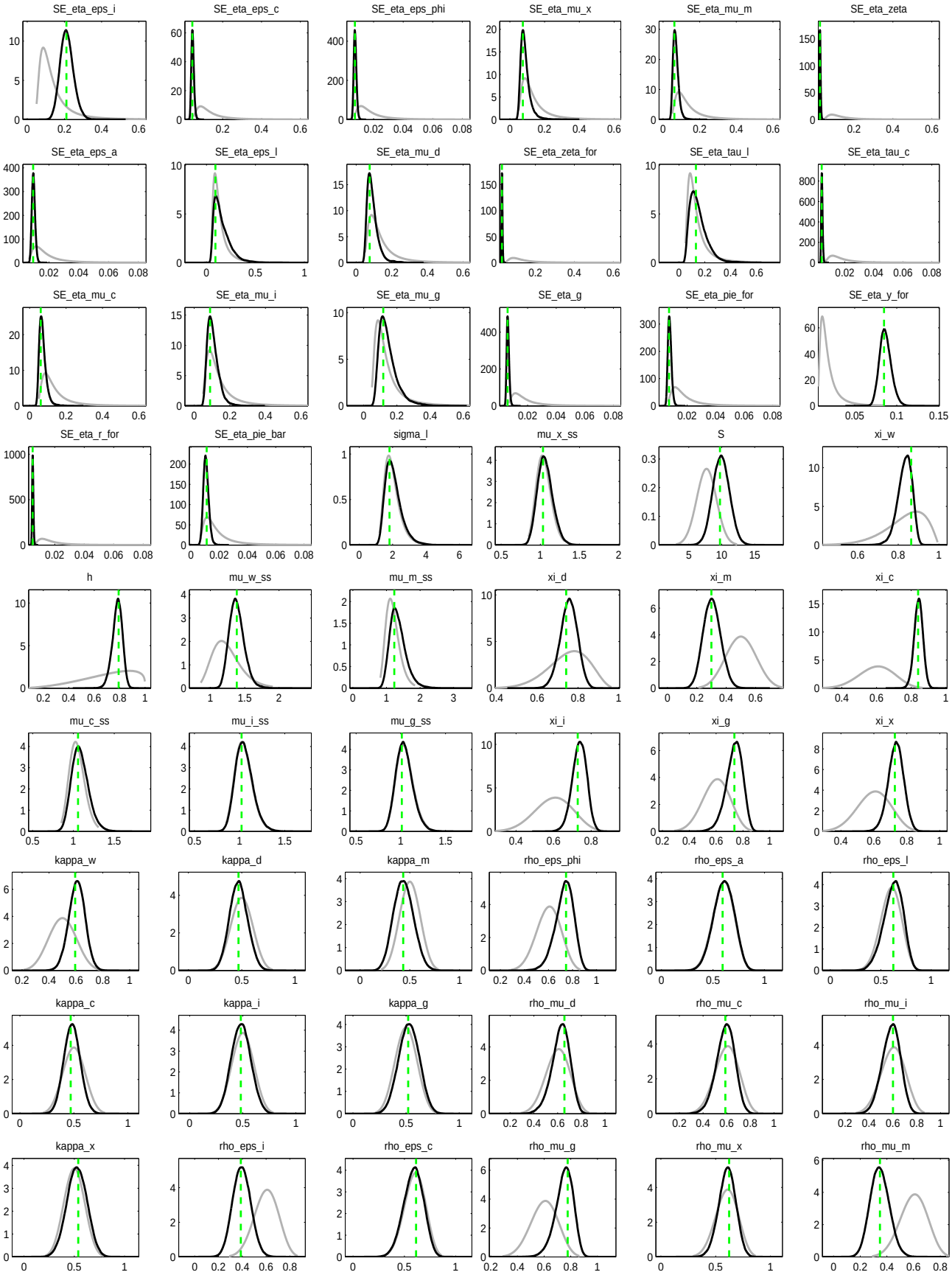


Figure 4: Priors and posteriors (cont.)

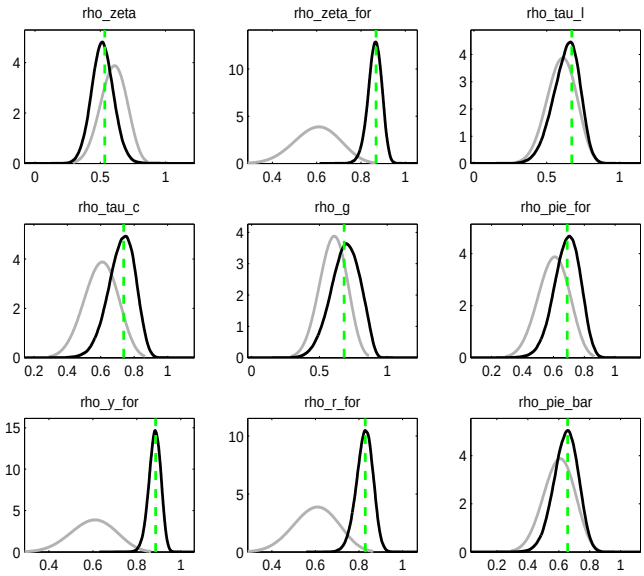


Figure 5: Estimated shocks

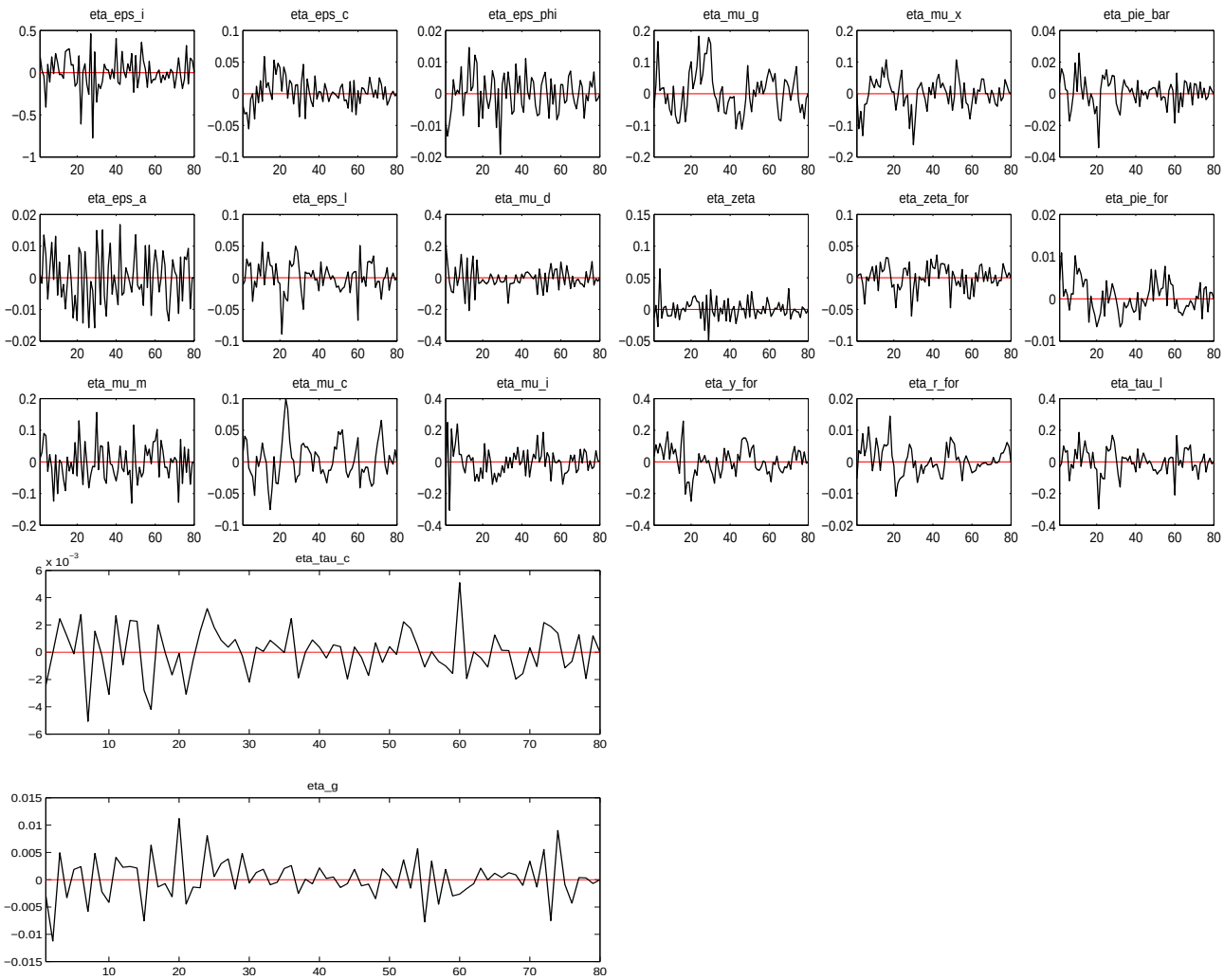
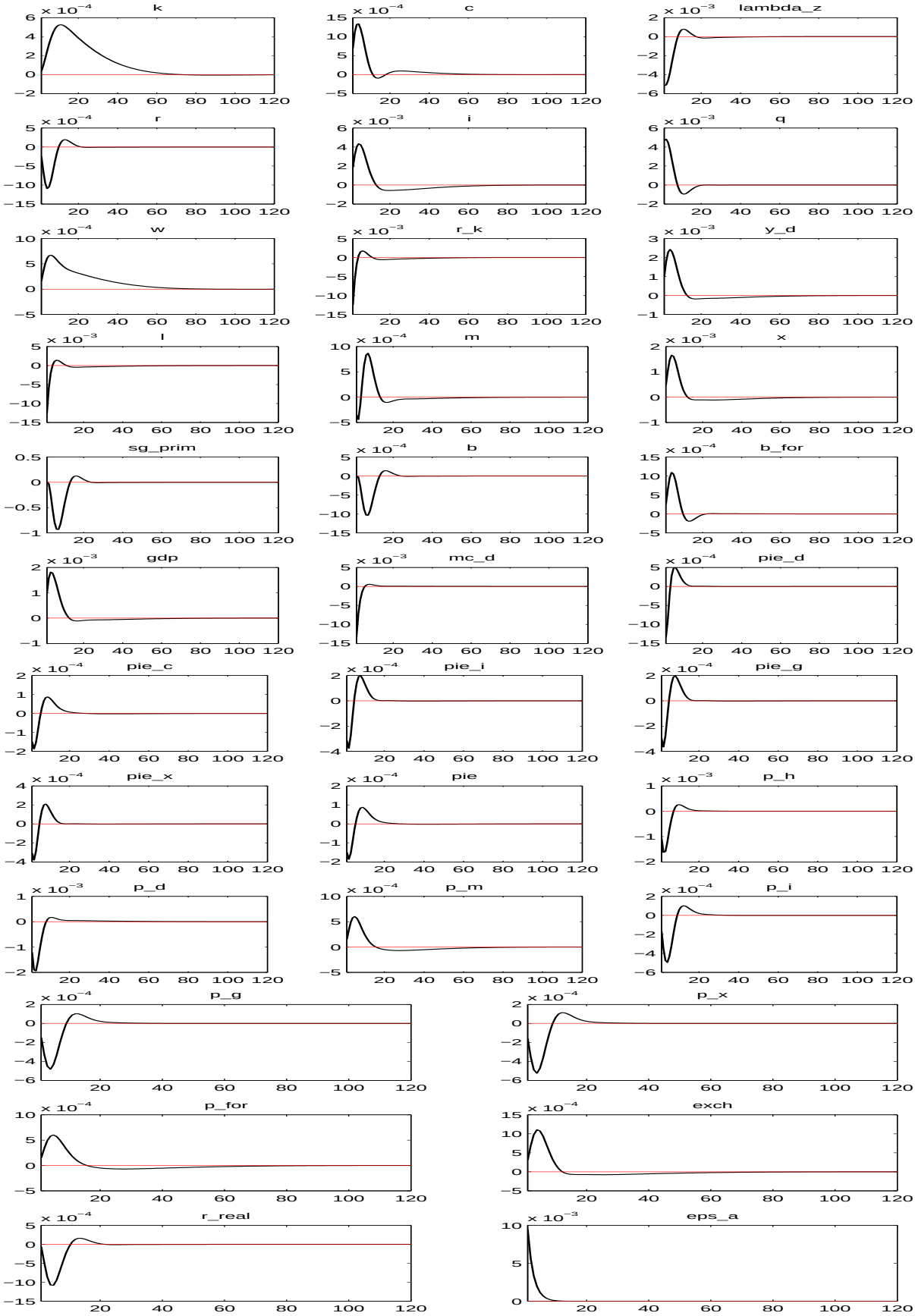


Figure 6: Impulse responses to a positive productivity shock



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