The Taylor Principle and (In-) Determinacy in a New Keynesian Model with hiring Frictions and Skill Loss

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Abstract

This paper extends the model of Blanchard and Gali (2008) to include skill loss among the unemployed along the lines of Pissarides (1992). The unemployed loose a fraction of their skills per quarter of their unemployment spell. If the central bank responds only to inflation and quarterly skill decay is sufficiently high, determinacy requires a coefficient on inflation smaller than one. This holds regardless of whether the central bank responds to lagged, current or future expected inflation. The critical skill decay percentage (i.e. the level of skill decay changing the determinacy requirement in the monetary policy rule) is low and plausible if the flow characteristics of the labour market are "Continental European" (little hiring and firing). In an American style labour market with much hiring and firing, the critical skill decay percentage is implausibly high.

Neither responding to the output gap in addition to inflation nor interest rate smoothing help to restore determinacy if skill loss is above its critical level. However, a modest response to unemployment robustly guarantees determinacy.

Furthermore, with skill loss above its critical level and the inflation coefficient in the interest feedback rule being larger than one, both an adverse sunspot shock and an adverse technology shock increase unemployment extremely persistently.

Contents

1 Introduction

The idea that skill loss among the unemployed might generate a relationship between the actual and the natural unemployment rate is an old one. In fact, Phelps (1972) himself emphasized this mechanism when developing the concept of the natural rate of unemployment. In this paper, we introduce skill loss among the unemployed along the lines of Pissarides (1992) into a New Keynesian model with hiring frictions developed by Blanchard and Gali (2008). We assume that workers who remain unemployed for one quarter or longer loose a fraction of their skills per quarter of their unemployment spell. The share of those unemployed for more than one quarter affects the willingness of firms to create jobs as firms are matched with different types of workers according to their share in the job seeking population. Our goal is to investigate the effects of introducing skill loss on macroeconomic stability under varying monetary policy rules and degrees of skill loss. As far as we are aware, this question has not been addressed so far within a state-of-the-art general equilibrium framework.

Our key results are as follows. Firstly, for sufficiently high levels of skill loss, a nominal interest rate feedback rule with a coefficient on inflation exceeding one does not guarantee determinacy if the quarterly skill loss percentage is large enough. If the central bank responds only to inflation, the coefficient on inflation has to be less than one. This does not depend on whether the central bank responds to current, expected future- or lagged inflation. Secondly, let us denote the level of skill decay above which determinacy requires the nominal interest rate to respond less than one for one to inflation as the "critical level". We find that the critical level of skill decay will be implausibly high if we adopt what Blanchard and Gali deem an "American" calibration of labour market flows, i.e. a high job finding probability and a high job destruction rate. By contrast, if we adopt Blanchard and Gali's "continental European" calibration, with little hiring and firing, the critical skill loss percentage will be a lot lower, about 2.5% per quarter. Thirdly, under the continental European calibration, with skill loss above its critical level and the inflation coefficient in the interest feedback rule being larger than one, both an adverse sunspot shock and an adverse uncorrelated technology shock have very persistent effects on unemployment.

Our results can be compared to an evolving literature on how the determinacy properties of monetary policy rules change when monetary policy has some indirect or direct effect on the supply side. These papers regularly find that some sort of restriction on the inflation coefficient in the interest feedback rule and/or some response to output are necessary to ensure determinacy. For instance, Korozumi and van Zandweghe (2008) and Carlstrom and Fuerst (2005) find in a New Keynesian model with capital, an interest feedback rule where the interest rate only responds to expected inflation limits the permissible inflation responses to an extremely small range above but very close to one. With such a rule, higher future inflation increases the ex ante real interest rate and thus the expected future capital rental via the no arbitrage condition. This in turn increases expected inflation. Kurozumi and van

Zandweghe (2008) also show that even a modest response to current output (as opposed to the output gap as used in this paper) substantially widens the permissible range. A response to the lagged interest rate above a certain threshold has a similar effect. Duffy and Xiao (2008) qualify their results by showing that in the presence of capital stock adjustment costs, even a modest response to expected future output is enough to guarantee determinacy.

Surico (2008) considers a New Keynesian model with a cost-channel along the lines of Ravenna and Walsh (2006), where the nominal interest rate has a direct positive effect on inflation since firms have to borrow working capital to pay wages. He shows that, if the interest rate responds to current inflation, determinacy requires an upper bound on the inflation coefficient, which, however, is too high to form a relevant constraint for monetary policy. Tuesta and Llosa (2009) investigate the same model with a purely forward looking rule and show that determinacy is unattainable if the central bank responds only to expected inflation.

All of the cited results have in common that the determinacy problem caused by a monetary policy rule responding to inflation alone is never caused by the active response to inflation per se, but to the timing subscript of inflation in the interest feedback rule. By contrast, in the model proposed here, it is the Taylor principle itself -the idea that an increase in inflation should sooner or later cause an increase in the real interest rate- which causes indeterminacy if the labour market flows are low and quarterly skill loss among the unemployed is too high.

The paper is structured as follows. Sections two discusses some empirical evidence for skill loss among the unemployed while section three derives the model. Section four analyses determinacy in the absence of skill loss, i.e. in the original Blanchard and Gali (2008) model. Section five derives the marginal cost equation in the presence of skill loss and shows how the effect of unemployment on marginal cost is affected by the introduction of skill loss. Section six analyses determinacy in the model with skill loss. Section seven discusses the response of the model under the European calibration to an adverse sunspot shock and an averse technology shock. Section eight concludes.

2 Evidence for Skill Loss

Direct, quantifiable evidence for skill loss during unemployment is difficult to obtain. An idea of the size of skill decay over time can be gained from the literature on wage loss upon worker displacement. This literature has produced evidence based on panel regressions showing that the wage upon reemployment depends negatively on the duration of the unemployment spell. Skill decay during unemployment is usually seen as one of the factors causing this relationship, although the evolution of reservation wage due to other factors (for instance depletion of an unemployed persons wealth) would be expected to have an impact as well. Evidence along these lines include Addison and Portugal (1989) for American male workers displaced and reemployed between 1979 and 1984, Pichelmann and Riedel (1993) for Austrian workers between 1972 and 1988, Gregory and Jukes (2001) for British male workers between 1984 and 1994 and Gangji and Plasman (2007) for Belgian workers. Their findings on the effect of a one year unemployment spell on the real wage are -39%, -24%, -11% and -8% respectively.[1] Pichelmann and Riedel (1993) explicitly ask whether the earnings penalty arising from duration diminishes over time and find that it is permanent. Furthermore, Nickell et al. (2001) look at three four year periods between 1982 to 1997. They ask how the earnings loss is changed if the unemployment spell exceeded 6 months and find an additional permanent earnings loss between 6.8% and 10.6%.[2] To which extent these numbers reflect skill depreciation depends on the movement of the reservation wage in general and in particular its responsiveness to the respective workers human capital evolution.

There is also evidence suggesting that unemployed workers become less attractive employees as their unemployment spell lengthens. Jackman et al. (1991) cite various studies showing that morale and motivation decline the longer a person remains unemployed.[3] The stylised fact that the probability of an unemployed person of leaving unemployment increases with the unemployment duration (see for instance Machin and Manning (1999)[4]) is also seen by some as evidence for skill loss among the unemployed. It is however a priori unclear whether this represents "true" duration dependence, i.e. the worsening of an individual's employment probability over time, or merely individual heterogeneity, possibly unobserved. In the later case different individuals have different hazard rates of leaving unemployment as a result of differing individual characteristics like their education. The individuals with higher hazard rates will leave the unemployment pool quickly, implying that the average hazard rate of a cohort of unemployed falls as the unemployment spell lengthens. However, Jackman et al (1991) argue that in the presence of pure individual heterogeneity, and under certain assumptions about its nature, the ratio of the average hazard rate and the hazard rate of new entrants into unemployment would have to be constant as the average hazard rate moves up or down. They find that for British data, the average hazard rate declines in fact much more than the hazard rate of new entrants. Van den Berg and van Ours confirm this result using other "eyeball" tests [5] and a more formal non-parametric estimation method[6]. Using the same method, they also find

[1]For Addison and Portugal (1989), we have calculated the annual earnings penalty using the lower coefficient on log(duration) in their two preferred specifications (Table 3, columns 5 and 6), p. 294. Duration is measured in weeks. For Pichelmann and Riedel (1993), we had to resort to the same procedure. Their coefficient estimates for the effect on the real wage is reported in table 2, p. 8. The results of Gregory and Jukes (2001) are reported on page F619, while the results of Gangji and Plasman (2007) are reported on page 18, table 2.

[2]See Nickel et al. (2001), p. 17.

[3]See Jackman, Layard and Nickell (1991), p. 259.

[4]See Machin and Manning (1999), p. 3100.

[5]van der Berg and van Ours (1994), p. 23.

[6]See Van der Berg and van Ours (1994b), p. 442.

negative duration dependence for the United States.[7] The model discussed below does not actually model duration dependence (although it could be extended to do so). However, overall, we view the evidence above as indicating that workers are less efficient at work the longer they have been unemployed.

3 The Model

In this section we add skill loss along the lines of Pissarides (1992) to the Blanchard and Gali (2008) model. We first go through the optimisation problems of households and firms and then show what the expressions for marginal cost and the Phillips Curve look like in the absence and in the presence of skill loss.

3.1 Households

The economy is populated by a continuum of representative and infinitely lived households. A household consists of a continuum of members who supply labour to firms. They might be employed or unemployed. The household derives income from wage payments, bond holdings, and firms' profits. It allocates its income to buying a CES basket of consumption goods and a riskless bond to maximise

$$E_t \sum_{t=0}^{\infty} \log C_t \tag{1}$$

where C_t denotes consumption, subject to the budget constraint

$$N_t W_t + \frac{B_{t-1}}{P_t}(1 + i_{t-1}) + F_t \geq C_t + \frac{B_t}{P_t}$$

where P_t, N_t, W_t, B_t, i_t and F_t denote the price level, hours worked by the members of the household, the real wage, bonds, the nominal interest rate and the profits of firms. Consumption is governed by the usual first order condition

$$\frac{1}{C_t} = [1 + i_t] \beta E_t \left[\frac{1}{1 + \pi_{t+1}} \frac{1}{C_{t+1}} \right]$$

where π_t denotes the inflation rate.

3.2 Firms

There are two types of firms. Final goods firms indexed by i produce a differentiated product using the intermediate good $X_t(i)$ in the linear technology

$$Y_t(i) = X_t(i)$$

[7] Van der Berg and van Ours (1996), p. 123.

They produce the varieties in the CES basket of goods consumed by households. The demand curve for variety i resulting from the household spreading it's expenditures across varieties in a cost minimising way is given by $c_t(i) = C_t \left(\frac{p_t(i)}{P_t}\right)^{-\theta}$, where $c_t(i)$, $p_t(i)$ and P_t denote consumption and price of variety i and the price level of the consumption basket, respectively. We will assume that final goods firms face nominal rigidities in the form of Calvo contracts, i.e. only a randomly chosen fraction $1-\omega$ of firms can re-optimise its price in a given period. They accordingly maximise

$$E_t \left[\sum_{i=0}^{\infty} (\omega\beta)^i \frac{C_t}{C_{t+i}} C_{t+i} \left[\left(\frac{p_t(j)}{P_{t+i}} \right)^{1-\theta} - mc_{t+i} \left(\frac{p_t(j)}{P_{t+i}} \right)^{-\theta} \right] \right]$$

where mc_t denotes real marginal costs. The price index evolves according to

$$P_t^{1-\theta} = (1-\omega) \left(p_t^*(j) \right)^{1-\theta} + \omega \left(P_{t-1} \right)^{1-\theta}$$

where $p_t^*(j)$ denotes the price set by those firms allowed to reset their price in period t. Taking first order approximations to both to the final goods first order condition and the law of motion of the price index and combining the resulting equations yields the familiar New Keynesian Phillips curve relating inflation in period t to expected t+1 inflation and period t marginal costs. The marginal cost of the final goods firms equals the real price of the intermediate good, $\frac{P_t^I}{P_t}$.

Intermediate goods firms operate under perfect competition and are owned by households. As is common in the matching literature, we assume that a fixed fraction δ of jobs is destroyed each period. This can be thought of as an idiosyncratic productivity shock and implies that even with constant employment, there are constantly flows in and out of employment. Thus employment of firm j evolves according to

$$N_t(j) = (1-\delta) N_{t-1}(j) + H_t(j)$$

Where $H_t(j)$ denotes the amount of hiring in firm j. Aggregate hiring is accordingly given by

$$H_t = N_t - (1-\delta) N_{t-1} \tag{2}$$

Note that the lower is δ, the more H_t will depend on the change as opposed to the level of employment.

The Intermediate good firms employ labour to produce intermediate goods $X_t(j)$, where j indexes the intermediate good firm. Following Pissarides (1992), we assume that the productivity of a newly hired worker is the product of exogenous technology A_t^P and the skill level of worker of type i A^i. Thus the productivity $prod_t^i$ of a worker of type i is given by

$$prod_t^i = A_t^P A^i$$

We follow Pissarides (1992) by making the following assumptions. A^i equals 1 if he is short term unemployed, i.e. if he lost his job in period t. Unemployed workers loose a fraction δ_s of their skills per quarter if the remain unemployed for 1 quarter or longer. Skill decay continues for the duration of the unemployment spell. We assume further, following Pissarides (1992), that the unemployed regain all their skills after one quarter of employment, that intermediate goods firms meet workers according to their share among job seekers and that they hire any worker they meet.[8] Finally, when firms decide whether to hire or not they know the state of exogenous technology A_t^P but not which type of worker they are going to meet with.

We denote the average skill level of the newly hired as A_t^L.The productivity of a newly hired worker expected by the firm when deciding whether to hire is denoted by A_t and is accordingly given by

$$A_t = A_t^P A_t^L \tag{3}$$

A_t^L is given by

$$A_t^L = \sum_{i=0}^{\infty} \beta_s^i s_t^i \tag{4}$$

where $\beta_s = 1 - \delta_s$ and s_t^i denotes the share of those unemployed i periods among job seekers. Note that A_t^L will be smaller than one if $\delta_s > 0$ and equal to one if $\delta_s = 0$. We will refer to A_t^L as the average skill level in period t rather than the expected skill level to avoid confusion when we refer to $E_t A_{t+1}^L$.

The shares of the various groups among the number of job seekers, denoted as U_t, are given by

$$s_t^i = \frac{\delta N_{t-1-i} \prod_{j=1}^{i} \left(1 - x_{t-i}\right)}{U_t} \tag{5}$$

where x_t denotes labour market tightness, defined as the ratio between aggregate hiring H_t and U_t, i.e.

$$x_t = \frac{H_t}{U_t} \tag{6}$$

We interpret labour market tightness x_t as the probability of an unemployed person to move into employment in period t. For instance, s_t^2 is calculated as follows: δN_{t-3} workers loose their jobs in period t-2. A fraction x_{t-2} moves right back into employment while a fraction $(1 - x_{t-2})$ remains unemployed and keeps looking for jobs in period t-1. Of those $\delta N_{t-3} (1 - x_{t-2})$ workers, a fraction $(1 - x_{t-1})$ does not find a job during t-1 and is still unemployed at the end of that period. Dividing those $\delta N_{t-3} (1 - x_{t-2}) (1 - x_{t-1})$ unemployed by U_t then gives the share of those unemployed for two periods among job seekers in period t.

U_t consists of those who did not find a job at the end of period t-1 and those whose jobs were destroyed at the beginning of t:

$$U_t = 1 - N_{t-1} + \delta N_{t-1} = 1 - (1 - \delta) N_{t-1} \tag{7}$$

[8] See Pissarides (1992), pp. 1371-1391. In contrast to Pissarides, skill loss does not stop after one quarter in our model.

As in the Blanchard Gali model, we assume that the real wage is rigid. We assume that the wage of a worker depends on his individual productivity in exactly the same way as in Blanchard Gali (2008): The wage W_t^i of a worker who has been unemployed for i periods is given by $W_t^i = \Theta' \left(A_t^P A_t^i\right)^{1-\gamma}$, with $0 \leq \gamma \leq 1$ This means that there are five different wage levels. Accordingly, the real wage the firm expects to pay when it decides to hire is given by

$$W_t = \Theta' \left(\sum_{i=0}^{\infty} \beta_s^{i(1-\gamma)} s_t^i \right) \left(A_t^P\right)^{1-\gamma} \tag{8}$$

Note that for $\delta^S = 0$, this collapses to $W_t = \Theta' \left(A_t^P\right)^{1-\gamma}$ as in Blanchard Gali. This is the wage a firm expects to pay when it decides whether to hire. Θ' is backed out to support a desired steady state combination of x, δ and n. This is shown in Appendix II. For future reference, we denote the skill dependent part of the real wage as

$$W_t^L = \left(\sum_{i=0}^{\infty} \beta_s^{i(1-\gamma)} s_t^i \right) \tag{9}$$

As in Blanchard and Gali (2008), we assume that every hire generates a cost G_t which is proportional to the productivity of a newly hired:

$$G_t = A_t B' x_t^{\alpha} \tag{10}$$

where B' denotes a constant.

The intermediate goods firms will hire additional workers until the hiring costs of an additional worker equal the present discounted value of the profits generated by this worker. However, unlike in the Blanchard and Gali model, we have to take account of the skill level of the workforce hired in period t as well as their wage schedule change in period t+1, as all hired workers who remain employed upgrade to the full skill level after one quarter. Thus we have

$$G_t = \frac{P_t^I}{P_t} A_t^P A_t^L - W_t + E_t \left[\sum_{i=1}^{\infty} \beta^i (1-\delta)^i \frac{C_t}{C_{t+i}} \left(\frac{P_{t+i}^I}{P_{t+i}} A_{t+i}^P - W_{t+i}^0 \right) \right] \tag{11}$$

where $\frac{P_t^I}{P_t}$ denotes the real price of intermediate goods. The terms $\frac{P_t^I}{P_t} A_t^P A_t^L - W_t$ and $E_t \left[\sum_{i=1}^{\infty} \beta^i (1-\delta)^i \frac{C_t}{C_{t+i}} \left(\frac{P_{t+i}^I}{P_{t+i}} A_{t+i}^P - W_{t+i}^0 \right) \right]$ represent the flow profit generated in period t (when the worker has just been hired) and the present discounted value of profits generated in period t+1 and after, respectively. Note that due to our assumption that the worker regains all his skills after one period, the expression for the flow profit in period t is different from the expression for the flow profit in period t+1 and after. Rewriting this equation as a difference equation and noting that the real price of intermediate goods firms equals the marginal cost of final goods firms (hence $\frac{P_t^I}{P_t} = mc_t$), we have

$$mc_t A_t^P A_t^L = W_t + G_t \tag{12}$$
$$-\beta (1-\delta) E_t \left[\frac{C_t}{C_{t+1}} \left(G_{t+1} + mc_{t+1} A_{t+1}^P - W_{t+1}^0 - \left(mc_{t+1} A_{t+1}^P A_{t+1}^L - W_{t+1} \right) \right) \right]$$

The left hand side represents the real marginal revenue product of labour, which depends on the period t average skill level among applicants. Clearly, an increase in the quality of the average period to job seeker A_t^L will reduce period t marginal cost. The right hand side features the period real wage W_t and the period t hiring costs G_t, and, with a negative sign, the present expected value of hiring costs saved (G_{t+1}) by hiring the worker in t rather than t+1. While an increase in hiring cost today means increasing production is more costly, an increase in future expected hiring costs will induce intermediate goods firms to shift hiring into the present, thus lowering the price of intermediate goods and thus marginal cost.

In addition, the right hand side also includes the present expected value of the t+1 difference between the real profit generated by a fully skilled worker (with productivity A_{t+1}^P and real wage W_{t+1}^0) and a t+1 newly hired worker (with productivity $A_{t+1}^P A_{t+1}^L$ and real wage W_{t+1}). This represents an additional benefit of hiring today rather than tomorrow not present in the Blanchard Gali model. For further reference note that this benefit decreases in A_{t+1}^L and increases in W_{t+1} and mc_{t+1}. Thus an expected higher t+1 skill level will increase marginal cost in period t (since it reduces the benefit from hiring today), while a higher expected average real wage for the t+1 newly hired and a higher expected t+1 price of intermediate goods (i.e. higher t+1 marginal cost) will decrease it.

While A_t is the relevant level of productivity at the margin, the average productivity of the whole workforce after adding the newly hired will be different because those employees who remained in employment from t-1 to t are all fully skilled. The average productivity level A_t^A is then given by

$$A_t^A = A_t^P \left[s_t^N A_t^L + \left(1 - s_t^N\right) \right] \tag{13}$$

where s_t^N denotes the share of the newly hired in period t employment, which is given by

$$s_t^N = \frac{H_t}{N_t} = \frac{N_t - (1-\delta) N_{t-1}}{N_t} \tag{14}$$

To set up the production function, we have to use $A_t^A N_t$ for gross output. Hence the production function becomes

$$C_t = A_t^A N_t - B' x_t^\alpha A_t^P A_t^L H_t = A_t^A N_t - B' x_t^\alpha A_t^P A_t^L \left(N_t - (1-\delta) N_{t-1}\right) \tag{15}$$

3.3 Marginal Cost and Phillips Curve the Absence of Skill Loss

The assumption of hiring costs made by Blanchard and Gali has interesting consequences for the Phillips Curve, which we would like to highlight next. It is well known that monopolistic competition and Calvo pricing as found in the final goods firms lead to, up to first order, the familiar New Keynesian Phillips curve relating inflation to expected future inflation and marginal costs (a lower case variable with a hat denotes the percentage-deviation of this variable from its steady state, unless otherwise stated):

$$\pi_t = \beta E_t \pi_{t+1} + \lambda \widehat{mc}_t, \ \lambda = \frac{(1-\beta\omega)(1-\omega)}{\omega} \tag{16}$$

Concerning marginal cost, in appendix V we show that combining log-linear approximations of equations (12) to (15) combined with log linear approximations to (6), (7) and (2) allows one to express the percentage deviation of marginal cost from its steady state as

$$\begin{aligned}
\widehat{mc}_t &= -a_1^L \widehat{a}_t^L + w_1^L \widehat{w}_t^L + a_2^L E_t \widehat{a}_{t+1}^L - w_2^L E_t \widehat{w}_{t+1}^L - p_0 \widehat{a}_t^P - p_1 E_t \widehat{a}_{t+1}^P \qquad (17) \\
&\quad + h_0^{'} \widehat{n}_t + h_L^{'} \widehat{n}_{t-1} + h_F^{'} E_t \widehat{n}_{t+1} - h_c E_t \widehat{mc}_{t+1} \\
&\quad where \\
h_c &= \beta (1-\delta) \frac{\left(1 - A^L\right)}{A^L}, \; g = B' x^{\alpha} \\
h_F' &= -\beta (1-\delta) \left(\frac{\alpha g M}{\delta} - \xi_0' X \right) \\
h_0^{'} &= \left(\frac{\alpha g M}{\delta} \right) \left(1 + \beta (1-\delta)^2 (1-x)\right) + \beta (1-\delta) \left(\xi_1^{'} - \xi_0^{'} \right) X \\
h_L' &= -\left(\frac{\alpha g M}{\delta} \right) (1-\delta)(1-x) - \beta (1-\delta) \xi_1' X \\
a_1^L &= 1 - gM + \beta (1-\delta) \frac{A^L \delta (1-g)}{A^A - A g \delta} X \\
a_2^L &= \beta (1-\delta) \left[1 - gM + \frac{A^L \delta (1-g)}{A^A - A g \delta} X \right] \\
w_1^L &= \frac{M}{A^L} W, \; w_2^L = \beta (1-\delta) \frac{M}{A^L} W \\
p_0 &= \Phi' + \beta (1-\delta) X, \; p_1 = \beta (1-\delta) \frac{\gamma M (\Theta' - W)}{A^L} \\
X &= gM + \frac{1 - A^L - M (\Theta' - W)}{A^L} \\
\xi_0^{'} &= \frac{A^L (1 - g (1+\alpha))}{A^A - A g \delta} \\
\xi_1^{'} &= \frac{(1-\delta) \left((1 + \alpha (1-x)) A^L g + \left(1 - A^L\right) \right)}{A^A - A^L g \delta} \\
\Phi' &= 1 - gM - (1-\gamma) \frac{M}{A^L} W
\end{aligned}$$

Smaller case variables with hats denote the percentage deviation of a variable from its steady state and M denotes the steady state mark-up of final goods firms.

We consider first the case of no skill loss, i.e. $\delta^s = 0$. In this case we have

$A^L = A^A = 1$, $\Theta' = W$ and $\widehat{a}_t^L = \widehat{w}_t^L$. This yields

$$\begin{aligned}
\widehat{mc}_t &= h_0\widehat{n}_t + h_L\widehat{n}_{t-1} + h_F E_t\widehat{n}_{t+1} - p_0\widehat{a}_t && (18)\\
h_0 &= \left(\frac{\alpha g M}{\delta}\right)\left(1+\beta\left(1-\delta\right)^2\left(1-x\right)\right) - \beta\left(1-\delta\right)gM\left(\xi_1-\xi_0\right)\\
h_L &= -\left(\frac{\alpha g M}{\delta}\right)\left(1-\delta\right)\left(1-x\right) - \beta\left(1-\delta\right)gM\xi_1\\
h_F &= -\beta\left(1-\delta\right)gM\left(\left(\frac{\alpha}{\delta}\right)-\xi_0\right)\\
\xi_0 &= \frac{1-g\left(1+\alpha\right)}{\left(1-\delta g\right)}\\
\xi_1 &= \frac{g\left(1-\delta\right)\left(1+\alpha\left(1-x\right)\right)}{\left(1-\delta g\right)}
\end{aligned}$$

Hence marginal cost depends positively on current employment but negatively on lagged employment. An increase in $\widehat{n}_t$ increases labour market tightness and thus marginal cost, while an increase in $\widehat{n}_{t-1}$ reduces the amount of hiring necessary to achieve a given amount of employment in period t and thus reduces marginal cost. Marginal cost also depends positively on $E_t\widehat{n}_{t+1}$, as higher t+1 employment in implies higher hiring costs in that period, thus increasing the benefit of creating jobs today and correspondingly lowering the price of intermediate goods.

Note that the effect of lagged and lead employment, relative to the effect of current employment, increases the less "fluid" the labour market is, i.e. the lower the separation rate δ and the steady state job finding rate x for a given level of employment.[9] Assume for instance, for the sake of example, that we have $N = 0.9$ and, unrealistically $x = 0.9$, implying a separation rate of $\delta = 1$. In this case, we have $h_L = h_F = 0$. In this scenario, all worker are fired at the beginning of the period. This implies that hiring and hence the cost of hiring depend only on N_t and that the cost of hiring in the future is irrelevant for job creation today because no job lasts longer than one period anyway. As we lower the job finding rate and by implication δ, the values of h_L and h_F increase.

Using the relationship $\widehat{n}_t = \frac{\widehat{u}_t}{-(1-u)}$ (where $\widehat{u}_t$ denotes the percentage point, not the percentage deviation of unemployment from its steady state) and (16), we arrive at the Phillips Curve:

$$\begin{aligned}
\pi_t &= \beta E_t\pi_{t+1} - \kappa_0\widehat{u}_t + \kappa_L\widehat{u}_{t-1} + \kappa_F E_t\widehat{u}_{t+1} - \lambda p_0\widehat{a}_t && (19)\\
\kappa_0 &= \frac{\lambda h_0}{1-u}, \kappa_L = \frac{-\lambda h_L}{1-u}, \kappa_F = \frac{-\lambda h_F}{1-u}
\end{aligned}$$

For future reference, we note that in the Blanchard Gali model we always have $\kappa_0 - \kappa_L - \kappa_F > 0$.[10] This means that a "permanent" increase in unemployment (i.e.

[9] Note the following steady state cross coefficient restriction between δ, x, and $N : \delta = \frac{x(1-N)}{N(1-x)}$, for values of $x \leq N$ and $N < 1$.

[10] This is easily shown: $\kappa_0 - \kappa_L - \kappa_F = \frac{\lambda}{1-u}\left(h_0 + h_L + h_F\right)$

an equal sized increase in $\widehat{u}_t, \widehat{u}_{t-1}$ and $\widehat{u}_{t+1}$) reduces inflation because the effect of current unemployment dominates the effect of lagged and lead unemployment.

The fact that lead as well as lagged unemployment have positive effects on price setting and thus inflation through their effect on marginal costs clearly distinguishes the Phillips Curve in the Blanchard and Gali model from its counterpart in the canonical New Keynesian model. The presence of a lagged unemployment term in the Phillips Curve is commonly associated with (partial) labour market hysteresis. Here the effect of lagged unemployment works through the effect on price setting. Jackman et al. (1991) jointly estimate a wage and a price setting equation featuring both the level and the change in the unemployment rate for 19 OECD countries, and find that the change in the unemployment rate has a positive effect on the real wage employers are willing to pay (given the change in the inflation rate and the level of unemployment) in all countries except for the United States.[11] This implies that lagged unemployment has a negative effect on the real wage employers are willing to pay and thus boosts inflation.

The difference between the United States and other, mostly European OECD economies concerning the role of lagged found by Jackman et al. is at least qualitatively reflected by (19) if the "American" and "European" calibrations of Blanchard and Gali are adopted, respectively. The two parameterisations are displayed in table 1. The two calibrations differ in that in the United states, steady state unemployment is lower and the labour market is more fluid, with a high steady state job finding probability x of 0.7, which (given u) backs out a high separation rate of 0.12. In continental Europe, unemployment is higher, with $u = 0.1$, and there are less flows in and out of unemployment, with $x = 0.25$ which backs out a separation rate δ of only 0.04. Plugging these parameters into (19), we get

$$\begin{aligned}
\pi_t &= 0.99E_t\pi_{t+1} - 0.083\widehat{u}_t + 0.02\widehat{u}_{t-1} + 0.056E_t\widehat{u}_{t+1} - \lambda\Phi\gamma\widehat{a}_t \text{ ["American"]} \\
\pi_t &= 0.99E_t\pi_{t+1} - 0.143\widehat{u}_t + 0.063\widehat{u}_{t-1} + 0.079E_t\widehat{u}_{t+1} - \lambda\Phi\gamma\widehat{a}_t \text{ ["Continental European"]}
\end{aligned}$$

The weight of lagged unemployment relative to the coefficients on current and lead unemployment is clearly higher under the continental European calibration than under the American one, as found by Jackman et al. The reduction in δ and x as we move from the American to the continental European calibration increases all three coefficients but the proportional increase is clearly the biggest for lagged unemployment.[12]

$= \frac{\lambda}{1-u}\frac{\alpha g M}{\delta}\Big(1 + \beta(1-\delta)^2(1-x) - (1-\delta)(1-x) - \beta(1-\delta)\Big) > 0$. using the fact that that $1 - \delta = \frac{N-x}{N(1-x)}$, this can be simplified to $(1-N)x^2 + (N-x)N(1-\beta) > 0$. This holds for all permissible values of x, β and N since the maximum value x can take without violating $\delta \leq 1$ is N.

[11] See Jackman et al (1991), pp. 401-408.

[12] This is due to the fact that the absolute value of the coefficient on $\widehat{n}_{t-1}$ in the equation relating $\widehat{x}_t$ to $\widehat{n}_{t-1}$ and $\widehat{n}_t$ equals $\frac{(1-\delta)(1-x)}{\delta}$. The coefficient on $\widehat{n}_t$, which equals the coefficient on $\widehat{n}_{t+1}$ in $\widehat{x}_{t+1}$, depends only on $1/\delta$. Once we substitute out $\widehat{x}_t$ and $\widehat{x}_{t+1}$ in the marginal cost equation, this

Table 1: Calibrations

Parameter	"American"	"European"
β	0.99	0.99
λ	0.08	0.08
θ	6	6
M	1.2	1.2
α	1	1
x	0.7	0.25
u	0.05	0.1
δ	0.12	0.04
B'	0.12	0.12
g	0.084	0.03
ξ_0	0.845	0.941
ξ_1	0.097	0.051

4 Determinacy in the Blanchard and Gali Model

We now explore under what conditions the Taylor principle ensures determinacy in the Blanchard Gali model. For that purpose, we first write our model as a system in $\pi_t, \widehat{u}_t, \widehat{c}_t, \widehat{i}_t$ and $\widehat{a}_t$ and close it by adding an interest feedback rule. The full model consists of

$$\begin{aligned}
\pi_t &= \beta E_t \pi_{t+1} - \kappa_0 \widehat{u}_t + \kappa_L \widehat{u}_{t-1} + \kappa_F E_t \widehat{u}_{t+1} - \lambda p_0 \widehat{a}_t \qquad (20) \\
\widehat{c}_t &= \widehat{a}_t - c_0 \widehat{u}_t - c_1 \widehat{u}_{t-1},\ c_0 = \frac{\xi_0}{1-u},\ c_1 = \frac{\xi_1}{1-u} \\
\widehat{c}_t &= E_t \widehat{c}_{t+1} - \left(\widehat{i}_t - E_t \pi_{t+1}\right) \\
\widehat{a}_t &= \rho_a \widehat{a}_{t-1} + e_t,\ e_t\ i.i.d. \sim \left(0, \sigma^2\right) \\
\widehat{i}_t &= \phi_\pi \pi_t + \phi_u \widehat{u}_t,\ \phi_\pi \geq 0,\ \phi_u \leq 0
\end{aligned}$$

The second equation is a log-linear approximation to equation (15) in the absence of skill loss. These equations can be reduced to system of three first order difference equations with variables π_t, $\widehat{u}_t$ and an auxiliary variable $\widehat{u}_t^L = \widehat{u}_{t-1}$ and the forcing process $\widehat{a}_t$:

$$\begin{pmatrix} E_t \pi_{t+1} \\ E_t \widehat{u}_{t+1} \\ \widehat{u}_{t+1}^L \end{pmatrix} = A \begin{pmatrix} \pi_t \\ \widehat{u}_t \\ \widehat{u}_t^L \end{pmatrix} + b \widehat{a}_t \qquad (21)$$

where A is a 3x3 coefficient matrix and b is a 3x1 coefficient vector. This system has one predetermined endogenous variable, $\widehat{u}_t^L$, and two endogenous jump variables, π_t

is multiplied with $(1-\delta)$ as the effect of future expected hiring costs depends on the likelihood that a job survives. Thus as δ and x both decrease, we will see a bigger increase of the coefficient on lagged employment (lagged unemployment) than on lead employment (lead unemployment).

and $\widehat{u}_t$. To check for determinacy, we can thus apply proposition C.2 from Woodford (2003) to matrix A.[13] This is done in the appendix. The result is summarised in the following proposition:

Proposition 1 *Consider the system described by* (20) *equilibrium is determinate if and only if* $\phi_\pi - \phi_u \frac{(1-\beta)}{\kappa_0 - \kappa_L - \kappa_F} > 1$ *and a set of other conditions discussed in the appendix are met, which however hold under reasonable restrictions on the parameters. Proof: Appendix I*

The interpretation of this condition is analogous to the one derived in Woodford (2003) for the canonical New Keynesian model, since it also says that in the long run, a one percentage point increase in inflation should trigger an increase in the nominal interest rate of more than one.[14] If inflation increases permanently by one percentage point, this will increase the nominal interest rate directly by ϕ_π and indirectly through the reduction in unemployment, which amounts to $\frac{(1-\beta)}{\kappa_0 - \kappa_L - \kappa_F}$, times the coefficient on unemployment in the interest feedback rule, ϕ_u (which is restricted to be negative). Hence it suffices for determinacy to set $\phi_\pi > 1$.

5 Marginal Cost and Phillips Curve in the Presence of Skill Loss

The main difference between (17) and (18) is the presence of the $-a_1^L \widehat{a}_t^L + a_2^L E_t \widehat{a}_{t+1}^L + w_1^L \widehat{w}_t^L - w_2^L E_t \widehat{w}_{t+1}^L$ term, the $-p_1 E_t \widehat{a}_{t+1}^P$ term and the $-h_c E_t \widehat{mc}_{t+1}$ term. The intuition for the impact of these on marginal costs was already provided in section 3.2. In this section we will express both the period t skill level of the average job seeker and the skill dependent real wage as a function of past employment alone. We also characterise the implied relationship between marginal cost and unemployment, and how the long run relationship between marginal cost and unemployment is shaped by the skill loss percentage δ_s and the job finding probability x.

To fully determine marginal cost, we will express both the skill level and the skill dependent component of the real wage as a function of past employment. In appendix III we show after linearising (4), (5) and (7), (6) , and (2), we can express the percentage deviation of the average skill level from its steady state $\widehat{a}_t^L$ as weighted infinite sum of past employment rates

$$\widehat{a}_t^L = \sum_{i=1}^{\infty} a_i^n \widehat{n}_{t-i}, \ \ a_i^n = \frac{1}{u} (1-x)^i \left(\beta_s^{i-1} - \beta_s^i \right) \tag{22}$$

[13] See Woodford (2003), p.672-673.
[14] Woodford (2003), p. 254.

and analogously for $\widehat{w}_t^L$ (using (9) instead of (4))

$$\widehat{w}_t^L = \sum_{i=1}^{\infty} w_i^n \widehat{n}_{t-i},\ w_i^n = \frac{1}{u}(1-x)^i \left(\beta_s^{(1-\gamma)(i-1)} - \beta_s^{(1-\gamma)i}\right) \tag{23}$$

For both equations, the coefficients on past employment a_i^n and w_i^n are zero for $\delta_s = 0$ and larger than zero for $\delta_s > 0$. Higher past employment means that the unemployment spell of the average job seeker will be shorter. This increases the average skill level and by implication also increases his real wage.

Furthermore, both a_i^n and w_i^n decrease in the steady state job finding probability x. If people move quickly out of unemployment, the effect of t-i employment on the average skill level in period t is lower since the additional worker employed in period t-i had a high probability to find a job in period t-i+1 or after that anyway. Analogously, the effect of employment on the skill dependent part of the real wage declines as well.

For the marginal cost of firms, what matters is not merely the direction of the effects of past employment on labour productivity and the real wage of the newly hired but also their relative magnitude. We would also like to know how the later depends on δ_s and x. Furthermore, what matters for our reasoning below will be the derivatives of the joint effects of past employment on the skill level and the real wage rather than the derivatives of the individual a_i^n coefficients. They are summarised by the following proposition. [15]

Proposition 2 *Let* $a^n = \sum_{i=1}^{\infty} a_i^n$ *and* $w^n = \sum_{i=1}^{\infty} w_i^n$*. Then* $a^n = \frac{1-x}{u}\frac{1-\beta_s}{1-(1-x)\beta_s}$, $w^n = \frac{1-x}{u}\frac{1-\beta_s^{1-\gamma}}{1-(1-x)\beta_s^{1-\gamma}}$ *and* $a^n > w^n$ *if and only if* $\gamma > 0$ *and* $\beta_s < 1$*. Furthermore,* $\frac{\partial a^n}{\partial \delta_s} = \frac{1-x}{u}\frac{x}{(1-(1-x)\beta_s)^2} > 0$ *and* $\frac{\partial w^n}{\partial \delta_s} = \frac{1-x}{u}(1-\gamma)\frac{x\beta_s^{-\gamma}}{\left(1-(1-x)\beta_s^{1-\gamma}\right)^2} > 0$. $\frac{\partial a^n}{\partial \delta^s} > \frac{\partial w^n}{\partial \delta^s}$ *if* β_s *is close to 1 and* $\gamma > 0$*. Furthermore,* $\frac{\partial a^n}{\partial x} = -\frac{1}{u}\frac{1-\beta_s}{(1-(1-x)\beta_s)^2} < 0$ *and* $\frac{\partial w^n}{\partial x} = -\frac{1}{u}\frac{1-\beta_s^{1-\gamma}}{\left(1-(1-x)\beta_s^{1-\gamma}\right)^2} < 0$*. Finally,* $\frac{\partial a^n}{\partial x} > \frac{\partial w^n}{\partial x}$ *if and only if* β_s *is close to one and* γ *is sufficiently large. Proof: Appendix IV.*

This proposition says that for positive skill loss and real wage rigidity, the joint effect of past employment levels on the quality of the average job seeker will always dominate the joint effect of past employment on the real wage. Furthermore, an increase in the quarterly skill loss percentage will increase both the joint effect of past employment on the quality of the average job seeker and the real wage. However, if skill loss is small and there is real wage rigidity, an increase in the quarterly skill loss percentage will have a larger impact on the joint effect of past employment on the average skill level than on the effect of employment on the real wage.

[15] As we show in the appendix, for $\delta_s > 0$, the relative magnitude of the a_i^n and w_i^n coefficients and in the case of $\frac{\partial a_i^n}{\partial \delta_s}$ and $\frac{\partial w_i^n}{\partial \delta_s}$ also the sign will depend on i.

Under qualitatively the same conditions, an increase in the job finding probability will reduce the joint effect of past employment on the quality of the average job seeker by more than the effect of past employment on the average real wage. These conditions are easily fulfilled for reasonable calibrations and in any case for the calibrations we will employ below.

The above implies that in the presence of real wage rigidity ($\gamma > 0$)

- with positive skill loss ($\delta_s > 0$) a "permanent" increase in unemployment (decrease in employment) increases the ratio between the (average) wage of the newly hired and their average productivity, while a decline in unemployment (an increase in employment) decreases this ratio. More formally, for a given increase in unemployment $\Delta\widehat{a}^L < \Delta\widehat{w}^L$

- the size of the increase of the ratio between productivity and the real wage increases in δ_s. Hence if δ_s is higher, $\Delta\widehat{w}^L - \Delta\widehat{a}^L$ will be higher as well.

- the size of the increase in the gap between productivity and the real wage decreases in x. Hence if x is higher, $\Delta\widehat{w}^L - \Delta\widehat{a}^L$ will be smaller.

We now turn to the meaning of all this for the relationship between unemployment and marginal cost. Note that $a_1^L > a_2^L$ and $w_1^L > w_2^L$ if $\delta, \beta > 0$, as will be the case for reasonable calibrations. Hence we can obtain from (17) that a permanent increase in the average skill level will lower marginal cost and an increase in the (skill dependent component of) the real wage will increase it. This is because the gain from hiring today rather than tomorrow originating from the skill appreciation is uncertain and is being discounted. The same is true for the effect of the factors affecting this gain on marginal cost.

Furthermore, as can be obtained from their definitions, $a_1^L - a_2^L$ and $w_1^L - w_2^L$ will be quite close for sensible calibrations. We have seen that if unemployment increases permanently, both $\widehat{a}_t^L$ and $\widehat{a}_{t+1}^L$ decline by a larger amount than $\widehat{w}_t^L$ and $\widehat{w}_{t+1}^L$ if $\gamma > 0$. This means that unemployment increases marginal cost via this channel, the more so higher the degree of skill loss δ_s. Thus we would expect an increase in δ_s to make the link between unemployment and marginal cost less negative.

We now turn to characterise the effect of unemployment on marginal costs and how this effect depends on δ_s more rigorously. First, we quasi-difference (22) and (23), which yields

$$\begin{aligned} \widehat{a}_t^L &= (1-x)\left(\frac{1}{u}(1-\beta_s)\widehat{n}_{t-1} + \beta_s \widehat{a}_{t-1}^L\right) \\ \widehat{w}_t^L &= (1-x)\left(\frac{1}{u}\left(1-\beta_s^{1-\gamma}\right)\widehat{n}_{t-1} + \beta_s^{1-\gamma}\widehat{w}_{t-1}^L\right) \end{aligned}$$

Substituting these equations into (17) and using $\widehat{n}_t = \frac{-\widehat{u}_t}{1-u}$ yields

$$\begin{aligned}
\lambda\widehat{mc}_t &= -a^*\widehat{a}_t^L + w^*\widehat{w}_t^L - \kappa_0^*\widehat{u}_t + \kappa_L^*\widehat{u}_{t-1} + \kappa_F^* E_t\widehat{u}_{t+1} \\
&\quad -h_c E_t\lambda\widehat{mc}_{t+1} - \lambda\left(p_0 + \rho_a p_1\right)\widehat{a}_t^P \\
\widehat{a}_t^L &= (1-x)\left(-\left(1-\beta_s\right)\frac{\widehat{u}_{t-1}}{u\left(1-u\right)} + \beta_s\widehat{a}_{t-1}^L\right) \\
\widehat{w}_t^L &= (1-x)\left(-\left(1-\beta_s^{1-\gamma}\right)\frac{\widehat{u}_{t-1}}{u\left(1-u\right)} + \beta_s^{1-\gamma}\widehat{w}_{t-1}^L\right) \\
a^* &= \lambda\left(a_1^L - a_2^L\left(1-x\right)\beta_s\right) \\
w^* &= \lambda\left(w_1^L - w_2^L\left(1-x\right)\beta_s^{1-\gamma}\right) \\
\kappa_0^* &= \lambda\frac{\left(h_0^{'} + (1-x)\left(a_2^L\frac{(1-\beta_s)}{u} - w_2^L\frac{\left(1-\beta_s^{1-\gamma}\right)}{u}\right)\right)}{1-u} \\
\kappa_L^* &= \frac{-\lambda h_L^{'}}{1-u},\ \kappa_F^* = \frac{-\lambda h_F^{'}}{1-u}
\end{aligned} \tag{24}$$

Setting $\widehat{mc}_{t+1} = \widehat{mc}_t = \widehat{mc}$, $\widehat{u}_{t+1} = \widehat{u}_t = \widehat{u}_{t-1} = \widehat{u}$, $\widehat{a}_t^L = \widehat{a}_{t-1}^L = \widehat{a}^L$ and $\widehat{w}_t^L = \widehat{w}_{t-1}^L = \widehat{w}^L$ and ignoring exogenous technology, we can write

$$\begin{aligned}
\lambda\widehat{mc} &= -\frac{\left[\kappa_0^* - \kappa_F^* - \kappa_{L1}^* - a^*\frac{(1-\beta_s)(1-x)}{u(1-u)(1-(1-x)\beta_s)} + w^*\frac{\left(1-\beta_s^{1-\gamma}\right)(1-x)}{u(1-u)\left(1-(1-x)\beta_s^{1-\gamma}\right)}\right]}{1+h_c}\widehat{u} \\
&= -\kappa\widehat{u} && (25) \\
\kappa &= \frac{h_0^{'} + h_L^{'} + h_F^{'} - \left[\frac{(1-x)}{u}\frac{(1-\beta_s)\left(a_1^L - a_2^L\right)}{(1-(1-x)\beta_s)} - \frac{(1-x)}{u}\frac{\left(1-\beta_s^{1-\gamma}\right)\left(w_1^L - w_2^L\right)}{\left(1-(1-x)\beta_s^{1-\gamma}\right)}\right]}{\left(1+h_c\right)\left(1-u\right)}\lambda && (26)
\end{aligned}$$

$-\kappa$ gives the effect of a "permanent" increase in unemployment on marginal cost.

Most conveniently, substituting the definitions of $h_0^{'}$, $h_L^{'}$ and $h_F^{'}$ yields

$$h_0^{'} + h_L^{'} + h_F^{'} = \frac{\alpha g M}{\delta}\left[1 + \beta\left(1-\delta\right)^2\left(1-x\right) - \left(1-\delta\right)\left(1-x\right) - \beta\left(1-\delta\right)\right]$$

which happens to be exactly the same as $h_0 + h_L + h_F$, is thus always positive and independent of δ^s.Hence in κ only the term in the squared brackets and h_c actually depend on skill loss. The squared bracket will be zero if there is no skill loss ($\beta_s = 1$), implying that $\kappa > 0$ and thus a negative effect of a "permanent" increase in unemployment on marginal cost.

With positive skill loss, the squared bracket represents the "skill loss channel" from unemployment to marginal cost. The first term gives the decline of the skill level of the average applicant caused by the decline in $\widehat{n}$ associated with the increase in $\widehat{u}$ (note that $\frac{(1-x)}{u}\frac{(1-\beta_s)}{(1-(1-x)\beta_s)} = a^n$) times the net effect of a permanent skill level decline on marginal cost $(\left(a_1^L - a_2^L\right))$. The second term gives the decline of the skill dependent real wage caused by the decline in $\widehat{n}$ associated with the increase in $\widehat{u}$ (Note that $\frac{(1-x)}{u}\frac{\left(1-\beta_s^{1-\gamma}\right)}{\left(1-(1-x)\beta_s^{1-\gamma}\right)} = w^n$) times the net effect of a permanent skill decline in the skill dependent real wage on marginal cost (-$\left(w_1^L - w_2^L\right)$).

As δ_s grows, we would expect the squared bracket to grow as well if the real wage is rigid. As was pointed out above, an increase in δ_s means that the gap between productivity and the real wage shrinks at a faster rate as unemployment increases. This would lower κ. To check our intuition, we take the derivative of κ with respect to δ_s and arrive at the following proposition:

Proposition 3 *Let κ be as in (25) and let δ_s close to zero. Then $\frac{\partial\kappa}{\partial\delta_s} < 0$ if $\gamma > \frac{B'x^\alpha M\beta(1-\delta)}{1-B'x^\alpha M(1-\beta(1-\delta))}$. Proof: Appendix VI.*[16]

Accepting the restriction on δ_s, the condition for $\frac{\partial\kappa}{\partial\delta_s} < 0$ is easily fulfilled for the calibrations adopted in this paper since $Bx^\alpha M\beta\left(1-\delta\right)$ is a small number, while $1 - B'x^\alpha M\left(1-\beta\left(1-\delta\right)\right)$ is very close to one. [17]

Thus an increase in δ_s indeed makes the effect of unemployment on marginal costs less negative. This raises the possibility of κ turning negative as δ_s increases. To put it differently, an increase in unemployment would then cause an increase rather than a decrease in marginal cost, and, by implication, inflation. This has consequences for the determinacy properties of the interest feedback rule of the central bank which we will come back to in the following section.

We are also interested in how a change in x for a given unemployment rate will affect κ and $\frac{\partial\kappa}{\partial\delta_s}$. It is easy to show that in the absence of skill loss, $\frac{\partial\kappa}{\partial x} > 0$. Hence in the absence of skill loss, the effect of a permanent increase in unemployment on marginal cost will be more negative. This is due to the reasons discussed earlier. Introducing skill loss adds two opposing forces of a change in x on both κ and $\frac{\partial\kappa}{\partial\delta_s}$. On the one hand, as was shown above, an increase in x will lower in absolute value the negative effect of past unemployment on the skill level a^n and, to a lesser extent, the

[16] A more general proof without restrictions on δ_s would have been desirable but struck us as impossible due to the complexity of the expression resulting from $\frac{\partial\kappa}{\partial\delta_s}$.

[17] One might wonder why the condition in the proposition does not simply say $\gamma > 0$. For better understanding, not first that this is merely a sufficient not a necessary and sufficient condition. As can be obtained from the appendix, the necessary and sufficient value of γ would be lower. Furthermore, it can obtained from (12) that even if there is no real wage rigidity and thus W_t would move by the same percentage as A_t^L the effects of a decline or increase in the average skill level would not be neutral. This is because the t+1 flow profit associated with hiring in t $mc_{t+1}A_{t+1}^P - W_{t+1}^0$ does not depend on the skill level of the average applicant. Thus a permanent decline in A_t^L affect mc_t in some way even if there is no real wage rigidity. The resulting effect can be obtained from (26) by setting $\gamma = 0$ in the squared bracket: $\left(\left(a_1^L - a_2^L\right) - \left(w_1^L - w_2^L\right)\right)\frac{(1-x)}{u}\frac{(1-\beta_s)}{(1-(1-x)\beta_s)}$.

effect of past unemployment on the real wage w^n. On the other hand, an increase in x given u will increase δ, implying that the gain associated with the skill appreciation of a worker hired today becomes more uncertain. This is reflected in the fact that both a_2^L and w_2^L decrease as δ increases, thus reducing the effect of the $\widehat{a}_{t+1}^L$ and $\widehat{w}_{t+1}^L$terms in (17). Concerning the effect on of a change of x on $\frac{\partial \kappa}{\partial \delta_s}$, we are able to prove the following proposition:

Proposition 4 *Let κ be as in (25), δ_s close to zero and α close to 1. Then $\frac{\partial^2 \kappa}{\partial \delta_s \partial x} > 0$ if $x < \frac{4-u-\sqrt{u^2+8u}}{4}$ Proof: Appendix VI*[18]

This condition holds a for a wide range of reasonable calibrations of x and u, including those considered in this paper. Hence and increase in the job finding probability x makes $\frac{\partial \kappa}{\partial \delta_s}$ less negative. To put it differently, if the job finding probability is higher, moving to a positive value of δ_s will still weaken the (negative) link between marginal cost and employment, but to a lesser extent than in a less fluid labour market.

The model developed above features multiple links between unemployment and marginal costs. To sum up what we have learned, a permanent increase in unemployment has the following four effects on marginal cost in period t:

1. The increase in period unemployment lowers period t hiring costs (-$h_0^{'}/\left(1-u\right)$), which tends to lower marginal cost. The strength of this channel increases in the job finding probability x.

2. The increase in period t+1 unemployment lowers period t+1 hiring cost, which tends to increase marginal cost $\left(-h_F^{'}/\left(1-u\right)\right)$. The strength of this channel decreases in x.

3. An increase in period t-1 unemployment increases period t hiring costs by increasing the amount of hiring necessary to reach a given level of employment $\left(h_L^{'}/\left(1-u\right)\right)$. The strength of this channel decreases in x.

4. An increase in period t to $t-\infty$ unemployment increases $\widehat{a}_t^L - \widehat{w}_t^L$ and $\widehat{a}_{t+1}^L - \widehat{w}_{t+1}^L$. The net effect of this is to increase marginal cost. $\left(\frac{(1-x)}{u(1-u)}\frac{(1-\beta_s)\left(a_1^L-a_2^L\right)}{(1-(1-x)\beta_s)} - \frac{(1-x)}{u(1-u)}\frac{\left(1-\beta_s^{1-\gamma}\right)\left(w_1^L-w_2^L\right)}{\left(1-(1-x)\beta_s^{1-\gamma}\right)(1-u)} > 0 \text{ if } \delta_s > 0\right)$. The strength of this channel increases in the skill loss percentage δ_s and decreases in x.

Note that effects 1-3 are already present in the model without skill loss, while the fourth effect arises from the introduction of skill loss among the unemployed.

[18] Again this is a sufficient condition not a necessary and sufficient one, which can be obtained from the appendix. For instance, the condition reported has γ set equal to zero. In fact the maximum value of x increases in γ.

6 Determinacy in the Model with Skill Loss

We now investigate which policy rules guarantee determinacy in the presence of skill loss. The first question we are interested in is whether $\phi_\pi > 1$ is still a sufficient condition to establish determinacy for varying levels of skill loss. Thus we consider current, forward and backward looking rules where the interest rate responds only to inflation. We are dealing with the following system:

$$
\begin{aligned}
\pi_t &= \beta E_t \pi_{t+1} + \lambda \widehat{mc}_t && \text{(M1)}\\
\lambda \widehat{mc}_t &= -a^* \widehat{a}_t^L + w^* \widehat{w}_t^L - \kappa_0^* \widehat{u}_t + \kappa_L^* \widehat{u}_{t-1} + \kappa_F^* E_t \widehat{u}_{t+1} && \text{(M2)}\\
&\quad - h_c E_t \lambda \widehat{mc}_{t+1} - \lambda \left(p_0 + \rho_a p_1\right) \widehat{a}_t^P \\
\widehat{a}_t^L &= (1-x)\left(-\left(1-\beta_s\right)\frac{\widehat{u}_{t-1}}{u\,(1-u)} + \beta_s \widehat{a}_{t-1}^L\right) && \text{(M3)}\\
\widehat{w}_t^L &= (1-x)\left(-\left(1-\beta_s^{1-\gamma}\right)\frac{\widehat{u}_{t-1}}{u\,(1-u)} + \beta_s^{1-\gamma} \widehat{w}_{t-1}^L\right) && \text{(M4)}\\
\widehat{c}_t &= \widehat{a}_t^P + c_L \widehat{a}_t^L - c_0^* \widehat{u}_t - c_1^* \widehat{u}_{t-1} && \text{(M5)}\\
c^L &= \frac{A^L \delta\,(1-g)}{A^A - A^L g \delta},\ c_0^* = \frac{\xi_0'}{1-u},\ c_1^* = \frac{\xi_1'}{1-u} \\
\widehat{c}_t &= E_t \widehat{c}_{t+1} - \left(\widehat{i}_t - E_t \pi_{t+1}\right) && \text{(M6)}\\
\widehat{a}_t^P &= \rho_a \widehat{a}_{t-1}^P + e_t,\ e_t\ i.i.d. \sim \left(0, \sigma^2\right) && \text{(M7)}\\
\widehat{i}_t &= \phi_\pi E_t \pi_{t+j},\ \phi_\pi \geq 0,\ -1 \leq j \leq 1 && \text{(M8)}
\end{aligned}
$$

$(M5)$ is derived in Appendix V. Unfortunately, unlike in the original Blanchard/ Gali model, we can not establish the conditions for determinacy analytically. Therefore we solve the model numerically using the software Dynare and perform a grid search for values of δ_s between 0 and 0.07 (step size: 0.005) and values of ϕ_π between 0 and 3 (step size: 0.1). All other parameters are set to meet Blanchard and Gali's "Continental European" calibration as reproduced in table 1. We then repeat the grid search for the "American" calibration.[19] The determinacy regions for the current looking rule are graphed in figures 1 and 2. The area between the two lines denotes the determinacy region in both graphs (including the points situated on these lines). For the European calibration, for values of $\delta_s \geq 0.025$, the standard requirement on ϕ_π to guarantee determinacy is reversed: A unique equilibrium now requires $\phi_\pi \leq 0.9$. The determinacy regions for the backward and forward looking rules (not shown) are almost identical. In particular, under the Continental European calibration, the drop of the maximum value of ϕ_π to 0.9 for $\delta_s \geq 0.025$ carries over. This

[19] Note that the following results discussed below also hold if we use the same lower unemployment rate under the American calibration as under the European calibration. For a given job finding probability, this implies a lower job destruction rate.

suggests that it is not the timing of the active response to inflation but the active response to inflation per se which induces indeterminacy.

By contrast, for the American calibration, $\phi_\pi > 1$ does guarantee determinacy for the whole range considered here. The determinacy regions for the current, forward and backward looking rule are completely identical. Experimentation suggest that for the current looking rule, the $\phi_\pi \leq 0.9$ requirement only becomes relevant at $\delta_s \geq 0.225$.

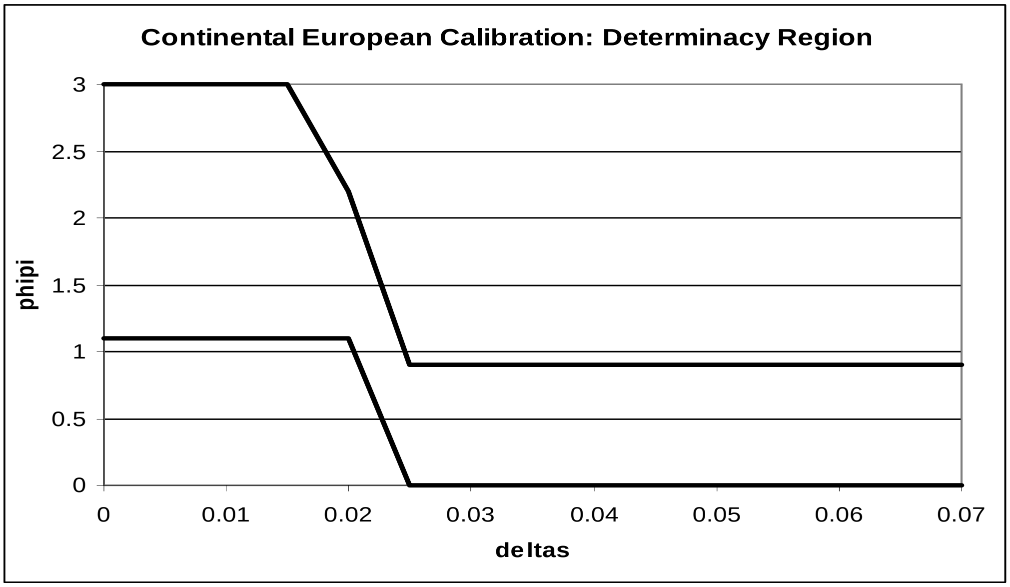

Figure 1

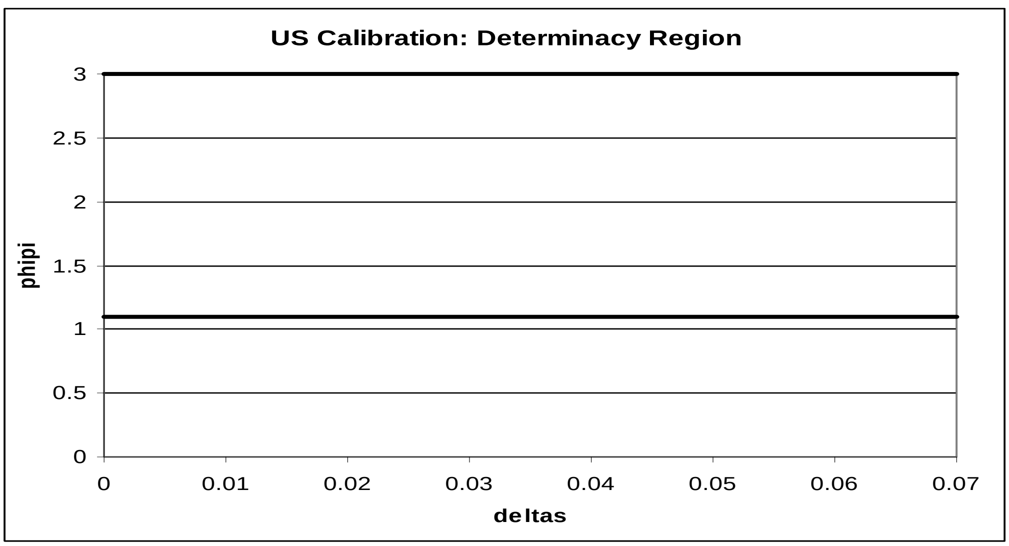

Figure 2

The intuition for this result can be gained by showing how the effect of a "permanent" increase in unemployment on marginal costs depends on δ_s. As can be seen from (25) , in the absence of skill loss this effect is negative since $h_0^{'}+h_L^{'}+h_F^{'}>0$. However, as we have shown in the previous section, $\frac{\partial\kappa}{\partial\delta_s}<0$. Thus as we increase δ_s, κ will ultimately turn negative. Figure 3 plots κ against δ_s for both the European (broken line) and the American (solid line) calibration. Note that under the continental European calibration, the level of skill loss for which this expression turns negative is the same for which the determinacy requirement switches to $\phi_\pi\leq 0.9$, i.e. 0.025. Thus if marginal costs and thus inflation increases in response to a persistent increase in the unemployment rate the central bank should lower the real interest rate. That makes sense as an increase in the real interest rate would further reduce demand and increase unemployment, marginal cost and inflation. Hence with $\phi_\pi\geq 1$, there is scope for sunspot equilibria if δ_s exceeds its respective critical value: An upward jump in unemployment will ultimately lead to an increase in inflation and (as $\phi_\pi\geq 1$) the real interest rate, which lowers demand and thus validates the increase in unemployment. In the next section, when we display the impulse response function to a sunspot shock, we show that this is in fact exactly what happens.

This leaves the question why this critical value is so much higher for the American than for the continental European calibration. The chief reason for this is that due to the more fluid labour market associated with the American calibration, for $\delta_s=0$, κ is a lot higher than under the continental European calibration. The intuition for that was discussed above. Furthermore, we have shown in the previous section that if x is higher, the effect of δ_s on κ will be less ($\frac{\partial^2\kappa}{\partial\delta_s\partial x}>0$) Therefore under the American calibration, κ decreases a little less as δ^s increases than under the European calibration.

We now check whether interest rate smoothing would help to restore determinacy. Therefore we replace M8 by $\widehat{i_t}=(1-\phi_i)\,\phi_\pi\pi_t+\phi_i\widehat{i}_{t-1}$ and perform a grid search over ϕ_π, ϕ_i and δ_s, with $\phi_\pi=[0,\ 3]$, $\phi_i=[0,\ 1]$ and $\delta_s=[0,\ 0.07]$. The determinacy requirement on ϕ_π remains almost unaffected.[20] In particular, determinacy requires $\phi_\pi\leq 0.9$ if $\delta_s\geq 0.025$ independently of the degree of interest rate smoothing. This result is in line with the intuition given above as even with interest rate smoothing, if $\phi_\pi>1$, an increase in inflation ultimately increases the real interest rate.

[20] Only for $\rho=0.8$ and $\rho=0.9$ does smoothing make a difference in that for $\delta^s=0.02$, the maximum value for ϕ_π increases to 2.3 and 2.5, respectively. For $\delta^s\geq 0.025$, the maximum value of ϕ_π th drops to 0.9, as for all other degrees of smoothing.

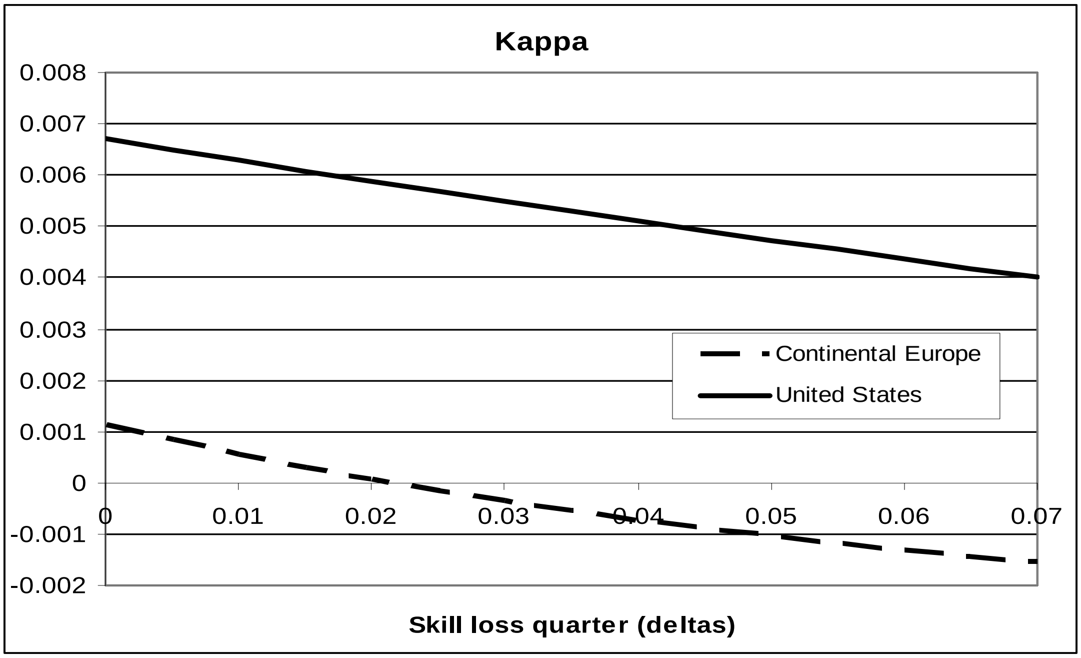

Figure 3

We investigate next whether responding to the output gap in addition to inflation helps to restore determinacy under the European calibration. As is standard in the New Keynesian literature, we define potential output Y_t^n as the output level including hiring costs at which final goods firms charge their desired mark-up, implying that marginal cost is at its steady state. The associated unemployment rate is denoted as u_t^n. As marginal cost is affected by both lead unemployment and lead marginal cost, when deriving u_t^n, we will further assume that if unemployment is at its natural level in period t, it will be expected to be at its natural level in period t+1 as well.[21] Thus we are dealing with the following system:

[21] The following results are broadly robust against relaxing this assumption.

$$
\begin{aligned}
\pi_t &= \beta E_t \pi_{t+1} + \lambda \widehat{mc}_t \\
\lambda \widehat{mc}_t &= -a^* \widehat{a}_t^L + w^* \widehat{w}_t^L - \kappa_0^* \widehat{u}_t + \kappa_L^* \widehat{u}_{t-1} + \kappa_F^* E_t \widehat{u}_{t+1} - h_c E_t \lambda \widehat{mc}_{t+1} - \lambda \left(p_0 + \rho_a p_1\right) \widehat{a}_t^P \\
\widehat{a}_t^L &= (1-x) \left(-\left(1-\beta_s\right) \frac{\widehat{u}_{t-1}}{u\,(1-u)} + \beta_s \widehat{a}_{t-1}^L \right) \\
\widehat{w}_t^L &= (1-x) \left(-\left(1-\beta_s^{1-\gamma}\right) \frac{\widehat{u}_{t-1}}{u\,(1-u)} + \beta_s^{1-\gamma} \widehat{w}_{t-1}^L \right) \\
\widehat{c}_t &= \widehat{a}_t^P + c_L \widehat{a}_t^P - c_0^* \widehat{u}_t - c_1^* \widehat{u}_{t-1},\ c^L = \frac{A^L \delta\,(1-g)}{A^A - A^L g \delta},\ c_0^* = \frac{\xi_0^{'}}{1-u},\ c_1^* = \frac{\xi_1^{'}}{1-u} \\
\widehat{c}_t &= E_t \widehat{c}_{t+1} - \left(\widehat{i}_t - E_t \pi_{t+1}\right) \\
\widehat{a}_t^P &= \rho_a \widehat{a}_{t-1}^P + e_t,\ e_t\ i.i.d. \sim \left(0, \sigma^2\right) \\
\widehat{u}_t^n &= \frac{\kappa_F^* E_t \widehat{u}_{t+1}^n + \kappa_L^* \widehat{u}_{t-1} + -a^* \widehat{a}_t^L + w^* \widehat{w}_t^L - \lambda p_0 \widehat{a}_t^P - \lambda p_1 E_t \widehat{a}_{t+1}^P}{\kappa_0^*} \\
\widehat{y}_t &= \widehat{a}_t^P + y^L \widehat{a}_t^L - y_0 \widehat{u}_t - y_1 \widehat{u}_{t-1},\ y_L = \frac{\delta A^L}{A^A},\ y_0 = \frac{A^L}{A^A\,(1-u)},\ y_1 = \frac{\left(1-A^L\right)(1-\delta)}{A^A\,(1-u)} \\
\widehat{y}_t^n &= \widehat{a}_t^P + y^L \widehat{a}_t^L - y_0 \widehat{u}_t^n - y_1 \widehat{u}_{t-1} \\
\widehat{i}_t &= \phi_\pi \pi_t + \phi_y \left(\widehat{y}_t - \widehat{y}_t^n\right),\ \phi_\pi, \phi_y \geq 0
\end{aligned}
$$

The equation for $\widehat{u}_t^n$ was derived by setting $\widehat{mc}_t = \widehat{mc}_{t+1} = 0$ and $\widehat{u}_{t+1} = \widehat{u}_{t+1}^n$ in the marginal cost equation, while the equation describing the deviation of output including hiring costs from its steady state is derived in the appendix. Clearly $\widehat{u}_t^n$ depends on past values of actual unemployment as well as its own future value.

We perform a grid search over ϕ_π, ϕ_y and δ_s, with $\phi_\pi = [0,\ 3]$, $\phi_y = [0,\ 3]$ (step size 0.1) and $\delta_s = [0,\ 0.07]$. We find that responding to the output gap extends the determinacy region if $\delta_s < 0.025$ but reduces it if $\delta_s \geq 0.025$. For example, figure 4 plots the lowest value of ϕ_π compatible with determinacy against ϕ_y for $\delta_s = 0$. Clearly the lower bound of ϕ_π declines as ϕ_y increases. By contrast, figure 5 plots the highest value of ϕ_π compatible with determinacy for the case of $\delta_s = 0.025$. The upper bound of ϕ_π is declining, thus reducing the determinacy region.

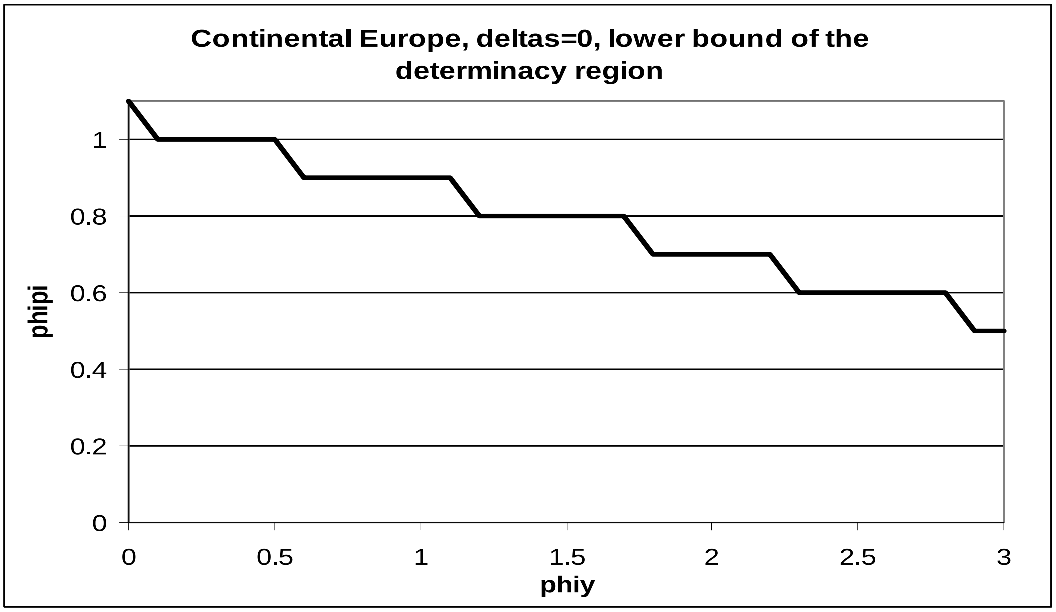

Figure 4

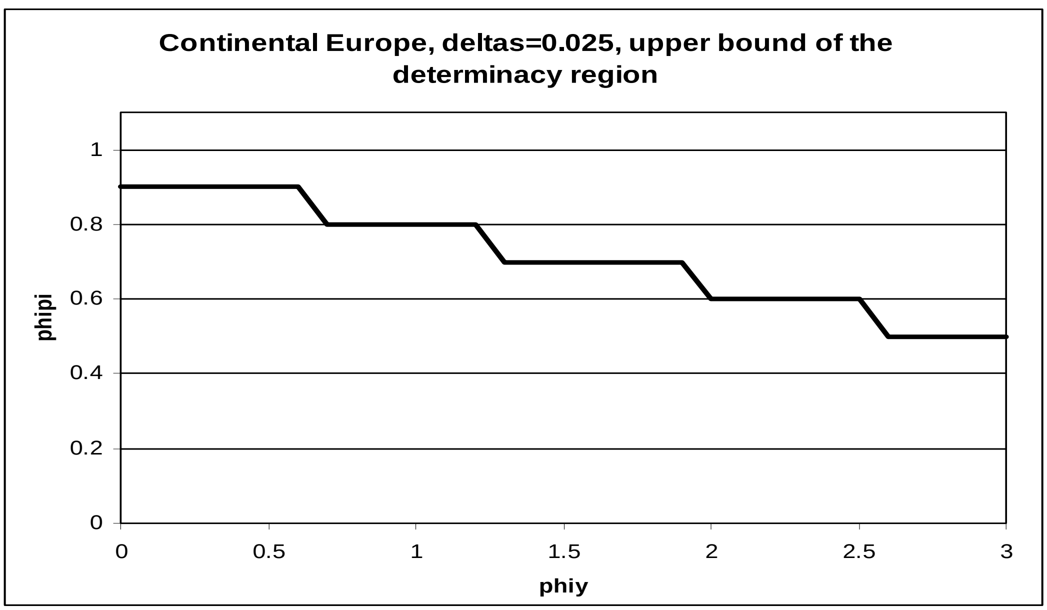

Figure 5

Intuition for this result can be gained from the effect of actual unemployment on natural unemployment. It is easy to see that $\widehat{y}_t - \widehat{y}_t^n = -y_0\left(\widehat{u}_t - \widehat{u}_t^n\right)$. Hence the output gap depends positively on $\widehat{u}_t^n$. Solving $\widehat{u}_t^n$ forward (ignoring the exogenous productivity process) yields $\widehat{u}_t^n = \sum_{i=0}^{\infty}\left(\frac{\kappa_F^*}{\kappa_0^*}\right)^i\left[\kappa_L^*\widehat{u}_{t-1} + a^*\widehat{a}_t^L - w^*\widehat{w}_t^L\right]$. Let us again assume for simplicity that $\widehat{u}_{t+i} = \widehat{u}$. We then have

$$\widehat{u}^n = \frac{\kappa_L^* + a^* \frac{(1-\beta_s)(1-x)}{u(1-u)(1-(1-x)\beta_s)} - w^* \frac{\left(1-\beta_s^{1-\gamma}\right)(1-x)}{u(1-u)\left(1-(1-x)\beta_s^{1-\gamma}\right)}}{\kappa_0^* - \kappa_F^*} \widehat{u}$$

If $\frac{\partial \widehat{u}^n}{\partial \widehat{u}} < 1$, then an increase in unemployment increases natural unemployment less than one for one. It thus lowers the output gap and tends to lower real interest rate. This should stabilise unemployment. By contrast, if $\frac{\partial \widehat{u}^n}{\partial \widehat{u}} > 1$, an increase in unemployment will increase $\widehat{u}^n$ more than one for one and thus tend to increase the real interest rate. In this case responding to the output gap is actually destabilising. Moreover, note that $\frac{\partial \widehat{u}^n}{\partial \widehat{u}} > 1 \Leftrightarrow \kappa_0^* - \kappa_F^* - \kappa_L^* - a^* \frac{(1-\beta_s)(1-x)}{u(1-u)(1-(1-x)\beta_s)} + w^* \frac{\left(1-\beta_s^{1-\gamma}\right)(1-x)}{u(1-u)\left(1-(1-x)\beta_s^{1-\gamma}\right)} < 0$, implying that $\kappa < 0$. As was shown above, this will be true if $\delta^s \geq 0.025$. Hence responding to the output gap will tend to destabilise the economy precisely when responding more than one for one to inflation tends to destabilise the economy as well.

If natural output tracks actual output too closely for the output gap to be a stabilising argument in the policy rule, then perhaps the deviation of unemployment from its steady state (rather than its natural) value will help to achieve determinacy. Thus we consider the policy rule $\widehat{i}_t = \phi_\pi \pi_t + \phi_u \widehat{u}_t$ and conduct a grid search over δ_s, ϕ_π and ϕ_u, with $\phi_\pi = [0,\ 3]$, $\phi_u = [0,\ -3]$ (step size 0.1) and $\delta_s = [0,\ 0.07]$. It turns out that responding to unemployment has a strong stabilising effect. Setting $\phi_u = -0.1$ guarantees determinacy for $\phi_\pi \geq 0.2$ if $\delta_s \leq 0.02$ and for the full interval of ϕ_π for $0 < \delta_s \leq 0.035$. For higher values of δ^s the upper bound of ϕ_π again begins to decline. For $\phi_u = -0.2$, determinacy is guaranteed for the full interval of ϕ_π as long as $\delta_s \leq 0.055$. Finally, for $\phi_u \leq -0.3$, the equilibrium is determinate for any combination of ϕ_π and δ_s. Thus a modest response to unemployment restores determinacy and in doing so is robust against variations in δ_s.

Let us assume that our model in its respective calibrations of u, x, and δ indeed captures major differences between the continental European and the US economy. Furthermore, note that the value of δ_s for which the value of ϕ_π begins to be bounded above under the American Calibration seems implausibly high. With $\delta_s = 0.225$, a worker would have lost about 64% after one year of unemployment. By contrast, the critical value of δ_s for the continental European calibration seems a lot more plausible. It would imply a skill loss of about 9.6% after one year of unemployment. Note also that estimates of interest feedback rules suggest that the Federal Reserve as well as the Bundesbank and the ECB respond more than one for one to inflation and pay some attention to the output gap as well.[22] Hence we conclude that indeterminacy is far more likely to happen in Europe than in the United States.

[22]See for instance Clarida et al. (1998), Orphanides (2001) and Clausen and Meier (2003) for reaction function estimates for the Fed and the Bundesbank and Gorter et al. (2008) and Sauer and Sturm (2003) for estimates for the ECB.

Moreover, in the 1970s, many central banks moved away from a "Keynesian" monetary policy which focuses on stabilising unemployment to a policy which aggressively targets inflation but pays little attention to unemployment. Within the model proposed here, with $\delta^s \geq 0.025$, this move would cause no determinacy problem with a fluid American labour market but would induce indeterminacy if labour market flows were low as in continental Europe. In the next section we will investigate the dynamics of unemployment and inflation for the European calibration with $\delta_s \geq 0.025$ and $\phi_\pi > 1$.

7 Dynamics under Indeterminacy

The previous section showed that a policy rule which increases the nominal interest rate more than one for one with inflation might quite likely imply indeterminacy if the flow - characteristics of the labour market are "continental European" in the sense that there is little hiring and firing and there is some skill loss among the unemployed. In this section we investigate the response of the model under the continental European calibration and with skill loss being at the critical level of 0.025 to a sunspot shock and a non-correlated technology shock.

To solve the indeterminate model, we follow a solution method proposed by Lubik and Schorfheide (2003). Their method builds on an approach by Sims (2000). Sims proposed to solve linear rational expectation (RE) models by solving for the vector of expectational errors $\eta_t = q_t - E_{t-1}q_t$, where q_t is a vector of variables over which agents form expectations. Thus the linear RE model is cast in the following form

$$\Gamma_0 y_t = \Gamma_1 y_{t-1} + \Psi \varepsilon_t + \Pi \eta_t \tag{27}$$

where ε_t denotes an i.i.d vector of structural shocks and all variables with a t and t-1 subscript are observable at time t, and all variables with a t-1 subscript are predetermined. Any system of first order difference equations can be brought into this form by replacing $E_t q_{t+1}$ with $y_t = E_t q_{t+1}$ and the adding an equation reading $q_t = y_{t-1} + \eta_t$. Thus there will be an expectation error for each forward looking variable. Note that all variables on the right hand side except for η_t are either predetermined or exogenous

The system has a stable solution if there exists a vector η_t as a function of the exogenous shocks ε_t to eliminate the explosive components of y_t. The solution is unique solution if the vector of structural shocks ε_t uniquely determines the vector of expectational errors η_t. The solution will not be unique if the number of expectation errors exceeds the number of explosive components in y_t.[23] This opens the door for sunspot shocks to affect the endogenous variables. Lubik and Schorfheide suggest to interpret these shocks as belief shocks that trigger reversion of forecasts of the

[23] See Lubik and Schorfheide (2003), pp. 276-277.

endogenous variable. Suppose that due to a sunspot the expectation of q_t between t and t-1 is revised by v_t. Hence

$$q_t = (E_{t-1}q_t + v_t) + \widetilde{\eta}_t$$

where the term in brackets denotes the revised forecast and $\widetilde{\eta}_t$ is the error associated with this revised forecast.[24] Thus (27) can be written as

$$\Gamma_0 y_t = \Gamma_1 y_{t-1} + \begin{bmatrix} \Psi & \Pi \end{bmatrix} \begin{bmatrix} \varepsilon_t \\ v_t \end{bmatrix} + \Pi \widetilde{\eta}_t \tag{28}$$

If the solution is unique, v_t will not appear in the solution.

We then assume that the effects of the sunspot shock v_t and the structural shock ε_t to the forecast error are orthogonal to each other. This is a standard assumption in the literature on indeterminate linear rational expectations models. It means we are restricting our attention to a subset of the set of solutions of the indeterminate model. This solution can be picked up easily by casting $M1$ to $M8$ in the form of (28). We thus have $y_t = \begin{bmatrix} x_t^{\pi} & x_t^{u} & x_t^{mc} & x_t^{n} & x_t^{c} & \widehat{a}_t^{P} & \pi_t & \widehat{u}_t & \widehat{mc}_t' & \widehat{u}_t^{n} & \widehat{c}_t & \widehat{a}_t^{L} & \widehat{w}_t^{L} & \widehat{i}_t \end{bmatrix}'$, $\varepsilon_t = e_t$ and $v_t = \begin{bmatrix} e_t & v_t^{\pi} & v_t^{u} & v_t^{mc} & v_t^{n} & v_t^{c} \end{bmatrix}'$ with $x_t^q = E_t q_{t+1}$, the v_t^q denoting the belief shock associated with the forecast of the t+1 value of variable q and $\widehat{mc}_t' = \lambda mc_t$. The matrices Γ_0, Γ_1, Ψ and Π are to be found in the appendix.

Note that the way the model is written, we have five belief shocks - one for each forward looking variable. However, the effects of those shocks on the forecast errors and thus on the endogenous variables will not generally be independent from each other. For instance, if there is one stable root too many as is the case under the calibration we are dealing with, there is one degree of freedom. That means we can choose the value of one endogenous variable and then the stable solution for the remaining ones will be pinned down as well. For instance, it will be possible to reproduce the dynamics produced by v_t^{π} with a suitable value of v_t^{u}, v_t^{mc}, v_t^{n} or v_t^{c}.

We assume that the central bank responds only to inflation and set $\phi_{\pi} = 1.5$. When looking at the impulse response of the technology shock, we set $\rho_a = 0$.

We first consider the effects of a -2% belief shock to consumption, i.e. $v_0^c = -0.02$. Figure 6 displays the deviation of unemployment from its steady state (in percentage points) and output net of hiring costs (in percent), i.e. consumption. Unemployment increases by about 0.9 percentage point, while consumption declines by a bit less than 0.9% and then declines somewhat further. The increase in unemployment is very persistent: after 10 years, unemployment is still about 0.72 percentage points above its steady state while after 25 years (100 quarters) it still exceeds its steady state by 0.51%.

[24] See Lubik and Schorfheide (2003), p. 279.

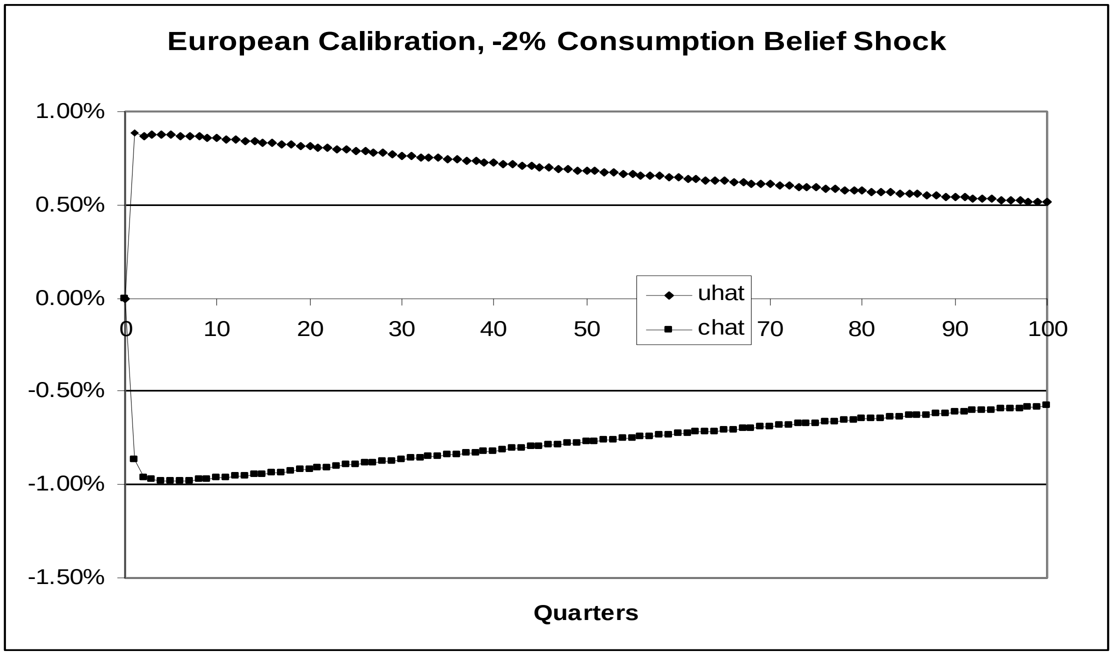

Figure 6

Figure 7 shows that $\lambda\widehat{mc}_t$ falls by 0.06% on impact and then starts increasing and turns positive in quarter 13. Since we have choosen a value of δ_s such that κ is smaller than zero (see Figure 3, the "Continental Europe" line), we would expect the persistent increase in unemployment to ultimately turn marginal cost positive. However, as long as the history of high unemployment is short, the skill loss among job seekers has not yet sufficiently build up to turn marginal cost positive. In terms of the four effects of an increase in unemployment on marginal cost listed at the end of section five, effect number 4 has not yet gained enough momentum such that the joint positive impact of effects 2, 3 and 4 can dominates the negative impact of effect 1. To illustrate how the dynamic of the skill decline matches with sign change and dynamic of $\lambda\widehat{mc}_t$, consider how the skill level evolves in response to a "permanent" change in the unemployment rate:

$$\widehat{a}_t^L = a^n \left((1-x)^t \beta_s^t - 1 \right) \frac{\widehat{u}}{(1-u)}$$

where a^n is the effect of a permanent increase in employment on the skill level of the average applicant and can be obtained from proposition 2. Note that for $t \to \infty$, as $(1-x)^t \beta_s^t \to 0$, this expression gives the effect of a permanent increases in unemployment on the skill level. In Figure 8, we plot $\widehat{a}_t^L$ (as defined in this equation) as a percentage of the change of $\widehat{a}_\infty^L$ after an infinite number of periods, i.e. $\left(1-(1-x)^t \beta_s^t\right) \times 100$. The curve is rather steep at the beginning but then flattens out. With an unemployment history of 12 quarters, which happens to be the case in quarter 13, the decline in $\widehat{a}_t^L$ has reached 97.7% of its total and the rate of change has decreased to about 0.5 percentage points. Thus $\lambda \widehat{mc}_t$ turns positive after the decline in the skill level resulting from the increase in unemployment has almost reached its maximum. Note also that the dynamics of $\lambda \widehat{mc}_t$ and $\widehat{a}_t^L$ are similar in that the rate of increase of $\lambda \widehat{mc}_t$ is at its highest during those first 13 quarters but then gradually declines.

Inflation declines to -0.08% on impact but turns positive in quarter 4. It then keeps rising until it reaches a maximum of 0.01% in quarter 17. Inflation is pushed faster above zero because it responds not just to current but also to expected future values of marginal costs. Correspondingly, we would expect the ex ante real interest rate to ultimately increase as well. Figure 9 shows that $(i_t - E_t \pi_{t+1})$ declines on impact but begins to increase in quarter two and begins to exceed its steady state value in quarter 5 and then remains persistently above it. The persistent increase in the real interest rate validates the initial decline in consumption and the associated increase in unemployment.

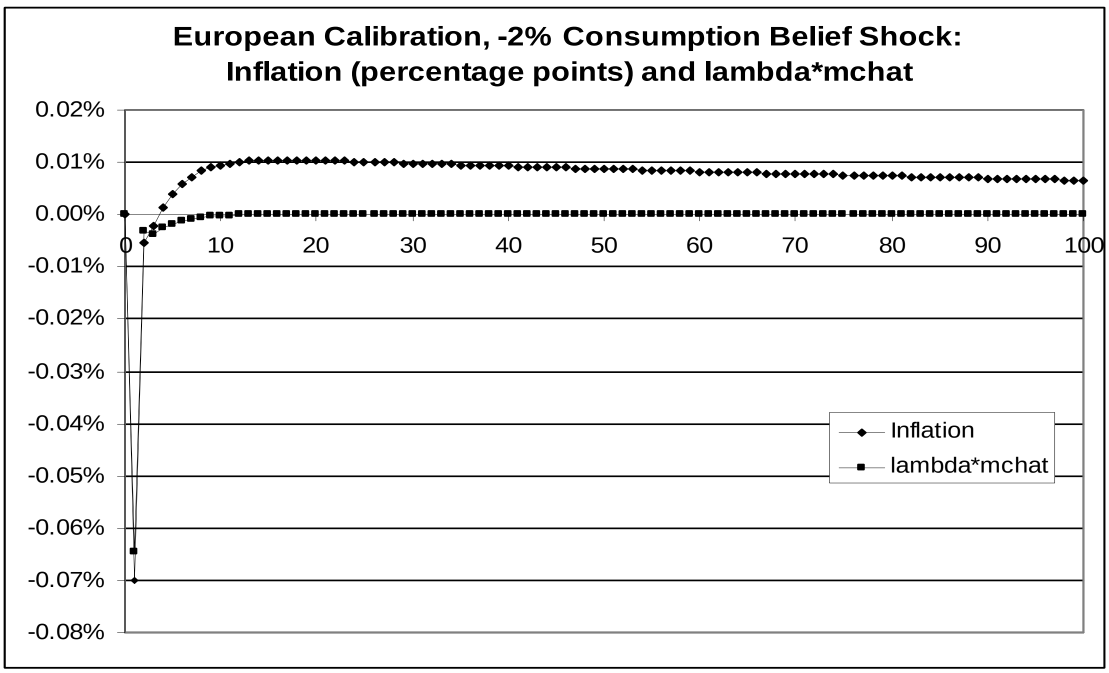

Figure 7

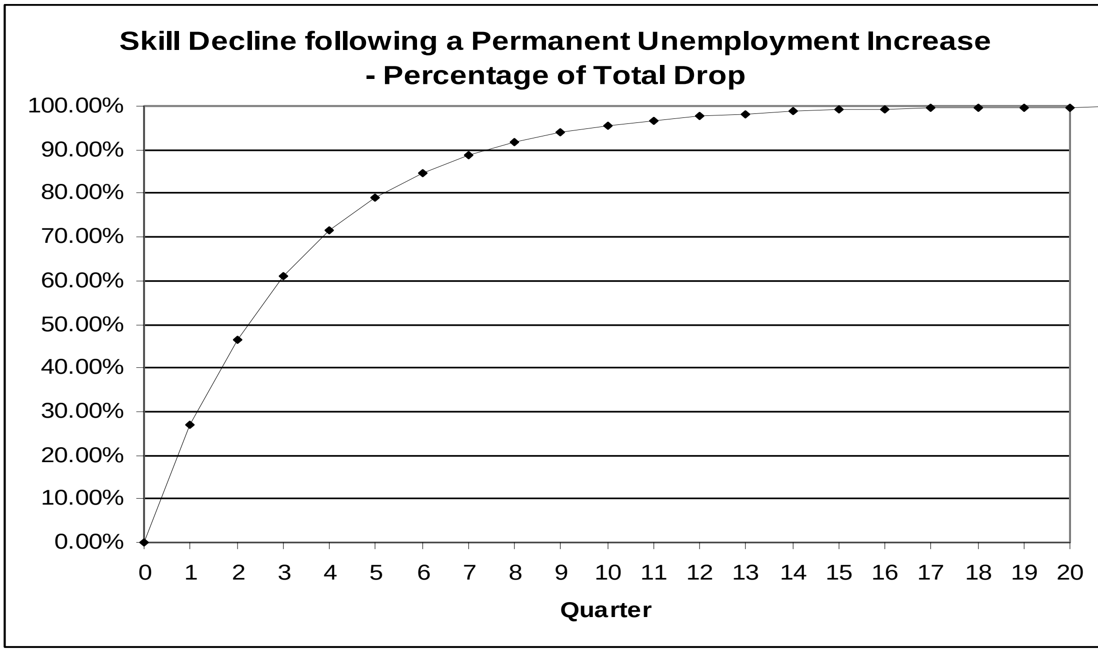

Figure 8

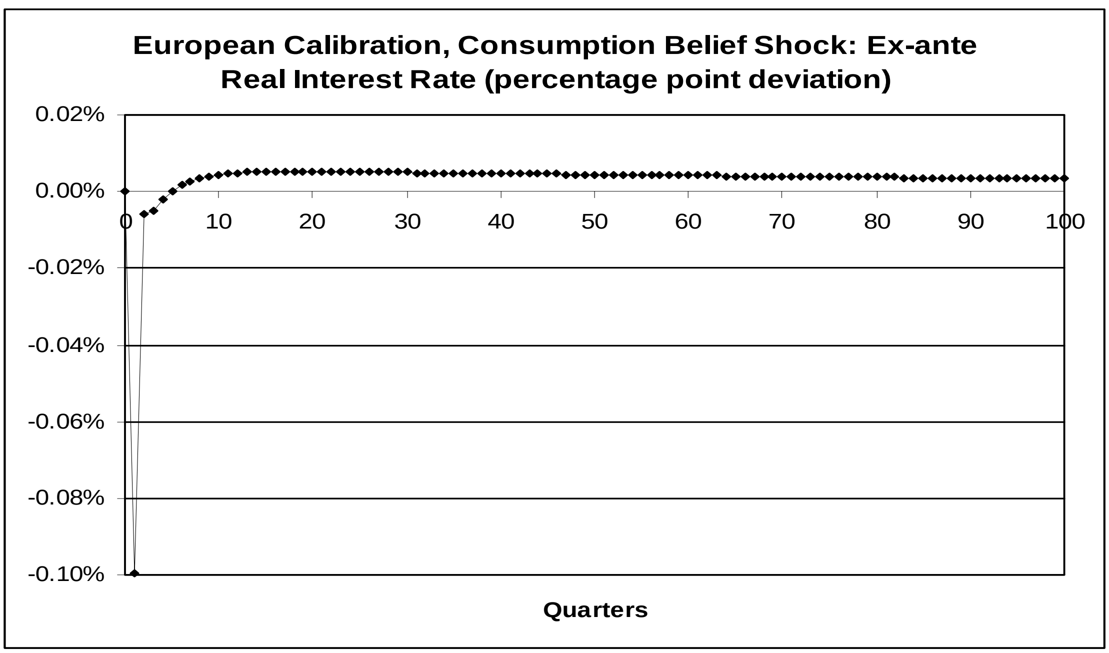

Figure 9

We now turn to the effects of a non-correlated technology shock of -2%, i.e. $e_0 = -0.02$. Figure 10 shows that unemployment and consumption both decline by about 1%, but in quarter 2 unemployment increases to about 0.65 percentage points above its steady state value. Unemployment and consumption then display a similar degree of persistence as in response to a consumption belief shock. Figure 11 shows that inflation and $\lambda\widehat{mc}_t$ both increase on impact. Both turn negative in the next period due to the increase in unemployment. $\lambda\widehat{mc}_t$ turns positive in quarter 15 due to the fact that unemployment persistently increases, while inflation again turns positive faster. This then ultimately implies an above steady state real interest rate.

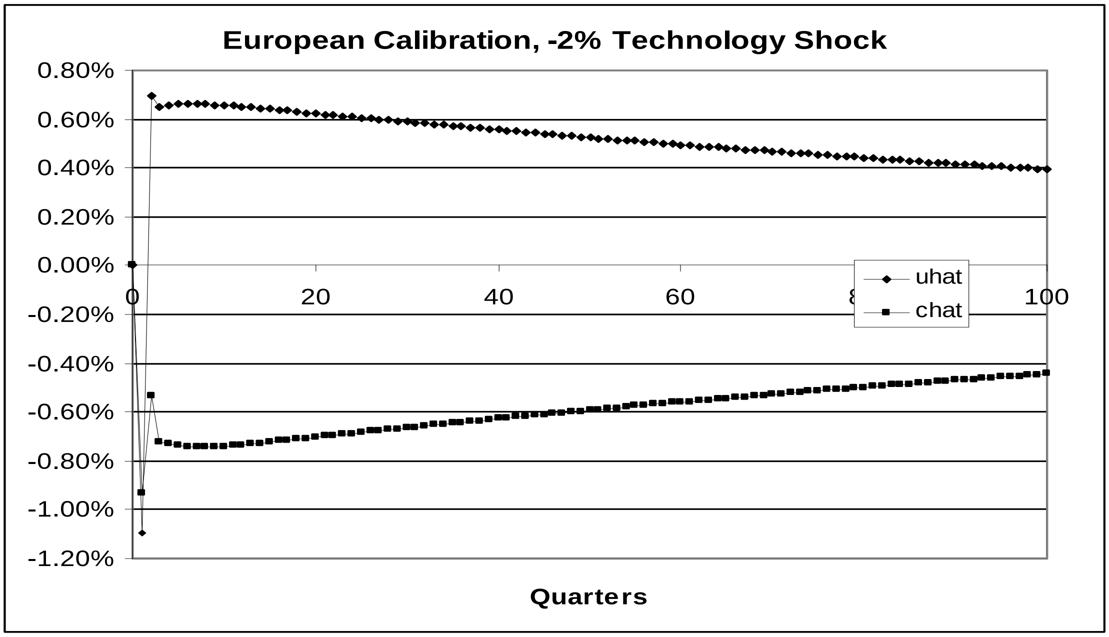

Figure 10

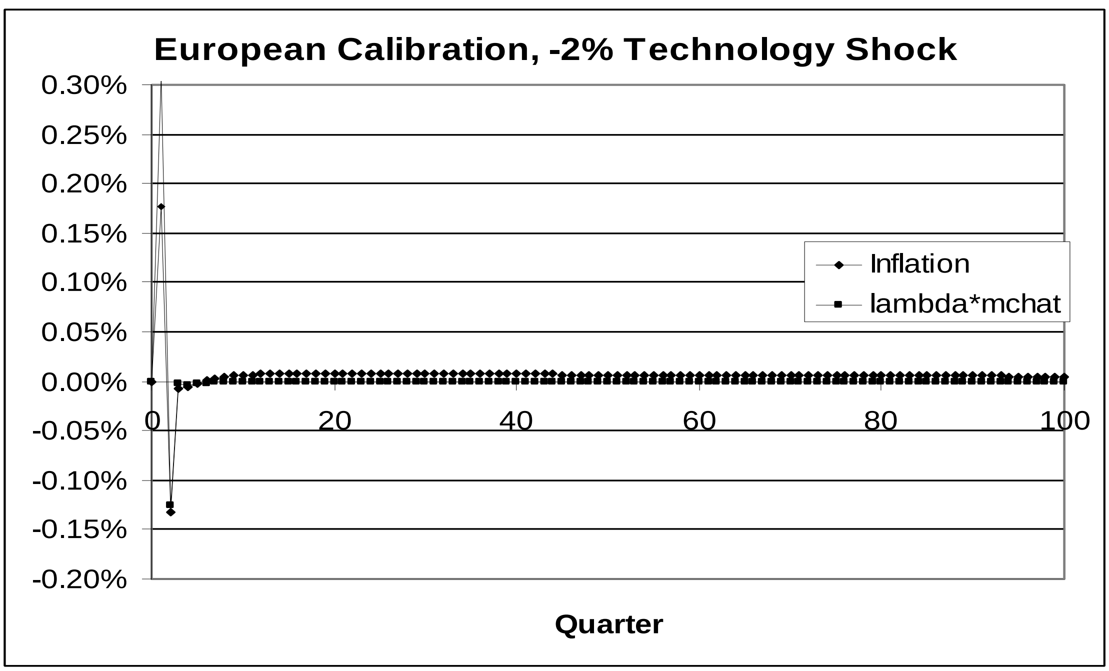

Figure 11

Thus both shocks can potentially trigger extremely persistent increases in unemployment under the continental European calibration if skill loss exceeds it's critical level and the central bank reacts more than one for one to inflation. This is clearly a very interesting result given the persistent increase in unemployment in many Western European countries which we have observed since the end of the 1970s.

8 Conclusion

This paper adds skill loss among the unemployed as an additional labour market friction to the model of Blanchard and Gali (2008) and shows the implications of this modification for determinacy. We assume that an unemployed person looses a set fraction of her skills during every quarter of her unemployment spell but regains all her skills after one quarter of employment. Firms who decide to hire meet workers according to their shares in the market. We first show that in the Blanchard and Gali (2008) model, a coefficient on inflation larger than one in an interest feedback for the nominal interest rate guarantees determinacy.

We then show that the introduction of skill loss increases the (positive) effect of past unemployment on marginal costs. An increase in past unemployment rates increases the share of the longer term unemployed and thus worsens the quality of the pool of job seekers. If the quarterly skill loss percentage is increased to or above a critical level, the combined positive effects of lagged and lead unemployment exceed the negative effect of current unemployment. In such a scenario, if the central bank responds only to inflation, determinacy requires a coefficient on inflation in the feedback rule smaller than one. This holds regardless of whether the central bank responds to current, lagged or expected future inflation.

We also show that the critical skill loss percentage is much lower, and a lot more plausible, if the flow characteristics of the labour market are "Continental European" (Blanchard and Gali (2008)) in the sense that there is little hiring and firing going on. By contrast, under and an "American" calibration of inflow and outflow rates, the implied critical skill loss percentage is implausibly high. This is largely due to the fact that even in the original Blanchard and Gali model lagged and lead unemployment matter a lot more for marginal costs under the continental European than under the American calibration.

Furthermore, neither interest rate smoothing nor responding to the output gap (as commonly modelled in the New Keynesian literature) help to restore determinacy under the continental European calibration if skill loss is above its critical level. As empirical estimates of interest feedback rules frequently find that both the Federal Reserve and Bundesbank and ECB respond more than one for one to inflation, this might mean that indeterminacy and thus sunspot driven dynamics are a much more likely phenomena in continental Europe than in the United States.

Finally, we compute the response of the model under the European calibration with skill loss above its critical level and the coefficient on inflation larger than one in the interest feedback rule to an adverse sunspot shock and an adverse non-correlated technology shock. It turns out that the response of unemployment is extremely persistent. Thus this admittedly quite stylised model potentially contributes to explaining the persistent increase in unemployment observed in continental Europe since the late 1970s. It also suggests the following story: The shift of monetary policy away from a "Keynesian" approach towards aggressive inflation targeting might have been unproblematic in the fluid labour market of the United States but might have been a source of instability and persistent unemployment fluctuations in Western continental Europe with its much less fluid labour market.

9 Appendix I: Determinacy in the Blanchard/ Gali model

We show in this section that, for reasonable calibrations, the condition stated in proposition one ensures determinacy in the Blanchard Gali model. Woodford (2003) derives conditions for determinacy for a linear rational expectations model of the form

$$\begin{pmatrix} E_t z_{t+1} \\ x_{t+1} \end{pmatrix} = A \begin{pmatrix} z_t \\ x_t \end{pmatrix} + b e_t \tag{29}$$

$$\text{where } A = \begin{pmatrix} \frac{1+\frac{\kappa_F \phi_\pi}{c_0}}{\beta+\kappa_F/c_0} & \frac{\kappa_0+\kappa_F\frac{c_1+\phi_u-c_0}{c_0}}{\beta+\kappa_F/c_0} & -\frac{\kappa_L+\frac{c_1}{c_0}}{\beta+\kappa_F/c_0} \\ \frac{-\beta\phi_\pi+1}{\kappa_F+c_0\beta} & \frac{-\beta(c_1-c_0+\phi_u)+\kappa_0}{\kappa_F+c_0\beta} & \frac{\beta c_1-\kappa_L}{\kappa_F+c_0\beta} \\ 0 & 1 & 0 \end{pmatrix},\ b = \begin{pmatrix} \frac{-\frac{\kappa_F\rho_a}{c_0}+\lambda\Phi\gamma}{\beta+\kappa_F/c_0} \\ \frac{-\beta(1-\rho_a)-\kappa_F/c_0+\lambda\Phi\gamma}{\kappa_F+c_0\beta} \\ 0 \end{pmatrix} \tag{30}$$

where z_t is a 2x1 vector of endogenous jump variables, x_t is single endogenous predetermined variable and e_t is a vector of disturbances. This is exactly the kind of system we are dealing with. The rational expectations equilibrium will be determinate if and only if the matrix A has exactly one eigenvalue inside the unit circle, i.e. with modulus smaller than 1 and the two other eigenvalues outside the unit circle. If the characteristic equation is written in the form

$$\mu^3 + A_2\mu^2 + A_1\mu + A_0 = 0$$

Woodford shows that it will have two roots outside and one root inside the unit circle if and only if

either (Case I)

$$\begin{aligned} 1 + A_2 + A_1 + A_0 &< 0 \text{ and} \\ -1 + A_2 - A_1 + A_0 &> 0 \end{aligned}$$

or (Case II)

$$\begin{aligned} 1 + A_2 + A_1 + A_0 &> 0 \\ -1 + A_2 - A_1 + A_0 &< 0 \\ A_0^2 - A_0A_2 + A_1 - 1 &> 0 \end{aligned}$$

or (Case III) the first two conditions of Case II and

$$\begin{aligned} A_0^2 - A_0 A_2 + A_1 - 1 &< 0 \\ |A_2| &> 3 \end{aligned}$$

As would be expected, some of the resulting expressions will be quite complicated functions of the deep parameters. We therefore do not aspire to give a completely general proof. Rather, we will make the assumption throughout that g is a very small number. $g = Bx^{\alpha}$, and B will be calibrated to such that the fraction of total hiring costs in GDP $\delta B x^{\alpha}$ does not exceed a small fraction of GDP (Blanchard and Gali set them equal to 1% of GDP for the "American" and even less for the continental European calibration). In Blanchard and Gali, it comes out as 0.03. This also implies that $\xi_1 < \xi_0$, and both c_1 and ξ_1 will be small. Furthermore, we will assume that $\kappa_F - \kappa_L > 0$, which will be the case if $\kappa_F - \kappa_L = Mg\lambda\frac{(1-\delta)}{1-u}\left[\frac{\alpha}{\delta}\left[1 - x - \beta\right] + \beta\left[\xi_1 + \xi_0\right]\right] > 0$. This condition holds for values of x and associated values of δ which are not too small. For the calibration considered in this paper, $\kappa_F - \kappa_L > 0$ for $x \geq 0.015$ and $\delta = 0.0017$, both of which is far below empirically reasonable values for these parameters.

Our first task is to derive the characteristic equation. To make the algebra easier, we first write our matrix A in a more general form:

$$A = \begin{pmatrix} \frac{1+\frac{\kappa_F\phi_\pi}{c_0}}{\beta+\kappa_F/c_0} & \frac{\kappa_0+\kappa_F\frac{c_1+\phi_u-c_0}{c_0}}{\beta+\kappa_F/c_0} & -\frac{\kappa_L+\frac{c_1}{c_0}}{\beta+\kappa_F/c_0} \\ \frac{-\beta\phi_\pi+1}{\kappa_F+c_0\beta} & \frac{-\beta(c_1-c_0+\phi_u)+\kappa_0}{\kappa_F+c_0\beta} & \frac{\beta c_1-\kappa_L}{\kappa_F+c_0\beta} \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 1 & 0 \end{pmatrix}$$

The characteristic equation is then given by

$$\mu^3 + \left(-a_{11} - a_{22}\right)\mu^2 + \left(-a_{23} + a_{11}a_{22} - a_{12}a_{21}\right)\mu + a_{11}a_{23} - a_{21}a_{13} = 0$$

Hence we can determine A_2, A_1 and A_0 as

$$\begin{aligned} A_2 &= -a_{11} - a_{22} = \frac{-c_0\left(1+\beta\right) - \kappa_F\phi_\pi - \kappa_0 + \beta\left(c_1 + \phi_u\right)}{\kappa_F + c_0\beta} \\ A_1 &= \frac{-\left(1+\beta\right)c_1 + \kappa_L + c_0 - \phi_u + \phi_\pi\kappa_0}{\kappa_F + c_0\beta} \\ A_0 &= \frac{c_1 - \phi_\pi\kappa_L}{\kappa_F + c_0\beta} \end{aligned}$$

We first look at the second condition of Case I. We have

$$-1 + A_2 - A_1 + A_0 > 0$$

implying

$$\frac{2}{1-u}\frac{\left(\xi_1 - \xi_0\right)\left(1+\beta\right)}{\kappa_0 + \kappa_L + \kappa_F} - 1 > \phi_\pi$$

or

$$\frac{2}{1-u}\frac{\left(g\left(1-\delta\right)\left(1+\alpha\left(1-x\right)\right)+g\left(1+\alpha\right)-1\right)\left(1+\beta\right)}{\kappa_0+\kappa_L+\kappa_F}-1>\phi_\pi \tag{31}$$

This condition will never be fulfilled by positive values of ϕ_π under the assumptions made.

Thus we conclude that Case 1 is not relevant and turn to Case 2. The first condition implies

$$\phi_\pi-\phi_u\frac{\left(1-\beta\right)}{\kappa_0-\kappa_L-\kappa_F}>1 \tag{32}$$

The second condition is implied by the fact that the second condition of Case 1 is violated, while the third condition implies

$$\begin{aligned}&\phi_\pi\left[-\phi_\pi\kappa_L\left[\kappa_F-\kappa_L\right]-c_1\kappa_L+c_0\left[\beta\kappa_0-\kappa_L\right]+\kappa_0\left[\kappa_F-\kappa_L\right]+\kappa_L\phi_u\beta\right]\\&+c_1\left[1-\beta+c_0\left(1-\beta^2\right)-\kappa_F\right]+\left[\kappa_L+c_0\left(1-\beta\right)-\kappa_F-\phi_u\right]\left[\kappa_F+c_0\beta\right]>0\end{aligned} \tag{33}$$

Not that if $\kappa_F<\kappa_L$, this expression will be monotonously increasing in ϕ_π. Hence in that case, if (33)) holds for $\phi_\pi=1$, it will hold for $\phi_\pi>1$ as well. Hence we set $\phi_\pi=1$ and $\phi_u=1$(since permissible, i.e. negative values of ϕ_u make (33)) more likely to be met), which allows us to write the condition as

$$\begin{aligned}&\kappa_0\left(\kappa_F-\kappa_L\right)+c_0\left[\beta\kappa_0-\kappa_L-\beta\kappa_F\right]+\kappa_L\left(\kappa_L-c_1\right)+c_1\left(1-\beta\right)\\&+c_0c_1\left(1-\beta^2\right)+c_0^2\left(1-\beta\right)\beta+c_0\kappa_F\left(1-\beta\right)+c_0\beta\kappa_L-\kappa_F^2-c_1\kappa_F>0\end{aligned}$$

This will usually be fulfilled. Given that c_0 slightly larger than 1 and that κ_L and κ_F are in the same order of magnitude but smaller than 1, and c_1 is quite small, $c_0\beta\kappa_L>\kappa_F^2+c_1\kappa_F$.

If we assume $\kappa_F-\kappa_L>0$, there is still an issue of (33) being violated for sufficiently large values of ϕ_π since ϕ_π^2 has a negative coefficient. We will now show that under the assumptions already made, if $A_0^2-A_0A_2+A_1-1>0$ becomes violated, we will already be in a situation where $|A_2|>3$ and thus Case III kicks in. Let us first consider the terms in (33) not involving ϕ_π. Those can be written as

$$c_1\left[1-\beta+c_0\left(1-\beta^2\right)\right]+\kappa_F\left[\kappa_L-c_1\right]+\kappa_Fc_0\left(1-\beta\right)+c_0\beta\kappa_L-\kappa_F^2+c_0^2\left(1-\beta\right)-c_0\beta\kappa_F>0.$$

The term $-\kappa_F^2$ is dominated by $c_0\beta\kappa_L$ under the assumptions already made and all the other terms but $-c_0\beta\kappa_F$ are positive. It is not clear that $-c_0\beta\kappa_F$ is being dominated by any of the other terms. Therefore, in the next step, we disregard all the other terms not involving ϕ_π except for $-c_0\beta\kappa_F$. If the modified condition is fulfilled, so will be (33) .Hence we look for which ϕ_π we have (still for $\phi_u=0$)

$$-\phi_\pi^2\kappa_L\left[\kappa_F-\kappa_L\right]+\phi_\pi\left[-c_1\kappa_L+c_0\left[\beta\kappa_0-\kappa_L\right]+\kappa_0\left[\kappa_F-\kappa_L\right]\right]-c_0\beta\kappa_F>0$$

or

$$\phi_\pi^2-\frac{\left[-c_1\kappa_L+c_0\left[\beta\kappa_0-\kappa_L\right]+\kappa_0\left[\kappa_F-\kappa_L\right]\right]}{\kappa_L\left[\kappa_F-\kappa_L\right]}\phi_\pi+\frac{c_0\beta\kappa_F}{\kappa_L\left[\kappa_F-\kappa_L\right]}<0$$

The polynomial on the left hand side has two solutions $\phi_{\pi 1}$ and $\phi_{\pi 2}$ and the inequality will be fulfilled if ϕ_π lies between. Hence we have

$$\begin{aligned}\phi_{\pi 1,2} &= \frac{-c_1\kappa_L + c_0\left[\beta\kappa_0 - \kappa_L\right] + \kappa_0\left[\kappa_F - \kappa_L\right]}{2\kappa_L\left[\kappa_F - \kappa_L\right]} \\ &\pm\sqrt{\frac{\left(-c_1\kappa_L + c_0\left[\beta\kappa_0 - \kappa_L\right] + \kappa_0\left[\kappa_F - \kappa_L\right]\right)^2}{4\kappa_L^2\left[\kappa_F - \kappa_L\right]^2} - \frac{c_0\beta\kappa_F}{\kappa_L\left[\kappa_F - \kappa_L\right]}}\end{aligned}$$

Since we now assume $\kappa_F > \kappa_L$, the expression under the root will always be positive, as will the expression outside of the root. This also implies that we can focus on the larger of the two solution since $\sqrt{\frac{\left(-c_1\kappa_L + c_0\left[\beta\kappa_0 - \kappa_L\right] + \kappa_0\left[\kappa_F - \kappa_L\right]\right)^2}{4\kappa_L^2\left[\kappa_F - \kappa_L\right]^2} - \frac{c_0\beta\kappa_F}{\kappa_L\left[\kappa_F - \kappa_L\right]}} > \frac{-c_1\kappa_L + c_0\left[\beta\kappa_0 - \kappa_L\right] + \kappa_0\left[\kappa_F - \kappa_L\right]}{2\kappa_L\left[\kappa_F - \kappa_L\right]}$ and thus the smaller solution will be will be negative. Hence the relevant lower bound is (32) The larger of the two roots will be at least as big as the term outside the brackets. Hence condition (33) will still be met under the assumptions made if

$$\phi_\pi < \frac{-c_1\kappa_L + c_0\left[\beta\kappa_0 - \kappa_L\right] + \kappa_0\left[\kappa_F - \kappa_L\right]}{2\kappa_L\left[\kappa_F - \kappa_L\right]} \tag{34}$$

We now turn to condition $|A_2| > 3$ from Case III to see what it implies for ϕ_π. For the "large" values of ϕ_π which are of interest here, A_2 will most likely be negative, so we consider the inequality $-A_2 > 3$, which can be written as

$$\phi_\pi > \frac{-c_0 + 2c_0\beta - \kappa_0 + c_1\beta}{\kappa_F} + 3 \tag{35}$$

We would like to check whether at the point (34) becomes violated (35) is already met. Hence we are asking whether

$$\frac{-c_1\kappa_L + c_0\left[\beta\kappa_0 - \kappa_L\right] + \kappa_0\left[\kappa_F - \kappa_L\right]}{2\kappa_L\left[\kappa_F - \kappa_L\right]} > \frac{-c_0 + 2c_0\beta - \kappa_0 + c_1\beta}{\kappa_F} + 3$$

holds. This can be written as

$$\begin{aligned}-c_1\kappa_L\kappa_F + c_0\left[\beta\kappa_0 - \kappa_L\right]\kappa_F + \kappa_0\left[\kappa_F - \kappa_L\right]\kappa_F & \\ -2c_0\kappa_L\left[\kappa_F - \kappa_L\right] + 4c_0\beta\kappa_L\left[\kappa_F - \kappa_L\right] & \\ -2\kappa_0\kappa_L\left[\kappa_F - \kappa_L\right] + c_1\beta 2\kappa_L\left[\kappa_F - \kappa_L\right] + 6\kappa_L\kappa_F\left[\kappa_F - \kappa_L\right] &> 0\end{aligned}$$

or

$$\begin{aligned}-c_1\kappa_L\kappa_F + c_0\left[\beta\kappa_0 - \kappa_L\right]\kappa_F + 2c_0\kappa_L\kappa_F - 2c_0\kappa_L^2 - 4\beta c_0\kappa_L\kappa_F & \\ +4\beta c_0\kappa_L^2 + \left[\kappa_F - \kappa_L\right]\left(\kappa_0\kappa_F + 2\kappa_L\kappa_0 - 6\kappa_L\kappa_F\right) & \\ -c_1\beta 2\kappa_L\left[\kappa_F - \kappa_L\right] > 0 &\end{aligned}$$

or

$$\begin{aligned} -c_1\kappa_L\kappa_F + c_0\kappa_F\left[\beta\kappa_0 - \kappa_L - \beta\kappa_L\right] + \kappa_L\left(2\beta c_0\kappa_L - \beta c_0\kappa_F\right) & \\ +2c_0\kappa_F\kappa_L - 2c_0\kappa_L^2 - 2c_0\beta\kappa_L\kappa_F + 2\beta c_0\kappa_L^2 & \\ +\left[\kappa_F - \kappa_L\right]\left(\kappa_0\kappa_F + 2\kappa_L\kappa_0 - 6\kappa_L\kappa_F\right) - c_1\beta 2\kappa_L\left[\kappa_F - \kappa_L\right] & > 0 \end{aligned}$$

or

$$\begin{aligned} -c_1\kappa_L\kappa_F + c_0\left[\beta\kappa_0 - \kappa_L\right]\kappa_F + 2c_0\kappa_L\kappa_F - 2c_0\kappa_L^2 - 4\beta c_0\kappa_L\kappa_F + 4\beta c_0\kappa_L^2 & \\ +\left[\kappa_F - \kappa_L\right]\left(\kappa_0\kappa_F + 2\kappa_L\kappa_0 - 6\kappa_L\kappa_F\right) - c_1\beta 2\kappa_L\left[\kappa_F - \kappa_L\right] & > 0 \end{aligned}$$

Note also that

$$\begin{aligned} & 2c_0\kappa_F\kappa_L - 2c_0\kappa_L^2 - 2c_0\beta\kappa_L\kappa_F + 2\beta c_0\kappa_L^2 - c_1\kappa_L\kappa_F - c_1\beta 2\kappa_L\left[\kappa_F - \kappa_L\right] \\ = & \ 2\kappa_L c_0\left(1-\beta\right)\left[\kappa_F - \kappa_L\right] - c_1\kappa_L\kappa_F - c_1\beta 2\kappa_L\left[\kappa_F - \kappa_L\right] \\ = & \ -2\kappa_L\left[\kappa_F - \kappa_L\right]\left[c_1\beta - c_0\left(1-\beta\right)\right] - c_1\kappa_L\kappa_F \end{aligned}$$

Thus we can write

$$\begin{aligned} c_0\kappa_F\left[\beta\kappa_0 - \kappa_L - \beta\kappa_L\right] + \beta c_0\kappa_L\left(2\kappa_L - \kappa_F\right) & \\ +\left[\kappa_F - \kappa_L\right]\left(\kappa_0\kappa_F + 2\kappa_L\kappa_0 - 6\kappa_L\kappa_F\right) - 2\kappa_L\left[\kappa_F - \kappa_L\right]\left[c_1\beta - c_0\left(1-\beta\right)\right] - c_1\kappa_L\kappa_F & > 0 \end{aligned}$$

Since (using $\kappa_0 > \kappa_F + \kappa_L$) $\kappa_0\kappa_F + 2\kappa_L\kappa_0 - 6\kappa_L\kappa_F > \left(\kappa_F - \kappa_L\right)^2 - \kappa_L\left(\kappa_F - \kappa_L\right) = \left(\kappa_F - \kappa_L\right)\left(\kappa_F - 2\kappa_L\right)$we can write

$$\begin{aligned} \kappa_F c_0\left(\beta\kappa_0 - \kappa_L - \beta\kappa_L\right) + \beta c_0\kappa_L\left(2\kappa_L - \kappa_F\right) + & \\ \left[\kappa_F - \kappa_L\right]^2\left(\kappa_F - 2\kappa_L\right) - 2\kappa_L\left[\kappa_F - \kappa_L\right]\left[c_1\beta - c_0\left(1-\beta\right)\right] - c_1\kappa_L\kappa_F & > 0 \end{aligned}$$

The first term is clearly positive. The second term will be positive as long as $\kappa_F < 2\kappa_L$. If $\kappa_F - \kappa_L$ increases, in that case the first term would increase and at a larger rate as both $(\beta\kappa_0 - \kappa_L - \beta\kappa_L)$ and κ_F would increase. In this case we would also see the third term switch from negative to positive, which would otherwise also be negative. The final two terms are negative. We believe it is safe to assume that this condition holds. For values of κ_L and κ_F which are close, $\kappa_F c_0\left(\beta\kappa_0 - \kappa_L - \beta\kappa_L\right) + \beta c_0\kappa_L\left(2\kappa_L - \kappa_F\right)$ will be in a higher order of magnitude than $\left[\kappa_F - \kappa_L\right]^2\left(\kappa_F - 2\kappa_L\right) - 2\kappa_L\left[\kappa_F - \kappa_L\right]\left[c_1\beta - c_0\left(1-\beta\right)\right] - c_1\kappa_L\kappa_F$. For values of κ_F substantially higher than κ_L, the order of magnitude of $\kappa_F c_0\left(\beta\kappa_0 - \kappa_L - \kappa_L\right)$ will increase and $\left[\kappa_F - \kappa_L\right]^2\left(\kappa_F - 2\kappa_L\right)$ would turn positive. Thus the second condition of case III will be satisfied for values of ϕ_π violating (33).

Thus we have shown, under the assumptions made, that $\phi_\pi - \phi_u\frac{(1-\beta)}{\kappa_0 - \kappa_L - \kappa_F} > 1$ guarantees the existence of a unique rational expectations equilibrium in the Blanchard/ Gali model.

10 Appendix II: Relevant steady state values in the model with skill Loss

As was mentioned in the text, we start by assuming values for u and x. This allows to write the steady state values of δ, s^i, A^L and A^A as

$$\begin{aligned}
\delta &= \frac{ux}{(1-u)(1-x)} \\
s^i &= x(1-x)^i \\
A^L &= \sum_{i=0}^{\infty} s^i \beta_s^i = \frac{x}{1-(1-x)\beta_s}
\end{aligned}$$

and

$$A^A = s^N A^L + \left(1 - s_t^N\right) = \delta A^L + 1 - \delta$$

This allows to back out Θ by first noting that in the steady state, we can write (12) as

$$A^L\left[\frac{1}{M} - g\left(1-\beta\left(1-\delta\right)\right)\right] + \beta\left(1-\delta\right)\left[\frac{1-A^L}{M}\right] = \Theta'\left[\beta\left(1-\delta\right) + \frac{W}{\Theta'}\left(1-\beta\left(1-\delta\right)\right)\right]$$

From (8), we have

$$W = \Theta' \sum_{i=0}^{\infty} s^i \beta_s^{i(1-\gamma)} = \Theta' \frac{x}{1-(1-x)\beta_s^{1-\gamma}} \tag{36}$$

and, for W^L

$$W^L = \sum_{i=0}^{\infty} s^i \beta_s^{i(1-\gamma)} = \frac{x}{1-(1-x)\beta_s^{1-\gamma}}$$

which we use to solve for Θ' as

$$\Theta' = \frac{1/M - g\left(1-(1-\delta)\beta\right) + \frac{(1-\delta)\beta}{M}\left(1-A^L\right)}{(1-\delta)\beta + \frac{x}{1-(1-x)\beta_s^{1-\gamma}}\left(1-(1-\delta)\beta\right)}$$

11 Appendix III: Deriving the Laws of Motion for $\widehat{a}_t^L$ and $\widehat{w}_t^L$

A log linear approximation to the skill level A_t^L is given by

$$\widehat{a}_t^L = \frac{\sum_{i=0}^{\infty} ds_t^i \beta_s^i}{A^L} \tag{37}$$

The shares of the various groups of the unemployed are given by

$$s_t^i = \frac{\delta N_{t-1-i} \prod_{j=1}^{i} (1 - x_{t-i})}{U_t}$$

This can be log-linearised as

$$ds_t^i = s^i \left[\widehat{n}_{t-1-i} - \widehat{U}_t + \sum_{j=1}^{i} \frac{-x}{1-x} \widehat{x}_{t-j} \right]$$

Log linear approximations to x_t and U_t are given by $\widehat{x}_{t-j} = \frac{\widehat{n}_{t-j} - (1-\delta)(1-x)\widehat{n}_{t-1-j}}{\delta}$ and $\widehat{U}_t = -\frac{(1-\delta)x}{\delta}\widehat{n}_t$ yields

$$\begin{aligned}
ds_t^i &= s^i \left[\widehat{n}_{t-1-i} + \frac{1-\delta}{\delta} x \widehat{n}_{t-1} - \sum_{j=1}^{i} \frac{-x}{1-x} \frac{\widehat{n}_{t-j} - (1-\delta)(1-x)\widehat{n}_{t-1-j}}{\delta} \right] \\
&= s^i \left[\widehat{n}_{t-1-i} + \frac{1-\delta}{\delta} x \widehat{n}_{t-1} - \frac{x}{1-x} \left[\sum_{j=1}^{i} \frac{\widehat{n}_{t-j}}{\delta} - \frac{(1-\delta)(1-x)}{\delta} \sum_{j=1}^{i} \frac{\widehat{n}_{t-j-1}}{\delta} \right] \right] \\
&= s^i \left[\widehat{n}_{t-1-i} + \frac{1-\delta}{\delta} x \widehat{n}_{t-1} - \frac{x}{1-x} \left[\begin{array}{c} \sum_{j=2}^{i} \frac{\widehat{n}_{t-j}}{\delta} + \frac{\widehat{n}_{t-1}}{\delta} \\ -\frac{(1-\delta)(1-x)}{\delta} \sum_{j=2}^{i} \frac{\widehat{n}_{t-j}}{\delta} - \frac{(1-\delta)(1-x)}{\delta} \widehat{n}_{t-j-1} \end{array} \right] \right] \\
&= s^i \left[\begin{array}{c} \widehat{n}_{t-1-i} \left[1 + \frac{x(1-\delta)}{\delta} \right] + \frac{1-\delta}{\delta} x \widehat{n}_{t-1} - \frac{x}{1-x} \frac{\widehat{n}_{t-1}}{\delta} \\ -\frac{x}{1-x} \left(\frac{1}{\delta} - \frac{(1-\delta)(1-x)}{\delta} \right) \sum_{j=2}^{i} \frac{\widehat{n}_{t-j}}{\delta} \end{array} \right] \\
&= s^i \left[\begin{array}{c} \widehat{n}_{t-1-i} \left[1 + \frac{x(1-\delta)}{\delta} \right] - \frac{x}{1-x} \left(\frac{1}{\delta} - \frac{(1-\delta)(1-x)}{\delta} \right) \widehat{n}_{t-1} \\ -\frac{x}{1-x} \left(\frac{1}{\delta} - \frac{(1-\delta)(1-x)}{\delta} \right) \sum_{j=2}^{i} \frac{\widehat{n}_{t-j}}{\delta} \end{array} \right] \\
&= s^i \left[\widehat{n}_{t-1-i} \left[1 + \frac{x(1-\delta)}{\delta} \right] - \left(\frac{x^2}{\delta(1-x)} + x \right) \sum_{j=1}^{i} \frac{\widehat{n}_{t-j}}{\delta} \right]
\end{aligned}$$

We now use $\delta = \frac{ux}{(1-x)(1-u)}$ and $(1-\delta) = \frac{1-u-x}{(1-x)(1-u)}$ to eliminate δ in the $1 + \frac{(1-\delta)x}{\delta}$ and $\frac{x^2}{\delta(1-x)} + x$. This yields

$$\begin{aligned}
1 + x\frac{(1-\delta)}{\delta} &= 1 + x\frac{(1-u-x)}{(1-x)(1-u)} \frac{(1-x)(1-u)}{ux} = 1 + \frac{1-u-x}{u} = \frac{1-x}{u} \\
\frac{x^2}{\delta(1-x)} + x &= \frac{x^2}{\frac{ux}{(1-x)(1-u)}(1-x)} + x = \frac{x}{u}(1-u) + x = \frac{x}{u}
\end{aligned}$$

Hence we can write

$$ds_t^i = s^i \left[-\frac{x}{u} \sum_{j=1}^{i} \widehat{n}_{t-j} + \frac{1-x}{u} \widehat{n}_{t-1-i} \right] \tag{38}$$

Substituting this into (37) yields

$$\begin{aligned}
\widehat{a}_t^L &= \frac{\sum\limits_{i=0}^{\infty} \beta_s^i s^i \left[-\frac{x}{u} \sum\limits_{j=1}^{i} \widehat{n}_{t-j} + \frac{1-x}{u} \widehat{n}_{t-1-i} \right]}{A^L} \\
&= \frac{1}{A^L} \left[\frac{1-x}{u} \sum_{i=0}^{\infty} \beta_s^i s^i \widehat{n}_{t-1-i} - \frac{x}{u} \sum_{i=0}^{\infty} \sum_{j=1}^{i} \beta_s^i s^i \widehat{n}_{t-j} \right] \\
&= \frac{1}{A^L} \left[\frac{1-x}{u} \sum_{i=0}^{\infty} \beta_s^i s^i \widehat{n}_{t-1-i} - \frac{x}{u} \sum_{q=1}^{\infty} \beta_s^q s^q \sum_{j=1}^{q} \widehat{n}_{t-j} \right] \\
&= \frac{1}{A^L} \left[\frac{1-x}{u} \sum_{i=0}^{\infty} \beta_s^i s^i \widehat{n}_{t-1-i} - \frac{x}{u} \sum_{q=1}^{\infty} \beta_s^q s^q \left(\widehat{n}_{t-1} + \widehat{n}_{t-2} + \widehat{n}_{t-3} ... + \widehat{n}_{t-q} \right) \right] \\
&= \frac{1}{A^L} \left[\frac{1-x}{u} \sum_{i=0}^{\infty} \beta_s^i s^i \widehat{n}_{t-1-i} - \frac{x}{u} \left[\beta_s^1 s^1 \widehat{n}_{t-1} + \beta_s^2 s^2 \left(\widehat{n}_{t-1} + \widehat{n}_{t-2} \right) + \beta_s^3 s^3 \left(\widehat{n}_{t-1} + \widehat{n}_{t-2} + \widehat{n}_{t-3} \right) ...... \right] \right] \\
&= \frac{1}{A^L} \left[\begin{array}{c} \left[\frac{1-x}{u} \sum\limits_{i=0}^{\infty} \beta_s^i s^i \widehat{n}_{t-1-i} \right] \\ -\frac{x}{u} \left[\left(\sum\limits_{q=1}^{\infty} \beta_s^q s^q \right) \widehat{n}_{t-1} + \left(\sum\limits_{q=2}^{\infty} \beta_s^q s^q \right) \widehat{n}_{t-2} + \left(\sum\limits_{q=3}^{\infty} \beta_s^q s^q \right) \widehat{n}_{t-3} + \left(\sum\limits_{q=4}^{\infty} \beta_s^q s^q \right) \widehat{n}_{t-4} ... \right] \end{array} \right] \\
&= \frac{1}{A^L} \left[\begin{array}{c} \frac{1-x}{u} \left[s^0 \widehat{n}_{t-1} + \beta_s^1 s^1 \widehat{n}_{t-2} + \beta_s^2 s^2 \widehat{n}_{t-3} + \beta_s^3 s^3 \widehat{n}_{t-4} ...... \right] \\ -\frac{x}{u} \left[\left(\sum\limits_{q=1}^{\infty} \beta_s^q s^q \right) \widehat{n}_{t-1} + \left(\sum\limits_{q=2}^{\infty} \beta_s^q s^q \right) \widehat{n}_{t-2} + \left(\sum\limits_{q=3}^{\infty} \beta_s^q s^q \right) \widehat{n}_{t-3} + \left(\sum\limits_{q=4}^{\infty} \beta_s^q s^q \right) \widehat{n}_{t-4} ... \right] \end{array} \right] \\
&= \frac{1}{A^L} \left[\begin{array}{c} \left[s^0 \frac{1-x}{u} - \frac{x}{u} \left(\sum\limits_{q=1}^{\infty} \beta_s^q s^q \right) \right] \widehat{n}_{t-1} + \left[s^1 \frac{1-x}{u} - \frac{x}{u} \left(\sum\limits_{q=2}^{\infty} \beta_s^q s^q \right) \right] \widehat{n}_{t-2} \\ + \left[s^2 \frac{1-x}{u} - \frac{x}{u} \left(\sum\limits_{q=3}^{\infty} \beta_s^q s^q \right) \right] \widehat{n}_{t-3} + \left[s^3 \frac{1-x}{u} - \frac{x}{u} \left(\sum\limits_{q=4}^{\infty} \beta_s^q s^q \right) \right] \widehat{n}_{t-4} \end{array} \right] \\
&= \frac{1}{A^L} \sum_{i=1}^{\infty} \left[\left[\beta_s^{i-1} s^{i-1} \frac{1-x}{u} - \frac{x}{u} \left(\sum_{q=i}^{\infty} \beta_s^q s^q \right) \right] \widehat{n}_{t-i} \right]
\end{aligned}$$

Using $A^L = \sum\limits_{i=0}^{\infty} s^i \beta_s^i = \frac{x}{1-(1-x)\beta_s}$ and $s^i = x(1-x)^i$ we can write $\left(\sum\limits_{q=i}^{\infty} \beta_s^q s^q \right) =$

$x\sum_{q=i}^{\infty}\beta_s^q(1-x)^q = \beta_s^i(1-x)^i x\sum_{q=0}^{\infty}\beta_s^q(1-x)^q = \beta_s^i(1-x)^i A^L$ and thus arrive at

$$\widehat{a}_t^L = \frac{x}{u}\sum_{i=1}^{\infty}\left[\left[\frac{\beta_s^{i-1}(1-x)^i}{A^L} - \beta_s^i(1-x)\right]\widehat{n}_{t-i}\right]$$

This can be rewritten as

$$\begin{aligned}\widehat{a}_t^L &= \sum_{i=1}^{\infty}\frac{1}{u}(1-x)^i\left(\beta_s^{i-1}-\beta_s^i\right)\widehat{n}_{t-i}\\ &= \sum_{i=1}^{\infty}a_i^n\widehat{n}_{t-i},\ a_i^n = \frac{1}{u}(1-x)^i\left(\beta_s^{i-1}-\beta_s^i\right)\end{aligned}$$

Thus in the presence of skill loss ($\beta_s < 1$), the deviation of the average skill level from its steady state depends positively on a weighted sum of past employment rates. The a_i^n coefficient depend on β_s and thus δ^s :

$$\frac{\vartheta a_i^n}{\vartheta\delta^s} = -\frac{1}{u}(1-x)^i\left((i-1)\beta_s^{i-2} - i\beta_s^{i-1}\right)$$

For $\beta_s = 1$ ($\delta^s = 0$), this is clearly positive. Thus the larger the quarterly skill decay among the unemployed, the larger is the effect of past employment on the skill level. For $\beta_s < 1$, we have

$$\frac{\vartheta a_i^n}{\vartheta\delta^s} \gtreqless 0 \Leftrightarrow i \lesseqgtr \frac{1}{1-\beta_s}$$

Hence the effect of δ^s on a_i^n will become negative if i is sufficiently large. Furthermore, $\frac{\vartheta a_i^n}{\vartheta x} < 0$ if $\beta_s < 1$.

To express the component of the real wage depending on the skill of the worker as a function of past employment rates, we follow an analogous process. A log linear approximation to W_t^L is given by

$$\widehat{w}_t^L = \frac{\sum_{i=0}^{\infty}ds_t^i\beta_s^{i(1-\gamma)}}{W^L} \tag{39}$$

Note that the only difference to (37) is that β_s and A^L are replaced by $\beta_s^{(1-\gamma)}$ and W^L, respectively. Substituting (38) and going through exactly the same process as before thus gives us

$$\widehat{w}_t^L = \sum_{i=1}^{\infty}w_i^n\widehat{n}_{t-i},\ w_i^n = \frac{1}{u}(1-x)^i\left(\beta_s^{(1-\gamma)(i-1)} - \beta_s^{(1-\gamma)i}\right)$$

and, as with the a_i^n coefficients,

$$\begin{aligned}
\frac{\partial w_i^n}{\partial \delta^s} &= -\frac{1}{u}(1-x)^i (1-\gamma)\left((i-1)\beta_s^{(1-\gamma)(i-1)-1} - i\beta_s^{(1-\gamma)i-1}\right) \\
\frac{\partial w_i^n}{\partial \delta^s} &> 0 \ \Leftrightarrow \delta^s = 0 \\
\frac{\partial w_i^n}{\partial \delta^s} &\gtreqless 0 \Leftrightarrow i \lesseqgtr \frac{1}{1-\beta_s} \\
\frac{\partial w_i^n}{\partial x} &< 0 \text{ iff } \beta_s < 1.
\end{aligned}$$

Furthermore, we have $\frac{\partial a_i^n}{\partial \delta^s} > \frac{\partial w_i^n}{\partial \delta^s}$ if $\delta^s = 0$ and $\gamma > 0$.

We now turn to express $\widehat{a}_t^L$ and $\widehat{w}_t^L$ as a function of their t-1 values and past employment. For $\widehat{a}_t^L$ we have

$$\begin{aligned}
\widehat{a}_t^L &= \frac{1}{u}(1-x)(1-\beta_s)\widehat{n}_{t-1} + \frac{1}{u}\sum_{i=2}^{\infty}(1-x)^i\left(\beta_s^{i-1} - \beta_s^i\right)\widehat{n}_{t-i} \\
&= \frac{1}{u}(1-x)(1-\beta_s)\widehat{n}_{t-1} + \frac{1}{u}\sum_{i=1}^{\infty}(1-x)^{i+1}\left(\beta_s^{i} - \beta_s^{i+1}\right)\widehat{n}_{t-i} \\
&= \frac{1}{u}(1-x)(1-\beta_s)\widehat{n}_{t-1} + \beta_s(1-x)\frac{1}{u}\sum_{i=1}^{\infty}(1-x)^i\left(\beta_s^{i-1} - \beta_s^i\right)\widehat{n}_{t-i}
\end{aligned}$$

and thus

$$\widehat{a}_t^L = (1-x)\left(\frac{1}{u}(1-\beta_s)\widehat{n}_{t-1} + \beta_s\widehat{a}_{t-1}^L\right) \tag{40}$$

Correspondingly for $\widehat{w}_t^L$ we have

$$\widehat{w}_t^L = (1-x)\left(\frac{1}{u}\left(1-\beta_s^{1-\gamma}\right)\widehat{n}_{t-1} + \beta_s^{1-\gamma}\widehat{w}_{t-1}^L\right) \tag{41}$$

12 Appendix IV: On the relative size of the a_i^n and w_i^n, $\frac{\partial a_i^n}{\partial \delta^s}$ and $\frac{\partial w_i^n}{\partial \delta^s}$ if $\delta_s > 0$

In the following we assume $\delta_s > 0 \Leftrightarrow \beta_s < 1$.

We have $a_i^n > w_i^n$ if

$$\left(\beta_s^{i-1} - \beta_s^i\right) > \left(\beta_s^{(1-\gamma)(i-1)} - \beta_s^{(1-\gamma)i}\right)$$

or

$$i < \frac{\ln(1-\beta_s) - \ln(\beta_s^{\gamma} - \beta_s)}{-\gamma \ln \beta_s}$$

Note that for this expression is always positive (as it should be).Thus a_i^n will turn smaller than w_i^n for large enough i.

The relative size $\frac{\partial a_i^n}{\partial \delta^s}$ and $\frac{\partial w_i^n}{\partial \delta^s}$ also depends on i. We have

$$\frac{\partial a_i^n}{\vartheta \delta^s} \gtreqless \frac{\partial w_i^n}{\partial \delta^s} \Leftrightarrow (1-\gamma)\, \beta_s^{(1-\gamma)i-1} \left((i-1)\, \beta_s^{-1} - i\right) \gtreqless \beta_s^{i-1} \left((i-1)\, \beta_s^{-1} - i\right)$$

Two cases have to be considered: $(i-1)\, \beta_s^{-1} - i\ (i-1)\, \beta_s^{-1} - i \lesseqgtr 0 \Leftrightarrow i \lesseqgtr \frac{1}{1-\beta_s}$. If $(i-1)\, \beta_s^{-1} - i\ (i-1)\, \beta_s^{-1} - i < 0$, this implies $\frac{\partial a_i^n}{\partial \delta^s} \gtreqless \frac{\partial w_i^n}{\partial \delta^s}$ if

$$\frac{1}{\gamma} \left(\frac{\ln(1-\gamma)}{\ln \beta_s} - 1 \right) \gtreqless i$$

Now given that we look at the case $i < \frac{1}{1-\beta_s}$, we ask whether $\frac{1}{\gamma} \left(\frac{\ln(1-\gamma)}{\ln \beta_s} - 1 \right) > i$ is implied by that assumption. Thus we ask whether

$$\frac{1}{\gamma} \left(\frac{\ln(1-\gamma)}{\ln \beta_s} - 1 \right) > \frac{1}{1-\beta_s}$$

This is not necessarily fulfilled but will be met for the range of β_s used in this paper if $\gamma > 0.38$. Thus if this hold, for $i < \frac{1}{1-\beta_s}$, we have $\frac{\partial a_i^n}{\partial \delta^s} > \frac{\partial w_i^n}{\partial \delta^s}$.

If $(i-1)\, \beta_s^{-1} - i\ (i-1)\, \beta_s^{-1} - i > 0$, we have $\frac{\partial a_i^n}{\partial \delta^s} \gtreqless \frac{\partial w_i^n}{\partial \delta^s}$ if

$$\frac{1}{\gamma} \left(\frac{\ln(1-\gamma)}{\ln \beta_s} - 1 \right) \lesseqgtr i$$

As for the calibration considered here, with $i = \frac{1}{1-\beta_s}$ we have $\frac{1}{\gamma} \left(\frac{\ln(1-\gamma)}{\ln \beta_s} - 1 \right) > i$, this means that as i becomes equal to $\frac{1}{1-\beta_s}$, we have $\frac{\partial a_i^n}{\partial \delta^s} < \frac{\partial w_i^n}{\partial \delta^s}$.However, it is also clear that as i increase, we will have $i > \frac{1}{\gamma} \left(\frac{\ln(1-\gamma)}{\ln \beta_s} - 1 \right)$ and hence $\frac{\partial a_i^n}{\partial \delta^s} > \frac{\partial w_i^n}{\partial \delta^s}$. Thus, for the minimum value of β_s considered here $\gamma > 0.38$, we have three different cases depending on the value of i. For sufficiently low values of i, we have $\frac{\partial a_i^n}{\partial \delta^s} > \frac{\partial w_i^n}{\partial \delta^s}$. There is then an intermediate range where $\frac{1}{1-\beta_s} < i < \frac{1}{\gamma} \left(\frac{\ln(1-\gamma)}{\ln \beta_s} - 1 \right)$ where we have $\frac{\partial a_i^n}{\partial \delta^s} < \frac{\partial w_i^n}{\partial \delta^s}$. Finally, for $i > \frac{1}{\gamma} \left(\frac{\ln(1-\gamma)}{\ln \beta_s} - 1 \right)$, we have again $\frac{\partial a_i^n}{\partial \delta^s} > \frac{\partial w_i^n}{\partial \delta^s}$.

Since the relative size of a_i^n and w_i^n as well as the effect of an increase in δ^s on them depends on i, it is interesting to look how the combined effect of past employment on the skill level instead to look at the "net" impact of an increase in past employment on the real wage and the skill level and how this impact is affected by δ^s. The sum of the a_i^n and w_i^n is given by

$$\begin{aligned} a^n &= \sum_{i=1}^{\infty} a_i^n = \frac{1-x}{u} \frac{1-\beta_s}{1-(1-x)\, \beta_s} \\ w^n &= \sum_{i=1}^{\infty} w_i^n = \frac{1-x}{u} \frac{1-\beta_s^{1-\gamma}}{1-(1-x)\, \beta_s^{1-\gamma}} \end{aligned}$$

Thus $a^n > w^n$ if $\frac{1-\beta_s}{1-(1-x)\beta_s} > \frac{1-\beta_s^{1-\gamma}}{1-(1-x)\beta_s^{1-\gamma}}$ or

$$\gamma > 0$$

Hence the combined effect of an increase in past employment on the average skill level is always higher than the impact on the real wage if there is some real wage rigidity. Turning towards $\frac{\partial a^n}{\partial \delta^s}$ and $\frac{\partial w^n}{\partial \delta^s}$, we have

This then gives

$$\begin{aligned}
\frac{\partial a^n}{\partial \delta^s} &= \frac{1-x}{u}\frac{x}{\left(1-(1-x)\beta_s\right)^2} > 0 \\
\frac{\partial w^n}{\partial \delta^s} &= \frac{1-x}{u}(1-\gamma)\frac{x\beta_s^{-\gamma}}{\left(1-(1-x)\beta_s^{1-\gamma}\right)^2} > 0
\end{aligned}$$

Thus the combined effect of past employment on the skill level of the average job seeker and the average real wage increases in δ^s Furthermore, for some real wage rigidity ($\gamma > 0$) and values of β_s not too much smaller than one, $\frac{\vartheta a^n}{\vartheta \delta^s} > \frac{\vartheta w^n}{\vartheta \delta^s}$.

Concerning the effect of a change in x, we have

$$\begin{aligned}
\frac{\partial a^n}{\partial x} &= -\frac{1}{u}\frac{1-\beta_s}{\left(1-(1-x)\beta_s\right)^2} < 0 \\
\frac{\partial w^n}{\partial x} &= -\frac{1}{u}\frac{1-\beta_s^{1-\gamma}}{\left(1-(1-x)\beta_s^{1-\gamma}\right)^2} < 0
\end{aligned}$$

We have $\frac{\partial a^n}{\partial x} < \frac{\partial w^n}{\partial x}$ if

$$2x\beta_s^{-\gamma} + \beta_s^{1-2\gamma} + 1 + \beta_s^{2-\gamma} > 2x + \beta_s^{2(1-\gamma)} + \beta_s + \beta_s^{-\gamma}$$

Comparing each of the terms on the left and right hand side, we see that for $\gamma > 0$, all terms on the left hand side are greater than corresponding term on the right hand side except for $\beta_s^{2-\gamma}$, which is smaller than $\beta_s^{-\gamma}$.The difference between the two will grow as β_s declines and thus at some point the inequality would be violated. However, it can be checked numerically that for the calibrations employed in this paper the condition is easily fulfilled.

13 Appendix V: Derivation of the marginal Cost Equation and the Output Equation in the model with Skill Loss

This appendix derives the Phillips Curve and the remaining linearised model equations. Linearising (12) yields

$$\widehat{mc}_t = -\left(1-Mg\right)\widehat{a}_t^P - \left(1-Mg\right)\widehat{a}_t^L + \frac{M}{A^L}W\widehat{w}_t + M\alpha g\widehat{x}_t \tag{42}$$

$$-\left(1-\delta\right)\beta E_t\left[\begin{array}{c} X\left(\widehat{c}_t - \widehat{c}_{t+1}\right) + \left(\frac{1-A^L}{A^L}\right)\widehat{mc}_{t+1} + \left[\left(\frac{1-A^L}{A^L}\right) - \frac{(1-\gamma)\Theta M}{A^L} + Mg\right]\widehat{a}_{t+1}^P \\ -\left(1-Mg\right)\widehat{a}_{t+1}^L + \frac{M}{A^L}W\widehat{w}_{t+1} + M\alpha g\widehat{x}_{t+1}\end{array}\right]$$

with $X = gM + \frac{1-A^L-M(\Theta'-W)}{A^L}$ and $g = Bx^\alpha$. From (8) and (9), we see that the average wage can be written up to first order as

$$\widehat{w}_t = \left(1-\gamma\right)\widehat{a}_t^P + \widehat{w}_t^L \tag{43}$$

where

$$\widehat{w}_t^L = \left(1-x\right)\left(\frac{1}{u}\left(1-\beta_s^{1-\gamma}\right)\widehat{n}_{t-1} + \beta_s^{1-\gamma}\widehat{w}_{t-1}^L\right) \tag{44}$$

Using (43) on (42) gives

$$\begin{aligned}\widehat{mc}_t &= -\left(1-Mg\right)\left(\widehat{a}_t^L - \beta\left(1-\delta\right)E_t\widehat{a}_{t+1}^L\right) + \frac{M}{A^L}W\left[\widehat{w}_t^L - \left(1-\delta\right)\beta E_t\widehat{w}_{t+1}^L\right] \qquad (45)\\ &\quad -\Phi'\widehat{a}_t^P - \beta\left(1-\delta\right)\left[\frac{1-\left(1-\gamma\right)\Theta' M}{A^L} - \Phi'\right]E_t\widehat{a}_{t+1}^P + M\alpha g\widehat{x}_t \\ &\quad -\beta\left(1-\delta\right)E_t\left[X\left(\widehat{c}_t - \widehat{c}_{t+1}\right) + \left(\frac{1-A^L}{A^L}\right)\widehat{mc}_{t+1} + M\alpha g\widehat{x}_{t+1}\right] \\ \Phi' &= 1 - gM - \left(1-\gamma\right)\frac{M}{A^L}W\end{aligned}$$

Linearising (15) yields

$$\widehat{c}_t = \frac{A^A}{A^A - A^L g\delta}\widehat{a}_t^A - \frac{Ag\delta}{A^A - A^L g\delta}\left(\widehat{a}_t^L + \widehat{a}_t^P\right) + \xi_0'\widehat{n}_t + \xi_1'\widehat{n}_{t-1} \tag{46}$$

with $\xi_0' = \frac{A^L(1-g(1+\alpha))}{A^A - Ag\delta}$ and $\xi_1' = \frac{(1-\delta)\left((1+\alpha(1-x))A^L g + \left(1-A^L\right)\right)}{A^A - A^L g\delta}$ Linearising (13) yields

$$\widehat{a}_t^A = \frac{A^L\delta}{A^A}\widehat{a}_t^L + \widehat{a}_t^P - \frac{\left(1-A^L\right)\left(1-\delta\right)}{A^A}\left(\widehat{n}_t - \widehat{n}_{t-1}\right) \tag{47}$$

Substituting this into (46) yields

$$\begin{aligned}\widehat{c}_t &= \widehat{a}_t^P + c^L\widehat{a}_t^L + \xi_0'\widehat{n}_t + \xi_1'\widehat{n}_{t-1} \qquad (48)\\ c^L &= \frac{A^L\delta\left(1-g\right)}{A^A - A^L g\delta}\end{aligned}$$

Substituting (48) into (45) yields

$$\begin{aligned}
\widehat{mc}_t &= a_1^L \widehat{a}_t^L + a_2^L E_t \widehat{a}_{t+1}^L + \frac{M}{A^L} W \left[\widehat{w}_t^L - \beta(1-\delta) E_t \widehat{w}_{t+1}^L \right] - p_0 \widehat{a}_t^P - p_1 E_t \widehat{a}_{t+1}^P + M\alpha g \widehat{x}_t \\
&\quad + \beta(1-\delta) \left[X \left(\xi_0^{'} - \xi_1^{'} \right) \widehat{n}_t + X \xi_0^{'} E_t \widehat{n}_{t+1} - \beta(1-\delta) \xi_1^{'} \widehat{n}_{t-1} - \left(\frac{1-A^L}{A^L} \right) \widehat{mc}_{t+1} - M\alpha g \widehat{x}_{t+1} \right] \\
a_1^L &= 1 - gM + \beta(1-\delta) \frac{A^L \delta (1-g)}{A^A - Ag\delta} X \\
a_2^L &= \beta(1-\delta) \left[1 - gM + \frac{A^L \delta (1-g)}{A^A - Ag\delta} X \right] \\
p_0 &= \Phi' + \beta(1-\delta) X \\
p_1 &= \beta(1-\delta) \frac{\gamma M (\Theta' - W)}{A^L}
\end{aligned}$$

Using $\widehat{x}_t = \frac{\widehat{n}_t - (1-\delta)(1-x)\widehat{n}_{t-1}}{\delta}$ then yields

$$\begin{aligned}
\widehat{mc}_t &= -a_1^L \widehat{a}_t^L + a_2^L E_t \widehat{a}_{t+1}^L + w_1^L \widehat{w}_t^L - w_2^L E_t \widehat{w}_{t+1}^L - p_0 \widehat{a}_t^P - p_1 E_t \widehat{a}_{t+1}^P \\
&\quad + h_0^{'} \widehat{n}_t + h_L^{'} \widehat{n}_{t-1} + h_F^{'} E_t \widehat{n}_{t+1} - h_c E_t \widehat{mc}_{t+1} \\
h_c &= \beta(1-\delta) \frac{\left(1 - A^L\right)}{A^L} \\
h_F^{\prime} &= -\beta(1-\delta) \left(\frac{\alpha g M}{\delta} - \xi_0^{'} X \right) \\
h_0^{'} &= \left(\frac{\alpha g M}{\delta} \right) \left(1 + \beta(1-\delta)^2 (1-x) \right) + \beta(1-\delta) \left(\xi_1^{'} - \xi_0^{'} \right) X \\
h_L^{\prime} &= -\left(\frac{\alpha g M}{\delta} \right) (1-\delta)(1-x) - \beta(1-\delta) \xi_1^{'} X
\end{aligned}$$

We now substitute (40) and (41), which, after rearranging, yields

$$\begin{aligned}
\widehat{mc}_t &= -\left(a_1^L - a_2^L (1-x) \beta_s \right) \widehat{a}_t^L + \left(w_1^L - w_2^L (1-x) \beta_s^{1-\gamma} \right) \widehat{w}_t^L \\
&\quad - \left[h_0^{'} + (1-x) \left(a_2^L \frac{(1-\beta_s)}{u} - w_2^L \frac{\left(1 - \beta_s^{1-\gamma}\right)}{u} \right) \right] \widehat{n}_t \\
&\quad + h_L^{'} \widehat{n}_{t-1} + h_F^{'} E_t \widehat{n}_{t+1} - h_c E_t \widehat{mc}_{t+1} - p_0 \widehat{a}_t^P - p_1 E_t \widehat{a}_{t+1}^P
\end{aligned}$$

Using $\widehat{n}_t = \frac{-\widehat{u}_t}{1-u}$ then yields

$$\begin{aligned}
\lambda\widehat{mc}_t &= -a^*\widehat{a}_t^L + w^*\widehat{w}_t^L - \kappa_0^*\widehat{u}_t + \kappa_L^*\widehat{u}_{t-1} + \kappa_F^* E_t\widehat{u}_{t+1} - \lambda h_c E_t\widehat{mc}_{t+1} - \lambda\left(p_0 + \rho_a p_1\right)\widehat{a}_t^P \\
a^* &= \lambda\left(a_1^L - a_2^L\left(1-x\right)\beta_s\right) \\
w^* &= \lambda\left(w_1^L - w_2^L\left(1-x\right)\beta_s^{1-\gamma}\right) \\
\kappa_0^* &= \lambda\left[h_0^{'} + (1-x)\left(a_2^L\frac{(1-\beta_s)}{u} - w_2^L\frac{\left(1-\beta_s^{1-\gamma}\right)}{u}\right)\right] \\
\kappa_L^* &= \frac{-\lambda h_L^{'}}{1-u},\ \kappa_F^* = \frac{-\lambda h_F^{'}}{1-u}
\end{aligned}$$

The equation for output including hiring costs is derived as follows. We have $Y_t = A_t^A N_t$. Linearising gives $\widehat{y}_t = \widehat{a}_t^A + \widehat{n}_t$ which, using (47) can be written as

$$\widehat{y}_t = \widehat{a}_t^P + \frac{1}{A^A}\left[A^L\delta\widehat{a}_t^L + A^L\widehat{n}_t + \left(1-A^L\right)\left(1-\delta\right)\widehat{n}_{t-1}\right]$$

Using $\widehat{n}_t = \frac{-\widehat{u}_t}{1-u}$ gives the equation used in the text.

14 Appendix VI Proof of Propositions on signs of $\frac{\partial\kappa}{\partial\delta_s}$ and $\frac{\partial^2\kappa}{\partial\delta_s\partial x}$

Marginal cost is given by

$$\lambda\widehat{mc}_t = -h_c E_t\lambda\widehat{mc}_{t+1} + \kappa_F^* E_t\widehat{u}_{t+1} - \kappa_0^*\widehat{u}_t + \kappa_{L1}^*\widehat{u}_{t-1} - a^*\widehat{a}_t^L + w^*\widehat{w}_t^L - \lambda p_0\widehat{a}_t^P - \lambda p_1 E_t\widehat{a}_{t+1}^P$$

If all variables stay constant over time and ignoring technology, we have

$$\begin{aligned}
\lambda\widehat{mc} &= -\frac{\left[\kappa_0^* - \kappa_F^* - \kappa_{L1}^* - a^*\frac{(1-\beta_s)(1-x)}{u(1-u)(1-(1-x)\beta_s)} + w^*\frac{\left(1-\beta_s^{1-\gamma}\right)(1-x)}{u(1-u)\left(1-(1-x)\beta_s^{1-\gamma}\right)}\right]}{1+h_c}\widehat{u} \\
&= -\frac{\begin{array}{c} h_0^{'} + h_L^{'} + h_F^{'} + \frac{(1-x)}{u} \\ \left[\begin{array}{c} a_2^L\left(1-\beta_s\right) - w_2^L\left(1-\beta_s^{1-\gamma}\right) - \left(a_1^L - a_2^L\left(1-x\right)\beta_s\right)\frac{(1-\beta_s)}{(1-(1-x)\beta_s)} \\ + \left(w_1^L - w_2^L\left(1-x\right)\beta_s^{1-\gamma}\right)\frac{\left(1-\beta_s^{1-\gamma}\right)}{\left(1-(1-x)\beta_s^{1-\gamma}\right)} \end{array}\right] \end{array}}{\left(1+h_c\right)\left(1-u\right)}\lambda\widehat{u} \\
&= -\frac{\begin{array}{c} h_0^{'} + h_L^{'} + h_F^{'} + \frac{(1-x)}{u} \\ \left[\left(1-\beta_s\right)\frac{a_2^L(1-(1-x)\beta_s) - a_1^L + a_2^L(1-x)\beta_s}{(1-(1-x)\beta_s)} + \left(1-\beta_s^{1-\gamma}\right)\frac{-w_2^L\left(1-(1-x)\beta_s^{1-\gamma}\right) + w_1^L - w_2^L(1-x)\beta_s^{1-\gamma}}{\left(1-(1-x)\beta_s^{1-\gamma}\right)}\right] \end{array}}{\left(1+h_c\right)\left(1-u\right)}\lambda\widehat{u} \\
&= -\frac{h_0^{'} + h_L^{'} + h_F^{'} + \frac{(1-x)}{u}\left[\frac{-(1-\beta_s)\left(a_1^L - a_2^L\right)}{(1-(1-x)\beta_s)} + \frac{\left(1-\beta_s^{1-\gamma}\right)\left(w_1^L - w_2^L\right)}{\left(1-(1-x)\beta_s^{1-\gamma}\right)}\right]}{\left(1+h_c\right)\left(1-u\right)}\lambda\widehat{u}
\end{aligned}$$

We can express $h_0^{'} + h_L^{'} + h_F^{'}$

$$h_0^{'} + h_L^{'} + h_F^{'} = \frac{\alpha g M}{\delta}\left[1 + \beta\left(1-\delta\right)^2\left(1-x\right) - \left(1-\delta\right)\left(1-x\right) - \beta\left(1-\delta\right)\right]$$

Using $\delta = \frac{ux}{(1-x)(1-u)}$ and $(1-\delta) = \frac{1-u-x}{(1-x)(1-u)}$, this can be rewritten as

$$\begin{aligned}
h_0^{'} + h_L^{'} + h_F^{'} &= \frac{\alpha g M\left(1-x\right)\left(1-u\right)}{ux}\left[1 + \beta\frac{\left(1-u-x\right)^2}{\left(1-x\right)\left(1-u\right)^2} - \frac{1-u-x}{\left(1-u\right)} - \beta\frac{1-u-x}{\left(1-x\right)\left(1-u\right)}\right] \\
&= \frac{\alpha g M}{ux}\left[\left(1-x\right)\left(1-u\right) + \beta\frac{\left(1-u-x\right)^2}{\left(1-u\right)} - \left(1-x\right)\left(1-u-x\right) - \beta\left(1-u-x\right)\right] \\
&= \frac{\alpha g M}{ux}\left[x\left(1-x\right) + \beta\left(1-u-x\right)\frac{\left(1-u-x\right) - \left(1-u\right)}{1-u}\right] \\
&= \frac{\alpha g M}{ux}\left[x\left(1-x\right) - \beta\left(1-u-x\right)\frac{x}{1-u}\right] \\
&= \frac{\alpha M B x^{\alpha}}{u\left(1-u\right)}\left[\left(1-u-x\right)\left(1-\beta\right) + ux\right] > 0
\end{aligned}$$

due to the restrictions on the parameters. Furthermore, we have $a_1^L - a_2^L = \left(1 - Bx^{\alpha}M\right)\left[1-\beta\left(1-\delta\right)\right]$ and $w_1^L - w_2^L = \frac{M}{A^L}W\left[1-\beta\left(1-\delta\right)\right]$. Hence we can now write:

$$\begin{aligned}
\lambda\widehat{mc} &= -\frac{\begin{array}{c}\frac{\alpha M B' x^{\alpha}}{(1-u)}\left[\left(1-u-x\right)\left(1-\beta\right) + ux\right] \\ +\left(1-x\right)\left[1-\beta\left(1-\delta\right)\right]\left[\frac{-(1-\beta_s)(1-Bx^{\alpha}M)}{(1-(1-x)\beta_s)} + \frac{\left(1-\beta_s^{1-\gamma}\right)\frac{WM}{A^L}}{\left(1-(1-x)\beta_s^{1-\gamma}\right)}\right]\end{array}}{u\left(1+h_c\right)\left(1-u\right)}\lambda\widehat{u} \\
&= -\kappa\widehat{u} \\
\kappa &= \frac{\left[\begin{array}{c}\frac{\alpha M B x^{\alpha}}{(1-u)}\left[\left(1-u-x\right)\left(1-\beta\right) + ux\right] + \\ \left(1-x\right)\left[1-\beta\left(1-\delta\right)\right]\left[\frac{-(1-\beta_s)(1-B'x^{\alpha}M)}{(1-(1-x)\beta_s)} + \frac{\left(1-\beta_s^{1-\gamma}\right)WM}{A^L\left(1-(1-x)\beta_s^{1-\gamma}\right)}\right]\end{array}\right]}{u\left(1+h_c\right)\left(1-u\right)}\lambda
\end{aligned}$$

We will now show that $\frac{\partial \kappa}{\partial \beta_s} > 0$ and thus $\frac{\vartheta \kappa}{\vartheta \delta^s} < 0$ if β_s is not too far away from 1. A more general proof seems impossible. We have

$$\frac{\partial \kappa}{\partial \beta_s} = \frac{-\frac{\partial h_c}{\partial \beta_s}\kappa\left(1+h_c\right)}{\left(1+h_c\right)^2}$$

$$+\frac{\lambda\left(1-x\right)}{u\left(1-u\right)}\left[\frac{\left[1-\beta\left(1-\delta\right)\right]}{\left(1+h_c\right)}\left[\begin{array}{c}\frac{(1-B'x^\alpha M)[(1-(1-x)\beta_s)-(1-\beta_s)(1-x)]}{(1-(1-x)\beta_s)^2}+M\\ \frac{\left[-\beta_s^{-\gamma}\left(1-\gamma\right)W+\left(1-\beta_s^{1-\gamma}\right)\frac{\vartheta W}{\vartheta \beta_s}\right]A^L\left(1-\left(1-x\right)\beta_s^{1-\gamma}\right)-\left(1-\beta_s^{1-\gamma}\right)W\left[\begin{array}{c}\frac{\vartheta A^L}{\vartheta \beta_s}\left(1-\left(1-x\right)\beta_s^{1-\gamma}\right)\\ -A^L\left(1-x\right)\left(1-\gamma\right)\beta_s^{-\gamma}\end{array}\right]}{\left(A^L\left(1-(1-x)\beta_s^{1-\gamma}\right)\right)^2}\end{array}\right]\right]$$

It is easily shown that $\frac{\partial h_c}{\partial \beta_s} = -\beta\left(1-\delta\right)\frac{\partial A^L}{\partial \beta_s}\frac{1}{\left(A^L\right)^2} < 0$. For $\kappa > 0$, this implies that $\frac{-\frac{\partial h_c}{\partial \beta_s}\kappa(1+h_c)}{(1+h_c)^2} > 0$. Furthermore, since the range of values of β_s are those for which κ is positive, or "just" negative, we can safely write $\frac{\vartheta \kappa}{\vartheta \beta_s} > 0$ if

$$\frac{\left(1-B'x^\alpha M\right)x}{\left(1-\left(1-x\right)\beta_s\right)^2}+MA^LW\frac{\begin{array}{c}\left[-\beta_s^{-\gamma}\left(1-\gamma\right)+\left(1-\beta_s^{1-\gamma}\right)\frac{\vartheta W}{\vartheta \beta_s}\frac{1}{W}\right]\left(1-\left(1-x\right)\beta_s^{1-\gamma}\right)\\ -\left(1-\beta_s^{1-\gamma}\right)W\left[\frac{\vartheta A^L}{\vartheta \beta_s}\frac{1}{A_L}\left(1-\left(1-x\right)\beta_s^{1-\gamma}\right)-\left(1-x\right)\left(1-\gamma\right)\beta_s^{-\gamma}\right]\end{array}}{\left(A^L\left(1-\left(1-x\right)\beta_s^{1-\gamma}\right)\right)^2} > 0$$

Further simplifying this yields

$$\frac{\left(1-Bx^\alpha M\right)x}{\left(1-\left(1-x\right)\beta_s\right)^2}+\frac{MW\left[-x\left(1-\gamma\right)\beta_s^{-\gamma}+\left(1-\left(1-x\right)\beta_s^{1-\gamma}\right)\left(1-\beta_s^{1-\gamma}\right)\left(\frac{\vartheta W}{\vartheta \beta_s}\frac{1}{W}-\frac{\vartheta A^L}{\vartheta \beta_s}\frac{1}{A_L}\right)\right]}{A^L\left(1-\left(1-x\right)\beta_s^{1-\gamma}\right)^2} > 0$$

Using $W = \Theta' W^L$,

We now set $\beta_s = 1$. This gives $W = \Theta' = \frac{1}{M} - g\left[1-\beta\left(1-\delta\right)\right]$ and, $\left(1-\beta_s^{1-\gamma}\right) = 0$ and $\left(1-\left(1-x\right)\beta_s^{1-\gamma}\right) = x$, means that our inequality becomes

$$\frac{\left(1-B'x^\alpha M\right)-\left[\frac{1}{M}-B'x^\alpha\left[1-\beta\left(1-\delta\right)\right]\right]M\left(1-\gamma\right)}{x} > 0$$

Or

$$\gamma > \frac{B'x^\alpha M\beta\left(1-\delta\right)}{1-B'x^\alpha M\left(1-\beta\left(1-\delta\right)\right)}$$

This is easily fulfilled under the calibrations considered in this paper.

For the case of $\beta_s = 1$, we now show that $\frac{\partial^2 \kappa}{\partial \beta_s \partial x} < 0$ if α is close to one (as we assume in the paper) and the other parameters have a calibration of "reasonable" magnitude. This means that the effect of the skill level on κ is weakened if the labour market becomes more fluid. For $\beta_s = 1$ and $\alpha = 1$ we have (noting that $\frac{\partial h_c}{\partial \beta_s} = -\frac{\beta(1-u-x)}{(1-u)x}$

$$\begin{aligned}
\frac{\partial \kappa}{\partial \beta_s} &= \kappa \frac{\beta(1-u-x)}{(1-u)x} + \frac{\lambda}{u(1-u)^2}\left[\frac{[(1-x-u)(1-\beta)+ux]}{\left[\frac{(1-B'x^{\alpha}M)}{x} - (1-\gamma)\left[\frac{1}{x} - MB\left[\frac{(1-\beta)(1-u)+x(u+\beta-1)}{(1-x)(1-u)}\right]\right]\right]}\right] \\
&= A_1 + A_2 \quad where \\
A_1 &= \kappa \frac{\beta(1-u-x)}{(1-u)x} = \frac{\lambda M B'\beta}{(1-u)^3 u}\left[(1-\beta)\left(1-2u-2x+u^2+2ux+x^2\right) + ux - ux^2 - u^2x\right] \\
A_2 &= \frac{\lambda}{u(1-u)^2}\left[\frac{[(1-x-u)(1-\beta)+ux]}{\left[\frac{(1-B'xM)}{x} - (1-\gamma)\left[\frac{1}{x} - MB'\left[\frac{(1-\beta)(1-u)+x(u+\beta-1)}{(1-x)(1-u)}\right]\right]\right]}\right]
\end{aligned}$$

Differentiating this with respect to x gives

$$\frac{\partial^2 \kappa}{\partial \beta_s \partial x} = \frac{\partial A_1}{\partial x} + \frac{\partial A_2}{\partial x}$$

$$\frac{\partial A_1}{\partial x} = \frac{\lambda M B'\beta}{(1-u)^3 u}\left[-2(1-\beta)(1-u-x) + u(1-2x-u)\right]$$

$$\begin{aligned}
\frac{\partial A_2}{\partial x} &= \frac{\lambda}{u(1-u)^2}\left[\frac{\begin{array}{c}(\beta+u-1)\left[\frac{(1-B'xM)}{x} - (1-\gamma)\left[\frac{1}{x} - MB'\left[\frac{(1-\beta)(1-u)+x(u+\beta-1)}{(1-x)(1-u)}\right]\right]\right] \\ +\left[(1-x-u)(1-\beta)+ux\right]\end{array}}{\left[\frac{-B'Mx-(1-B'Mx)}{x^2} - (1-\gamma)\left[\frac{-1}{x^2} - \frac{MB'}{(1-u)}\left[\frac{(u+\beta-1)(1-x)+(1-\beta)(1-u)+x(u+\beta-1)}{(1-x)^2}\right]\right]\right]}\right] \\
&= \frac{\lambda}{u(1-u)^2}\left[\begin{array}{c}(\beta+u-1)\left[\frac{(1-B'xM)}{x} - (1-\gamma)\left[\frac{1}{x} - MB'\left[\frac{(1-\beta)(1-u)+x(u+\beta-1)}{(1-x)(1-u)}\right]\right]\right] \\ +\left[(1-x-u)(1-\beta)+ux\right]\left[\frac{-1}{x^2} - (1-\gamma)\left[\frac{-1}{x^2} - \frac{MB'u\beta}{(1-u)(1-x)^2}\right]\right]\end{array}\right]
\end{aligned}$$

Note that

$$\begin{aligned}
&\frac{(1-B'xM)}{x} - (1-\gamma)\left[\frac{1}{x} - MB'\left[\frac{(1-\beta)(1-u)+x(u+\beta-1)}{(1-x)(1-u)}\right]\right] \\
&= \frac{(\gamma - B'xM)}{x} + (1-\gamma)MB'\left[\frac{(1-\beta)(1-u)+x(u+\beta-1)}{(1-x)(1-u)}\right] \\
&= \frac{(\gamma - B'xM)(1-x)(1-u) + x(1-\gamma)MB'\left[(1-\beta)(1-u)+x(u+\beta-1)\right]}{x(1-x)(1-u)}
\end{aligned}$$

and that

$$\begin{aligned}
&\frac{-1}{x^2} - (1-\gamma)\left[\frac{-1}{x^2} - \frac{MB'u\beta}{(1-u)(1-x)^2}\right] \\
&= -\frac{\gamma}{x^2} + \frac{(1-\gamma)MBu\beta}{(1-u)(1-x)^2} \\
&= \frac{-\gamma(1-u)(1-x)^2 + x^2(1-\gamma)MBu\beta}{x^2(1-u)(1-x)^2}
\end{aligned}$$

Thus

$$\frac{\partial A_2}{\partial x}=\frac{\lambda}{u\left(1-u\right)^2}\left[\begin{array}{c}\frac{(\beta+u-1)}{x(1-x)(1-u)}\left[\begin{array}{c}\left(\gamma-BxM\right)\left(1-x\right)\left(1-u\right)\\+x\left(1-\gamma\right)MB'\left[\left(1-\beta\right)\left(1-u\right)+x\left(u+\beta-1\right)\right]\end{array}\right]\\+\left[\frac{(1-x-u)(1-\beta)+ux}{x^2(1-u)(1-x)^2}\left(-\gamma\left(1-u\right)\left(1-x\right)^2+x^2\left(1-\gamma\right)MB'u\beta\right)\right]\end{array}\right]$$

We can then write

$$\begin{aligned}\frac{\partial^2\kappa}{\partial\beta_s\partial x} &= \frac{\partial A_1}{\partial x}+\frac{\partial A_2}{\partial x}\\ &= \frac{\lambda}{u\left(1-u\right)^3}\left[\begin{array}{c}MB'\beta\left[-2\left(1-\beta\right)\left(1-u-x\right)+u\left(1-2x-u\right)\right]\\+\frac{(\beta+u-1)}{x(1-x)}\\ \left[\begin{array}{c}\left(\gamma-B'xM\right)\left(1-x\right)\left(1-u\right)\\+x\left(1-\gamma\right)MB'\left[\left(1-\beta\right)\left(1-u\right)+x\left(u+\beta-1\right)\right]\end{array}\right]\\+\left[\frac{(1-x-u)(1-\beta)+ux}{x^2(1-x)^2}\left(-\gamma\left(1-u\right)\left(1-x\right)^2+x^2\left(1-\gamma\right)MB'u\beta\right)\right]\end{array}\right]\end{aligned}$$

As can be easily checked, setting $\beta=1$ makes $\frac{\partial^2\kappa}{\partial\beta_s\partial x}$ more positive. Thus if $\frac{\partial^2\kappa}{\partial\beta_s\partial x}<0$ for $\beta=1$, then $\frac{\partial^2\kappa}{\partial\beta_s\partial x}<0$ for $\beta<1$ as well. Hence $\frac{\partial^2\kappa}{\partial\beta_s\partial x}<0$ if

$$MBu\left(1-2x-u\right)+\frac{u\left[\left(\gamma-B'xM\right)\left(1-x\right)\left(1-u\right)+x^2\left(1-\gamma\right)MB'u\right]}{x\left(1-x\right)}$$
$$+\frac{u\left(-\gamma\left(1-u\right)\left(1-x\right)^2+x^2\left(1-\gamma\right)MB'u\right)}{x\left(1-x\right)^2}<0$$

$$\begin{aligned}MB'\left(1-2x-u\right)x\left(1-x\right)^2+\left[\left(\gamma-B'xM\right)\left(1-x\right)\left(1-u\right)+x^2\left(1-\gamma\right)MB'u\right]\left(1-x\right)&\\ -\gamma\left(1-u\right)\left(1-x\right)^2+x^2\left(1-\gamma\right)MB'u &< 0\\ MB'\left(1-2x-u\right)x\left(1-x\right)^2+\left(\gamma-B'xM\right)\left(1-x\right)^2\left(1-u\right)+x^2\left(1-\gamma\right)MB'u\left(1-x\right)&\\ -\gamma\left(1-u\right)\left(1-x\right)^2+x^2\left(1-\gamma\right)MB'u &< 0\\ MB'\left(1-2x-u\right)x\left(1-x\right)^2-MBx\left(1-x\right)^2\left(1-u\right)&\\ +x^2\left(1-\gamma\right)MB'u\left(1-x\right)+x^2\left(1-\gamma\right)MB'u &< 0\\ \left(1-2x-u\right)\left(1-x\right)^2-\left(1-x\right)^2\left(1-u\right)+x\left(1-\gamma\right)u\left(1-x\right)+x\left(1-\gamma\right)u &< 0\\ -2x\left(1-x\right)^2+x\left(1-\gamma\right)u\left(1-x\right)+x\left(1-\gamma\right)u &< 0\\ -2\left(1-x\right)^2+\left(1-\gamma\right)u\left(1-x\right)+\left(1-\gamma\right)u &< 0\\ -2\left(x^2-2x+1\right)+\left(1-\gamma\right)u\left(1-x\right)+u\left(1-\gamma\right) &< 0\\ -2x^2+4x-2+\left(1-\gamma\right)u-x\left(1-\gamma\right)u+u\left(1-\gamma\right) &< 0\\ -2x^2+x\left(4-\left(1-\gamma\right)u\right)+2u\left(1-\gamma\right)-2 &< 0\\ x^2-\frac{x\left(4-\left(1-\gamma\right)u\right)}{2}-u\left(1-\gamma\right)+1 &> 0\end{aligned}$$

The polynomial on the left hands side has two solutions and the inequality will hold for values of x to the left of the smaller solution or to the right of the larger one. We have

$$x_{1,2} = \frac{4 - (1-\gamma)\, u \pm \sqrt{(1-\gamma)^2 u^2 + 8u\,(1-\gamma)}}{4}$$

Since the root will be larger than $(1-\gamma)\, u$, we have $x_1 > 1$, which is outside the permissible range for x. For x_2, we have

$$x_2 = \frac{4 - (1-\gamma)\, u - \sqrt{(1-\gamma)^2 u^2 + 8u\,(1-\gamma)}}{4}$$

Clearly x_2 increases in γ. Thus the larger γ, the larger is the maximum value of x consistent with $\frac{\partial^2 \kappa}{\partial \beta_s \partial x} < 0$. Setting $\gamma = 0$, we have

$$x_2 = \frac{4 - u - \sqrt{u^2 + 8u}}{4}$$

Thus for $x < x_2 = \frac{4-u-\sqrt{u^2+8u}}{4}$, which is easily fulfilled for the range of parameters we consider in this paper, we have $\frac{\partial^2 \kappa}{\partial \beta_s \partial x} < 0$ and thus $\frac{\partial^2 \kappa}{\partial \delta_s \partial x} > 0$.

15 Appendix VII

We first use the interest feedback rule to substitute $\widehat{i}_t$ out of the Euler equation (not the policy rule employed here is $\widehat{i}_t = \phi_\pi \pi_t + \phi_u \widehat{u}_t$). We can then write the system in the form

$$\Gamma_0 y_t = \Gamma_1 y_{t-1} + \left[\begin{array}{cc} \Psi & \Pi \end{array} \right] \left[\begin{array}{c} \varepsilon_t \\ v_t \end{array} \right] + \Pi \widetilde{\eta}_t$$

with $y_t = \left[\begin{array}{c} x_t^\pi \\ x_t^u \\ x_t^{mc} \\ x_t^n \\ x_t^c \\ \widehat{a}_t^P \\ \pi_t \\ \widehat{u}_t \\ \widehat{mc}_t' \\ \widehat{u}_t^n \\ \widehat{c}_t \\ \widehat{a}_t^L \\ \widehat{w}_t^L \\ \widehat{i}_t \end{array} \right]$, $\Gamma_0 = [\Gamma_0^1 \; \Gamma_0^2]$

$$\Gamma_0^1 = \begin{pmatrix} \beta & 0 & 0 & 0 & 0 & 0 \\ 0 & -\kappa_F^* & h_c & 0 & 0 & \lambda\left(p_0+\rho_a p_1\right) \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & \kappa_F^* & 0 & -\lambda\left(p_0+\rho_a p_1\right) \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\Gamma_0^2 = \begin{pmatrix} -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \kappa_0^* & 1 & 0 & 0 & a^* & -w^* & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & c_0^* & 0 & 0 & 1 & -c_L & 0 & 0 \\ (1-\rho)\,\phi_\pi & (1-\rho)\left(\phi_u-\phi_y y_0\right) & 0 & (1-\rho)\,\phi_y y_0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & -\kappa_0^* & 0 & -a^* & w^* & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

$$\Gamma_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \kappa_{L1}^{*} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{-(1-x)(1-\beta_s)}{u(1-u)} & 0 & 0 & 0 & (1-x)\,\beta_s & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{-(1-x)\left(1-\beta_s^{1-\gamma}\right)}{u(1-u)} & 0 & 0 & 0 & 0 & (1-x)\,\beta_s^{1-\gamma} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -c_1^{*} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\rho \\ 0 & 0 & 0 & 0 & 0 & \rho_a & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\kappa_{L1}^{*} & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\Psi = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \Pi = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

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