Long Run Risks in the Term Structure of Interest Rates: Estimation

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Abstract

Using Bayesian methods, this paper estimates a model in which persistent fluctuations in expected consumption growth, expected inflation, and their time-varying volatility determine asset price variation. The model features Espstein-Zin recursive preferences that determine the market price of these macro risk factors as in the existing literature on long-run risks. The analysis of the U.S. nominal term structure data from 1953 to 2006 shows that agents dislike high uncertainty and demand compensation for volatility risks, and that the time variation of the term premium is driven by the compensation for fluctuating inflation volatility. The central role of inflation volatility is consistent with empirical evidence from statistical models and survey data but in contrast to the existing long-run risks literature which emphasizes consumption volatility based on model calibration. The finding suggests that once estimated by full-information methods, a long-run risks model allows only a limited role for consumption volatility risk in fitting the time series of the U.S. nominal term structure data.

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1 Introduction

Understanding sources of risks implied in nominal bond yields is an important issue in asset pricing. In particular, recent empirical research points out that term premia of long-term bonds are positive on average, time varying (e.g., Campbell and Shiller (1991)) and highly related with macro factors (e.g., Ludvigson and Ng (2009), Barillas (2010) and Joslin et. al. (2009)). To rationalize positive term premia of nominal bonds in an equilibrium asset pricing framework requires that the real payoffs of nominal bonds should vary negatively with investors' marginal utility.

In the context of consumption-based asset pricing models, this requirement is often satisfied by a negative covariance between consumption growth and inflation. The negative covariance implies that nominal bonds pay less in real terms when consumption growth is low and investors' marginal utility is high.[1] And the negative covariance should move in a counter-cyclical way to generate counter-cyclical term premia. While counter-cyclical risk-aversion through habit formation (Wachter (2006)), learning of long run expected consumption growth and inflation (Piazzesi and Schneider (2006)), or time-varying volatility of consumption growth (Bansal and Shaliastovich (2010)) can create this property, time-varying volatility of expected inflation can be another potential source of time-varying term premia.

This paper specifies and estimates an equilibrium term structure model in which time-varying inflation volatility as well as persistent fluctuations in expected consumption growth, expected inflation, and time-varying consumption volatility drive asset price variation. Drawing on the long-run risks model developed by Bansal and Yaron (2004) and Bansal and Shaliastovich (2010), I combine persistent fluctuations in consumption growth and inflation with Epstein-Zin (1989) preferences. This combination allows compensation for long-run risks in expected consumption growth, expected inflation, their volatilities, and also the short-run unexpected fluctuations in consumption growth and inflation. Using a Bayesian approach, I estimate the model with U.S. nominal term structure data from 1953:Q1 to 2006:Q4. From the

[1]For example, Bansal and Shaliastovich (2010) , Piazzesi and Schneider (2006), and Wachter (2006) explicitly introduce this feature.

estimation, I recover the expectations and volatility of consumption growth and inflation implied by nominal bonds data. Two main findings emerge out of the empirical analysis.

First, posterior distributions of parameters indicate that i) expected consumption growth and expected inflation are highly persistent but the persistence of expected consumption growth is not well identified by solely consumption growth and inflation data and ii) agents dislike high volatility and prefer the early resolution of uncertainty. Second, inflation volatility is a predominant risk factor in explaining the time variation of the term premia. And estimates of inflation risk factors are in line with survey data evidence.

The previous empirical studies on long-run consumption risks highlight the difficulty in identifying the persistent component of consumption growth solely based on consumption data.[2] The inclusion of asset price data in the estimation alleviates this problem and provides a tight posterior interval for the persistence parameter in spite of a wide prior interval. The estimated risk aversion and the intertemporal elasticity of substitution (IES) are both higher than one, implying that agents are averse to volatility risks.[3]

In contrast to Bansal and Shaliastovich (2010), consumption volatility risk plays only a limited role in explaining the time-variation of term premia. However, their model allows time-varying volatility for only consumption growth and time-varying inflation uncertainty is explained by the exposure of inflation to consumption volatility risk. This paper uses a more flexible set-up that incorporates inflation volatility risk as well as consumption volatility risk. In addition, Bansal and Shaliastovich (2010) calibrate their model and do not provide estimates of consumption volatility risk that can be checked with survey data evidence. When estimates of volatility are compared with survey data, I find a significant correlation between the estimates of inflation volatility and inflation forecast uncertainty from survey data but only

[2] See Ma (2007) for this point.

[3] Estimates of preference parameters reported in the empirical analysis of fully specified general equilibrium models with term structure data (e.g., van Binsberg et al. (2010)) also imply investors' aversion to high uncertainty.

a weak correlation for consumption volatility. This finding suggests that in fitting the time-series of the U.S. nominal term structure data, consumption volatility risk often emphasized in the long-run risks literature is not so important.

The central role of inflation volatility in determining the time variation of term premia is consistent with empirical evidence from statistical models. Using the regression analysis of an international panel dataset, Wright (2011) argues that inflation uncertainty measured by survey data explains a substantial part of the time variation of term premia in nominal government bonds.[4] While Wright (2011) does not impose any equilibrium restrictions on the relation between inflation uncertainty and term premia in the analysis, I reach a similar conclusion by estimating an equilibrium term structure model. Since inflation volatility is heavily dependent on the way that monetary policy responds to inflationary pressures, this finding can be regarded as preliminary evidence for the connection between term premia and monetary policy.[5]

I proceed as follows: Section 2 describes the model economy and derives equilibrium bond yields. Section 3 explains the econometric methodology. Section 4 provides estimation results based on the empirical analysis of U.S. data. Section 5 contains concluding remarks. The appendix explains the construction of empirical measures of consumption and inflation uncertainty based on survey data.[6].

[4]Barillas (2010), Joslin et. al. (2009), and Ludvigson and Ng (2009) suggest that real factors rather than inflation explain variations in term premia. Because their models do not explicitly consider time varying volatility of macro variables, their results are not in conflict with the empirical evidence in Wright (2011) .

[5]Gallmeyer et al. (2008) endogeneize the inflation process given a monetary policy rule and show that term premium dynamics can be highly sensitive to monetary policy. They argue that a more aggressive policy response to inflation reduces both inflation volatility and long term nominal bond yields.

[6]Details of the model solution and econometric methodology are given in a separate web appendix available on www.taeyoung-doh.net

2 Model

2.1 Preference and Shocks

I consider a discrete-time endowment economy. As in Bansal and Yaron (2004), investors have Epstein-Zin (1989) recursive preferences.

$$U_t = [(1-\delta)C_t^{\frac{1-\gamma}{\theta}} + \delta(E_t U_{t+1}^{1-\gamma})^{\frac{1}{\theta}}]^{\frac{\theta}{1-\gamma}}. \tag{1}$$

The time discount factor (δ), the risk-aversion ($\gamma \geq 0$), and the intertemporal elasticity of substitution (IES : $\psi \geq 0$) characterize preferences. Here, θ is equal to $\frac{1-\gamma}{1-\frac{1}{\psi}}$. The standard expected utility is a special case of the above recursive preferences when γ is equal to $\frac{1}{\psi}$.

Epstein-Zin (1989) shows that the logarithm of the real stochastic discount factor has the following form:

$$m_{r,t+1} = \theta \log \delta - \frac{\theta}{\psi} g_{c,t+1} + (\theta - 1) r_{c,t+1}. \tag{2}$$

Here, $g_{c,t+1}$ is the log growth rate of aggregate consumption and $r_{c,t+1}$ is the log of the return on an asset that pays aggregate consumption as its dividends.

Then we can construct the log of the nominal discount factor in this economy by subtracting the logged inflation rate from the log of the real discount factor

$$m_{t+1} = m_{r,t+1} - \pi_{t+1}. \tag{3}$$

π_{t+1} is the logged inflation rate at $t+1$. While $r_{c,t+1}$ is not directly observable, we can approximate it as a function of state variables that drive the dynamics of $g_{c,t+1}$ and π_{t+1} by using the following no-arbitrage restriction,[7]

$$E_t(e^{m_{t+1}+\pi_{t+1}+r_{c,t+1}}) = 1. \tag{4}$$

[7] Following Bansal and Yaron (2004), I conjecture that the log price consumption ratio of an asset which pays per-period consumption as its' dividend is affine with respect to long-run risks in order to approximate the return on consumption claims. This strategy results in constant market prices of risks. In contrast, Le and Singleton (2010) propose that the price consumption ratio can be expressed as a quadratic function of state variables governing the evolution of consumption and inflation. While they incorporate time-varying market prices of risks in this way, they do not assign specific economic meanings to the state variables.

Exogenous processes for consumption growth and inflation contain predictable components which correspond to expected consumption growth and expected inflation. As in Piazzesi and Schneider (2006), I assume that expected consumption growth and expected inflation are both dynamically and contemporaneously correlated. Furthermore, I allow time-varying volatilities for both consumption growth and inflation, and assume that there is regime-dependent heteroskedasticity in innovations of volatility processes. The following equations describe stochastic processes for the evolution of consumption growth and inflation.

$$\begin{aligned}
\begin{pmatrix} g_{c,t+1} \\ \pi_{t+1} \end{pmatrix} &= \mu + X_t + \Sigma_t \eta_{t+1} \ , \ \Sigma_t = \begin{pmatrix} \sigma_{1,t} & 0 \\ 0 & \sigma_{2,t} \end{pmatrix} , X_t = \begin{pmatrix} x_{1,t} \\ x_{2,t} \end{pmatrix} . & (5) \\
X_{t+1} &= \rho X_t + \Phi \Sigma_t e_{t+1} \ , \ \Phi = \begin{pmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{pmatrix} , \rho = \begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix} . & (6) \\
\sigma^2_{i,t+1} &= (1-\nu_i)\sigma^2_i + \nu_i \sigma^2_{i,t} + \sigma_{i,w}(S_{t+1}) w_{i,t+1} \ , \ (i=1,2). & (7) \\
\begin{pmatrix} \eta_{t+1} \\ e_{t+1} \\ w_{t+1} \end{pmatrix} &\sim iid\mathcal{N}\left(0, I \right) , S_t = \begin{cases} 1, & \text{with probability } \alpha \\ 2, & \text{with probability 1-}\alpha \end{cases}
\end{aligned}$$

where μ is a vector consisting of the unconditional mean of consumption growth and inflation, ρ and Φ govern the persistence and the volatility of long run risk components X_t. S_{t+1} is an indicator for volatility regimes. ν_i and $\sigma_{i,w}(S_{t+1})$ control the persistence and conditional volatility of shocks to consumption growth volatility and inflation volatility.[8] For analytical tractability, I assume that all the innovations are independent from each other. In this model, different regimes can distinguish periods of volatility spikes from more tranquil periods.

A large body of empirical research has provided evidence of substantial changes in the volatility of US macroeconomic variables over the postwar period, although

[8] Since the stochastic volatility terms are assumed to be normally distributed, there is a possibility to hit the zero bound, although chances are very small (less than 5 %) for the range of parameters considered in the empirical analysis. However, simulated moments of observed variables are virtually the same even if we do not truncate stochastic volatilities at zero. Also, while not all the parameters in Φ are exactly identified because only $\Phi\Sigma_t\Sigma_t'\Phi'$ shows up in the likelihood function, data can still provide information on the most probable area of these parameters.

there are still debates on the sources of these changes.[9] Changing macroeconomic volatility has direct implications for macro risks priced in financial assets. In the model, not just realized consumption growth and inflation, but also expected consumption growth and expected inflation exhibit time varying volatility. Therefore, the model should be consistent with the available evidence for the volatility of expected macro variables. Forecast uncertainty from the survey data provides an empirical proxy for the time-varying volatility of expected macro variables and can be used to test the relevance of the model.

By restricting some parameters in the above specification, we can obtain simpler models which are close to Bansal and Shaliastovich (2010) and Piazzesi and Schneider (2006). For example, if we assume that inflation does not affect real variables and time-varying volatility exists only for consumption growth but not for inflation, the specification is close to Bansal and Shaliastovich (2010). On the other hand, if we assume that volatility of both consumption growth and inflation is constant but allow the real impacts of inflation, the model is close to Piazzesi and Schneider (2006).

2.2 Equilibrium Bond Yields

We can derive equilibrium bond yields based on the stochastic discount factor implied by the model. In the model, the exact form of the return on consumption claims is not known. As in Bansal and Yaron (2004), I draw on the standard log-linearization of returns using the log price consumption ratio (z_t) to get an ap-

[9]Stock and Watson (2002) provide a survey of the literature. The role of monetary policy in volatility changes is controversial. While Sims and Zha (2006) and Justiano and Primiceri (2008) argue that policy shifts were not main factors of changes in the volatility of US macro variables, Boivin and Giannoni (2006) stress the role of monetary policy shifts.

proximate form for the return on consumption claims.[10]

$$r_{c,t+1} = \kappa_0 + \kappa_1 z_{t+1} - z_t + g_{c,t+1} \ , \ z_t = A_0 + A_1 X_t + A_{2,1}\sigma_{1,t}^2 + A_{2,2}\sigma_{2,t}^2 \ , \ A_1 = [A_{11}, A_{12}]. \tag{8}$$

Expected inflation ($x_{2,t}$) affects the real economy because it predicts the future expected consumption growth ($x_{1,t+1}$) in the case of $\rho_{12} \neq 0$. For the same reason, the price consumption ratio is also affected by expected inflation. As mentioned in Bansal, Kiku, and Yaron (2007), κ_0 and κ_1 are constants determined by the mean log price consumption ratio $\overline{z}$, and given by,

$$\kappa_1 = \frac{exp(\overline{z})}{1 + exp(\overline{z})} \ , \ \kappa_0 = \ln(1 + exp(\overline{z})) - \kappa_1 \overline{z}. \tag{9}$$

We first plug equation (8) into equation (4) and obtain three restrictions for A_0, A_1, and $A_{2,i}\,(i = 1, 2)$. This gives A_0 and A_1 as functions of $\overline{z}$ and parameters determining preferences and shock processes. Then $\overline{z}$ can be found numerically by solving the fixed point problem $\overline{z} = A_0(\overline{z}) + A_{2,1}(\overline{z})\sigma_1^2 + A_{2,2}(\overline{z})\sigma_2^2$. This nonlinear equation can be transformed into an equation with respect to κ_1. Since κ_1 stays in the open interval $(0, 1)$, we can check the uniqueness of the solution by checking its existence in fine grids over the unit interval. Once the unique solution is found, we can derive the following expressions for A_1, and $A_{2,i}$,[11]

$$\begin{aligned} A_{11} &= \frac{(1 - \kappa_1\rho_{22})(1 - \frac{1}{\psi})}{\kappa_1^2(\rho_{11}\rho_{22} - \rho_{12}\rho_{21}) - (\rho_{11} + \rho_{22})\kappa_1 + 1} \ , \ A_{12} = \frac{\kappa_1\rho_{12}(1 - \frac{1}{\psi})}{\kappa_1^2(\rho_{11}\rho_{22} - \rho_{12}\rho_{21}) - (\rho_{11} + \rho_{22})\kappa_1 + 1} \\ A_{21} &= \frac{\theta^2((1 - \frac{1}{\psi})^2 + (\kappa_1^2[A_{11}\phi_{11} + A_{12}\phi_{21}]^2))}{2\theta(1 - \kappa_1\nu_1)} \ , \ A_{22} = \frac{\theta^2\kappa_1^2[A_{11}\phi_{12} + A_{12}\phi_{22}]^2}{2\theta(1 - \kappa_1\nu_2)}. \end{aligned} \tag{10}$$

It follows that a positive shock to expected consumption growth increases the price consumption ratio only if the IES is greater than 1. Moreover, a positive

[10]The accuracy of this approximation turns out to be reasonably good as discussed in Bansal, Kiku, and Yaron (2007) and Beeler and Campbell (2008) once the mean price consumption ratio is found in a model-consistent way. I compare the first and second moments of the log price consumption ratio from the log-linearization with the counterparts obtained from a numerical method to check the accuracy of the approximation. The web technical appendix shows that the two methods lead to fairly similar moments.

[11]The details of the derivation can be found in the web technical appendix.

shock to expected inflation decreases the price consumption ratio if a high expected inflation predicts a low expected consumption growth (i.e. $\rho_{12} < 0$) and the IES is greater than 1. For volatility risk, high volatility decreases the price consumption ratio only if θ is negative. When the IES is greater than 1, θ is negative only if γ is greater than 1. This configuration of parameters implies that agents prefer the early resolution of uncertainty because γ is bigger than $\frac{1}{\psi}$.[12] If other things are equal, an increase in the persistence of shocks to expected consumption growth, expected inflation, or volatility lead to an increase in the absolute values of coefficients A_1 , $A_{2,1}$ and $A_{2,2}$. Hence the price consumption ratio is more sensitive to persistent risk factors.

Using the approximate return on consumption claims, we can express the negative log-stochastic discount factor in terms of risk factors and their innovations,

$$\begin{aligned} -m_{t+1} &= \Gamma_0 + \Gamma_1' x_t + \Gamma_2' \sigma_t^2 + \Lambda' \zeta_{t+1} \qquad (11) \\ \zeta_{t+1} &= [\sigma_{1,t}\eta_{1,t+1}, \sigma_{2,t}\eta_{2,t+1}, \sigma_{1,t}e_{1,t+1}, \sigma_{2,t}e_{2,t+1}, \sigma_{1,w}(S_{t+1})w_{1,t+1}, \sigma_{2,w}(S_{t+1})w_{2,t+1}]', \end{aligned}$$

where Γ_i and Λ are factor loadings and market prices of risks, respectively. The market prices of risks determine the magnitude of risk compensation. We can express these market prices of risks in terms of parameters governing preferences and shock processes, as follows:

$$\begin{aligned} \Lambda &= [\lambda_{\eta,1}, \lambda_{\eta,2}, \lambda_{e,1}, \lambda_{e,2}, \lambda_{w,1}, \lambda_{w,2}], \\ \lambda_{\eta,1} &= \gamma\,,\, \lambda_{\eta,2} = 1, \\ \lambda_{e,1} &= (1-\theta)\kappa_1(A_{11}\Phi_{11} + A_{12}\Phi_{21})\,,\, \lambda_{e,2} = (1-\theta)\kappa_1(A_{11}\Phi_{12} + A_{12}\Phi_{22}), \\ \lambda_{w,1} &= (1-\theta)\kappa_1 A_{21}\,,\, \lambda_{w,2} = (1-\theta)\kappa_1 A_{22}. \qquad (12) \end{aligned}$$

In the special case of power utility, $\gamma = \frac{1}{\psi}$ and $\theta = 1$. Therefore, shocks to expected consumption growth, expected inflation, and volatility are not priced risk factors. The separation of the risk aversion from the inverse of the IES in Epstein-Zin (1989) preferences allows separate compensation for these shocks.

[12] If γ is equal to $\frac{1}{\psi}$ as in the power utility case, agents are indifferent about the timing of the resolution of uncertainty. And they prefer the late resolution of uncertainty if γ is less than $\frac{1}{\psi}$.

The covariance of inflation and the real stochastic discount factor determines the inflation risk premium. In the model, unexpected short run fluctuations in inflation are not related to the real economy and hence there is no inflation risk premium for the short rate since $cov_t(\pi_{t+1}, m^r_{t+1}) = cov_t(\eta_{2,t+1}, m^r_{t+1}) = 0$. However, long-term bonds command inflation risk premia because variations in expected inflation are correlated with the real stochastic discount factor. This specification is consistent with the observation that the component of inflation priced in bond yields is the persistent component of inflation, which is a shock to expected inflation.[13]

Using the log nominal stochastic discount factor, we can compute arbitrage-free nominal bond prices from the Euler equation. This calculation can be done relatively easily as shown below, because the nominal stochastic discount factor obtained from the log-linearization of $r_{c,t+1}$ is normally distributed.

$$e^{p_{n,t}} = E_t(e^{m_{t+1}+p_{n-1,t+1}}) \implies p_{n,t} = E_t(m_{t+1} + p_{n-1,t+1}) + \frac{V_t(m_{t+1} + p_{n-1,t+1})}{2}, \tag{13}$$

where $p_{n,t}$ is the log of the price of a nominal bond whose time to maturity is n periods. Since m_{t+1} is affine with respect to risk factors, we can also express $p_{n,t}$ as an affine function of risk factors. Hence, the model implied bond yields are also affine functions of risk factors, given by the following relation,

$$y_{n,t} = -\frac{p_{n,t}}{n} = a_n + b_n X_t + c_n \sigma^2_t \, , \; \sigma^2_t = [\sigma^2_{1,t}, \sigma^2_{2,t}]'. \tag{14}$$

Since volatility regimes of $\sigma^2_{i,t}$ are i.i.d., the current regime does not provide any information about the future volatility. Hence, coefficients in equilibrium bond yields do not depend on the current regime.

[13]See Kim (2008) for evidence supporting this view. And D'Amico, Kim, and Wei (2008) show that inflation risk premium accounts for only 1% of the variance in the short rate while the portion increases for long-term bonds.

3 Data and Econometric Methodology

In this section, I will describe the dataset and explain the Bayesian estimation methods used in this paper.[14]

3.1 Data

We use the same dataset as Piazzesi and Schneider (2006) except for a slight change in the sample period.[15] Aggregate consumption growth is from the quarterly National Income and Product Account (NIPA) data on nondurables and services. Following Piazzesi and Schneider (2006), we use the price index for NIPA data. The three month treasury bill rate from the CRSP Fama risk-free rate file is used for the short term interest rate. One, two, three, four, and five year bond yields are extracted from the CRSP Fama-Bliss discount bond files. Figure 1 shows time series plots of all the observed variables used in the estimation.

3.2 Econometric Methodology

Since bond yields are affine functions of the four risk factors, it follows that we can perfectly recover expected consumption growth, expected inflation, and volatility if we have observations for bond yields of four different maturities. This is possible because agents in the model economy have full information on long-run risks as well as time-varying volatility, and they use that information to price financial assets. Of course, in reality, this is a very strong assumption;[16] however, under the assumption that the above model is a good approximation to the true data generating process,

[14] More details related to the econometric methodology are discussed in the web technical appendix.

[15] The sample period in Piazzesi and Schneider (2006) is from 1952:Q2 to 2005:Q4 while here it is from 1953:Q1 to 2006:Q4.

[16] Indeed, Joslin et. al. (2009) and Kim (2008) point out that macro risks are not spanned by bond yields of different maturities. I introduce bond-specific pricing errors so that macro risks are not completely spanned by the cross-sectional yield curve data. Nonetheless, the yield curve can still provide information on macro risks.

bond yields of different maturities can provide rich information about expected consumption growth, expected inflation, and volatility. Based on this idea, Bansal, Kiku, and Yaron (2007) estimate expected consumption growth by regressing realized consumption growth on the asset market data. While this approach is very easy to implement, it does not link parameters governing preferences and shock processes with the estimates of risk factors.

In this paper, I jointly estimate parameters and risk factors based on the following state space representation,

$$\begin{aligned}
F_{1,t} &= (I - T_1(\vartheta))\overline{F_1}(\vartheta) + T_1(\vartheta)F_{1,t-1} + Q_1(\vartheta)F_{2,t-1}e_t, \\
F_{2,t} &= (I - T_2(\vartheta))\overline{F_2}(\vartheta) + T_2(\vartheta)F_{2,t-1} + Q_2(\vartheta, S_t)w_t, \\
Z_t &= A_z(\vartheta) + B_z(\vartheta)F_t + C_z F_t \xi_t \qquad (15) \\
F_{1,t} &= [X_t, X_{t-1}]\,,\ F_{2,t} = \sigma_t^2\,,\ F_t = [F_{1,t}, F_{2,t}] \\
\vartheta &= [\rho, \Phi, \sigma_i^2, \nu_i, \sigma_{w,ij}, \alpha, \mu_i, \delta, \psi, \gamma, \sigma_{u,k}]\,,\ (i,j = 1,2)\,,\ (k = 1, \cdots, 6)
\end{aligned}$$

where Z_t is a vector of observed variables including consumption growth, inflation, and bond yields. ϑ is a vector of structural parameters in the model and ξ_t denotes a vector consisting of transitory shocks to consumption growth and inflation, and bond-specific measurement errors. Time-varying volatility introduces nonlinearities into the state transition equation through the term $F_{2,t-1}e_t$. While agents in the model are assumed to have full information on current and past state variables, an econometrician does not have such a knowledge and has to solve a filtering problem to recover state variables from the observed variables. The presence of nonlinearities complicates the filtering problem; however, the above model has a linear and Gaussian state space representation once we condition on a series of stochastic volatilities.[17] Also, conditional on parameters and regimes governing the variance of innovations, volatilities follow Gaussian processes. Finally, conditional on parameters and volatilities, we can recover the volatility regimes of innovations to stochastic volatility by applying the Hamilton (1989) filter.

[17]For a similar reason, conditional on the information set of agents, the log stochastic discount factor follows a normal distribution. I appreciate the comment from an anonymous referee to clarify this issue.

We use Bayesian methods which draw parameters, volatilities, and the volatility regimes of innovations to stochastic volatility iteratively.[18] By doing so, we can characterize the joint posterior distributions of parameters and volatilities which are updated from prior distributions, reflecting new information given by data.

4 Estimation Results

4.1 Prior Distributions of Parameters

There are two sets of parameters in the model. For the set of parameters related to the stochastic processes for consumption growth and inflation, we set the prior distributions to be roughly consistent with i) sample moments of consumption growth and inflation and ii) calibrated values in the existing literature. For preference parameters, we set prior means close to calibrated values in Bansal and Yaron (2004). Prior standard deviations of risk aversion and the IES are set wide enough to cover values commonly reported in other studies. Table 1 summarizes the prior information for all the parameters.

4.2 Posterior Analysis

4.2.1 Posterior Distribution

We can revise our prior beliefs about the parameters by using new information from the data. Table 2 illustrates how the data refine our beliefs about the parameters by contrasting prior distributions with posterior distributions. To identify the additional information from including term structure data, I also report posterior distributions of parameters from the estimation using only macro data.

[18] Jacquier, Polson, and Rossi (1994) propose Bayesian methods to draw volatilities conditional on parameters by using a Metropolis-Hastings algorithm. While they can compute the exact conditional distributions of parameters, this is not feasible in the model considered here. I run another Metropolis-Hastings algorithm to draw parameters conditional on volatilities and regimes. The details of the algorithm are explained in the appendix.

For persistence parameters of expected consumption growth and expected inflation, the posterior intervals are much narrower than prior intervals when term structure data are used in the estimation. The finding indicates that there is a lot of information about these parameters in the data. In contrast, when we use only the data on consumption growth and inflation, the posterior intervals are as wide as the prior intervals. This finding suggests that identifying persistence parameters of expected consumption growth and expected inflation is difficult with information from only the macro data.[19] A similar observation can be made for the persistence of volatility.

Compared to the prior distribution, the posterior intervals for risk aversion (γ) and the IES (ψ) are much narrower, suggesting that the data provide rich information on these parameters. In particular, the posterior distribution of the IES is slightly higher than 1 with a tight interval. Risk aversion is moderately high, with a posterior distribution around 9.5, which is comparable to 10 used in the calibration of Bansal and Shaliastovich (2010). The posterior distributions of risk aversion and the IES together imply that agents dislike high uncertainty and prefer the early resolution of uncertainty. With this configuration of preference parameters, agents may demand sizeable compensation for taking volatility risk. Interestingly, when volatility processes are homoskedastic, the estimates of risk aversion and the IES imply much lower market prices of volatility risks as shown in Figure 2.[20]

For some parameters, posterior mean values are quite different from prior mean values. For example, the probability of a high variance regime of volatility process is very low in the posterior distribution with the term structure data. The posterior mean is about 0.023 indicating that we can observe a high regime once in eleven years on average. However, the probability increases to 0.653 in the posterior distribution from the estimation with only the macro data. In the estimation results with the

[19] This may be a motivation for Hansen and Sargent (2009) who argue that the difficulty in distinguishing a consumption growth process with a small but highly persistent component from an i.i.d process generates model uncertainty premia in asset prices.

[20] While regime-dependent volatility processes affect the amount of volatility risks, they do not change market prices of volatility risks. The estimation results illustrate a nontrivial interaction between preference parameters and parameters governing shock processes.

term structure data, the differences across regimes are much starker than those estimated by using only the macro data. The finding implies that a volatility process with infrequent large spikes is in line with the term structure data.

4.2.2 Macro Implications

To asses the model's fit for macro variables, we compute the posterior moments for the macro variables, which can be compared with sample moments from the data. Table 3 provides information about the model's implications for dynamics of consumption growth and inflation. We compute the average level, volatility, and persistence of consumption growth and inflation as well as the correlation of the two variables using posterior draws of parameters and volatilities.

We observe overlaps between confidence intervals of sample moments and the corresponding posterior intervals for moments other than the sample correlation between inflation and consumption growth. The model estimates create a slightly positive correlation between consumption growth and inflation, in spite of the fact that posterior distributions of parameters governing dynamic and contemporaneous correlation between expected consumption growth and expected inflation are concentrated around negative values. The time series plot of estimates for expected consumption growth and expected inflation in Figure 3 provides a hint to the cause of this mismatch. While recessions during the 1970s were characterized by a spike in expected inflation and a drop in expected consumption growth, such a negative comovement is much less pronounced in periods since the early 1980s. In fact, when we use estimates of long-run risks up to only the late 1970s, the model implies a strongly negative correlation between consumption growth and inflation. By contrast, the negative correlation is smaller if we use a subsample after the early 1980s. Estimates of long-run risks imply a significantly positive correlation between consumption growth and inflation in the second subsample, resulting in the mismatch of the full-sample moment. To fix this mismatch may require the generalization of the model structure by allowing the time variation of parameters governing shock processes, although such an extension can be very challenging in solving and estimating the model.

Estimates of expected consumption growth and expected inflation are functions of observed variables used in the estimation. Therefore, they may be sensitive to the data we include in the estimation. One way to check if the model implied estimates reasonably capture agents' expectations is to connect these estimates with observed proxies for expectations that are not directly used in the estimation.

Table 4 provides results from regressing the median one-quarter ahead forecasts of consumption growth and CPI inflation from the Survey of Professional Forecasters (SPF, hereafter) on estimates of expected consumption growth and expected inflation from the model.[21] R^2 statistics reported in Table 4 show that model-implied expectations explain survey data for inflation well, but not for consumption growth. A similar pattern is observed when we extract information on uncertainty about consumption growth and inflation from survey data. We construct two proxies for uncertainty from the SPF. The first measure is obtained by averaging uncertainty in the density forecast of each individual forecaster.[22] The second measure is simply the dispersion in the point forecast of each individual forecast.[23] For both measures, the model does a good job in explaining inflation uncertainty. However it does a relatively poor job in explaining consumption uncertainty as shown in Table 5.

While the poor correlation of estimates of consumption risk factors with survey data clearly suggests that a more general specification of shock processes might be necessary, some part of it can be attributed to the Federal Reserve's prolonged easing policy after the 2001 recession. As evident in Figure 1, the short rate remained low until 2004:Q2 after the 2001 recession, whereas realized consumption growth started to rebound in 2002. Indeed, Smith and Taylor (2009) argue that long term interest rates became less responsive to macro variables during this period as a result of the Federal Reserve's policy that deviated a lot from the Taylor rule prescription.[24]

[21] Using the median one-year forecasts and the corresponding model-implied expectations delivers virtually same results.

[22] The appendix describes the details of the construction for this measure.

[23] I get rid of outliers which are more than two standard deviation away from the mean forecast.

[24] However, Bernanke (2010) argues that if we consider real-time inflation forecasts rather than the realized inflation data to measure the inflation gap, the policy was not excessively loose compared

Indeed, if we use observations up to 2001:Q4, estimates of consumption risk factors such as expected consumption growth and consumption volatility are moderately correlated with survey data as shown in Tables 1 $\sim$ 2.

4.2.3 Term Structure Implications

To evaluate the model's fit for term structure, we compare posterior means of yield curve moments with sample moments from the data. Table 6 shows that the unconditional moments of level, volatility, and persistence of the yield curve from sample data are very close to the corresponding posterior means. Moreover, the mean absolute pricing errors for bond yields of maturities longer than 1 year are pretty small, ranging from 4.3 basis points to 6.1 basis points. These numbers are comparable to average pricing errors reported in the literature on estimating no-arbitrage macro-finance term structure models (e.g., Bikbov and Chernov (2010)).

In the model, time-varying term premia can be determined by either consumption volatility or inflation volatility. To determine which factor is more important, we run the following counterfactual exercise.[25] We compute the model-implied term premium for the ten-year bond yield by keeping inflation volatility constant at the time-series average of the posterior mean estimates and compare it with the counterpart based on posterior mean estimates of inflation volatility. In both cases, we use posterior mean estimates of consumption volatility and parameters. The model-implied term premium for the ten-year bond yield in Figure 5 shows that there is a huge difference in the time-variation of the term premium when the variation of inflation volatility is suppressed. In particular, the decline of the term premium after the Volcker period of the early 1980s documented in the reduced-form empirical studies of the U.S. yield curve (e.g., Wright (2011)), cannot be detected in the case of the counterfactual constant inflation volatility.

The key role played by inflation volatility in explaining the time-varying term premium seems to be at odds with the emphasis on consumption volatility risk in

to the benchmark Taylor rule.

[25] I am grateful for an anonymous referee who suggested me to perform this exercise.

the existing long-run risks literature. In particular, Bansal and Shaliastovich (2010) show that consumption volatility risk in a standard long-run risks model can explain the predictability of term premia. The analysis in this paper differs from Bansal and Shaliastovich (2010) in two important aspects. First, this paper allows a separate volatility risk for inflation which can be independent of consumption risk. By contrast, in Bansal and Shaliastovich (2010), time-varying inflation volatility is perfectly correlated with consumption volatility risk. Therefore, their measure of consumption volatility will mix both consumption volatility and inflation volatility in this paper. Second, they calibrate their model and do not provide estimates of consumption volatility risk. It is difficult to know if the model explains the predictability puzzle by consumption volatility risk that is reasonably well matched by empirical proxies for consumption uncertainty. In this paper, I obtain estimates of the model-implied inflation volatility that can be checked with inflation forecast uncertainty from survey data. The reasonably high correlation between two measures of inflation uncertainty supports the importance of inflation volatility.

4.2.4 Robustness

The central role of inflation volatility in the time-variation of the term premium is not driven by the fact that we use a particular measure of term premium based on the model estimates. We regress two estimates of the term premium constructed by Wright (2011) on the estimates of volatilities in the model. Since these term premium estimates are not used in estimation, they can be used to check the robustness of the relation between inflation volatility and the term premium.[26] The first measure (statistical term premium) is constructed based on the estimation of a no-arbitrage three factor affine term structure model, with monthly data from January 1990 to December 2007. In this case, the first three principal components of the yield curve are used as risk factors explaining the yield curve movement and the

[26] I also estimated the model using the ten-year bond yield data. Estimates of parameters and volatilities are not much different. Details of estimation results are available on the web technical appendix. I am thankful for an anonymous referee who suggested this exercise.

five-to-ten-year forward term premium is computed. The second measure (survey-based term premium) is obtained by estimating expected future short rates from the regression of the short rate on survey data on inflation and real GDP growth. Table 7 reports regression results of the two measures of the forward term premium on the estimates of volatilities. R^2 statistics show that inflation volatility explains the term premium better than consumption volatility and the three statistical yield curve factors.[27] The relationship between the estimated inflation uncertainty and the term premium in Figure 6 indicates that the rise and fall of the term premium is consistent with changes in inflation volatility. These results imply that risk compensation for inflation volatility is a key economic determinant of the term premium implied in the long-term bond yield.

In general, posterior estimates of parameters and volatilities are influenced by all the features of the model, not just by the presence of the time-varying volatility. So it might be the case that if we suppress the time-variation of inflation volatility, consumption volatility from such a restricted model could explain the time-variation of term premia and fit macro and term structure data as good as the model that allows the time-varying inflation volatility. Furthermore, even a simpler model with constant volatility may fit the data well. To check this possibility, we estimate the two simpler models. The first one allows stochastic volatility only for consumption growth and assumes that inflation does not affect consumption growth while it can be affected by consumption growth much like Bansal and Shaliastovich (2010). The second one assumes constant volatility, as in Piazzesi and Schneider (2006), but allows real impacts of expected inflation on expected consumption growth. To compare the fit of different models, we use marginal data density which is defined by,

$$mdd(M_i) = \int p(Z^T|\vartheta, M_i)p(\vartheta|M_i)d\vartheta. \tag{16}$$

[27]There might be a concern that this result may be spurious due to the relatively poor correlation of consumption volatility with survey data. However, Wright (2011) shows that inflation uncertainty matters more than consumption uncertainty even if both measures from survey data are used in term premium regressions.

Once marginal likelihood is obtained, we can calculate the posterior probability of S different models by,

$$p(M_i|Z^T) = \frac{mdd(M_i)}{\sum_{j=1}^{S} mdd(M_j)}. \tag{17}$$

The baseline model of the paper has a much higher marginal data density than simpler models as shown in Table 8. The finding suggests the importance of allowing time-varying inflation volatility to fit macro and term structure data jointly.

5 Conclusion

In this paper, we estimate an equilibrium term structure model in which agents have recursive preferences and persistent fluctuations in expected consumption growth, expected inflation, and their volatilities drive the time variation of bond yields. Parameter estimates suggest that agents dislike volatility risks and demand a sizeable compensation for taking these risks. Unlike the calibration exercises common to the existing literature on long-run risks models, this paper takes the long-run risks model seriously to the time series data of macro variables and nominal bond yields using full information methods. By linking the estimates of volatilities with term premium measures, we find that risk compensation for inflation volatility is central in explaining the time variation of term premia. This finding is consistent with empirical evidence from statistical models and survey data. However, it is different from the emphasis on consumption volatility risk in the existing long-run risks literature based on model calibration.

This paper does not provide an answer to the sources of fluctuations in inflation volatility. While changes in monetary policy can be a potential source, investigating further into this issue requires endogeneizing the inflation process. In addition, a more general specification of shock processes might be necessary to capture the time-varying relationship between consumption risk and inflation risk. These works are left for future research.

6 Appendix

The Survey of Professional Forecasters, published by the Federal Reserve Bank of Philadelphia, contains probability range for annual real GDP growth and inflation assessed by each individual forecaster. Separate information for real and nominal GDP is available from 1981:Q3 onwards. Using the midpoints of the intervals for probability assessment, we can compute moments associated with each individual forecaster's probability assessment. Let $\Omega_{i,t}^{j}$ be the forecast uncertainty for the i_{th} forecaster at time t for the j_{th} variable. Averaging $\Omega_{i,t}^{j}$ across forecasters, I obtain the following measure for the average forecast uncertainty.

$$\Omega_t^j = \frac{\sum_{i=1}^{N_t} \Omega_{i,t}^j}{N_t} \tag{18}$$

For real GDP growth, I eliminate one individual forecaster who puts more than 90% probability for the interval to which no one else puts more than 5% probability because this forecaster is a clear outlier.

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Table 1: PRIOR DISTRIBUTION

Parameters	Domain	Density	Para(1)	Para(2)
ρ_{11}	[0,1)	Beta	0.92	0.05
ρ_{12}	$\mathbb{R}$	Normal	0	0.1
ρ_{21}	$\mathbb{R}$	Normal	0	0.1
ρ_{22}	[0,1)	Beta	0.92	0.05
ϕ_{11}	$\mathbb{R}^+$	Gamma	0.3	0.05
ϕ_{12}	$\mathbb{R}$	Normal	0	0.05
ϕ_{21}	$\mathbb{R}$	Normal	0	0.05
ϕ_{22}	$\mathbb{R}^+$	Gamma	1.26	0.15
σ_1	$\mathbb{R}^+$	Inverse Gamma	0.004	4
σ_2	$\mathbb{R}^+$	Inverse Gamma	0.005	4
ν_1	[0,1)	Beta	0.8	0.1
ν_2	[0,1)	Beta	0.8	0.1
$\sigma_{w,11}$	$\mathbb{R}^+$	Gamma	8×10^{-6}	4
$\sigma_{w,12}$	$\mathbb{R}^+$	Gamma	2×10^{-6}	4
$\sigma_{w,21}$	$\mathbb{R}^+$	Gamma	8×10^{-6}	4
$\sigma_{w,22}$	$\mathbb{R}^+$	Gamma	2×10^{-6}	4
α	[0,1)	Uniform	0.0001	0.9999
μ_1	$\mathbb{R}$	Normal	0.008	0.0005
μ_2	$\mathbb{R}$	Normal	0.009	0.0005
δ	[0,1)	Beta	0.997	0.002
ψ	$\mathbb{R}^+$	Gamma	1.5	0.5
γ	$\mathbb{R}^+$	Gamma	7	5
$\sigma_{u,i}$	$\mathbb{R}^+$	Inverse Gamma	8×10^{-4}	4

Notes: Para (1) and Para (2) list the means and the standard deviations for Beta, Gamma, and Normal distributions; s and ν for the Inverse Gamma distribution, where $p_{\mathcal{IG}}(\sigma|\nu, s) \propto \sigma^{-\nu-1}e^{-\nu s^2/2\sigma^2}$, a and b for the Uniform distribution from a to b.

Table 2: Posterior Distribution

Parameter	Prior	Posterior: Joint		Posterior: Macro	
	90% Interval	Mean	90% Interval	Mean	90% Interval
ρ_{11}	[0.850, 0.994]	0.967	[0.956, 0.977]	0.858	[0.780, 0.948]
ρ_{12}	[-0.167, 0.162]	-0.020	[-0.025, -0.016]	-0.029	[-0.069, 0.011]
ρ_{21}	[-0.163, 0.166]	-0.064	[-0.075, -0.050]	0.075	[-0.045, 0.197]
ρ_{22}	[0.848, 0.992]	0.947	[0.939, 0.956]	0.930	[0.881, 0.979]
ϕ_{11}	[0.219, 0.382]	0.229	[0.211, 0.252]	0.296	[0.222, 0.367]
ϕ_{12}	[-0.082, 0.083]	-0.015	[-0.034, 0.007]	-0.051	[-0.114, 0.017]
ϕ_{21}	[-0.083, 0.082]	-0.058	[-0.086, -0.031]	-0.026	[-0.097, 0.045]
ϕ_{22}	[1.015, 1.506]	0.718	[0.667, 0.778]	0.850	[0.707, 1.002]
σ_1	[0.0021, 0.0079]	0.0058	[0.0049, 0.0068]	0.0039	[0.0034, 0.0044]
σ_2	[0.0026, 0.0099]	0.0029	[0.0026, 0.0034]	0.0026	[0.0023, 0.0029]
ν_1	[0.647, 0.958]	0.977	[0.965, 0.987]	0.823	[0.677, 0.949]
ν_2	[0.650, 0.959]	0.960	[0.952, 0.969]	0.680	[0.519, 0.873]
$\sigma_{w,11}$	[4.26, 15.77] $\times 10^{-6}$	12.09 $\times 10^{-6}$	[9.46, 14.15] $\times 10^{-6}$	5.16 $\times 10^{-6}$	[3.44, 6.83] $\times 10^{-6}$
$\sigma_{w,12}$	[1.06, 3.96] $\times 10^{-6}$	3.48 $\times 10^{-6}$	[3.03, 3.85] $\times 10^{-6}$	2.25 $\times 10^{-6}$	[1.09, 3.46] $\times 10^{-6}$
$\sigma_{w,21}$	[4.25, 15.88] $\times 10^{-6}$	8.30 $\times 10^{-6}$	[7.01, 9.88] $\times 10^{-6}$	3.43 $\times 10^{-6}$	[2.37, 4.43] $\times 10^{-6}$
$\sigma_{w,22}$	[1.05, 3.94] $\times 10^{-6}$	1.56 $\times 10^{-6}$	[1.26, 1.93] $\times 10^{-6}$	1.67 $\times 10^{-6}$	[1.09, 2.27] $\times 10^{-6}$
α	[0.0934, 0.9921]	0.0224	[0.0001, 0.0477]	0.653	[0.276, 0.999]
μ_1	[0.0072, 0.0088]	0.0074	[0.0071, 0.0076]	0.0081	[0.0074, 0.0089]
μ_2	[0.0082, 0.0098]	0.0091	[0.0088, 0.0093]	0.0089	[0.0080, 0.0097]
δ	[0.9942, 0.9998]	0.9982	[0.9974, 0.9991]		
ψ	[0.6985, 2.2721]	1.053	[1.021, 1.079]		
γ	[0.2354, 13.7730]	9.518	[8.234, 11.778]		
$\sigma_{u,1}$	[0.00043, 0.00160]	0.0011	[0.00098, 0.00119]		
$\sigma_{u,4}$	[0.00043, 0.00160]	0.00038	[0.00034, 0.00042]		
$\sigma_{u,8}$	[0.00043, 0.00159]	0.00023	[0.00021, 0.00026]		
$\sigma_{u,12}$	[0.00042, 0.00158]	0.00021	[0.00019, 0.00023]		
$\sigma_{u,16}$	[0.00042, 0.00158]	0.00023	[0.00021, 0.00025]		
$\sigma_{u,20}$	[0.00042, 0.00159]	0.00024	[0.00021, 0.00027]		

Notes: Macro stands for the estimation results using only macro data and Joint for the estimation results including term structure data. Posterior distribution is based on 50,000 (80,000) posterior draws after discarding the initial 10,000 (20,000) draws in the joint(macro) estimation.

Table 3: MACRO IMPLICATIONS

Moment	**Data**		**Macro**		**Joint**	
	estimate	standard error	mean	90% interval	mean	90% interval
$E(g_{c,t})$	3.23	0.2	3.23	[3.14, 3,33]	2.91	[2.71, 3.12]
$\sigma(g_{c,t})$	1.84	0.16	1.73	[1.60, 1.85]	2.28	[2.12, 2.42]
$AR_1(g_{c,t})$	0.34	0.062	0.15	[0.09, 0.21]	0.23	[0.18, 0.28]
$AR_4(g_{c,t})$	0.07	0.057	0.10	[0.06, 0.14]	0.19	[0.14, 0.24]
$E(\pi_t)$	3.71	0.48	3.73	[3,67, 3.77]	3.88	[3.73, 4.04]
$\sigma(\pi_t)$	2.52	0.36	2.43	[2.36, 2.51]	3.08	[2.94, 3.22]
$AR_1(\pi_t)$	0.84	0.048	0.81	[0.78, 0.84]	0.72	[0.70, 0.75]
$AR_4(\pi_t)$	0.71	0.084	0.71	[0.68, 0.74]	0.62	[0.59, 0.65]
$Corr(g_{c,t}, \pi_t)$	-0.34	0.150	-0.18	[-0.10, -0.26]	0.08	[0.03, 0.15]

Notes: Macro stands for the estimation results using only macro data and Joint for the estimation results including term structure data in the estimation. Means and standard deviations of consumption growth and inflation are expressed in terms of annualized percentage. I compute posterior moments using 50,000 posterior draws. Standard errors are Newey and West (1987) corrected using 10 lags.

Table 4: Regressions of Consumption Growth and Inflation on the Estimates of the Model-implied Expectations

Regressors	SPF Con.	SPF Inf.	Realized Con.	Realized Inf.
constant	2.484	3.393	3.075	4.006
	[2.288, 2.679]	[3.264, 3.522]	[2.839, 3.311]	[3.839, 4.173]
expected con.	0.192	0.184	0.475	-0.796
	[0.011, 0.373]	[0.065, 0.304]	[0.242, 0.708]	[-0.961, -0.632]
expected inf.	-0.041	0.510	-0.259	0.843
	[-0.148, 0.066]	[0.440, 0.581]	[-0.346, -0.172]	[0.781, 0.904]
R^2	0.045/*0.240*	0.796/*0.601*	0.163/*0.369*	0.776/*0.865*
	(0.366/*0.284*)	(0.782/*0.776*)		

Notes: SPF Con. and SPF Inf. denote one-quarter ahead median forecasts of consumption growth and inflation from the survey of professional forecasters from 1981:Q3 to 2006:Q4. Realized consumption growth and inflation from 1953:Q1 to 2006:Q4 are also regressed on the model implied expectations computed at the posterior means of parameters and volatilities. Entries in square brackets are the 95 percent confidence intervals for coefficients. Numbers in the parentheses denote R^2s in regressions using data from 1981:Q3 to 2001:Q4. Italicized numbers are from the corresponding regressions using estimates obtained by only macro data.

Table 5: Regressions of Forecast Uncertainty and Dispersion of Forecasts on the Estimates of Time-varying Volatility

Regressors	Uncertainty Real GDP	Uncertainty Inf.	Dispersion Con.	Dispersion Inf.
constant	1.096 [0.939, 1.252]	0.246 [0.116, 0.377]	0.615 [0.364, 0.866]	-0.258 [-0.532, 0.016]
consumption vol.	0.024 [-0.067, 0.116]		0.120 [-0.026, 0.266]	
inflation vol.		0.471 [0.389, 0.553]		0.656 [0.484, 0.829]
R^2	0.003/*0.347*	0.565/*0.061*	0.026/*0.264*	0.364/*0.248*
	(0.264/*0.295*)	(0.531/*0.269*)	(0.414/*0.224*)	(0.498/*0.324*)

Notes: Forecast uncertainty is constructed from probability forecasts in the SPF from 1981:Q3 to 2006:Q4. Dispersion of one quarter ahead forecasts of CPI inflation and consumption growth are also obtained from the survey of professional forecasters. Numbers in parentheses denote R^2 in regressions using data from 1981:Q3 to 2001:Q4. Italicized numbers are from the corresponding regressions using estimates obtained by only macro data.

Table 6: Posterior Mean of Yield Curve Moments

	$E(y_{1,t})$	$E(y_{4,t})$	$E(y_{8,t})$	$E(y_{12,t})$	$E(y_{16,t})$	$E(y_{20,t})$
data	5.188	5.596	5.797	5.964	6.090	6.169
model	5.308	5.565	5.804	5.965	6.080	6.174
	$\sigma(y_{1,t})$	$\sigma(y_{4,t})$	$\sigma(y_{8,t})$	$\sigma(y_{12,t})$	$\sigma(y_{16,t})$	$\sigma(y_{20,t})$
data	2.882	2.885	2.846	2.773	2.743	2.699
model	2.899	2.868	2.832	2.792	2.744	2.686
	$AR_1(y_{1,t})$	$AR_1(y_{4,t})$	$AR_1(y_{8,t})$	$AR_1(y_{12,t})$	$AR_1(y_{16,t})$	$AR_1(y_{20,t})$
data	0.94	0.95	0.95	0.96	0.97	0.97
model	0.94	0.95	0.96	0.96	0.97	0.97
	$400E(\|u_{1,t}\|\|Y^T)$	$400E(\|u_{4,t}\|\|Y^T)$	$400E(\|u_{8,t}\|\|Y^T)$	$400E(\|u_{12,t}\|\|Y^T)$	$400E(\|u_{16,t}\|\|Y^T)$	$400E(\|u_{20,t}\|\|Y^T)$
	0.328	0.099	0.053	0.043	0.050	0.061

Notes: All the estimates are in annualized percentage terms. Posterior moments are computed based on every 25th draw among 50,000 posterior draws.

Table 7: Regressions of Term Premium on Various Factors

Regressors					
			Panel A : Statistical Term Premium		
constant	2.976	2.873	2.959	2.989	-4.0111
	[2.590, 3.361]	[2.679, 3.067]	[2.665, 3.253]	[2.004, 3.974]	[-5.097, -2.924]
level	0.046				
	[-0.045, 0.137]				
slope		1.649			
		[1.339, 1.958]			
curvature			-6.807		
			[-10.440, -3.174]		
con. vol.				-0.075	
				[-0.64, 0.489]	
inf. vol.					5.139
					[4.337, 5.941]
R^2	0.015	0.632	0.175	0.001	0.713
			Panel B : Term Premium from Survey Data		
constant	1.923	1.617	1.678	2.533	-2.097
	[1.638, 2.208]	[1.340, 1.893]	[1.399, 1.957]	[1.727, 3.339]	[-3.354, -0.840]
level	0.126				
	[0.058, 0.193]				
slope		0.303			
		[-0.137, 0.743]			
curvature			-2.734		
			[-5.949, 0.481]		
con. vol				-0.554	
				[-1.019, -0.089]	
inf. vol.					2.784
					[1.855, 3.714]
R^2	0.317	0.060	0.088	0.160	0.546

Notes: The level, slope, and curvature are the first, second, and third principal components of the yield curve. The last two regressors are the posterior mean values of the estimated stochastic volatilities of consumption growth and inflation. Term premium is a 5 to 10 year forward premium computed by Wright (2011) in two different ways. Entries in square brackets are the 95 percent confidence intervals for coefficients.

Table 8: Log Marginal Data Densities

Model	Log Marginal Data Density
M_1 (benchmark model)	9,679.8
M_2 (homoskedastic volatility process)	9,642.2
M_3 (homoskedastic volatility process, no real impacts of expected inflation)	9,456.3
M_4 (constant volatility)	9,278.1

Notes I compute marginal data densities based on the simulation methods in Chib and Jeliazkov (2001).

Figure 1: Consumption Growth, Inflation, and Bond Yields

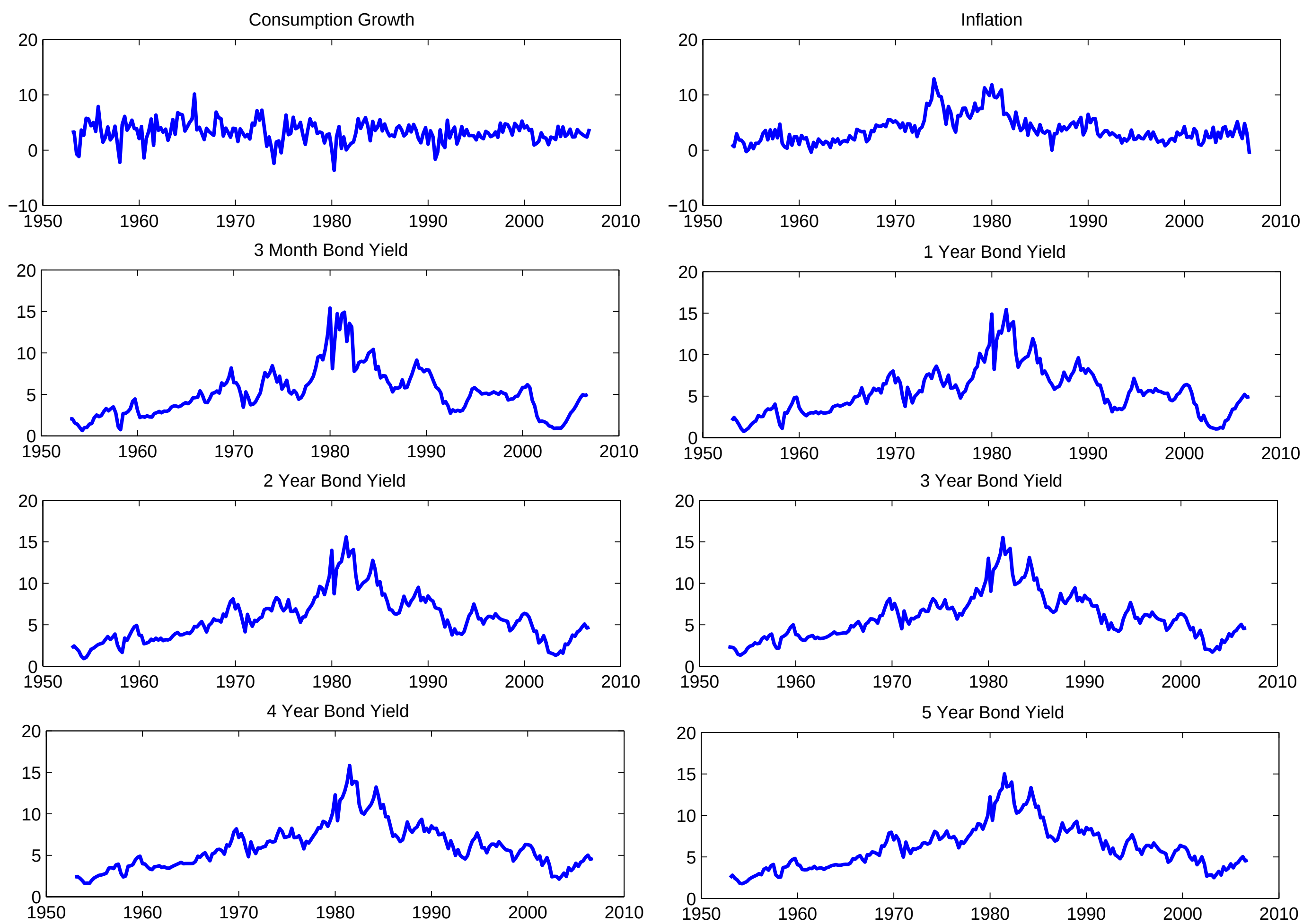


Figure 2: Posterior Distribution of Market Price of Risk

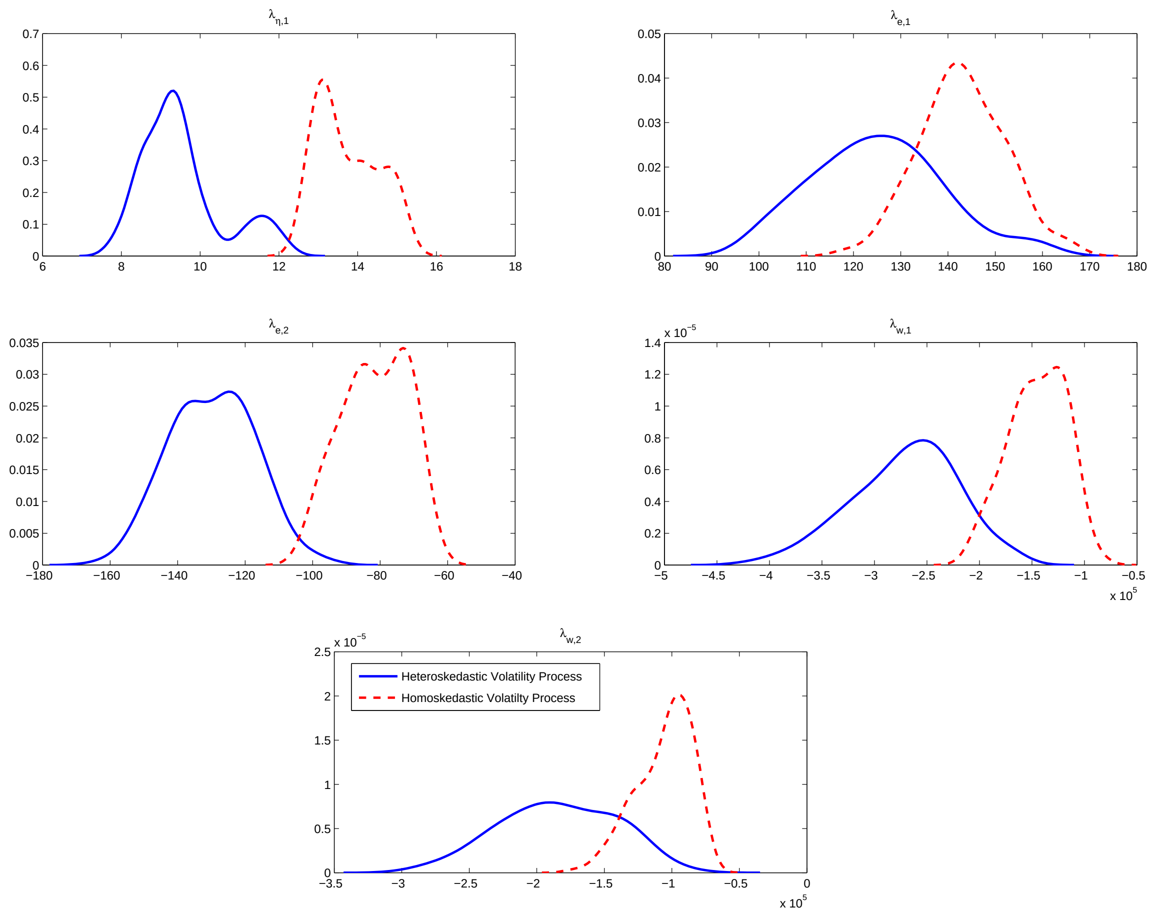
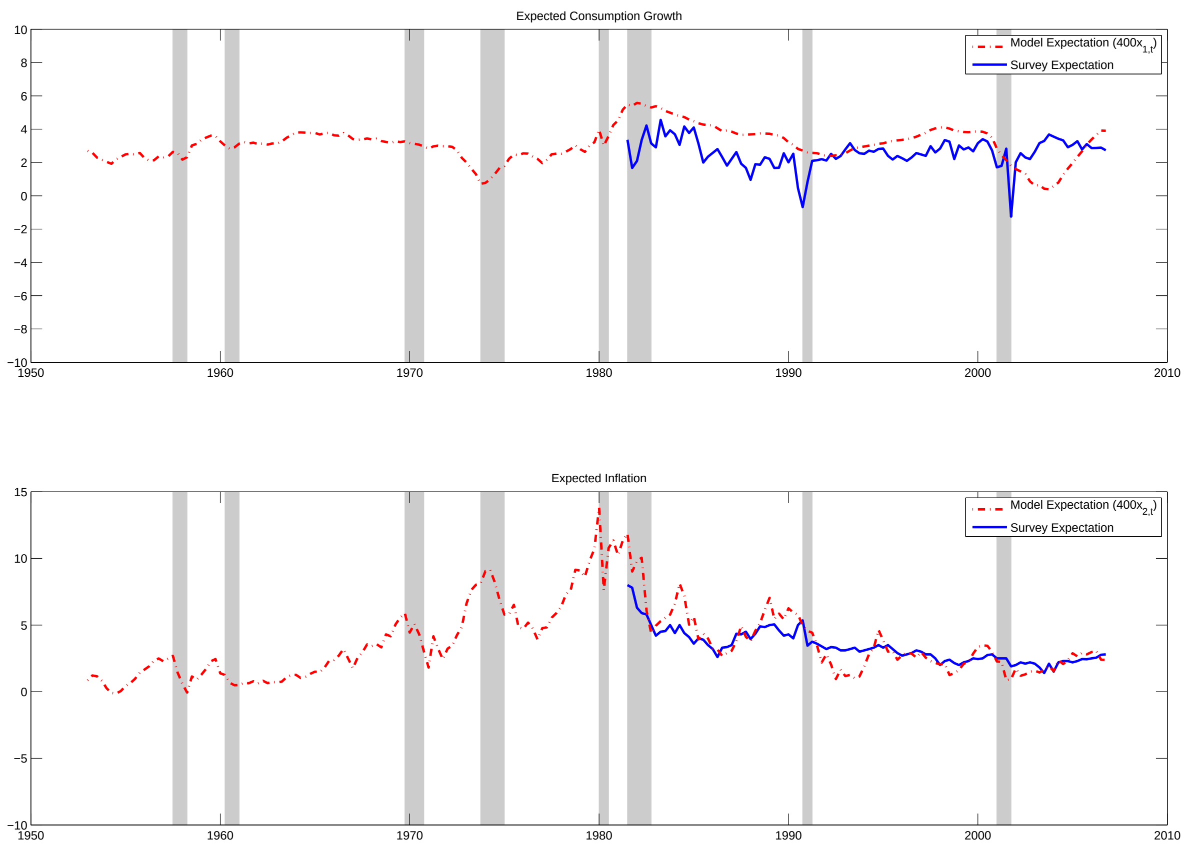
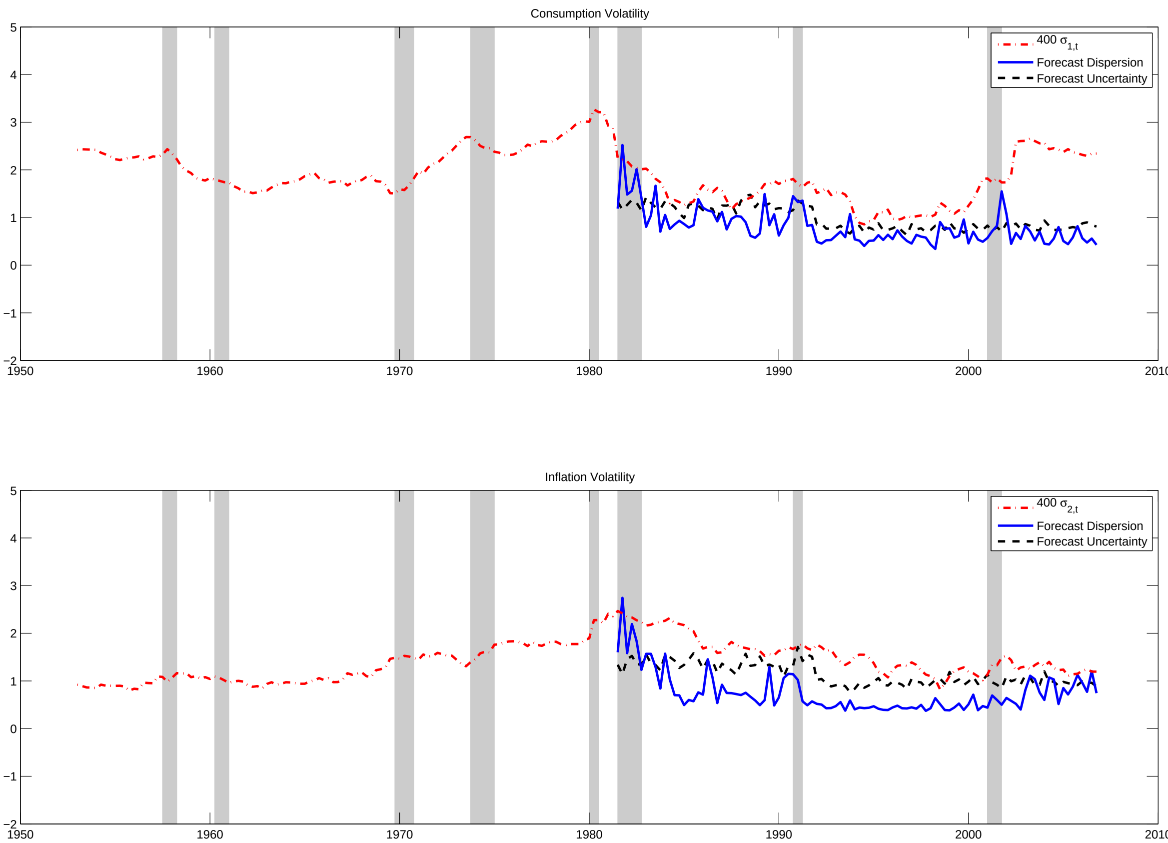


Figure 3: Estimates of Expected Consumption Growth and Inflation



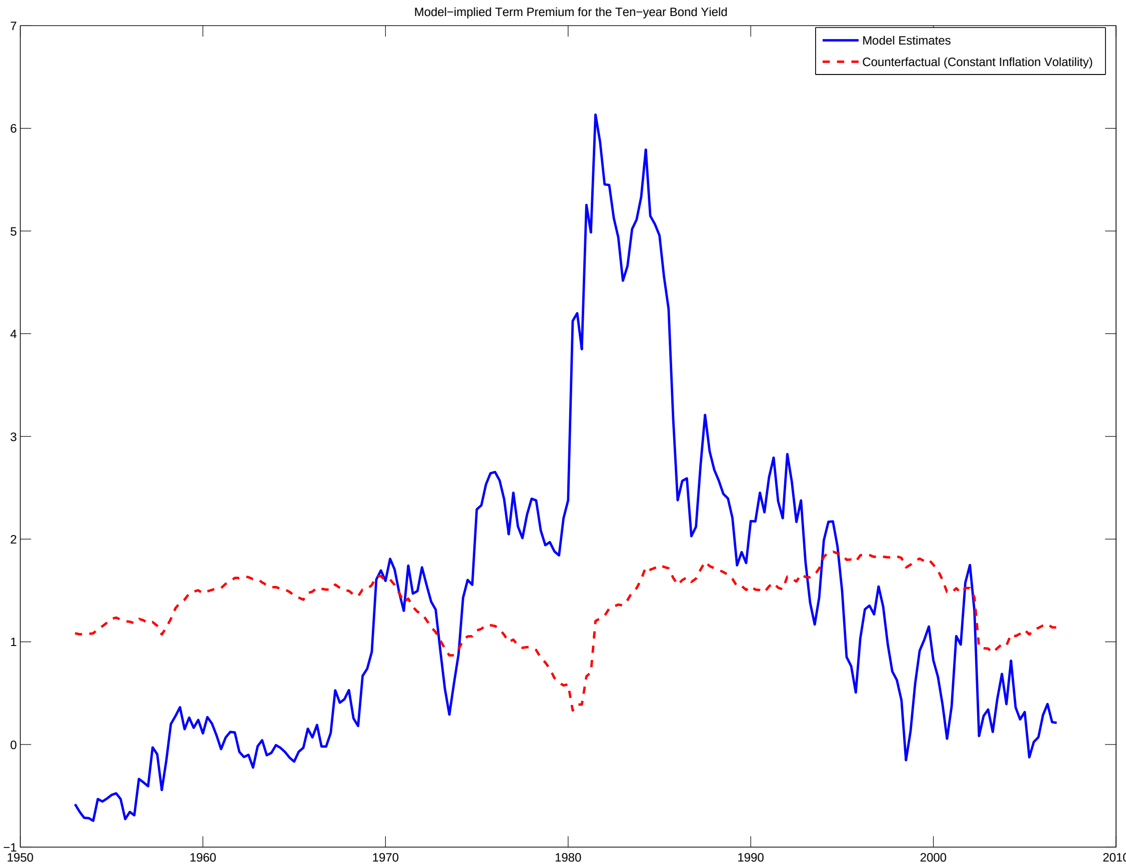
I run Kalman smoothing at the mean of parameters and stochastic volatilities based on 50,000 posterior draws in order to compute model implied expectations. Survey-based expectations are one quarter ahead median forecasts of CPI inflation and consumption growth from the SPF. The sample period is from 1981:Q3 to 2006:Q4.

Figure 4: ESTIMATES OF TIME-VARYING VOLATILITY



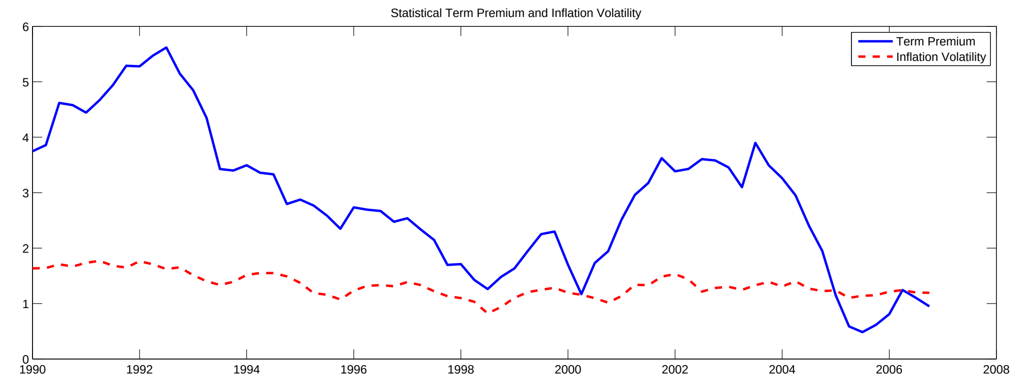
The posterior means of stochastic volatilities from 50,000 posterior draws are plotted. Dispersion of one quarter ahead forecasts of CPI inflation and consumption growth are obtained from the SPF. Forecast uncertainty is constructed from probability forecasts in the SPF. The sample period is from 1981:Q3 to 2006:Q4.

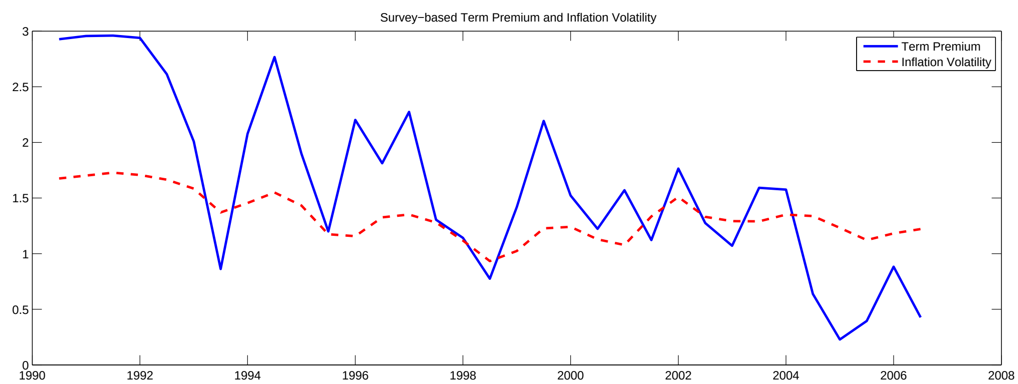
Figure 5: Model-implied Term Premium for the Ten-year Bond Yield



Term premium is computed by $y_{n,t} - \frac{E_t(\sum_{j=0}^{n-1} i_{t+j})}{n}$. Model estimates use posterior means of parameters and stochastic volatilities. The counterfactual exercise keeps inflation volatility constant at the time-series average of posterior mean estimates.

Figure 6: Inflation Volatility and Term Premium





Term premium (five-to-ten-year forward premium) measures are from Wright (2011) . The statistical measure of the term premium is obtained by estimating a three factor no-arbitrage model using data from 1990:Q1 to 2006:Q4. The survey based measure of the term premium uses expected short rates from survey data to compute term premium from the second half of 1990 to the second half of 2006.