

# Markov Switching Monetary Policy in a two-country DSGE Model.

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## Abstract

Estimates from a structural VAR model for the Eurozone and the US provide evidence in favor of regime shifts in the monetary policy of the Fed. Based on this finding I construct a two-country DSGE model in which only foreign monetary policy changes regimes. This may result in higher inflation volatility in the home country, even though the latter follows a time invariant policy rule. Moreover, in this case private sector expectations in the home country are enough to destabilize both home inflation and output. This suggests that home monetary policy should change over time, conditional on foreign monetary policy, in order to minimize the destabilizing effects of regime shifts.

Keywords: Markov-switching DSGE, monetary policy, Dynamic programming, optimized coefficients, SVAR, real-time data.

JEL Classification: E52, F41, F42.

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## 1 Introduction

Regime changes in the conduct of monetary policy have been documented largely over the last ten years. They refer to changes in the way the Central Bank of a country reacts to the key macroeconomic variables, i.e. inflation and output. An example of such a change in monetary policy is that of the US. In particular, Clarida et al. (2001), Lubik and Schorfheide (2004) and Boivin and Giannoni (2006) showed that the reaction of the Fed towards inflation fluctuations until the late '70s was less aggressive compared to that from the early '80s onwards. As a result many authors attribute high inflation volatility in the US during the '70s to the way the Fed was reacting over that period to inflation fluctuations<sup>1</sup>. Moreover, according to these authors, changes in monetary policy are the main reason for the changes in the impulse responses of inflation and output.

A weakness of the above papers is that they assume that the private sector is naive. That is, they do not take into account the expectations formation effects. The latter refers to the way expectations are formed and how they affect the dynamics of inflation and output. In a rational expectations framework where monetary policy switches regimes over time, the private sector will always take those shifts into account. For example, if there is the expectation the Central Bank will react more aggressively to inflation in the near future, agents will incorporate this into their expectations about future inflation. Consequently, such an information is able to start stabilizing inflation and output even before the actual future policy takes place.

A popular way of modelling this behavior of the monetary authority and the private sector is by introducing Markov switching in monetary policy. Davig and Leeper (2007), Liu et al. (2008, 2009) and Farmer et al (2011), relying on the empirical estimates about the way monetary policy was conducted in the US from 1970 until recently construct closed economy DSGE models where the coefficients in the interest rate rule of the Central Bank change over time according to a Markov switching process. All the three papers conclude

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<sup>1</sup>There is a huge literature over the causes of a change in inflation volatility in the US. Some authors, such as Stock and Watson (2003), attribute that change to different shock sizes, rather than to changes in the way monetary policy was conducted. In other words, according to these authors, heteroskedasticity in shocks variances seems to be the main reason for changes in the volatility of the key macroeconomic variables.

that the expectation of a future regime shift in monetary policy has significant effects on inflation and output today. Those can be either stabilizing or destabilizing depending on what is the expected future policy.

A weakness, though, of the existing literature is that it is restricted to a closed economy framework. As a result, so far, in the literature on Markov-switching DSGE models, the cross country effects of regime shifts in monetary have not been analyzed. The monetary policy of one country has effects on other countries as well. Therefore, it is important that we have an open economy framework, so that to analyze the effects that a change in the monetary policy in one country has on another country. Changes in the volatilities and the impulse responses of key macroeconomic variables may be the result of changes in domestic conditions. But, it may be that the volatility of inflation or output changes as a result of a shift in monetary policy of a foreign country. Furthermore, it is likely that the impulse responses of those two variables change as well, as the foreign monetary policy changes and even though the domestic monetary policy has not changed at all. For this reason I construct a two country DSGE model, in order to extend the existing analysis on Markov-switching DSGE models from the closed economy, to the open economy framework.

This paper has three objectives. The first is to find whether there have been changes in the monetary policy of the Eurozone or the US or both at the same time. Then, if this is the case, to analyze what are the cross country effects of a change in the monetary policy of at least one of the two countries. In particular, the objective in the empirical part is to find the extent to which changes in the dynamics of inflation and output in one country are the result of the change in the monetary policy of the other. The second objective, given the empirical findings, is to construct an open economy structural model, in order to disentangle the different forces that lead to variations in the dynamics of inflation and output in the home country, when the foreign country changes its policy. The third objective is to find what the optimal reaction of the home Central Bank should be when the monetary policy of the foreign country switches regimes over time, based on a welfare criterion. Should it follow a time invariant policy rule, or not?

As a first step I provide some motivation for the construction of the theoretical model.

A SVAR model for the Eurozone and the US is estimated, using real time monthly data spanning from 1999 through 2010. The empirical model includes seven variables, namely inflation, output gap and the nominal interest rate for both the Eurozone and the US, as well as the real exchange rate. I perform parameter stability tests using the Andrews sup-Wald test, as in Boivin and Giannoni (2002) and the Andrews-Ploberger test<sup>2</sup>. Both tests find that there have been statistically significant changes in the coefficients in the US interest rate equation. The Andrews-Ploberger test locates the date of the break in June 2004. However, coefficients in the Eurozone interest rate equation seem to be more stable. Therefore, I split the sample into two subsamples, namely before and after that date. The impulse response analysis showed that the responses of Eurozone inflation are completely different in the two samples. The same holds for output gap in the Eurozone. The importance of this result rests on the fact that the responses of those two variables have changed even though the coefficients in the Eurozone interest rate rule equation seem to be fairly stable. Additionally, the responses of inflation and output in the Eurozone in the second subsample are more pronounced. Keeping this finding I proceed to the construction of an open economy DSGE model, in order to provide some intuition behind the driving forces of this result.

I construct a two country DSGE model as in Benigno and Benigno (2001) and Benigno (2004). I extend their approach by allowing the coefficients in the interest rate rule to change over time. I assume, initially, that it is only the foreign country whose interest rate rule coefficients change over time. Moreover, the home country is initially assumed to not optimally react to foreign monetary policy. It rather adopts the standard Taylor rule with some interest rate smoothing. I show that even though the home Central Bank does not change its interest rate rule, a shift in the foreign monetary policy affects the volatilities of home inflation and output, as well as their responses to alternative shocks. Therefore, even though domestic monetary policy is constantly (and with a constant coefficient) aggressive towards inflation fluctuations, home inflation may exhibit increasing or decreasing volatility over time. Specifically, if there is a non zero probability that the

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<sup>2</sup>I use the Andrews-Ploberger test because of its virtue of identifying the break date.

coefficient on inflation in the foreign interest rate rule will be less than one in the near future, then not only foreign inflation will be more volatile, but also home inflation. This is so because, both home and foreign agents incorporate this probability in their future inflation expectations<sup>3</sup>. Committing, thus, to a specific, regime independent interest rate rule proves not to be enough to stabilize the economy of the home country.

Hence, as a next step, I solve the dynamic programming problem of the home Central Bank conditional on foreign monetary policy switching regimes over time, as in Zampolli (2006). I show that commitment to a time invariant interest rate rule is suboptimal for the home country. Home central bank must be always aggressive towards inflation fluctuations. That is, it must always have a coefficient on inflation in its interest rate rule that is greater than one. I find that as the probability that the foreign Central Bank decreases its coefficient on inflation to less than one rises, the home Central Bank should become more aggressive towards inflation. The opposite holds as the probability of the foreign coefficient on inflation becoming greater than one increases<sup>4</sup>. The intuition behind that result is consistent with initial finding of an increasing volatility of home inflation, as the probability of foreign monetary policy becoming dovish<sup>5</sup> in the future rises. Additionally, the coefficient on output gap must increase as well, when the foreign monetary policy becomes hawkish. The importance of finding rests on the fact that, when one country changes its policy, then the other country must do the same. Markov switching policy, thus, proves to be Pareto superior for the home country.

The paper is organized as follows. In section 2 a SVAR model is estimated using real time data for the Eurozone and the US, in order to motivate the theoretical model. In section 3 a two country DSGE model is constructed, allowing for regime switching in monetary policy of the foreign country. In section 4, the model is presented in its loglinear form. In section 5, the welfare criterion for the Home central bank is derived. In section 6 the solution technique of the Markov-Switching DSGE (MSDSGE) is described. In section

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<sup>3</sup>Throughout the paper I assume that the probability of a regime switch is the same for both home and foreign agents.

<sup>4</sup>The way the dynamic programming problem is constructed implies that there is a 'leader' (i.e. the foreign Central Bank) and a 'follower' (i.e. the home Central Bank). One, however, must not confuse this with the timing of the decisions. See also Benigno and Benigno (2001) for a similar characterization.

<sup>5</sup>As is standard in the literature, from now on dovish regime will refer to the case where the foreign coefficient on inflation is less than one. Hawkish regime will refer to the case where the foreign coefficient on inflation is larger than one.

7 the model is calibrated and simulated. In section 8 the dynamic programming problem of the Home central bank is solved, in order to find what the optimal reaction of the latter should be, conditional on the fact that foreign monetary policy switches regimes. Section 9 concludes.

## 2 Stylized facts

### 2.1 A SVAR model for the Eurozone and the US

In this section I present a structural VAR model for the Eurozone and the US. The objective is to show how key macroeconomic variables respond to various kinds of shocks in both areas. Since the focus of this paper is on changes in monetary policy of either one or, at the same time, both countries I perform paramtere stability tests, as in Boivin and Giannoni (2002, 2006).

The SVAR model consists of seven variables, namely output gap, inflation rate and nominal interest rates in the Eurozone and the US, and the real exchange rate. Such a model may lead to better policy implications. This is so, because the regions under consideration are close trade partners and, hence, it is likely that changes or shocks in the monetary policy of one country have important effect on the key macroeconomic variables of the other. Additionally, this allows us to draw inference on how each Central Bank should react with respect to the foreign monetary policy. The SVAR model receives the following form.

$$A_0 X_t = \Gamma_0 + \sum_{i=1}^p \Gamma_i X_{t-i} + u_t \quad (1)$$

where  $A_0$  is nonsingular, while the variance-covariance matrix of the fundamental disturbances  $\Sigma_u = E(u_t, u_t')$  is assumed to be diagonal. Since I am testing also the importance of real exchange rate targeting in the Eurozone, given that the Fed follows a Taylor-type rule<sup>6</sup>, the restrictions imposed allow for contemporaneous effects of the real exchange rate

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<sup>6</sup> I assume that the Taylor rule fits well US monetary policy until today. Clarida, Gali and Gertler (1998) estimating

in the policy rate, apart from those by CPI rate and the output gap. Therefore, the complete representation of the SVAR model is summarized as follows.

$$\begin{pmatrix} 1 & a_{12} & 0 & a_{14} & 0 & a_{16} & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ a_{31} & a_{32} & 1 & 0 & 0 & 0 & 0 \\ a_{41} & 0 & 0 & 1 & a_{45} & 0 & 0 \\ 0 & a_{52} & 0 & a_{54} & 1 & a_{56} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & a_{75} & a_{76} & 1 \end{pmatrix} \begin{pmatrix} CPI_{Euro} \\ Gap_{Euro} \\ i_{Euro} \\ RER \\ CPI_{US} \\ Gap_{US} \\ i_{US} \end{pmatrix}_t = \\
= \begin{pmatrix} \gamma_{10} \\ \gamma_{20} \\ \gamma_{30} \\ \gamma_{40} \\ \gamma_{50} \\ \gamma_{60} \\ \gamma_{70} \end{pmatrix} + \begin{pmatrix} \gamma_{11} & \gamma_{12} & \gamma_{13} & \gamma_{14} & \gamma_{15} & \gamma_{16} & \gamma_{17} \\ \gamma_{21} & \gamma_{22} & \gamma_{23} & \gamma_{24} & \gamma_{25} & \gamma_{26} & \gamma_{27} \\ \gamma_{31} & \gamma_{32} & \gamma_{33} & \gamma_{34} & \gamma_{35} & \gamma_{36} & \gamma_{37} \\ \gamma_{41} & \gamma_{42} & \gamma_{43} & \gamma_{44} & \gamma_{45} & \gamma_{46} & \gamma_{47} \\ \gamma_{51} & \gamma_{52} & \gamma_{53} & \gamma_{54} & \gamma_{55} & \gamma_{56} & \gamma_{57} \\ \gamma_{61} & \gamma_{62} & \gamma_{63} & \gamma_{64} & \gamma_{65} & \gamma_{66} & \gamma_{67} \\ \gamma_{71} & \gamma_{72} & \gamma_{73} & \gamma_{74} & \gamma_{75} & \gamma_{76} & \gamma_{77} \end{pmatrix} \begin{pmatrix} CPI_{Euro} \\ Gap_{Euro} \\ i_{Euro} \\ RER \\ CPI_{US} \\ Gap_{US} \\ i_{US} \end{pmatrix}_{t-1} + \begin{pmatrix} u_{1,t} \\ u_{2,t} \\ u_{3,t} \\ u_{4,t} \\ u_{5,t} \\ u_{6,t} \\ u_{7,t} \end{pmatrix}$$

The reduced form of the VAR model, thus, is specified as

$$X_t = A_0^{-1}\Gamma_0 + A_0^{-1}\sum_{i=1}^p \Gamma_i X_{t-i} + \varepsilon_t$$

where  $\varepsilon_t = A_0^{-1}u_t$  are the reduced form errors with a variance-covariance matrix  $\Sigma_\varepsilon = E(\varepsilon_t, \varepsilon_t') = A_0^{-1}E(u_t, u_t')A_0^{-1} = A_0^{-1}\Sigma_u A_0^{-1}$ .

The target of this paper is to figure out whether there have been changes in the way monetary policy was conducted until today by both the ECB and the Fed, and, if so, what does this imply for what the optimal monetary policy of the home country, i.e. the Eurozone, should be. Therefore, in the empirical model stability tests are performed. The policy rules for the US find that the real exchange rate or foreign variables are not statistically significant in the interest rate rule.

strategy followed is similar to that in Boivin and Giannoni (2002). In particular, I am interested in testing for the stability of the parameters of the SVAR model throughout the sample. For each equation of the SVAR model, the stability of all the coefficients is tested<sup>7</sup>. I test for parameter stability using two tests. The first test is the Wald version of the Quandt test, or the Andrews sup-Wald test. The second is the Andrews-Ploberger test<sup>8</sup>. The former has the virtue that it has power against various alternatives, as far as the process of the structural parameters is concerned. The latter is able to locate the timing of the break, if there is one. If there is evidence of parameter instability, then the impulse responses computed using the model estimated for the whole sample are no longer valid. Therefore, if this is the case, we split the sample in smaller subsamples, depending on the timing of the break, estimated by the Andrews-Ploberger test.

Given that some authors have argued in favour of changes in the size of shocks hitting the economy, rather than changes in the structural parameters, being the reason for changes in the transmission of monetary policy, heteroskedasticity tests in the estimated residuals are also performed. For each equation specific estimated residual the *LM* test for *ARCH* effects is used.

## 2.2 Data

Monthly real time data were gathered from the ECB statistical warehouse and the Federal Reserve Bank of Philadelphia. The dataset spans from 1999:1 through 2010:6. GDP is proxied by total industrial production. CPI for each region is used as the inflation rate. As far as the policy rates are concerned, the Federal Funds rate for the US and the interbank overnight rate for the Eurozone are used. Finally, the nominal exchange rate is proxied by the end of period euro-dollar rate.

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<sup>7</sup>Evidence of parameter instability in monetary VAR models is mixed. Boivin and Giannoni (2002), Bernanke, Gertler and Watson (1997) and Boivin (2006) find evidence of parameter instability, while Christiano, Eichenbaum and Evans (1999) find the opposite.

<sup>8</sup>Note that the heteroskedasticity robust version of both tests was used.

## 2.3 Empirical results

### 2.3.1 Stability and heteroskedasticity tests

In this section I estimate the SVAR model<sup>9</sup>. For each equation's coefficients the above mentioned stability tests are performed. At table 1 below the  $p$  – values from both tests are reported<sup>10</sup>.

Table 1: Stability Tests on Reduced-form VAR coefficients

Dep. vrb	Regressors						
	$CPI_{Euro}$	$Gap_{Euro}$	$i_{Euro}$	$RER$	$CPI_{US}$	$Gap_{US}$	$i_{US}$
$CPI_{Euro}$	0.0181	0.9491	0.0189	0.0415	0.0174	0.4007	0.0353
$Gap_{Euro}$	0.7225	0.2944	0.7338	0.7030	0.7407	0.3018	0.6947
$i_{Euro}$	0.0508	0.6871	0.1231	0.0432	0.0497	0.5500	0.0825
$RER$	0.0008	0.5122	0.0002	0.0015	0.0007	0.7031	0.0047
$CPI_{US}$	0.5558	0.4223	0.2338	0.6056	0.5608	0.4859	0.1903
$Gap_{US}$	0.0112	0.0561	0.0132	0.0429	0.0112	0.1491	0.0388
$i_{US}$	0.0025	0.6122	0.0000	0.0030	0.0026	0.2339	0.1093

Notes:  $p$  – values reported. Red: Significant at 1% s.l., Blue: Significant at 5% s.l.

Stability tests at Table 1 show that at 1% significance level, monetary policy in the US seems to have changed over the sample considered. Four out of seven coefficients in the equation for the Fed Funds rate have changed over time. On the other hand monetary policy in the Eurozone has not changed at 1% significance level. At 5% significance level, though, the coefficients on lagged foreign inflation and the real exchange rate appear to have changed. As for the output gap in the Eurozone, it seems to be fairly stable. I derive the same result for CPI in the US. On the other hand the coefficients in the Eurozone CPI and the US output gap equations are subject to breaks at 5 %significance levels. Although, it is easy to interpret breaks in the coefficients in the interest rate equations

<sup>9</sup>The lag length of the VAR model was chosen based on the  $AIC$  and the  $BIC$  criterion. Both criteria showed that 2 lags is optimal.

<sup>10</sup>We report  $p$  – values obtained only from the Andrews-Ploberger test in order to save space. The results from the Andrews-Quandt test lead to the same conclusions.

as changes in the way monetary policy is conducted, breaks in the CPI and the output gap equations are less easy to interpret. As regards Eurozone CPI, it is shown that the coefficients on lagged domestic and foreign CPI rates are subject to breaks. This could be attributed to changes in the degree of openness in the Eurozone, or home bias. Taking into account the structure of a New-Keynesian Phillips curve, the break in the coefficient on lagged interest rate in the Eurozone CPI equation, could be due to either a change in the frequency of price adjustments, or a change in the degree of backward lookingness in price setting behaviour, or a change in the degree of risk aversion, or change in the degree of habits in consumption, or a combination of all the above. Finally, the changes in the coefficients on lagged Eurozone CPI rate, on lagged Eurozone interest rate, on lagged real exchange rate, on lagged US CPI rate and on lagged US interest rate in the US output gap equation could be attributed to changes in the degree of openness of the US economy, the degree of risk aversion, the degree of endogenous persistence in output, or to a combination of those three factors. I keep, however, the fact that monetary policy seems to have changed, particularly in the US, which is the principle motivation of this paper.

Finally, the Andrews-Ploberger test showed that the break in the US interest rate equation took place in June 2006. I, thus, use this estimate to split the initial sample into two subsamples when I will be doing the impulse response analysis in the next section.

The last testing performed was on the variance of the estimated equation specific residuals. As already mentioned, I tested for this using the *LM* test for *ARCH* effects. The results are shown at table 2 below.

Table 2: Heteroskedasticity tests

	<i>p - values</i>
$CPI_{Euro}$	0.6088
$Gap_{Euro}$	0.1550
$i_{Euro}$	0.0105
$RER$	0.5734
$CPI_{US}$	0.2365
$Gap_{US}$	0.4856
$i_{US}$	0.4261

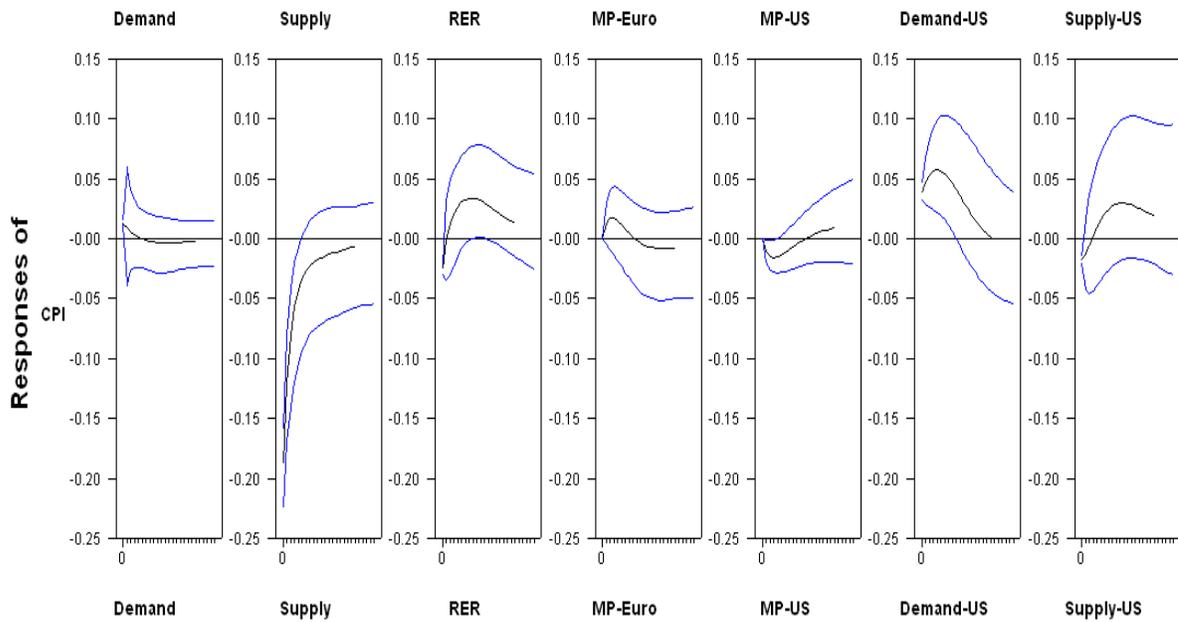
Results at table 2 above show that at 5% significance level only the variance of the residuals from the Eurozone interest rate equation seems to have changed over time. In the two country DSGE model presented in subsequent sections on the paper, I will consider the case where there is such change, as well, apart from the changes in the coefficients in the foreign country's interest rate rule.

### 2.3.2 Impulse responses

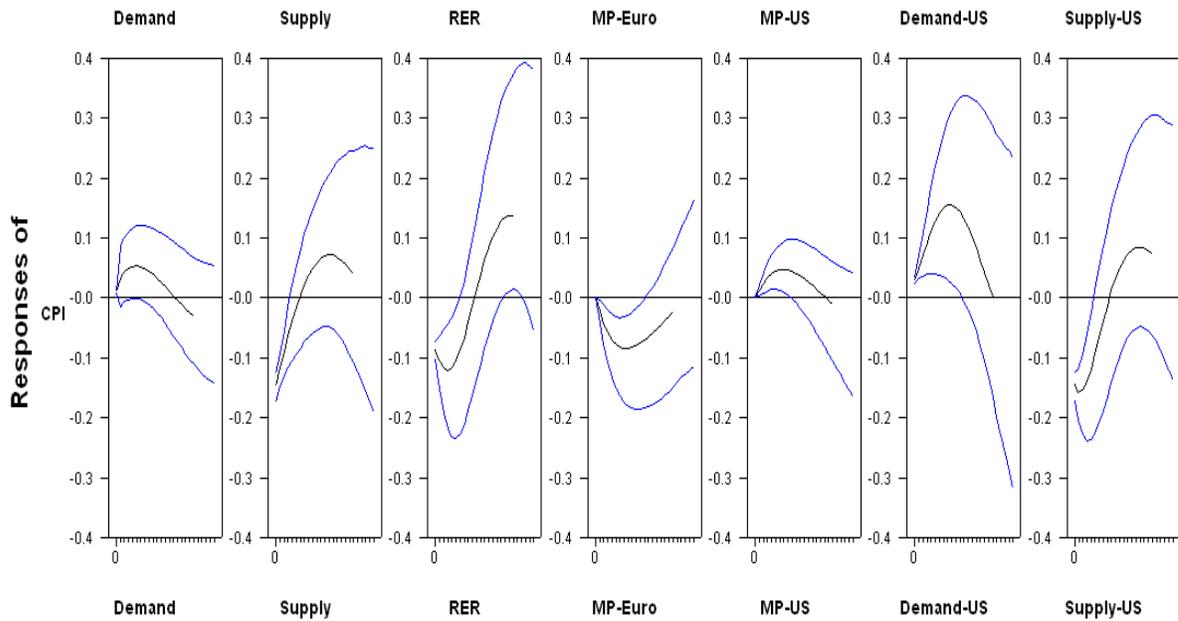
In this section the impulses responses are computed. Since the Andrews-Ploberger stability test suggested that there has been a change in the US monetary policy at 1% significance level, the sample was split into two subsamples. Namely, before and after June 2004. The impulse responses of the variables of the VAR model are computed for each subsample. Given the presence of breaks in the whole sample, those should be expected to differ depending on the sample used in the estimation in each case. At figure 2 below I present the responses of CPI in the Eurozone following a contractionary monetary policy shock, a positive cost-push shock, a positive demand shock and a positive RER shock in both the Eurozone and the US.

**Figure 2: Impulse Responses of Eurozone CPI to alternative shocks.**

*Sample: 1999:1 - 2004:6*



*Sample: 2004:7 - 2010:6*



Notes: Blue lines: 95% posterior confidence interval.

From the impulse responses I derive the following results. The impulse responses are different in the two samples. In particular, CPI inflation is more volatile and persistent in the second sample for all kinds of shocks considered<sup>11</sup>. Moreover, the sign of the initial impact seems to change as well. For example, CPI initially jumps in the first sample, after a monetary policy shock in the Eurozone. On the contrary, it falls in the second sample. We derive the same result when looking at the impulses following a demand shock in the US. Given that we found evidence against a switch in the monetary policy of the ECB, higher inflation volatility in the Eurozone in the second sample could be attributed to the cumulative effect of the US monetary policy being less aggressive against inflation fluctuations for a long period of time. This is so, because it takes time for the Eurozone CPI to incorporate the effects of the foreign monetary policy. At table 3 below, the standard deviation of CPI rates in the Eurozone and the US is computed<sup>12</sup>.

Table 3: Inflation Volatility

	<i>Sample 1</i>	<i>Sample 2</i>
<i>Eurozone</i>	0.5175	0.7023
<i>US</i>	0.7486	1.2730

Notes: Sample 1: 1991:1 to 2004:6 while  
Sample 2: 2004:7 to 2009:12 using ex-post data

### 2.3.3 Robustness checks

In order to check the sensitivity of the results found so far, various robustness exercises are implemented. The first one considers alternative measures for the output gap. The procedure followed is similar to that in CGG (2000). In particular, instead of using the *hp – filter*, the output gap was measured as the deviation of *log* industrial output from a fitted quadratic function of time. The results do not differ significantly<sup>13</sup>. Both the *AIC* and the *BIC* information criteria show that two is the optimal choice of lags in the

<sup>11</sup>Impulse responses of the output gap lead to the same conclusion. The latter is less volatile and persistent after all kinds of shocks, in the first sample.

<sup>12</sup>Volatilities were also computed for sample 2 until the recent financial crisis. The results are the same (i.e. higher volatility in the second sample). Not surprisignly though, volatilities in this smaller second sample are smaller than our original sample 2.

<sup>13</sup>We do not show the results of the robustness exercise here, in order to save space.

VAR model. The parameter stability tests do not differ significantly from those reported at table 1 above. The Andrews-Ploberger test locates a break in the parameters in the Federal Funds rate equation in June 2004, as was the case when the *hp* – *filter* was used instead. However, what seems to change now is the coefficients only on the lags of the Euro-rate at 1% significance level. The coefficients on the rest the parameters remain unchanged<sup>14</sup>. The *LM* test for *ARCH* effects provides the same results as before. That is, only the the variance of the errors in the Euro-rate equation changes at 1% significance level. Finally, the impulse responses lead to the same conclusion as above. Both the CPI and the output gap in the Eurozone are more volatile and persistent, following any kind of shock, in the second sample.

As a second exercise, a more parsimonious SVAR model was constructed. Given that the dataset is small, it is likely that the impulse responses may not be accurate, the higher the number of the free parameters to be estimated in matrix *A* in (1). Therefore, a new SVAR model was estimated allowing for  $a_{31}, a_{32}, a_{75}, a_{76}$  to be the only free parameters to be estimated. The key results, found so far, do not change. The impulse responses of the CPI and the the output gap in the Eurozone show that both are more volatile and persistent in sample 2<sup>15</sup>.

Moreover, the importance of additional targets in the interest rate rule of both central banks was tested. That is, it was assumed that the each of rest the variables in the system has a contemporaneous effect on the interest rate of each region. At first, the strategy followed was to test the importance of each of the parameters in matrix *A* individually, so that to avoid the cost of loosing degrees of freedom. Then, the case where both banks react to foreign variables or the RER, jointly, was considered. Targeting the RER is

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<sup>14</sup>Remember that when the *hp* – *filter* was used, the Andrews-Ploberger test found that the coefficients on the US and the Euro CPI, the Eurozone output gap change and the real exchange rate, as well, apart from those on the lags of the Euro-rate.

<sup>15</sup>Setting  $a_{12} = a_{16} = a_{52} = a_{56} = a_{75} = a_{76} = 0$  does has negligible effects on the impulse responses. Setting, though,  $a_{14} = a_{54} = 0$  has nonnegligible effects on the impulse responses. That is, allowing for a contemporaneous effect of real exchange rate shocks on the CPI in either country changes the behavior of both the output gap and inflation. In the first subsample, the Eurozone output gap is less volatile after a shock to the RER than when  $a_{14}, a_{54} \neq 0$ . The same holds for the Eurozone CPI. In the second subsample, the Eurozone CPI is much less volatile after a shock to the RER. Following a demand shock, though, the latter is more volatile. The output gap in the Eurozone is more volatile after a RER shock whenever  $a_{14} = a_{54} = 0$ . However, as regards the rest of the shocks, the effects of not allowing for contermpraneous effects of RER shocks to the CPI are negligible. Finally, note that still the main conclusion does not change. All variables are more volatile in the secong subsample.

beneficial for both central banks only in sample 1. It is enough that only one of the two banks adopts a target for the real exchange rate. However, the opposite holds in sample 2, where RER targeting does worse than the initial specification in matrix  $A$ . Reacting to foreign inflation yields nonnegligible gains<sup>16</sup> to both regions. But this holds only for sample 1. Moreover, the sign of the initial responses of some variables, after some shocks, seems to be reversed. When both banks react to the foreign interest rate, there are significant gains regarding inflation fluctuations, in sample 1, especially after a monetary policy shock in the Euro-rate. On the contrary, this no longer holds in sample 2, where reacting to the foreign rate seems not preferable. Finally, foreign output gap targeting allows for lower inflation and output fluctuations in both regions, regardless of the sample.

The possibility, though, of both central banks targeting at the same time foreign variables and/or the real exchange rate was also considered. The results do not change importantly.

#### 2.3.4 A Markov switching interest rate rule for the US

Taking into account the stability test results of section 2.4.1, I now estimate an interest rate rule for the US, which receives the following form

$$i_t = \alpha_0(s_t) + \alpha_\pi(s_t)\pi_t + \alpha_x(s_t)x_t + \varepsilon_t \quad (2)$$

where  $\pi_t$  is inflation and  $x_t$  is the output gap.  $s_t$  indicates the monetary policy regime and follows a two-state Markov chain. The sample I use is the same as that used for the estimation of the structural VAR model above. Table 4 reports the parameter estimates.

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<sup>16</sup>By gains, we mean lower inflation and output gap fluctuations.

Table 4

Monetary policy rule estimates

<i>States</i>	<i>Active</i>	<i>Passive</i>
	$s_t = 1$	$s_t = 2$
$\alpha_\pi$	1.4562(0.00)	0.3798(0.02)
$\alpha_x$	0.5934(0.01)	0.4803(0.02)
$\sigma_\varepsilon$	1.8785e-003	2.2436e-003

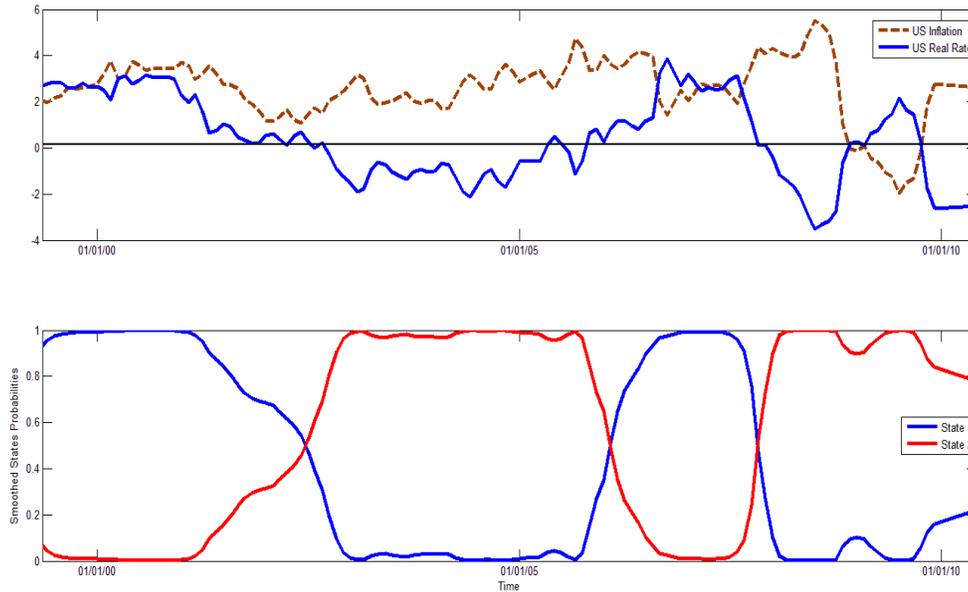
Log likelihood value = -235.8437. P-values in parentheses

the estimated transition matrix is as follows:

$$P = \begin{bmatrix} 0.91 & 0.09 \\ 0.09 & 0.91 \end{bmatrix} \quad (3)$$

Figure 3 below consists of two panels. The upper panel plots the US real interest rate and inflation. The bottom panel plots the estimated transition probabilities for each regime.

Figure 3



The estimated Markov-switching interest rate rule seems to capture the monetary policy of the fed quite well. The real interest rate starts to fall more than inflation from late 2001

onwards, implying a switch to a more accommodative monetary policy. Indeed, looking at the transition probabilities over the same period, it seems that the probability of staying in state 1 (hawkish) falls (blue line), while the probability of state 2 (dovish) rises (red line). Throughout the period from 2002 to mid-late 2006, where the real rate is much lower than inflation and negative, the probability of the dovish regime (red line) is close to one. On the other hand the real rate exceed inflation for the period from late 2006 to late 2007. During that period there is a switch as the Markov switching rule indicates. That is, the probability of switching and then staying in state 1 (hawkish) goes up. Overall, the estimated rule captures fed's policy well. The only exception is the year 2009 where the real rate exceed inflation.

The results shown above are in line with the existing literature. Bekaert et al. (2011) estimate a closed economy New-Keynesian model where the parameters in the interest rate rule change according to a Markov chain. Using quarterly data for the US, they find that from 1999, which is the start of the sample in this paper, until late 2004, the monetary policy of the fed has been active<sup>17</sup>. From mid 2005, the US monetary policy is accommodative, while the probability of a switch to a high inflation regime rises as well. In particular Bekaert et al. note that "*in 2000 there was a switch to the activist regime, as interest rates rapidly declined*". Moreover, regarding the accommodative policy during 2002-2006, they note that "*the recent credit crisis starting in 2007 is preceded by a passive monetary policy*".

The finding of an accommodative US monetary policy from early 2003 until mid to late 2006 is in line with description of the fed's policy by Ben Bernanke (2010) at the Annual Meeting of the American Economic Association in Atlanta stating the following:

*"The low policy rates during the 2002-06 period were accompanied at various times by "forward guidance" on policy from the Committee. For example, beginning in August 2003, the FOMC noted in four post-meeting statements that policy was likely to remain accommodative for a considerable period"*.

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<sup>17</sup>In their paper active is equivalent to the hawkish regime in my paper.

### 2.3.5 Key Results

From the empirical analysis above, we keep the following key messages. The first is that, there were changes in US monetary policy since the adoption of the common currency in Europe, which seem to have affected the volatility of key macroeconomic variables not only in the US, but also in the Eurozone. Moreover, this change in US monetary policy has affected the way macroeconomic aggregates react to various kinds of domestic and foreign shocks. Therefore, changes in the way monetary policy is conducted in the foreign country have important implications on the behavior of the Home country macroeconomic variables, even though domestic monetary policy does not change. The degree of openness and, hence, terms of trade effects are likely to be one of the main driving forces for this result. The second is that, there were changes in the behavior of the private sector, as well. This implies that the way expectations were formed, as well as the price setting decisions might have changed over the time considered in this analysis. Finally, empirical results show that changes in the shock variances do not seem to be the cause for the differences in impulse responses and volatility observed in the two subsamples. Keeping, thus, those facts I proceed to the construction of a two country DSGE model, in order to decompose the effects of those results, theoretically, and to figure out what the optimal policy of the Home country should be, given that foreign monetary policy switches regimes. Throughout, the Eurozone is assumed to be the home country, while the US is the foreign.

## 3 The model

### 3.1 *Households*

In this section, I specify the structure of the baseline, two country stochastic general equilibrium model. Each country is populated by a continuum of infinitely lived and identical households in the interval  $[0, 1]$ . Foreign variables are denoted with an asterisk. There are two kinds of households as in Amato and Laubach (2003). We allow  $\psi$  to denote the probability that the household is able to choose its consumption optimally, and which

is independent of the household's history. Therefore, by the law of large numbers, in each period a fraction  $\psi$  of households will reoptimize, whereas the remaining fraction  $1 - \psi$  will not. The latter will choose its consumption in period  $t$  according to the following rule of thumb

$$C_t^R = C_{t-1} \quad (4)$$

where  $C_t$  denotes aggregate per capita consumption in period  $t$ . The remaining  $1 - \psi$  of households choose  $C_t^O$  so as to maximize their utility. Thus, per capita consumption in period  $t$  is given by

$$C_t = \psi C_t^O + (1 - \psi) C_t^R \quad (5)$$

As in Laubach and Amato, this modification to the consumer's problem is based on the assumption that it is costly to reoptimize every period<sup>18</sup>. The households who choose consumption optimally choose  $C_t^O$  to maximize their utility function. They derive utility from consumption and disutility from labor supply. The utility function, thus, is specified as

$$U_t = E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[ \frac{(C_s)^{1-\sigma}}{1-\sigma} - \frac{(L_s)^{1+\gamma}}{1+\gamma} \right] \quad (6)$$

where  $\sigma$  is the degree of relative risk aversion.  $C_t$  is a composite consumption index described as

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<sup>18</sup>Amato and Laubach note that Rule (1) has the important feature that rule-of-thumb consumers learn from optimizing households with one period delay. Hence, although Rule (1) is not optimal, it has three important properties. First agents are not required to compute anything. Second, rule-of-thumb households learn from optimizing ones, because lat period's decisions by the latter are part of  $C_{t-1}$ . Third, the differences between  $C_t^R$  and  $C_t^O$  are bounded, and will be zero in the steady state.

$$C_t = \left[ \delta^{\frac{1}{\rho}} C_{H,t}^{\frac{\rho-1}{\rho}} + (1-\delta)^{\frac{1}{\rho}} C_{F,t}^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}} \quad \rho > 1 \quad (7)$$

$$C_t^* = \left[ (\delta^*)^{\frac{1}{\rho}} (C_{F,t}^*)^{\frac{\rho-1}{\rho}} + (1-\delta^*)^{\frac{1}{\rho}} (C_{H,t}^*)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}}$$

where  $\rho$  captures the intratemporal elasticity of substitution between home and foreign goods.  $\delta > \frac{1}{2}$  is a parameter of home bias in preferences.  $C_H$  is the home consumption index.  $C_F$  is the foreign consumption index. Consumption indices in the home and the foreign country are defined as

$$C_{H,t} = \left[ \int_0^1 c_t(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}, \quad C_{F,t} = \left[ \int_0^1 c_t(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}} \quad (8)$$

$$C_{H,t}^* = \left[ \int_0^1 c_t^*(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}, \quad C_{F,t}^* = \left[ \int_0^1 c_t^*(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}$$

Money deflator is given by the aggregate consumption price index for the home and foreign country respectively, which is specified as

$$P_t = \left[ \delta (P_{H,t})^{1-\rho} + (1-\delta) P_{F,t}^{1-\rho} \right]^{\frac{1}{1-\rho}} \quad (9)$$

$$P_t^* = \left[ \delta^* (P_{F,t}^*)^{1-\rho} + (1-\delta^*) P_{H,t}^{*1-\rho} \right]^{\frac{1}{1-\rho}}$$

where  $P_H$  and  $P_F$  are price indices for home and foreign goods, expressed in the domestic currency and  $\tau_t$  captures the time varying transaction cost assumed to follow a stationary  $AR(1)$ ,  $\tau_t = \rho_\tau \tau_{t-1} + \nu_t$ ,  $\nu_t \sim N(0, \sigma^2)$ . The price indices for the Home and Foreign country are defined as

$$P_{H,t} = \left[ \int_0^1 p_t(z)^{1-\theta} dz \right]^{\frac{1}{1-\theta}}, \quad P_{F,t} = \left[ \int_0^1 p_t(z)^{1-\theta} dz \right]^{\frac{1}{1-\theta}} \quad (10)$$

$$P_{H,t}^* = \left[ \int_0^1 p_t^*(z)^{1-\theta} dz \right]^{\frac{1}{1-\theta}}, \quad P_{F,t}^* = \left[ \int_0^1 p_t^*(z)^{1-\theta} dz \right]^{\frac{1}{1-\theta}}$$

In each period  $t$  the economy experiences one of the finitely many events  $s^t \in \Omega$  ( $\Omega$  being the set of the finitely many states). Let  $h^t$  denote the history of realized states until period  $t$  included. The probability of particular state to occur is defined as  $\pi(s^{t+1}|h^t)$ . The initial realization  $s_0$  is given.

Capital markets are complete. The consumers of both countries purchase state contingent bonds denominated in the domestic currency,  $B_t$  for domestic agents and  $B_t^*$  for foreign agents at price  $Q_t$ . That is  $B_t$  denotes the home agent's holdings of a one period nominal bond paying one unit of the home currency.

The home agent maximizes her utility subject to the period budget constraint

$$P_t C_t + Q_{t,t+1} B_{t+1} = B_t + W_t L_t + \Pi_t \quad (11)$$

where  $W_t$  is the nominal wage and  $\Pi_t$  are nominal profits the individual receives.

### 3.2 *First order conditions*

Maximizing the utility function (1) subject to the budget constraint (6) yields the following first order conditions

$$Q_{t,t+1} = \frac{\beta P_t}{P_{t+1}} \left( \frac{C_t^O}{(C_{t+1}^O + x)} \right)^\sigma \quad (12)$$

$$L_t = (C_t^O)^{-\frac{\sigma}{\gamma}} w_t^{\frac{1}{\gamma}} \quad (13)$$

where the first equation is the usual Euler equation, the second determines the labor supply schedule and the third the demand for real money balances.

Individual demands for each good  $z$  produced in the home and in the foreign country respectively are expressed as

$$c_{h,t}(z) = \left( \frac{p_t^h(z)}{P_{H,t}} \right)^{-\theta} \left( \frac{P_{H,t}}{P_t} \right)^{-\rho} \delta C_t \quad (14)$$

$$c_{f,t}(z) = \left( \frac{p_t^f(z)}{P_{F,t}} \right)^{-\theta} \left( \frac{P_{F,t}}{P_t} \right)^{-\rho} (1 - \delta) C_t \quad (15)$$

### 3.3 Risk sharing

The fraction of foreign households who choose their consumption optimally ( $\psi^*$ ), maximize their utility subject to their budget constraint specified as

$$P_t^* C_t^* + \frac{Q_{t,t+1} B_{t+1}^*}{z_t} = \frac{B_t^*}{z_t} + W_t^* L_t^* + \Pi_t^* \quad (16)$$

where  $z_t$  is the nominal exchange rate defined as the domestic currency price of the foreign currency.

Therefore, the Euler equation from the foreign agent's maximization problem is

$$Q_{t,t+1} = \frac{\beta P_t^* z_t}{P_{t+1}^* z_{t+1}} \left( \frac{C_t^{O*}}{C_{t+1}^{O*}} \right)^\sigma \quad (17)$$

International financial markets are complete. Domestic and foreign households trade in the state contingent one period nominal bonds denominated in the domestic currency. Therefore, combining (9) and (14), we receive the following optimal risk sharing condition

$$\left( \frac{C_t^{O*}}{C_t^O} \right)^{-\sigma} = \varpi q_t \quad (18)$$

where  $\varpi \equiv \left( \frac{C_0^f + x}{C_0^h + x} \right)^{-\sigma} \frac{P_0}{z_0 P_0^*}$  depends on initial conditions and  $q_t = \frac{z_t P_t^*}{P_t}$  is the real exchange rate.

### 3.4 Price setting

There are two types of firms, the backward looking and the forward looking. As a result, inflation will depend on both its lagged and forward values. Prices are sticky with a price setting behavior *à la* Calvo (1983). At each date, each firm changes its price with a probability  $1 - \omega$ , regardless of the time since it last adjusted its price. The probability of not changing the price, thus, is  $\omega$ . The probability of not changing the price in the subsequent  $s$  periods is  $\omega^s$ . Consequently, the price decision at time  $t$  determines profits for the next  $s$  periods. The price level for home goods at date  $t$  will be defined as

$$P_{H,t} = [\omega P_{H,t-1}^{1-\theta} + (1 - \omega) \tilde{p}_t(h)^{1-\theta}]^{\frac{1}{1-\theta}} \quad (19)$$

Firms that are given the opportunity to adjust their prices will either follow a rule of thumb (backward looking firms) or will chose the price that maximizes their expected discounted profits (forward looking firms). The price  $\tilde{p}_t(h)$  that will be set at date  $t$  is specified as

$$\tilde{p}_t(h) = \zeta p_t^B(h) + (1 - \zeta) p_t^F(h) \quad (20)$$

where  $\zeta \in (0, 1)$  is the fraction of backward looking firms,  $p_t^B(h)$  and  $p_t^F(h)$  is the price set by the backward and the forward looking firms, respectively. A continuum of firms is assumed for the home economy indexed by  $z \in [0, 1]$ . Each firm produces a differentiated good, with a technology

$$Y_t(z) = A_t L_t(z) \quad (21)$$

where  $A_t$  is a country specific productivity shock at date  $t$  which is assumed to follow a log stationary process

The structure of productivity shocks across the two countries receives the following form

$$\begin{bmatrix} \alpha_t \\ \alpha_t^* \end{bmatrix} = \begin{bmatrix} \rho_{\alpha_t} & \rho_{\alpha_t \alpha_t^*} \\ \rho_{\alpha_t^* \alpha_t} & \rho_{\alpha_t^*} \end{bmatrix} \begin{bmatrix} \alpha_{t-1} \\ \alpha_{t-1}^* \end{bmatrix} + \begin{bmatrix} \varepsilon_{\alpha,t} \\ \varepsilon_{\alpha^*,t} \end{bmatrix}$$

where  $\begin{bmatrix} \varepsilon_{\alpha,t} \\ \varepsilon_{\alpha^*,t} \end{bmatrix} \sim N(0, \Sigma^2)$ , with  $\Sigma^2 = \begin{bmatrix} \sigma_{\varepsilon_a}^2 & 0 \\ 0 & \sigma_{\varepsilon_{\alpha^*}}^2 \end{bmatrix}$ .

Each firm chooses a price for the home market and a price for the foreign market.

*Backward looking firms.*

Backward looking firms set their prices according to the following rule

$$p_t^B(h) = P_{H,t-1} + \pi_{H,t-1} \quad \text{and} \quad p_t^{B^*}(h) = P_{H,t-1}^* + \pi_{H,t-1}^* \quad (22)$$

*Forward looking firms.*

Forward looking firms set their prices by maximizing their expected discounted profits. Their maximization problem comprises of two decisions. The one concerns the price for the domestic market and the other the price charged in the foreign market, when it exports. Hence their maximization problem is described as

$$\max E_t \sum_{s=0}^{\infty} \omega^s Q_{t,t+s} \left\{ \tilde{p}_t(h) y_{t+s}^h(h) + \varepsilon_t \tilde{p}_t^*(h) y_{t+s}^f(h) - W_{t+s}^h L_{t+s}^h \right\} \quad (23)$$

where  $y_t^i(h)$ ,  $i = h, f$  is the demand for the home good for home and foreign agents specified as

$$y_t^h(p_t(h)) = \left( \frac{p_t(h)}{P_{H,t}} \right)^{-\theta} \left( \frac{P_{H,t}}{P_t} \right)^{-\rho} \delta^* C_t, \quad (24)$$

$$y_t^f(p_t^*(h)) = \left( \frac{p_t^*(h)}{P_{H,t}^*} \right)^{-\theta} \left( \frac{P_{H,t}^*}{P_t^*} \right)^{-\rho} (1 - \delta^*) C_t^* \quad (25)$$

The firm maximizes its objective function (25) subject to (26) in order to find the optimal price for the Home good in the Home economy. It maximizes subject to (27), in order to find the optimal price for the Home good in the Foreign economy. The firm chooses a price for the Home good in the Home economy that satisfies the first order condition

$$E_t \sum_{s=0}^{\infty} \omega^s Q_{t,t+s} y_{t+s}(p_t(h)) \left\{ p_t(h) - \frac{\theta}{\theta-1} MC_{t+s} \right\} = 0$$

where  $MC_{t+s} = \frac{W_{t+s}}{A_{t+s}}$  denotes the nominal marginal cost and  $\frac{\theta}{\theta-1}$  captures the optimal markup.

The optimal price, thus, for the Home good in the Home country is specified as

$$p_t(h) = \frac{\theta}{\theta-1} \frac{E_t \sum_{s=0}^{\infty} \omega^s Q_{t,t+s} MC_{t+s} y_{t+s}^h(p_t(h))}{E_t \sum_{s=0}^{\infty} \omega^s Q_{t,t+s} y_{t+s}^h(p_t(h))} \quad (26)$$

Respectively, the optimal price for the Home good in the Foreign country is specified as

$$p_t^*(h) = \frac{\theta}{\theta-1} \frac{E_t \sum_{s=0}^{\infty} \omega^s Q_{t,t+s} MC_{t+s} y_{t+s}^f(p_t^*(h))}{E_t \sum_{s=0}^{\infty} \omega^s Q_{t,t+s} y_{t+s}^f(p_t^*(h)) z_{t+s}} \quad (27)$$

Finally, dividing (16) by  $P_{H,t-1}$ :

$$\Pi_{H,t}^{1-\theta} = \omega + (1 - \omega) \left( \frac{\tilde{p}_t(h)}{P_{H,t-1}} \right)^{1-\theta} \quad (28)$$

where  $\Pi_{H,t} \equiv \frac{P_{H,t}}{P_{H,t-1}}$ .

Similarly, for the foreign goods consumed in the home economy:

$$\Pi_{F,t}^{1-\theta} = \omega + (1 - \omega) \left( \frac{\tilde{p}_t(f)}{P_{F,t-1}} \right)^{1-\theta} \quad (29)$$

The aggregate price level dynamics are specified, thus, as

$$\Pi_t^{1-\rho} = \delta \left[ \left( \frac{P_{H,t-1}}{P_{t-1}} \right) \Pi_{H,t} \right]^{1-\rho} + (1 - \delta) \left[ \left( \frac{P_{F,t-1}}{P_{t-1}} \right) \Pi_{F,t} \right]^{1-\rho} \quad (30)$$

## 4 Markov Switching Monetary Policy

Monetary policy is conducted through nominal interest rate rules by the Central Bank. A weakness of the existing literature on monetary policy in open economy models is that it fails to take into account potential regime switches in the way monetary policy is conducted in either one of the two or in both countries. I allow for such changes. I first show that even though domestic monetary policy may not switch, a switch in the foreign monetary policy has effects on the volatility of domestic output and inflation, given the structure of the model. In section 8, it is shown that optimal monetary policy for the home country suggests it changes the coefficients in its interest rate rule, depending on which regime foreign monetary policy lies in and, of course, on the probabilities of a switch.

### 4.1 Policy rules

In this subsection I describe how Markov switching is introduced into the model. A markov switching interest rate rule for the foreign country is specified as

$$i_t^* = i_{t-1}^{*\rho_{s_t}^*} \left( \xi_{s_t}^* \left( \frac{\pi_t^*}{\tilde{\pi}^*} \right)^{\phi_{\pi^*,s_t}^*} \tilde{y}_t^{*\phi_{y^*,s_t}^*} \right)^{1-\rho_{s_t}^*} e^{\varepsilon_t^*} \quad (31)$$

where  $s_t$  captures the realized policy regime taking values 1 or 2. Regime follows a Markov process with transition probabilities  $p_{ji} = P[s_t = i | s_{t-1} = j]$ , where  $i, j = 1, 2$ .  $\xi_t$  is a scale parameter,  $\tilde{\pi}^*$  is the inflation target and  $\tilde{y}_t^*$  is the output gap. This specification implies that the policy maker and the private sector observes the current regime. Therefore, private sector expectations about future inflation, for example, are specified as  $E[\pi_{t+1} | \Omega_t^{-s}]$ , where  $\Omega_t^{-s} = \{s_{t-1}, \dots, \varepsilon_t, \varepsilon_{t-1}, \dots, \varepsilon_t^*, \varepsilon_{t-1}^*, \dots\}$  captures its information set. Having, thus, assumed a two regime markov process for monetary policy, the transition probability matrix  $P$  receives the form

$$P = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}$$

where  $p_{11}$  measures the probability of staying at date  $t$  in regime 1 and  $p_{12}$  the probability of moving to regime 2 at date  $t$  while being in regime 1 at date  $t - 1$ .  $p_{22}$  measures the probability of staying in regime 2 at date  $t$  and  $p_{21}$  the probability of moving to regime 1 at date  $t$  while being in regime 2 at date  $t - 1$ .

Monetary policy may switch because of various reasons. One of them could be the switch of the interests of the Central banker. There may be periods, for example, that he is more interested in fighting unemployment than inflation. As a result, the weight on inflation in the interest rate rule could be lower. A monetary policy switch may also be justified by the change of the Central banker. As already mentioned, there is a high number of papers arguing that the US monetary policy has been more tolerant as regards inflation fluctuations in the pre-Volcker period.

The empirical findings in section 2 showed that there was a change in the volatility of inflation in the Eurozone, even though its monetary policy remained unchanged. I keep this finding, at first, and assume that the interest rate of home central bank has time

invariant coefficients. A standard Taylor rule with interest rate smoothing is adopted which can be summarized as

$$i_t = i_{t-1}^\rho \left( \left( \frac{\pi_t}{\tilde{\pi}} \right)^{\phi_\pi} \tilde{y}_t^{\phi_y} \right)^{1-\rho} e^{\varepsilon_t} \quad (32)$$

## 5 Log linearized model

A log linearized version of the relationships found in the previous section serves in providing us with a way to deal with the problem of no closed form solution. The model is, thus, loglinearized around a specific steady state. Given the markov switching nature of the model, it is necessary that we provide the necessary and sufficient conditions which guarantee that the steady state of the model is unique, and, thus, independent of regime changes. This can be summarized in the following proposition, which is a simple extension to that in Liu, Waggoner and Zha (2008) for the closed economy case

**Proposition:** *The steady state equilibrium values of aggregate output, consumption and the real wage are independent of monetary policy and are thus invariant to monetary policy regime shifts. Moreover, as long as domestic monetary policy does not change regimes, it is enough that*

$$\xi_{s_t}^* = \frac{\lambda^*}{\beta} \tilde{\pi}^* \bar{y}^{*-\phi_{y^*,s_t}^*},$$

where  $\lambda^*$  is the exogenous trend growth rate of productivity and  $\bar{y}$  is the steady state output gap, so that the steady state nominal variables are given by  $\pi = \tilde{\pi}$ ,  $\pi^* = \tilde{\pi}^*$ ,  $R = \frac{\lambda^*}{\beta} \tilde{\pi}$  and  $R^* = \frac{\lambda^*}{\beta} \tilde{\pi}^*$ , and which are independent of regime changes as well.

## 5.1 Supply side

We use a first order Taylor approximation around the steady state of zero inflation rate. Log linearized variables are denoted with a hat.

After loglinearizing the first order condition (11), the production function (19) the demand schedules faced by each firm (22) and (23) and optimal price setting rules (24) and (25), we receive the two relations describing the domestically consumed home goods inflation rate and the respective of the home goods consumed in the Foreign country

$$\begin{aligned} \pi_{H,t} = & b_{\pi_{H,-1}} \pi_{H,t-1} + b_{\pi_{H,-1}^*} \pi_{H,t-1}^* + \beta E_t \pi_{H,t+1} + b_{\pi_H^*} \pi_{H,t}^* + b_C \hat{C}_t + \dots \\ & \dots + b_T \hat{T}_t + b_{T^*} \hat{T}_t^* + b_q \hat{q}_t + b_a a_t + \varepsilon_{H,t} \end{aligned} \quad (33)$$

$$\begin{aligned} \pi_{H,t}^* = & b_{\pi_{H,-1}} \pi_{H,t-1} + b_{\pi_{H,-1}^*} \pi_{H,t-1}^* + \beta E_t \pi_{H,t+1}^* + b_{\pi_H}^* \pi_{H,t} + b_C^* \hat{C}_t + \dots \\ & \dots + b_T^* \hat{T}_t + b_{T^*}^* \hat{T}_t^* + b_q^* \hat{q}_t + b_a^* a_t + \varepsilon_{H,t}^* \end{aligned} \quad (34)$$

where  $\varepsilon_{H,t}$  and  $\varepsilon_{H,t}^*$  are *i.i.d.* cost push shocks.  $T_t = \frac{P_{F,t}}{P_{H,t}}$  and  $T_t^* = \frac{P_{H,t}^*}{P_{F,t}^*}$  captures the terms of trade for the Home and Foreign country respectively.

The log linearized aggregate price level relation (22) is specified as

$$\pi_t = \pi_{H,t} + (1 - \delta)(\pi_{F,t} - \pi_{H,t}) \quad (35)$$

which can be further simplified as<sup>19</sup>

$$\pi_t = \pi_{H,t} + (1 - \delta)\Delta \hat{T}_t$$

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<sup>19</sup>To end up to that expression, we used equation  $\hat{T}_t = \hat{T}_{t-1} + \pi_{F,t} - \pi_{H,t}$  for the relative which is reported later in the text.

## 5.2 Demand side

In this section we proceed to the loglinearization of the Euler equation

$$C_t^O = \kappa(i_t - E_t\pi_{t+1}) + E_t C_{t+1}^O \quad (36)$$

where  $\kappa = -\frac{1}{\sigma}$ , and using (2) the Euler equation receives the forward form, which includes both backward and forward looking elements

$$C_t = \frac{\kappa\psi}{2-\psi}(i_t - E_t\pi_{t+1}) + \frac{1}{2-\psi}E_t C_{t+1} + \frac{1-\psi}{2-\psi}C_{t-1} \quad (37)$$

Goods market clearing assumes the following two conditions

$$Y = C_H + C_H^* + G_t \quad \text{and} \quad Y^* = C_F + C_F^* + G_t^*$$

where  $G_t$  and  $G_t^*$  capture government expenditures for home and foreign country respectively, assumed to follow an exogenous stationary  $AR(1)$  process  $g_t = \rho_g g_{t-1} + \varepsilon_{g,t}$  and  $g_t^* = \rho_g^* g_{t-1}^* + \varepsilon_{g,t}^*$ ,  $\varepsilon_{g,t} \sim N(0, \sigma_{\varepsilon_g}^2)$  and  $\varepsilon_{g,t}^* \sim N(0, \sigma_{\varepsilon_g^*}^2)$ .

Combining equation (35) and the market clearing conditions, I derive the aggregate demand equation:

$$\begin{aligned} \hat{Y}_t = & \eta_1 \hat{Y}_{t-1} + \eta_2 E_t \hat{Y}_{t+1} + \eta_3 (i_t - E_t \pi_{t+1}) + \eta_4 \Delta \hat{Y}_t^* + \eta_5 E_t \Delta \hat{Y}_{t+1}^* + \eta_6 \Delta \hat{T}_t + \\ & \dots \eta_7 E_t \Delta \hat{T}_{t+1} + \eta_8 \Delta \hat{T}_t^* + \eta_9 E_t \Delta \hat{T}_{t+1}^* \end{aligned} \quad (38)$$

where  $\eta_i$ ,  $i = 1, \dots, 9$  are defined in detail in the appendix.

### 5.3 Real exchange rate and relative prices

The real exchange rate dynamics are specified by the following relationship

$$\Delta \hat{q}_t = \Delta z_t + \pi_t^* - \pi_t \quad (39)$$

In the Home country the price of imported goods relative to that of Home goods is specified as  $T_t = \frac{P_{F,t}}{P_{H,t}}$ , whereas in the Foreign country the relative price of Home exported goods to Foreign goods is specified as  $T_t^* = \frac{P_{H,t}^*}{P_{F,t}^*}$ . Loglinearizing those two expressions we receive the following

$$\hat{T}_t = \hat{T}_{t-1} + \pi_{F,t} - \pi_{H,t} \quad \hat{T}_t^* = \hat{T}_{t-1}^* + \pi_{H,t}^* - \pi_{F,t}^*$$

### 5.4 Flexible price equilibrium

At the flexible price equilibrium firms adjust their prices at each period. Each firm will set its marginal cost equal to the optimal marginal cost (i.e.  $-\log\left(\frac{\theta}{\theta-1}\right)$ ) which is constant over time and equal across firms. Since firms adjust their prices every period, monetary policy will not have any real effects into the economy. The real marginal cost is specified by the following equations

$$mc_t = -\log\left(\frac{\theta}{\theta-1}\right) = -\mu$$

$$mc_t = w_t - \alpha_t - \nu$$

where  $w_t$  is the real wage,  $\alpha_t$  (log) productivity and  $\nu$  a subsidy to labor. Solving for the case with flexible prices, we receive the following set of equations describing the equilibrium

processes for output, consumption, labor, real interest rate and real exchange rate, given by:

$$y_t^n = \psi_c \bar{c}_{t-1} + \psi_\zeta \zeta + \psi_a \alpha_t + \psi_{a^*} \alpha_t^* + \psi_\tau \tau_t + \psi_{\tau^*} \tau_t^* + \psi_g g_t + \psi_{g^*} g_t^* \quad (40)$$

$$c_t^n = \tilde{\psi}_c \bar{c}_{t-1} + \psi_\zeta \zeta + \left( \frac{\gamma \delta^* + \sigma}{\delta(\gamma + \sigma) - \gamma(1 - \delta^*)} \right) \psi_\alpha \alpha_t - \left( \frac{\gamma}{\sigma} \psi_{\alpha^*} \right) \alpha_t^* - \left( \frac{\gamma}{\sigma} \psi_\tau \right) \tau_t - \left( \frac{\gamma}{\sigma} \psi_{\tau^*} \right) \tau_t^* - \left( \frac{\gamma}{\sigma} \psi_g \right) g_t - \left( \frac{\gamma}{\sigma} \psi_{g^*} \right) g_t^* \quad (41)$$

$$l_t^n = \tilde{\psi}_c \bar{c}_{t-1} + \psi_\zeta \zeta + \left( \frac{\gamma(\delta^*(1 - \sigma) - (1 - \delta)) - \sigma(1 - \delta)\psi_\alpha}{\delta(\gamma + \sigma) - \gamma(1 - \delta^*)} \right) \alpha_t - \psi_{a^*} \alpha_t^* + \psi_\tau \tau_t + \psi_{\tau^*} \tau_t^* + \psi_g g_t + \psi_{g^*} g_t^* \quad (42)$$

$$r_t^n = \tilde{\psi}_c \bar{c}_{t-1} + \left( \frac{(\gamma \delta^* + \sigma)(1 - \rho_a) \psi_a}{\kappa \delta(\gamma + \sigma) - \gamma(1 - \delta^*)} \right) \alpha_t - \left( \frac{\gamma(1 - \rho_{a^*}) \psi_{a^*}}{\kappa \sigma} \right) \alpha_t^* - \left( \frac{\gamma(1 - \rho_\tau) \psi_\tau}{\kappa \sigma} \right) \tau_t + \left( \frac{\gamma(1 - \rho_{\tau^*}) \psi_{\tau^*}}{\kappa \sigma} \right) \tau_t^* - \dots \\ \dots - \left( \frac{\gamma(1 - \rho_g) \psi_g}{\kappa \sigma} \right) g_t - \left( \frac{\gamma(1 - \rho_{g^*}) \psi_{g^*}}{\kappa \sigma} \right) g_t^* \quad (43)$$

## 5.5 Welfare

The Central Bank sets the interest rate in such a way to minimize a measure of social loss derived by a second order Taylor expansion of the consumer's utility function as in Rotemberg and Woodford (1998), Amato and Laubach (2003), Pappa (2004) and Benigno and Benigno (2006). It is summarized as<sup>20</sup>

$$W_t = -\frac{1}{2} u_c C \{ \lambda_1 (y_t - y_t^n)^2 + \lambda_2 (y_t - y_{t-1})^2 + \lambda_3 (y_t^* - y_t^{*n})^2 + \lambda_4 (y_t^* - y_{t-1}^*)^2 + \dots \\ + \lambda_5 \pi_{H,t}^2 + \lambda_6 (\pi_{H,t} - \pi_{H,t-1})^2 + \lambda_7 (\pi_{H,t}^* )^2 + \lambda_8 (\pi_{H,t}^* - \pi_{H,t-1}^*)^2 + \lambda_9 (q_t - q_t^n)^2 + \dots \\ + \lambda_{10} (q_t - q_{t-1})^2 + \lambda_{11} (q_t + y_t)^2 + \lambda_{12} (q_t + y_t^*)^2 + \lambda_{13} (q_{t-1} + y_t)^2 + \lambda_{14} (q_{t-1} + y_t^*)^2 + \dots \\ + \lambda_{15} (y_t + y_{t-1}^*)^2 + \lambda_{16} (y_{t-1} + y_t^*)^2 + \lambda_{17} (y_{t-1} - y_{t-1}^n) (q_{t-1} - q_{t-1}^n) + \dots \\ + \lambda_{18} (y_{t-1}^* - y_{t-1}^{*n}) (q_{t-1} - q_{t-1}^n) + \lambda_{19} (y_{t-1} - y_{t-1}^n) (y_{t-1}^* - y_{t-1}^{*n}) + \dots \\ + \lambda_{20} (c_{t-1}^* - c_{t-1}^{*n}) (q_{t-1} - q_{t-1}^n) + \lambda_{21} (c_t - c_t^n) (q_t - q_t^n) \} + t.i.p. + O(\|\xi\|^3) \quad (44)$$

<sup>20</sup>The derivation of the loss function is given in detail in the Appendix.

where the coefficients  $\lambda_i$ ,  $i = 1, \dots, 21$  are functions of the structural parameters and are defined in detail in the appendix.

## 6 Model Solution

Given the Markov-Switching structure of the model, standard solution techniques cannot be applied in order to find a solution. In the recent literature on Markov-Switching DSGE models, various alternative techniques of solving those models have been suggested (Farmer, Waggoner and Zha, 2011; Farmer, Waggoner and Zha, 2008; Dacig and Leeper, 2007; Svensson and Williams, 2005). The technique I use is that of Farmer, Waggoner and Zha (2011). The virtue of that technique is that it is able to find all possible minimal state variable (MSV) solutions. Moreover, the algorithm is able to find whether the MSV solution is stationary (Mean square stable) in the sense of Costa, Fragoso and Marques (2004)<sup>21</sup>. The model can be written in the following state space form

$$A(s_t)X_t = B(s_t)X_{t-1} + \Psi(s_t)\varepsilon_t + \Pi(s_t)\eta_t \quad (45)$$

where  $X_t = [y_{t+1}, y_{t+1}^*, \pi_{t+1}, \pi_{t+1}^*, \pi_{H,t+1}, \pi_{H,t+1}^*, \pi_{F,t+1}, \pi_{F,t+1}^*, q_t, z_{t+1}, T_{t+1}, T_t, y_t, y_t^*, \pi_{H,t}, \dots, \pi_{H,t}^*, \pi_{F,t}, \pi_{F,t}^*, q_t, z_t, T_{t+1}^*, T_t^*, i_t, i_t^*, \varepsilon_{i,t}, \varepsilon_{i,t}^*, g_t, g_t^*, v_t, v_t^*]$ ,  $\varepsilon_t$  is a  $6 \times 1$  vector of i.i.d. stationary exogenous shocks and  $\eta_t$  is a  $10 \times 1$  vector of endogenous random variables.

According to that technique the MSV equilibrium of the model takes the form

$$X_t = g_{1,s_t}X_{t-1} + g_{2,s_t}\varepsilon_t \quad (46)$$

In order for the above minimal state variable solution to be stationary it must be that the the eigenvalues of

$$(P \otimes I_{30^2})diag[\Gamma_1 \otimes \Gamma_1, \Gamma_2 \otimes \Gamma_2] \quad (47)$$

where  $\Gamma_j = A(j)V_j$  for  $j = 1, 2$ . And where  $V_j$  is a  $30 \times 20$  matrix resulting from the

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<sup>21</sup>For an extensive argument regarding the merits of the solution technique used in this paper over the alternative ones see Farmer et al. (2011) and the references therein.

Schur decomposition of  $A(j)^{-1}B(j)$ . In the present model the largest eigenvalue was found to be equal to 0.9893, implying, thus, that our MSV solution is stationary. The impulse responses and the moments of the variables of interest were then derived by that stationary solution.

## 7 Parameterization

In this section, the model is simulated so that to explore what regime switching implies about the dynamic behavior of the key macroeconomic variables. In order to make our argument clearer the impulse responses of inflation and output are compared to those when there is no regime switching, as in Liu et al. (2009). Throughout this section I assume that it is only the foreign central bank switching regimes. The home central bank is assumed to (naively) always follow the Taylor rule, independently of what the foreign central bank does. Therefore, whenever I refer to the hawkish regime, we mean an inflation coefficient in the interest rate rule of the foreign central bank that is greater than one. Whenever we refer to the dovish regime, we mean an inflation coefficient in the interest rate rule of the foreign country that is less than one.

Since it is only the foreign central bank that switches regimes in its monetary policy I have to choose four different parameters for its interest rate rule, depending on the regime. The values assigned are those from the Markov-switching interest rate rule estimated in section 2 (2004)<sup>22</sup>. That is  $\phi_{\pi,1}^* = 1.4562$ ,  $\phi_{\pi,2}^* = 0.3798$ ,  $\phi_{x,1}^* = 0.5934$ ,  $\phi_{x,2}^* = 0.4803$ . I also assume some interest rate smoothing with  $\rho_1 = \rho_2 = 0.6$ <sup>23</sup>.

As far as the rest of the parameters in the model are concerned, they are regime invariant. Those parameters are the subjective discount factor  $\beta$ , the degree of relative risk aversion  $\sigma$ , the elasticity of substitution between goods produced domestically  $\theta$ , the elasticity of substitution between home and foreign goods  $\rho$ , the Frisch elasticity of labor supply  $1/\gamma$ , the degree of price stickiness for the home and the foreign country respectively

<sup>22</sup>I also tried values for the parameters like those estimated by CGG (2000). This combination, though, leads to unstable minimum state variable solutions. Hence they were ruled out.

<sup>23</sup>Note that the results presented in this section hold also for  $\rho_1 = \rho_2 = 0$

$\omega$  and  $\omega^*$ , the fractions of rule of thumb firms for each country  $\zeta$  and  $\zeta^*$ , the fractions of rule of thumb consumers  $1 - \psi$  and  $1 - \psi^*$ , the home bias parameters  $\delta$  and  $\delta^*$  and the coefficients on the home country interest rate rule  $\phi_\pi$ ,  $\phi_x$  and  $\rho_i$ . The values of the parameters are chosen according to the existing empirical and theoretical literature in models similar to ours. They are summarized at table 5.

Table 5: Parameter Values

<i>Structural parameters</i>		
$\beta$	0.99	
$\sigma$	1.5	
$\theta$	10	(Obstfeld & Rogoff, 2000)
$\rho$	3	(Obstfeld & Rogoff, 2000)
$\gamma$	1	(Boivin, Giannoni & Mojon, 2008)
$\omega = \omega^*$	0.9	(Adjemian, Paries & Smets, 2008)
$\delta = \delta^*$	0.7	
$\zeta = \zeta^*$	0.5	(Gadzinski & Orlandi, 2004)
$\psi$	0.4	(Adjemian, Paries & Smets, 2008)
$\psi^*$	0.5	(Adjemian, Paries & Smets, 2008)
$\phi_\pi$	1.5	
$\phi_x$	0.5	
$\rho$	0.65	
$p11 = 0.91$	$p22 = 0.91$	
<i>Interest rate weight</i>		
$\lambda_r$	0.237	(Amato & Laubach, 2003)

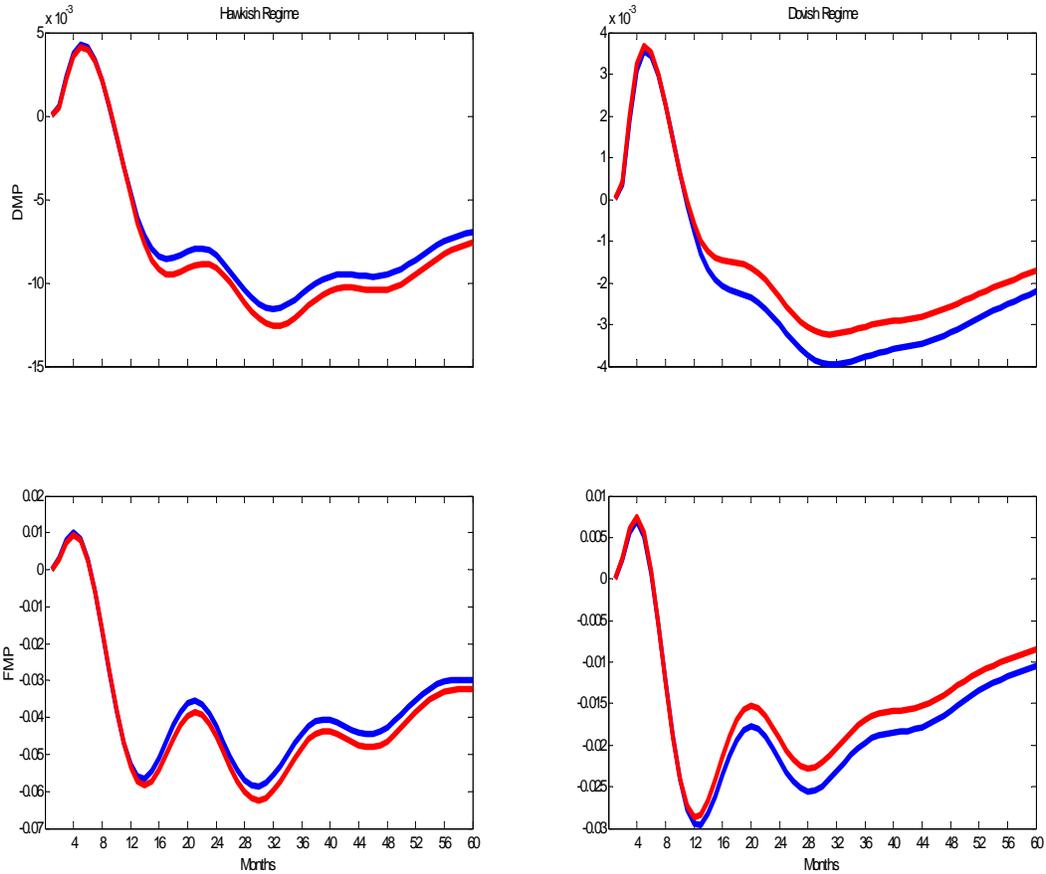
## 7.1 Impulse responses

To gauge how the possibility of a future switch in foreign monetary affects the dynamics of the domestic macroeconomic variables, I first compute the impulse responses following a one standard deviation shock in monetary policy, demand, productivity in both the domestic and foreign country. In order, thus, to emphasize the importance of expectation effects, the impulse responses of the regime switching model are compared to those of

the constant parameter model<sup>24</sup>. That is, the impulse responses when there is a non-zero probability of a change in the regime (red lines) are compared to the absorbing state case (blue line). In the latter case the way agents form their expectations is much simpler, because they do not have to incorporate in their expectations the probability of a future change in regime. On the other hand, when the probability of switching regime in the future is not zero, expectations are affected by the probabilities assigned to each regime. The comparison, thus, of the impulse responses from those two cases allows us to analyze the effects of regime switching on expectations.

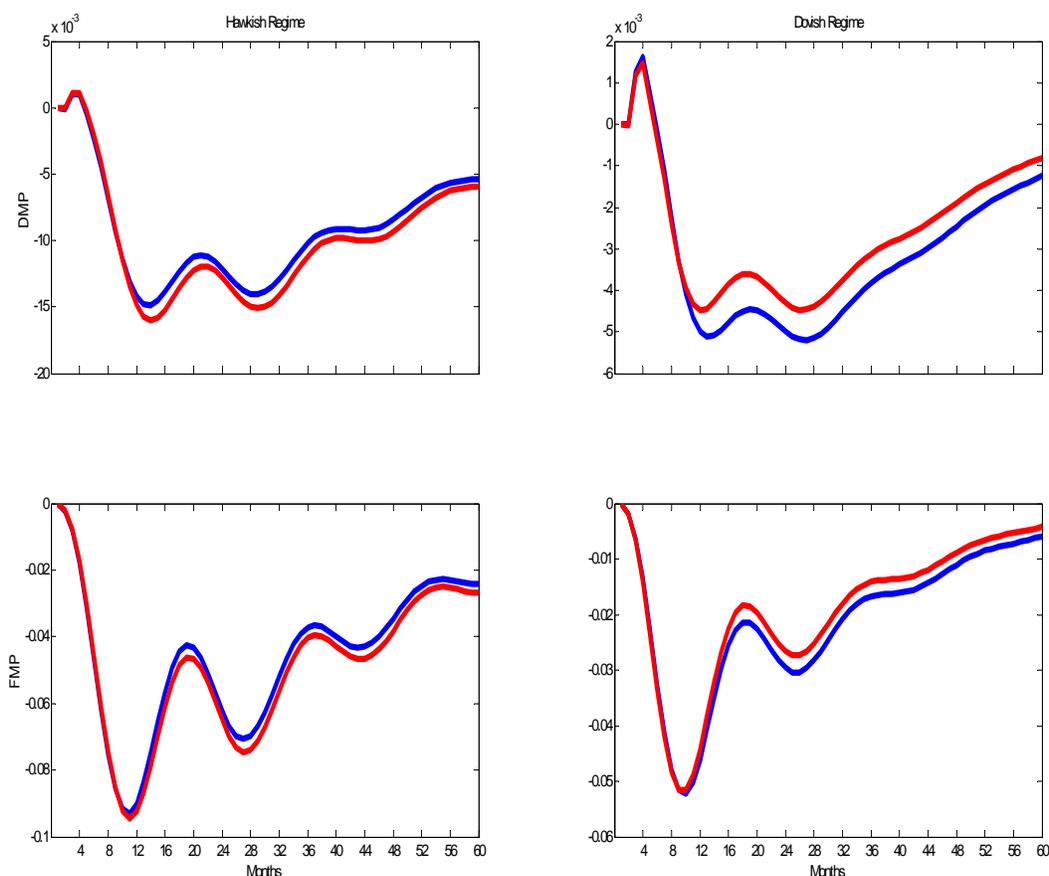
**Figure 5: Home and Foreign inflation responses to a MP shock**

(a) Home CPI



<sup>24</sup>As already mentioned, by constant parameter, we mean the absorbing state, i.e. when there is a zero probability of switching to either the dovish or the hawkish regime.

(b) Foreign CPI



Notes: The red line impulse responses are from the Markov switching model. The blue line responses are from the constant parameter model. DMP and FMP refer to a home and a foreign monetary policy shock respectively. Impulse responses in the hawkish regime are illustrated on the left panel in each graph. Impulse responses in the dovish regime are illustrated on the right panel in each graph.

In figure 5 the impulse responses of the CPI rate are plotted for each of the two regimes. As it is evident, inflation responses, in both countries are dampened<sup>25</sup> in the dovish regime when the probability of a switch to the hawkish regime in the future becomes non zero (redline) after both a home (MP) and a foreign monetary policy shock (FMP). Inflation fluctuates at slightly lower levels than in the absorbing state (blue line). This change in the behavior of inflation, as the probability of switching to the hawkish regime in the future increases, is due to the expectations formation effect. Agents in both countries assign a positive probability on the foreign monetary policy becoming hawkish in the

<sup>25</sup>From now on, I will use the term "stabilizing effect" for the case where the effects of a shock, as measured by the impulse responses, are magnified, and the term "destabilizing effect" when the effects of a shock are amplified.

future, affecting, thus, the dynamic behavior of inflation in the home (and the foreign) country. Domestic and foreign inflation are better controlled. As far as home inflation is concerned, this result is brought about solely, by agents expectations, without any change in the policy of the home central bank. This is one of the key results in this paper.

**Result 1:** *In the dovish regime, inflation in the Home country can be better controlled without any intervention from the home Central Bank. This result is purely expectations driven. It is enough, that agents in the home country assign a positive probability on the foreign monetary policy becoming hawkish in the future, while it being currently dovish.*

On the other hand, there is an amplifying effect on inflation in the hawkish regime. However, following a domestic monetary policy shock, the effects on either Home or Foreign inflation are weaker than those in the dovish regime. Inflation responses in both countries seem to be slightly amplified. It is evident that the stabilizing effect, generated in the dovish regime, is bigger than the amplifying effect, generated in the hawkish regime. This can be easily observed by looking at the distance between the red and the blue impulse responses in the hawkish and the dovish regime, respectively. However, as I am showing later, this does not imply that the overall stabilizing effect on either home or foreign inflation is stronger than the amplifying effect. The conclusion drawn until here concerns the two monetary policy shocks only. The dynamics of the model are rich enough and one cannot derive any inference by focusing only on one shock. In order to make this point clearer, I compute the changes in volatilities on inflation and output relative to the absorbing state, at table 6 below. Note, however, the asymmetry in the responses of inflation in each regime, for both countries. The asymmetry in expectation effects causes this asymmetry in inflation responses. The asymmetric expectation effects arise because of the existence of the hawkish regime. Additionally, the possibility of a future switch to that regime helps anchor agent's expectations (Liu et al., 2009).

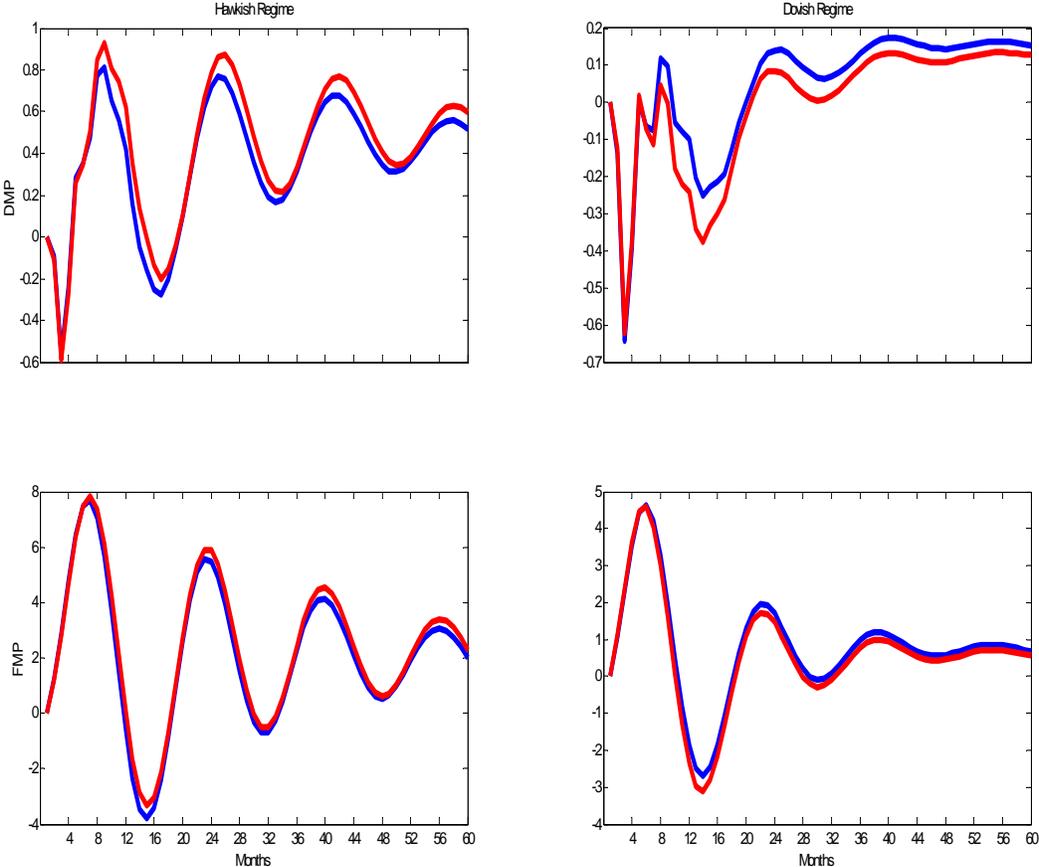
Similarly, following a foreign monetary policy shock, inflation responses in both countries

are amplified in the hawkish regime and dampened in the the dovish regime.

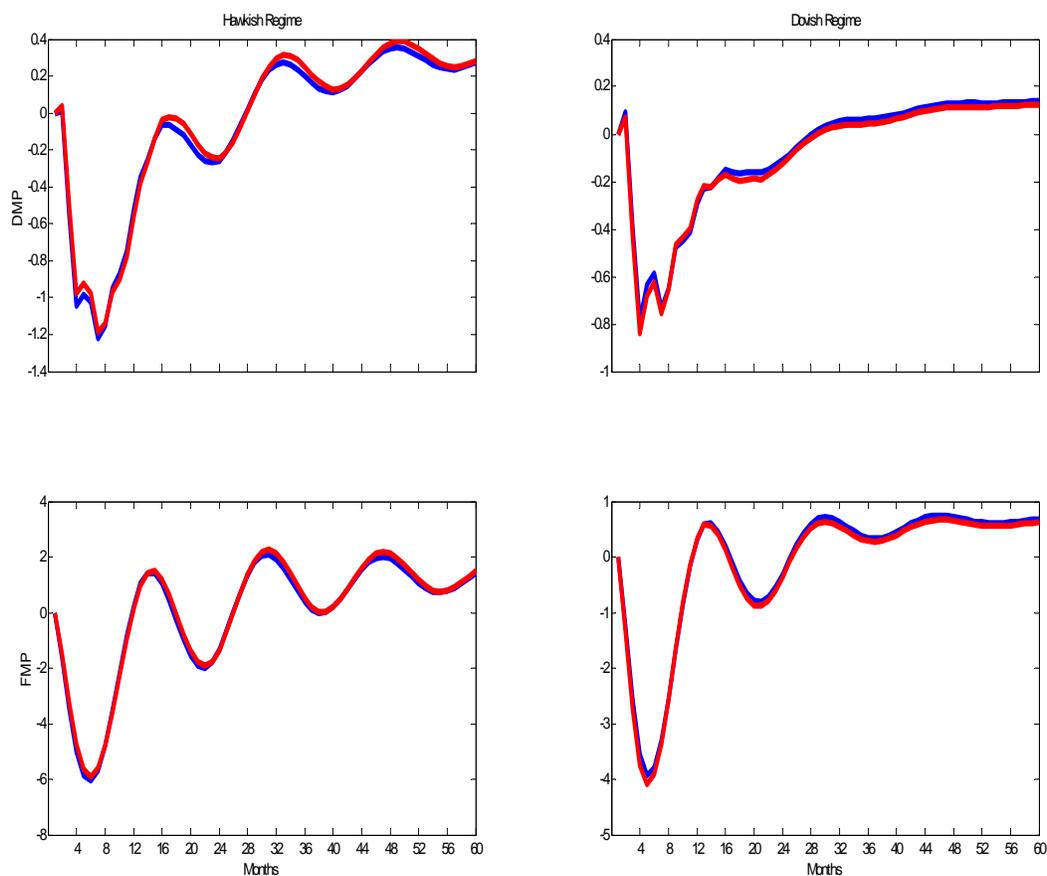
The same reasoning applies to output responses, illustrated in figure 6. The asymmetric effects on output responses, though, are less pronounced, which is consistent with the closed economy MSDSGE models. Following a domestic monetary policy shock, home output response, in the dovish regime, is dampened, as it fluctuates at a lower level than if foreign monetary policy stayed dovish forever. On the other hand, in the hawkish regime, its response is slightly amplified. Foreign output responses are not subject to asymmetric expectations effects.

**Figure 6: Output responses to a MP shock**

(a) Home output



(b) Foreign output



Notes: The red line impulse responses are from the Markov switching model. The blue line responses are from the constant parameter model. DMP and FMP refer to a home and a foreign monetary policy shock respectively.

Home country's output impulse responses exhibit a pattern similar to those of inflation. Following a foreign monetary policy shock, home and foreign output fluctuate less in the dovish regime for a non zero probability of regime switch, compared to the absorbing state. Output in either country is clearly less volatile in the dovish regime for a positive probability of moving to the hawkish regime (red line). The stabilizing effect is clearly stronger. Home output fluctuations are controlled better when home agents attach a positive probability to the foreign monetary policy becoming hawkish in the future, while being currently dovish. In order to make the comparison better, at table 6 we compute the volatility of each variable in each regime relative to the absorbing state. Global welfare

losses are also shown<sup>26</sup>.

Table 6: Inflation and Output relative volatilities

	<i>Inflation</i>			<i>Output</i>			<i>Global Loss</i>
	<i>Home</i>	<i>Foreign</i>	<i>Global</i>	<i>Home</i>	<i>Foreign</i>	<i>Global</i>	
<i>Hawkish</i>	1.0203	1.0485	1.0397	1.0524	1.0154	1.0388	1.0165
<i>Dovish</i>	0.9977	0.9544	0.9624	0.9514	0.9887	0.9689	0.9787

Table 6 shows that there are significant decreases in inflation and output volatility, relative to the absorbing state (i.e. no regime switching case), when foreign monetary policy is dovish. In particular, home country's inflation is 0.9977 times or 0.23% lower than in the case where the probability of staying in the dovish regime forever is one. This fall is larger for the foreign country, 0.9544 times or 4.76% lower. On the other hand, a positive probability of a switch to the dovish regime increases home inflation relative to the absorbing state by 2%, while foreign inflation is increased by 4.85%. The stabilizing effect, thus, on home inflation is much smaller than the amplifying effect. As for the foreign inflation the amplifying effect is marginally greater than the stabilizing effect.

The amplifying effect seems to dominate, marginally, though, in output fluctuations, as well. In particular, home output is 5.24% more volatile in the hawkish regime relative to the absorbing state, while it is 4.86% less volatile in the dovish regime. Foreign output is 1.54% higher more volatile in the hawkish regime and 1.13% less volatile in the dovish regime.

The interesting result is that both effects are greater for home output. The reason for this is that the home central bank is assumed to follow a time-invariant Taylor rule, which causes the home real interest rate to be more sensitive to regime changes.

As already shown, a coefficient on inflation that is less than one causes foreign inflation volatility to increase. But this also implies that the nominal interest rate is not as volatile as it would otherwise be. That is, inflation is more sensitive to the various exogenous

<sup>26</sup>Relative global losses are computed in the same way as relative volatilities. That is, the loss associated with the Markov-switching model relative to that in the absorbing state.

shocks hitting the economy, under the dovish regime. Consequently, since the interest rate is less responsive to inflation, it will be less volatile relative to what it would have been, had the coefficient on inflation been greater than one. That is why the foreign real interest rate is less sensitive to regime changes, compared to the home real rate. In fact, the home real rate is 1.0177 more volatile in the hawkish regime relative to the absorbing state, while the foreign is 1.0125 times more volatile. The former is 0.9849 times less volatile in the dovish regime relative to the absorbing state, while the latter is 0.9887 times less volatile. In other words, the domestic nominal and, hence, real interest rate is a volatility absorber.

The higher sensitivity of the home real interest rate to regime changes is transmitted to output. As a result, home output is more sensitive to regime changes as well, which explains the results illustrated at table 6 for the home and foreign output. That effect questions the appropriateness of a time invariant interest rate rule for the home country<sup>27</sup>.

Markov-switching closed economy models examine the effectiveness of regime switching monetary policy by looking at the change in volatilities of inflation and output only. Given the structure of those models, judging such a policy relying on change in volatility, or on changes in a welfare measure leads to the same conclusions. In an open economy model, as the one in this paper, judging Markov-switching monetary policy by simply looking at the changes in volatilities on inflation and output could lead to the wrong conclusions. As the welfare measure (44) shows the dynamics in the model are far more rich than those in a closed economy model. Therefore, alternative policies would be better compared based on an appropriate welfare measure, rather than by observing changes in volatilities of some variables. I use, thus, the relative changes in the welfare measure (44) as a guide, in order to figure out whether Markov-switching monetary policy is beneficial for both economies. As is clear in table 6, the relative fall in global welfare loss in the dovish regime is larger, in absolute terms, than its relative increase in the hawkish regime. In particular, in the dovish regime, a non-zero probability of a switch to the hawkish regime

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<sup>27</sup> As I am showing in section 8, by solving the dynamic programming problem of the home central bank, it follows that optimality requires that the coefficients on inflation and output be regime sensitive and not time invariant. Changes in coefficients, conditional on foreign monetary policy, allow the home central bank to smooth the changes in volatility better, resulting from regime changes in foreign monetary policy.

causes global welfare loss to be 0.9787 times or 2.13% lower relative to the absorbing state. On the other hand, it is 1.0165 times or 1.65% higher relative to the absorbing state, in the hawkish regime. Therefore, Markov-switching monetary policy is globally beneficial. The above arguments can be summarized in the following three results.

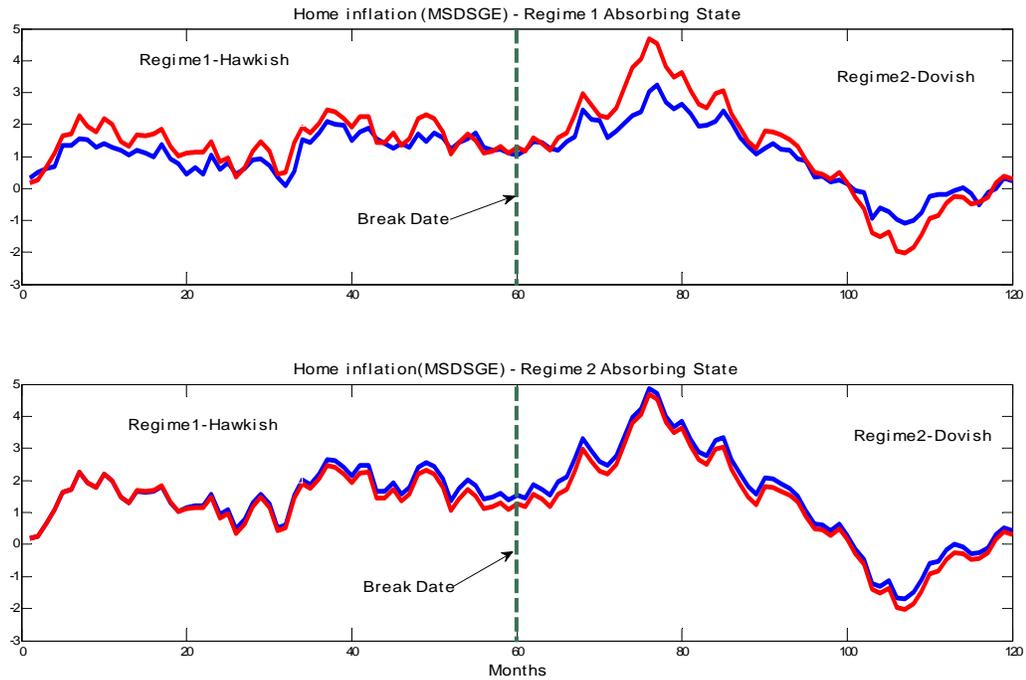
**Result 2:** *Markov switching monetary policy in the foreign country generates a stabilizing (dovish regime) and an amplifying (hawkish regime) effect on output and inflation. The stabilizing effect is weaker than the destabilization effect. This result holds for global output and inflation, as well.*

**Result 3:** *Both the stabilization and the amplifying effects are more pronounced on home than on foreign output.*

**Result 4:** *Markov-switching monetary policy is beneficial, based on a global welfare measure.*

So far I have shown that changes in the volatilities and the impulse responses of key macroeconomic variables of the home country may be caused by changes in the way monetary policy is conducted in the foreign country only. In figures 7 and 8 below I show the simulated paths of inflation in each country. The model was simulated for 120 periods allowing for a random date of regime switching in foreign monetary policy. I assume that the initial regime is the hawkish one. The regime changing date is 60 (switch to the dovish regime). For convenience a green dashed line is drawn on the regime changing date. In the upper panel in both figures, along with inflation in the MSDSGE model (red line) I plot home (foreign) inflation, had foreign monetary policy stayed in the hawkish regime forever (blue line). In the bottom panel inflation in the MSDGE model (red line) is compared to inflation, had foreign monetary policy been always dovish (blue line).

Figure 7: Home country's inflation



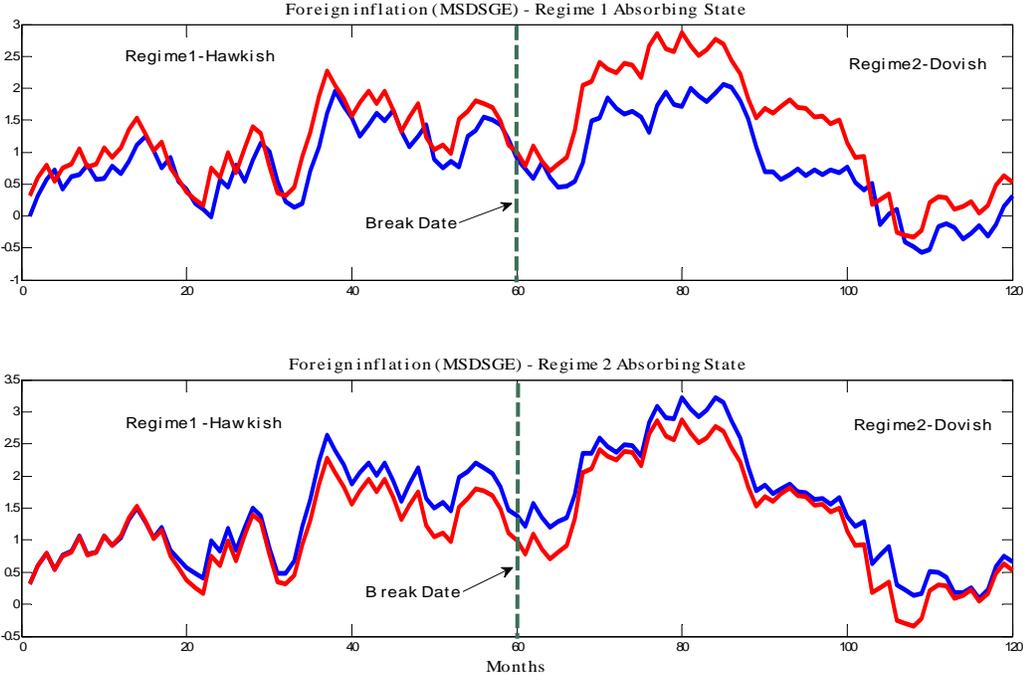
As the upper panel in figure 7 illustrates, inflation in the home country appears to be fluctuating within a certain band while still being in regime 1. For a long period in the regime 1 (until date 60), home inflation behavior resembles that when there is not any regime change (blue line). However, from date 60 onwards, home inflation has higher peaks than it would have, had there not been a change in the foreign monetary policy. The reason for this is the expectations formation effect. As the probability of a future switch in foreign monetary policy rises, inflation in the hawkish regime starts to fluctuate at higher levels, than before. After the regime change date home country's inflation fluctuates constantly at higher levels than it would have fluctuated, otherwise. This implies that the home central should change its policy as well, in order to eliminate as much as possible the additional volatility on domestic inflation.

At the lower panel in figure 7, inflation in the MSDGE model (red line) is illustrated along with inflation when the dovish regime is the absorbing state (blue line). Home inflation fluctuates at slightly lower levels than it would have otherwise, had foreign monetary

policy been always dovish. Comparing the two panels above, it is clear that the destabilizing effect is stronger, since inflation in the MSDGE moves much closer to inflation when the dovish regime is the absorbing state than when the hawkish regime is the absorbing state.

Foreign inflation exhibits the same pattern, as shown in figure 8. Not surprisingly, the expectation effect of a regime change is stronger. Foreign inflation fluctuates at constantly higher levels for most of the period in regime 1 (i.e. until date 60). As already mentioned, the reason for this effect is the expectation formation effect becoming stronger as the probability of a regime switch increases and as the regime change date approaches. From date 60 onwards (Regime 2), foreign inflation keeps fluctuating at constatly higher levels than otherwise. Again the blue line shows how inflation fluctuates when the foreign central bank stays in the hawkish regime forever. The red line shows how inflation behaves when the foreign central bank switches from being hawkish to dovish.

**Figure 8: Foreign country’s inflation**



## 8 The Dynamic Programming Problem

So far in the analysis, the parameters in the interest rate rule of the Home country have been assumed to be naively constant over time, independently of what the foreign monetary is and have been set arbitrarily, corresponding to the standard Taylor rule suggested by Taylor (1993). In this section optimized coefficients are computed. The reason for this, is to find how the Home central bank should react for a given policy rule of the foreign country. In other words, I am looking for the optimal coefficients in the interest rate rule conditional on the coefficients in the interest rate rule of the foreign country. I am not interested, thus, in the cooperative allocation as in Benigno and Benigno (2006).

### 8.1 Formulation

The procedure followed in this section is similar to that in Zampolli (2006). The policy maker chooses the control  $i_t$  (i.e. the interest rate rule) which minimizes the expected value of the intertemporal loss function, stated in the previous section and summarized as

$$\sum_{t=0}^{\infty} \beta^t L(h_t, i_t) \quad (48)$$

subject to  $h_0, s_0$  given, and the model describing the economy

$$h_{t+1} = A(s_{t+1})h_t + B(s_{t+1})i_t + C\varepsilon_{t+1} \quad t \geq 0 \quad (49)$$

where  $L(h_t, i_t)$  is the period loss function,  $\beta$  is the discount factor,  $h_t$  is a  $29 \times 1$  vector of state variables,  $i_t$  is the control variable (i.e. the interest rate) and  $\varepsilon_t$  is a  $6 \times 1$  vector of white noise shocks with variance covariance matrix  $\Sigma_\varepsilon$  and  $C$  is a  $29 \times 6$  block matrix specified as

$$C = \begin{pmatrix} C_{11} \\ I \end{pmatrix}$$

where  $C_{11}$  is a  $23 \times 6$  matrix of zeros,  $I$  is a  $6 \times 6$  identity matrix.

The loss function, thus, can be conveniently expressed as follows

$$L(h_t, i_t) = h_t' R h_t + i_t Q i_t \quad (50)$$

where  $R$  is a  $29 \times 29$  positive definite matrix and  $Q$ , in our case, is a scalar. The matrices  $A$  and  $B$ , as already mentioned, are stochastic and take on different values depending on the regime  $s_t$ ,  $t = 1, 2$ .

## 8.2 The Bellman equation

The policy maker in a markov switching environment needs to find the interest rate rule that is state-contingent. This rule describes the way that the control variable, the interest rate, should be set as a function of both the state variables and the regime occurring at date  $t$ . Therefore, as in Zampolli (2006) a Bellman equation is associated with each regime. In other words, the policy maker solves her minimization problem conditional on the regime. The regime  $j$  dependent Bellman equation is specified, thus, as follows

$$V(h_t, j) = \max_{i_t} \{L(h_t, i_t) + \beta \sum_{i=1}^2 p_{ji} E_t [V(h_{t+1}, i)]\} \quad (51)$$

where  $V(h_t, j)$  is a function of the state variables  $h_t$ , the regime prevailing at date  $t$  and represents the continuation value of the optimal dynamic programming problem at  $t$ .

The value function for this problem is

$$V(h_t, j) = h_t' P_j h_t + d_j, \quad j = 1, 2 \quad (52)$$

where  $P_j$  is a  $29 \times 29$  symmetric positive semidefinite matrix, while  $d_j$  is a scalar. The optimal policy is given by

$$i(h_t, j) = -F_j h_t, \quad j = 1, 2 \quad (53)$$

where  $F_j$  is a  $29 \times 1$  matrix, depending on  $P_j$ . That is, matrix  $F_j$  specifies the coefficients in the policy rule of the central bank. Those coefficients are regime specific. Maximizing, thus, the Bellman subject to the constraints, the matrix  $F_j$  is specified as

$$F_j = \left( Q + \beta p_{j1} B_1' P_i B_1 + \beta p_{j2} B_2' P_i B_2 \right)^{-1} \beta \left( p_{j1} A_1' P_i B_1 + p_{j2} A_2' P_i B_2 \right) \quad (54)$$

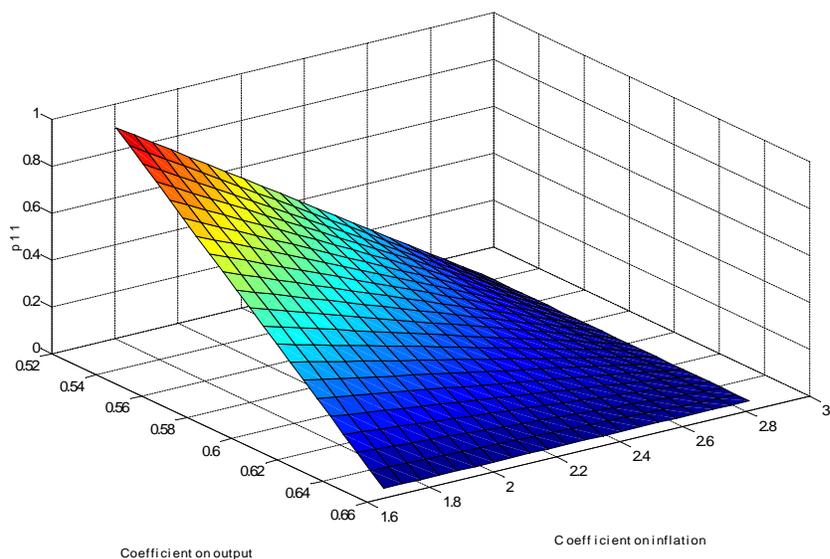
where matrix  $P_i$  has been already determined by a set of interrelated Riccati equations, which specify a system with the following form

$$P_j = R + \beta p_{j1} A_1' P_i A_1 + \beta p_{j2} A_2' P_i A_2 - \dots \\ - \beta^2 \left( p_{j1} A_1' P_i B_1 + p_{j2} A_2' P_i B_2 \right) \left( Q + \beta p_{j1} B_1' P_i B_1 + \beta p_{j2} B_2' P_i B_2 \right)^{-1} \left( p_{j1} B_1' P_i A_1 + p_{j2} B_2' P_i A_2 \right) \quad (55)$$

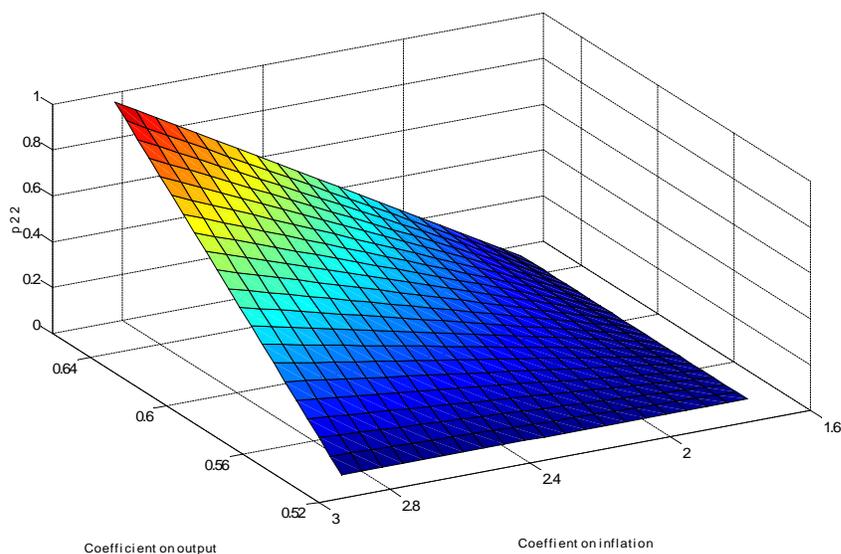
### 8.3 How should home central bank react?

Given foreign monetary policy, I find in this section, the optimized coefficients for the interest rate rule of the home central bank. That is, the coefficients on the output gap and inflation that minimize welfare conditional on both foreign monetary policy and the regime the economy stands. Figures 9 and 10 summarize our key results.

**Figure 9: Coefficients in the hawkish regime**



**Figure 10: Coefficients in the Dovish regime**



The first result from the two figures above is that the home central bank must change the coefficients in its interest rate rule as the probabilities of staying in one regime or the other change. Therefore, it is not optimal to adopt a regime invariant interest rate rule. The second is that, the weight on inflation must increase as the probability of switching to the dovish regime increases. The opposite holds as the probability of switching to the

hawkish regime increases. In this case the weight on inflation falls. On the other hand, the weight on the output gap seems to change only slightly. Its behavior is similar to that on inflation. That is, it rises as the probability of switching to the dovish regime increases, and falls as the probability of moving to the hawkish regime increases. Hence, I end up to the following two results.

**Result 4:** *As the probability of the foreign monetary policy switching to the dovish regime increases, the home central bank should become more aggressive to inflation fluctuations. As the probability of the foreign monetary policy switching to the hawkish regime increases, the home central bank should become less aggressive to inflation fluctuations.*

**Result 5:** *The home central bank must attach a weight on inflation that is always greater than one. That is, it must be always hawkish. Moreover, it must even more aggressive to inflation fluctuations, as the foreign central bank becomes dovish.*

Finally, as a last exercise we compare the welfare losses when the home central bank optimally changes the coefficients in its interest rate rule with the case where it naively allows the coefficients in its rule to be time invariant<sup>28</sup>. In this case, the welfare losses and volatilities are computed relative to those in the baseline calibration (and not to those in absorbing state). That is, the welfare loss in the hawkish regime is compared to that which would have resulted had foreign monetary policy been always hawkish. On the other hand, welfare loss in the dovish regime is compared to that which would have resulted had foreign monetary policy been always dovish.

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<sup>28</sup>The constant parameters used in this exercise are the ones used in the baseline calibration.

Table 7: Inflation and Output relative volatilities

	<i>Inflation</i>			<i>Output</i>			<i>Global Loss</i>
	<i>Home</i>	<i>Foreign</i>	<i>Global</i>	<i>Home</i>	<i>Foreign</i>	<i>Global</i>	
<i>Hawkish</i>	0.9820	0.8716	0.8740	0.8302	0.9689	0.8598	0.5872
<i>Dovish</i>	0.9939	0.8330	0.8181	0.7614	0.9574	0.8375	0.9499

As the results at table 7 show, there are important gains to both countries when the home central bank changes its policy over time conditional on foreign monetary policy. The most significant gains are on global loss in the hawkish regime. It is 0.5872 times lower relative to its level when the home central bank follows the Taylor rule used in the baseline calibration, and whose coefficients are constant over time. Finally, there is a strong improvement on output volatility in both regimes for the home country. This is due to the aggressive response of the nominal interest rate to inflation fluctuations in either regime. This results in a dampened response of output to real interest rate fluctuations in both regimes.

## 9 Concluding remarks

Even though existing literature on MSDSGE modelling focuses on closed economy models, I constructed a two country DSGE model in which foreign monetary switches regimes over time. I gave further insight regarding the effects of regime switching in monetary policy both domestically and abroad. The assumption that it is only the foreign monetary policy that switches regimes was introduced after having estimated a SVAR model for the Eurozone and the US, in which stability tests showed that only the US monetary policy has changed since the adoption of the common currency. I then estimated a Markov-switching interest rate rule for the US. This rule seems to capture the monetary policy of the US for the last 10 years quite well. In the baseline calibration, home monetary policy was assumed to be naively time invariant and follow the Taylor rule with some interest rate smoothing. I made this initial assumption in order to emphasize on the fact that inflation volatility may grow over time, even if domestic monetary policy is stable.

Domestic inflation was shown to be affected both in terms of volatility and in terms of its response to alternative shocks, the only reason being the regime shifts in foreign monetary policy, and, hence, the change in the way the expectations of the private sector are formed. Foreign monetary policy regime shifts generate a stabilization and an amplifying effect on output and inflation, both in the foreign and home country. Which effect arises depends on which regime the foreign monetary policy lies in. When the latter is dovish there is a stabilization effect, given a non-zero probability of the foreign monetary policy becoming hawkish in the future. When foreign monetary policy is hawkish there is an amplifying effect in both countries. Moreover, there seems to be an asymmetry on the size of each effect. In particular, I showed that the stabilizing effect is stronger both in the foreign country as in Davig and Leeper (2007) and Liu et al. (2008) and globally, based on a welfare measure, derived by a second order approximation of the agents utility function. Finally, through the solution of the dynamic programming problem of the home central banker, conditional on foreign monetary policy switching regimes over time, it was shown that it is optimal for the latter to follow a time varying interest rate rule. The relative benefits of the time varying rule, tend to increase as the degree of endogenous persistence increases.

## References

- Benigno, G. (2004), "Real exchange persistence and monetary policy rules", *Journal of Monetary Economics*, 51(3): 473-502.
- Benigno, G. and P. Benigno (2001), "Monetary policy rules and the exchange rate", CEPR discussion paper No. 2807.
- Benigno, G. and P. Benigno (2006), "Designing targeting rules for international monetary policy cooperation", *Journal of Monetary Economics*, 53(3): 473-506.
- Boivin, J and M. Giannoni (2006), "Has monetary policy become more effective?", *Review of Economics and Statistics*, 88(3): 445-462.
- Boivin, J and M. Giannoni (2002), "Assessing the changes in monetary transmission mechanism: A VAR approach", *Federal Reserve Bank of New York Policy Review* 8:1, 97-112.
- Calvo, G., (1983), "Staggered Prices in a Utility Maximizing Framework", *Journal of Monetary Economics*, 12: 383-398.
- Clarida, R., J. Gali and M. Gertler, (2001), "Monetary policy rules and macroeconomic stability: Evidence and some theory", *Quarterly Journal of Economics*, 147-180.
- Coagley, T. and T. Sargent, (2005), "Drifts and volatilities: Monetary policies and outcomes in the Post-WWII US", *Review of Economic Dynamics*, 8: 262-302.
- Davig, T. and E. Leeper (2007), "Endogenous monetary policy regime change", *American Economic Review*.
- Farmer, R., D. Waggoner and T. Zha (2011), "Minimal state variable solutions to Markov-switching rational expectations models", *Journal of Economic Dynamics and Control*, forthcoming.
- Gali, J., (2002). "New perspectives on monetary policy, inflation and the business Cycle". NBER working paper.
- Gali, J. and T. Monacelli, (2005). "Monetary policy and exchange rate volatility in a small open economy". *Review of Economic Studies*, 72: 707-734.
- Galí, J., M. Gertler, and D. López-Salido (2001): "European Inflation Dynamics", *European Economic Review*, 45(7): 1237-1270.
- Galí, J. and M. Gertler (1999): "Inflation Dynamics: A Structural Econometric Analysis", *Journal of Monetary Economics*, 44(2): 195-222.
- Granger, C. and T. Terasvirta, (1993). "Modelling Nonlinear Economic Relationships". (Oxford: Oxford University Press).
- Levin, A., V. Wieland and J. C. Williams (1999), "Robustness of simple monetary policy rules under uncertainty". In: Taylor J. B. (ed), *Monetary policy rules*, University of Chicago press, Chicago.
- Liu, Z., D. Waggoner and T. Zha (2009), "Asymmetric expectation effects of regime shifts in monetary

policy”, *Review of Economic Dynamics*, 12, 284-303.

Lubik, T. and F. Schorfheide (2004), "Testing for indeterminacy: An application to US monetary Policy", *American Economic Review*, 94: 190-217.

Obstfeld, M. and K. Rogoff, (1998), "Risk and exchange rates", NBER working paper 6694.

Orphanides, A. (2001), "Monetary policy rules based on real time data", *American Economic Review*, 964-985.

Rotemberg, J, and M. Woodford, (1998). " An optimization based framework for the evaluation of monetary policy", NBER Technical Working Paper, NO 233.

Sbordone, A., (1998), "Prices and Unit Labor Costs: Testing Models of Pricing Behavior", mimeo

Zampolli F. (2006), "Optimal monetary policy in a regime switching economy: The response to abrupt shifts in exchange rate dynamics", *Journal of Economic Dynamics and Control*, 30: 1527-1567.