Optimal Fiscal and Monetary Policy With Occasionally Binding Zero Bound Constraints

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Question: How should the government conduct fiscal policy at the zero lower bound (ZLB) in the face of uncertainty?
Many new research on fiscal policy at the zero lower bound:

- Christiano, Eichenbaum, and Rebelo (2011), Eggertsson (2010) and Woodford (2011) show that government spending multiplier is large at the ZLB.

- Eggertsson (2001 and 2006), Nakata (2011), and Werning (2011) show that it is also optimal to increase government spending at the ZLB.
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- They look at deterministic models.
This paper characterizes optimal fiscal and monetary policy when the nominal interest rate is subject to the ZLB constraint in a stochastic environment.

In the model:

- Two policy instruments: the nominal interest rate and government spending.
- The government makes decisions sequentially (i.e. no commitment).
- An exogenous variation in the household’s discount rate occasionally forces the government to lower nominal interest rates to zero.
In the stochastic environment,

- optimal increase in the government spending is larger.
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A key mechanism:

- Policy functions for allocations and prices are highly concave or convex due to the ZLB constraint.
- A mean-preserving spread in the shock distribution increases expected real interest rates and decreases expected real marginal costs.
- Thus, we see larger declines in consumption, output, and inflation in the stochastic environment.
In the stochastic environment,

- the access to government spending policy decreases welfare.
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A key mechanism:

- The government reduces nominal interest rate more aggressively before reaching the ZLB if it does not have access to government spending policy.
- This creates a temporary increase in consumption.
Model

- Discrete Time, Infinite Horizon. Economy starts at $t=1$.
- A representative household.
- A final good producer.
- A continuum of intermediate-good producers, indexed by $i \in [0, 1]$
- The government.
Model

- Discrete Time, Infinite Horizon. Economy starts at $t=1$.

- A representative household.
  - Discount factor shocks: $\{\delta_t\}_{t=1}^{\infty}$.
  - $\delta_t - 1 = \rho(\delta_{t-1} - 1) + \epsilon_t, \epsilon_t \sim N(0, \sigma^2_\epsilon)$.

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- A final good producer.

- A continuum of intermediate-good producers, indexed by $i \in [0, 1]$
  - Linear Production Technology: $Y_{i,t} = N_{i,t}$
  - Quadratic Price Adjustment Costs: $RC_{i,t} = \frac{\varphi}{2} \left[ \frac{P_{i,t}}{P_{i,t-1}} - 1 \right]^2 Y_t$

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- The government.
  - Lump-sum tax available. Supply of the government bond is zero.
  - $G_t = T_t$
Symmetric Equilibrium

Given any $P_0$ and $\{\delta_t\}_{t=1}^\infty$, a set of symmetric implementable equilibria is characterized by $\{C_t, N_t, Y_t, w_t, \Pi_t, R_t, G_t\}_{t=1}^\infty \equiv \{d_t\}_{t=1}^\infty$ satisfying

$$C_t^{-\chi_c} = \beta \delta_t R_t E_t C_{t+1}^{-\chi_c} \Pi_{t+1}^{-1}$$

$$w_t = N_t^{\chi_n} C_t^{\chi_c}$$

$$\frac{Y_t}{C_t^{\chi_c}} \left[ \varphi(\Pi_t - 1) \Pi_t - (1 - \theta) - \theta w_t \right] = \beta \delta_t E_t \frac{Y_{t+1}}{C_{t+1}^{\chi_c}} \varphi(\Pi_{t+1} - 1) \Pi_{t+1}$$

$$Y_t = C_t + G_t + \frac{\varphi}{2} [\Pi_t - 1]^2 Y_t$$

$$Y_t = N_t$$

$$R_t \geq 1$$

where $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ and $w_t = \frac{W_t}{P_t}$. 
Government’s Optimization Problem

Given \( \{ V_{t+1}(\cdot), C_{t+1}(\cdot), N_{t+1}(\cdot), \Pi_{t+1}(\cdot), w_{t+1}(\cdot), R_{t+1}(\cdot), G_{t+1}(\cdot) \} \), the problem of the government at time \( t \) is

\[
V_t(\delta_t) = \max_{\{d_t\}} \left[ \frac{C_t^{1-\chi_c}}{1 - \chi_c} - \frac{N_t^{1+\chi_n}}{1 + \chi_n} + \chi_g,0 \frac{G_t^{1-\chi_g}}{1 - \chi_g,1} \right] + \beta \delta_t E_t V_{t+1}(\delta_{t+1})
\]

subject to the equations characterizing symmetric implementable equilibria.

A Markov-Perfect Equilibrium consists of a set of time-invariant value and policy functions, \( \{ V(\cdot), C(\cdot), N(\cdot), \Pi(\cdot), w(\cdot), R(\cdot), G(\cdot) \} \) that solves the problem above.
### Table: Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Calibrated Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Discount rate</td>
<td>$\frac{1}{1+0.0075} \approx 0.9925$</td>
</tr>
<tr>
<td>$\chi_c$</td>
<td>Inverse intertemporal elasticity of substitution for $C_t$</td>
<td>1.0</td>
</tr>
<tr>
<td>$\chi_n$</td>
<td>Inverse labor supply elasticity</td>
<td>1.0</td>
</tr>
<tr>
<td>$\chi_{g,0}$</td>
<td>Utility weight on $G_t$</td>
<td>0.25</td>
</tr>
<tr>
<td>$\chi_{g,1}$</td>
<td>Intertemporal elasticity of substitution for $G_t$</td>
<td>1.0</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Elasticity of substitution among intermediate goods</td>
<td>10</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>Price adjustment cost</td>
<td>150</td>
</tr>
<tr>
<td>$\rho$</td>
<td>AR(1) coefficient for the discount factor shock</td>
<td>0.8</td>
</tr>
<tr>
<td>$\sigma_\epsilon$</td>
<td>The standard deviation of shocks</td>
<td>$[0, \frac{0.42}{100}]$</td>
</tr>
<tr>
<td></td>
<td>to the discount factor shock</td>
<td></td>
</tr>
</tbody>
</table>
Global Solution Method

Time-iteration method ("policy function iteration").
Markov-Perfect Policy Without Uncertainty

Nominal Interest Rate (Annualized Percentage)

Inflation (Annualized Percentage)

Government Spending

Labor Supply/Output

Consumption

Red Line: Deterministic ($\sigma_\epsilon = 0$)
Markov-Perfect Policy With/Without Uncertainty

Red Line: Deterministic ($\sigma_\varepsilon = 0$). Black Line: Stochastic ($\sigma_\varepsilon = \frac{42}{100}$)
Allocations With/Without Fiscal Policy

Given
\{V_{c,t+1}(\cdot), C_{c,t+1}(\cdot), N_{c,t+1}(\cdot), \Pi_{c,t+1}(\cdot), w_{c,t+1}(\cdot), R_{c,t+1}(\cdot), G_{c,t+1}(\cdot)\},
the problem of the government at time t is

\[ V_{c,t}(\delta_t) = \max\{d_t\} \left[ \frac{C_{c,t}^{1-\chi_c}}{1-\chi_c} - \frac{N_{c,t}^{1+\chi_n}}{1+\chi_n} + \chi_g,0 \frac{G_{c,t}^{1-\chi_g,1}}{1-\chi_g,1} \right] + \beta \delta_t E_t V_{c,t+1}(\delta_{t+1}) \]

subject to

- the equations characterizing symmetric implementable equilibria
- \( G_{c,t} = \bar{G} \)
### Table: Welfare Gains from Fiscal Policy

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<th>Welfare Gains from Fiscal Policy*</th>
<th>-0.07 %</th>
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</table>

*Perpetual consumption transfer (expressed as a percentage of the steady-state consumption) required to make the agent in the economy without fiscal policy as well-off as the agent in the economy with fiscal policy.
Table: Welfare Gains from Fiscal Policy

<table>
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<th>Welfare Gains from Fiscal Policy*</th>
<th>With production subsidy</th>
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<td>-0.07 %</td>
<td>0.08 %</td>
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*Perpetual consumption transfer (expressed as a percentage of the steady-state consumption) required to make the agent in the economy without fiscal policy as well-off as the agent in the economy with fiscal policy.
Summary

In the stochastic environment,

- Optimal increase in the government spending is larger.
- the access to government spending policy decreases welfare.
Extra slides
Household

\[
\max_{\{C_t, N_t, B_t\}_{t=1}^\infty} \quad E_1 \sum_{t=1}^\infty \beta^{t-1} \left[ \prod_{s=0}^{t-1} \delta_s \right] \left[ \frac{C_t^{1-\chi_c}}{1 - \chi_c} - \frac{N_t^{1+\chi_n}}{1 + \chi_n} + \chi_{g,0} \frac{G_t^{1-\chi_g,1}}{1 - \chi_{g,1}} \right]
\]

subject to

\[
P_t C_t + R_t^{-1} B_t \leq W_t N_t + B_{t-1} - P_t T_t + P_t \Phi_t
\]

where \(B_0 = 0\) and \(\delta_1\) is given.

\(\beta \delta_t\) is the relative weight the agent puts on the future utility flows at time \(t\):

\[
\beta \delta_0 U(C_1, N_1, G_1) + \beta^2 \delta_0 \delta_1 U(C_2, N_2, G_2) + \beta^3 \delta_0 \delta_1 \delta_2 U(C_3, N_3, G_3) + ...$

A final-good firm aggregates intermediate goods by CES technology.

Intermediate-good firms:

\[
\max_{\{P_{i,t}\}_{t=1}^{\infty}} E_1 \sum_{t=1}^{\infty} \beta^{t-1} \left[ \prod_{s=0}^{t-1} \delta_s \right] \lambda_t \left[ P_{i,t} Y_{i,t} - W_t N_{i,t} - P_t \frac{\varphi}{2} \left[ \frac{P_{i,t}}{P_{i,t-1}} - 1 \right]^2 Y_t \right]
\]

subject to

\[
Y_{i,t} = \left[ \frac{P_{i,t}}{P_t} \right]^{-\theta} Y_t \quad \& \quad Y_{i,t} = N_{i,t}
\]

\[
P_{i,0} = P_0 \text{ for some given constant } P_0 > 0
\]
Government’s Policy Instruments

- Supply of the government bond is zero.
- Lump-sum taxation available. No distortionary taxation.
- The government budget constraint: $G_t = T_t$
- The nominal interest rate is subject to the ZLB constraint: $R_t \geq 1$. 
Market-Clearing

\[ Y_t = C_t + G_t + \int_0^1 \frac{\varphi}{2} \left[ \frac{P_{i,t}}{P_{i,t-1}} - 1 \right]^2 Y_t \, di \]

\[ N_t = \int_0^1 N_{i,t} \, di \]

\[ B_t = 0 \]
Symmetric Equilibrium (1)

Given $P_0$ and $\{\delta_t\}_{t=1}^{\infty}$, a symmetric implementable equilibrium consists of

- **Allocations:** $\{C_t, N_t, N_{i,t}, Y_t, Y_{i,t}\}_{t=1}^{\infty}$
- **Prices:** $\{W_t, P_t, P_{i,t}\}_{t=1}^{\infty}$
- **Policy Instruments:** $\{R_t, G_t, T_t\}_{t=1}^{\infty}$

such that

- Allocations solve the problem of the household given prices and policies.
- $P_{i,t}$ solves the problem of firm $i$.
- $P_{i,t} = P_{j,t}$ for all $i \neq j$.
- Markets clear.
- Government budget constraint is satisfied.