Fiscal Consolidations and Heterogeneous Expectations

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Abstract

We analyze fiscal consolidations using a New Keynesian model where agents have heterogeneous expectations and are uncertain about the composition of consolidations. Heterogeneity in expectations may amplify expansions, stabilizing thus the debt-to-GDP ratio faster under tax based consolidations, in the short run. In the medium to long-run though, the magnitude of the consolidation may favor spending cuts as long as agents anticipate tax increases. Interestingly, wrong beliefs about the composition of fiscal consolidation may improve or harm the effectiveness of the latter, depending on the degree of heterogeneity.

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1 Introduction

The recent financial crisis gave rise to various types of government interventions. These interventions led to rising government debt levels in most advanced economies (see IMF (2011)). Consequently, the majority of governments of those economies started the implementation of consolidation policies. In the Eurozone, such policies are still pursued in an effort to overcome a long lasting debt crisis in the countries of the Periphery. The debt crisis reshaped the way governments within and outside the Monetary Union think in terms of fiscal policy. In particular, the necessity for fiscal sustainability has arisen with the ultimate purpose of enhancing growth.

In the literature, there is substantial evidence on the effects of fiscal consolidations. In particular, there has been much research over the effects of different types of fiscal consolidations. That is, spending-based and tax-based consolidations. Moreover, the duration of consolidations is of substantial importance. There is a number of empirical studies arguing in favor of spending cuts as they tend to boost growth, contrary to tax increases (see Alesina and Perotti (1995),Perotti (1996),Alesina and Ardagna (1998, 2010), Ardagna (2004)).

Following the recent empirical facts, on the theoretical side, the analysis of fiscal consolidations has regained interest. Bi et al. (2013) construct a New Keynesian model and analyze the effects of different types of fiscal consolidations. Moreover, they look at the effects of persistence in those, as well as of uncertainty of economic agents over the type of the upcoming fiscal consolidation. Accounting for the monetary policy stance as well, they find that spending and tax-based consolidations can be equally successful in stabilizing government debt at low debt levels. Nevertheless, at high debt levels, spending based consolidations are expected to be expansionary and more successful in stabilizing debt, especially when agents anticipate a tax-based consolidation.

The theoretical literature on consolidations, so far, has assumed that agents are fully rational. The failure of traditional rational expectations models to capture some key facts in the data, especially after the recent financial crisis, raised the need for a richer modeling of economic behavior. In fact, there is very little evidence about the effects of fiscal consolidations when agents are not behaving fully rational at all times. In this paper, we try to bridge this gap by building a New Keynesian model with distortionary taxes where agents are boundedly rational in the spirit of Brock and Hommes (1997). In particular, we assume that there are two

types of agents in the economy. Namely, the Fundamentalists and the Naive agents. The first fraction uses policy (e.g. monetary and fiscal) announced rules when it forms its expectations about inflation and output. In particular, Fundamentalists take into account the commitment of the central bank to price stability. Moreover, when forming their expectations, they take into account the commitment of fiscal authority to stabilize debt-to-GDP when the latter exceeds a certain limit. On the contrary, Naive agents ignore the commitments of the two authorities when forming their expectations. Hence, they naively use past information, only, in order to predict inflation and output. Agents can switch between those two types according a endogenous fitness measure. Agents choose the type with the higher fitness measure (i.e. lower past forecast error).

Following Bi et al. (2013), we introduce uncertainty about the nature of fiscal consolidations. In particular, agents are uncertain about whether consolidation will be tax-based or spending-based once debt exceeds a known debt limit. As a result, they assign a probability to the occurrence of each type of consolidation. Given that the fiscal authority implements consolidations with a certain lag, this type of uncertainty affects expectations of fundamentalists. This is due to the assumption that those agents are forward looking and take into account the future monetary/fiscal policy stance.

We find that tax-based consolidations lead to a quick drop in debt-to-GDP ratio in the short-run, as opposed to spending based consolidations when fundamentalists anticipate the latter. Interestingly, in the period before consolidation, output expands due to the expected spending cuts. We show that, in this case, once the tax-based consolidation is implemented fundamentalists realize that they were wrong when forming their expectations. This leads to a rise in the fraction of naive agents the next period, which in turn leads to longer expansions following the consolidation. On the contrary, when consolidation is spending-based, output contracts abruptly upon consolidation and continues to contract throughout the consolidation.

Tax-based consolidations continue to be more successful in stabilizing debt in the short-run, when fundamentalists believe that they are more likely to happen. We show that, in this case, output contracts in the period before implementation. Once implemented, the rise in distortionary taxes increases marginal costs and, hence, inflation. Since monetary policy satisfies the Taylor principle, the rise in inflation leads to an increase in real interest rates which intensifies the initial contraction in output. However, given that the debt ratio

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falls faster below the debt limit under tax-based consolidations, the duration of consolidation lasts less which stops output from contracting. In the medium to long-run, spending-based consolidations might outperform tax-based, when the latter is anticipated, leading to lower a debt ratio, as long as they have a high enough magnitude. This is achieved, though, at the expense of a longer contraction in output. This is because it takes longer for the debt-to-GDP to fall below the debt limit which raises the duration of consolidation.

Our findings contribute to the existing literature in many ways. To the best of our knowledge, this is the first paper to analyze fiscal consolidations when agents are boundedly rational. We distinguish between the short and long-run effects of fiscal consolidations in terms of their performance in stabilizing debt. In line with the existing literature, we show that the magnitude, the duration, the composition and the likelihood of consolidation matter in determining the extend to which a specific type of consolidation is successful in stabilizing debt and/or expansionary. Our major contribution, though, is that the assumption of boundedly rational agents leads to much richer dynamics and policy implications that may differ to those under rational expectations, substantially.

The next section outlines the model and the fiscal consolidations that may occur. Section 3 describes introduces heterogeneity in the way agents form expectations. Section 4 illustrates the complete model after aggregation. Section 5 describes how effective the two types of consolidations are under different private sector expectations. Section 6 concludes.

2 Model

2.1 Households

There is a continuum of households that differ in the way they form expectations. In particular, a household can be either naive or fundamentalist. Households of the same type have the same preferences and make identical decisions. The intratemporal problem of each household i, consists of choosing consumption over different goods to minimize expenditure. This implies

$$C_t^i(j) = \left(\frac{p_t(j)}{P_t}\right)^{-\theta} C_t^i, \tag{1}$$

with C_t^i and P_t total consumption of the household and the aggregate price level, defined by

$$C_t^i = \left(\int_0^1 C_t^i(j)^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}}, \tag{2}$$

$$P_t = \left(\int_0^1 P_t(j)^{1-\theta} dj \right)^{\frac{1}{1-\theta}}, \tag{3}$$

where θ is the elasticity of substitution between the different goods.

Even though the horizon of the model is infinite, the households are short-sighted. That is, they form expectations about one period ahead, only. The household i chooses consumption (C_t^i), labor (H_t^i), and nominal bond holdings (B_t^i). Hence, the maximization problem of the household is summarized as

$$\max_{C,H,B} u(C_t^i, H_t^i) + \beta \tilde{E}_t^i u(C_{t+1}^i, H_{t+1}^i), \tag{4}$$

subject to its budget constraint

$$P_t C_t^i + B_t^i \leq (1 - \tau_t) W_t H_t^i + (1 - i_{t-1}) B_{t-1}^i + \Xi_t, \tag{5}$$

where W_t is the nominal wage, τ_t is the labor tax rate, and i_t is the nominal interest rate and. Ξ_t represents profits received by firms, and $\tilde{E}_t^i$ is the type specific expectation operator of household i (which can either be naive or fundamentalist).

The first order conditions with respect to C_t^i, H_t^i and b_t^i give

$$(C_t^i)^{-\sigma} = \lambda_t^i,$$

$$(H_t^i)^\eta = \lambda_t (1 - \tau_H) w_t,$$

$$\lambda_t^i = \beta \tilde{E}_t^i \frac{(1 + i_t) \lambda_{t+1}^i}{\Pi_{t+1}},$$

where Π_t is the gross inflation rate, and λ the Lagrange multiplier. Solving for this multiplier, we can rewrite these conditions to the Euler equation and an expression for the real wage, which, together with the budget constraint (5), must hold in equilibrium

$$(C_t^i)^{-\sigma} = \beta \tilde{E}_t^i \left[\frac{(1 + i_t)(C_{t+1}^i)^{-\sigma}}{\Pi_{t+1}} \right], \tag{6}$$

$$w_t = \frac{(H_t^i)^\eta (C_t^i)^\sigma}{(1 - \tau_t)}. \tag{7}$$

2.2 Firms

There is a continuum of firms producing the final differentiated goods. There is monopolistic competition and each firm is run by a household and follows the same heuristic for prediction of future variables as that household in each period. We assume Rotemberg pricing. Each monopolistic firm j faces a quadratic cost of adjusting nominal prices, which can be measured in terms of the final good and given by

$$\frac{\phi}{2} \left(\frac{P_t(j)}{P_{t-1}(j)} - 1 \right)^2 Y_t, \tag{8}$$

where ϕ measures the degree of nominal price rigidity. The adjustment cost, accounting for the negative effects of price changes on the customer-firm relationship, is increasing in the size of the price change and in the overall scale of economic activity. Each firm has a linear technology with labor as its only input

$$Y_t(j) = A_t H_t(j), \tag{9}$$

where A_t is an aggregate productivity shock following a stationary AR(1) process. Since firms are owned by households, they will be short sighted. That is, they will form expectations about their marginal costs and the demand for their product for one period ahead only. The problem of firm j is then to maximize the value of its profits discounted one period ahead only,

$$\max_{\{P_t(j)\}_{t=0}^1} \sum_{s=0}^1 Q_{t,t+s}^i \Xi_{t+s}, \tag{10}$$

where

$$Q_{t,t+s}^j = \beta^s \left(\frac{C_{t+s}^j}{C_t^j} \right)^{-\sigma} \frac{P_t}{P_{t+s}}. \tag{11}$$

is the stochastic discount factor of the household (j) that runs firm j. Aggregate nominal profits are defined as

$$\Xi_t = P_t(j) Y_t(j) - mc_t Y_t(j) P_t - \frac{\phi}{2} \left(\frac{P_t(j)}{P_{t-1}(j)} - 1 \right)^2 Y_t P_t \tag{12}$$

$$= P_t(j)^{1-\theta} P_t^{\theta} Y_t - mc_t P_t(j)^{-\theta} P_t^{1+\theta} Y_t - \frac{\phi}{2} \left(\frac{P_t(j)}{P_{t-1}(j)} - 1 \right)^2 Y_t P_t \tag{13}$$

Each firm resets the price of its good at each period, subject to the payment of the adjustment cost. The first order condition is

$$(1-\theta)Y_t(j) + \theta mc_t \frac{P_t}{P_t(j)} Y_t(j) - \phi \left(\frac{P_t}{P_{t-1}(j)} \right) (\Pi_t(j) - 1) Y_t + \phi \tilde{E}_t^j \left[Q_{t,t+1}^j Y_{t+1} \frac{P_{t+1}}{P_t(j)} \Pi_{t+1}(j)(\Pi_{t+1}(j) - 1) \right] = 0, \tag{14}$$

where $\Pi_t(j)$ is the (gross) price inflation of the good produced by firm j. From firm's cost minimization problem we can write the marginal cost as

$$mc_t = \frac{w_t}{A_t}. \tag{15}$$

Multiplying (14) by $\frac{P_t(j)}{P_t Y_t}$ and rearranging gives

$$(\theta-1)\frac{P_t(j)Y_t(j)}{P_t Y_t} + \phi \Pi_t(j)(\Pi_t(j) - 1) = \theta mc_t \frac{Y_t(j)}{Y_t} + \phi \tilde{E}_t^j \left[Q_{t,t+1}^j \frac{Y_{t+1}}{Y_t} \Pi_{t+1} \Pi_{t+1}(j)(\Pi_{t+1}(j) - 1) \right]. \tag{16}$$

Finally, plugging in the stochastic discount factor gives

$$(\theta-1)\frac{P_t(j)Y_t(j)}{P_t Y_t} + \phi \Pi_t(j)(\Pi_t(j) - 1) = \theta mc_t \frac{Y_t(j)}{Y_t} + \phi \beta \tilde{E}_t^j \left[\left(\frac{C_{t+1}^j}{C_t^j} \right)^{-\sigma} \frac{Y_{t+1}}{Y_t} \Pi_{t+1}(j)(\Pi_{t+1}(j) - 1) \right]. \tag{17}$$

2.3   Government

The government issues bonds and levies labor taxes (τ) to finance its spending (G_t). Its budget constraint is given by

$$B_t = P_t G_t - \tau_t w_t H_t + (1 + i_{t-1}) B_{t-1}, \tag{18}$$

with $H_t = \int H_t^i di$ and $B_t = \int B_t^i di$ aggregate labor and aggregate bond holdings respectively. Dividing by $Y_t P_t$ gives

$$b_t = g_t - \tau_t w_t \frac{H_t}{Y_t} + \frac{(1 + i_{t-1})b_{t-1}}{\Pi_t} \frac{Y_{t-1}}{Y_t} = g_t - \tau_t mc_t + \frac{(1 + i_{t-1})b_{t-1}}{\Pi_t} \frac{Y_{t-1}}{Y_t}, \tag{19}$$

where $b_t = \frac{B_t}{P_t Y_t}$ and $g_t = \frac{G_t}{Y_t}$ are the ratios of debt to GDP and government expenditure to GDP, respectively.

We assume constant taxes τ_1 and constant government spending g_1. We assume that these variables will only change when the debt level rises above a certain threshold, in which case fiscal consolidation is implemented. This consolidation can not immediately be implemented. If the government sees in period $t-1$ that the last debt level (b_{t-2}) was above the threshold (DL_{t-2}), it will decide to implement a consolidation. Their is however an implementation lag so that the consolidation will only take place in period t. Fiscal policy is defined by

$$g_t = g_1 - \zeta \gamma_1 \max(0, b_{t-2} - DL_{t-2}), \tag{20}$$

and

$$\tau_t = \tau_1 + (1 - \zeta)\gamma_2 \max(0, b_{t-2} - DL_{t-2}) \tag{21}$$

Here ζ is the fraction of the consolidation that is spending based. Below we will only consider the two extreme cases of fully spending based consolidation ($\zeta = 1$) and fully tax based consolidation ($\zeta = 0$).

2.4 Market Clearing

The aggregate resource constraint of the economy is summarized as

$$Y_t = C_t + G_t + \frac{\phi}{2}\left(\frac{P_t}{P_{t-1}} - 1\right)^2 Y_t, \tag{22}$$

Since the budget constraint of the government is expressed in per output terms and since its instruments to stabilize debt can be either tax revenues and/or government expenditure as a fraction of output we may write the above market clearing condition in the following form

$$Y_t = C_t + g_t Y_t + \frac{\phi}{2}\left(\frac{P_t}{P_{t-1}} - 1\right)^2 Y_t. \tag{23}$$

2.5 Zero inflation, low debt steady state

In this section we derive the steady state of the non-linear model, where gross inflation equals 1, and government spending and taxes do not respond to debt. We furthermore assume constant technology at $A = 1$

Evaluating (17) at the zero inflation steady state gives

$$\bar{mc} = \frac{\theta - 1}{\theta} \tag{24}$$

From (6) it follows that in this steady state we must have

$$1 + \bar{i} = \frac{1}{\beta} \tag{25}$$

Furthermore, from (9) it follows that

$$\bar{H} = \bar{Y} \tag{26}$$

Next, we solve the steady state aggregate resource constraint, (23), for consumption, and write

$$\bar{C} = \bar{Y}(1 - \bar{g}) \tag{27}$$

Plugging in these steady state labor and consumption levels in the steady state version of (7) gives

$$\bar{w} = \frac{\bar{Y}^{\eta}(\bar{Y}(1 - \bar{g}))^{\sigma}}{1 - \bar{\tau}} = \frac{\bar{Y}^{\eta + \sigma}(1 - \bar{g})^{\sigma}}{1 - \bar{\tau}} = \frac{\theta - 1}{\theta} \tag{28}$$

Where the last equality follows from (15) and (24). We can thus write

$$\bar{Y} = \left(\frac{\theta - 1}{\theta} \frac{1 - \bar{\tau}}{(1 - \bar{g})^{\sigma}}\right)^{\frac{1}{\eta + \sigma}} \tag{29}$$

Then we turn to the government budget constraint. In steady state (19) reduces to

$$\bar{b} = \bar{g} - \bar{\tau}\frac{\theta - 1}{\theta} + (1 + \bar{i})\bar{b}, \tag{30}$$

which gives

$$\bar{b} = \beta\frac{(\bar{\tau}\frac{\theta - 1}{\theta} - \bar{g})}{1 - \beta}, \tag{31}$$

where we used (25) to substitute for the interest rate.

Steady state government spending and taxes are given by

$$\bar{g} = g_1 - \zeta\gamma_1 \max(0, \bar{b} - \bar{D}L), \tag{32}$$

and

$$\bar{\tau} = \tau_1 + (1 - \zeta)\gamma_2 \max(0, \bar{b} - \bar{D}L), \tag{33}$$

Assuming that the steady state debt limit equals steady state debt this reduces to

$$\bar{g} = g_1 \tag{34}$$

and

$$\bar{\tau} = \tau_1 \tag{35}$$

2.6 Log linearized equilibrium

The Euler equation, (6), can be log linearized around the zero inflation steady state to get

$$\hat{C}(i)_t = \tilde{E}_t^i[\hat{C}(i)_{t+1}] - \frac{1}{\sigma}(i_t - \tilde{E}_t^i[\pi_{t+1}]), \tag{36}$$

where $\hat{C}_t = \frac{C_t - \bar{C}}{\bar{C}}$, with $\bar{C}$ the steady state value of consumption, and π_t is net inflation.

Assumption 1. *Agents realize they may switch to another heuristic in the future, and that other agents may do so as well. They furthermore assume that the probability to follow a particular heuristic next period is the same across agents.*

Agents with the same expectations will make the same consumption decision. It therefore follows from Assumption 1 that, from the perspective of any agent, its own expected future consumption is the same as expected aggregate future consumption, that is, $\tilde{E}_t^i[\hat{C}(i)_{t+1}] = \tilde{E}_t^i[\hat{C}_{t+1}]$, with $\hat{C}_{t+1} = \int_0^1 \hat{C}(i)_{t+1}di$. Agents therefore realize they should base their current period consumption decision on expectations about *aggregate* consumption. The Euler equation can then be written as

$$\hat{C}(i)_t = \tilde{E}_t^i[\hat{C}_{t+1}] - \frac{1}{\sigma}(i_t - \tilde{E}_t^i[\pi_{t+1}]), \tag{37}$$

Log linearizing the market clearing condition, (23), around the zero inflation steady state gives

$$\hat{Y}_t = \hat{C}_t + \frac{\hat{g}_t}{1 - \bar{g}} \tag{38}$$

where $\hat{Y}_t$ is the defined in the same way as $\hat{C}_t$, and $\hat{g} = g_t - \bar{g}$ (since g_t is already a rate). Agents know about market clearing and their forecasts satisfy $\tilde{E}_t^i[\hat{C}_{t+1}] = \tilde{E}_t^i[\hat{Y}_{t+1}] - \frac{1}{1-\bar{g}}\tilde{E}_t^i[\hat{g}_{t+1}]$. Therefore, (37) can be written as

$$\hat{C}(i)_t = \tilde{E}_t^i[\hat{Y}_{t+1}] - \frac{1}{\sigma}(i_t - \tilde{E}_t^i[\pi_{t+1}]) - \frac{1}{1 - \bar{g}}\tilde{E}_t^i[\hat{g}_{t+1}], \tag{39}$$

Aggregating this equation over all agents, and using the period t market clearing condition then gives

$$\hat{Y}_t = \bar{E}_t[\hat{Y}_{t+1}] - \frac{1}{\sigma}(i_t - \bar{E}_t[\pi_{t+1}]) - \frac{1}{1 - \bar{g}}(\bar{E}_t[\hat{g}_{t+1}] - \hat{g}_t) \tag{40}$$

Here $\bar{E}_t$ is the aggregate expectation operator defined by $\bar{E}_t[X_{t+1}] = n_t^N \tilde{E}_t^N[X_{t+1}] + (1 - n_t^N)\tilde{E}_t^F[X_{t+1}]$, with n_t^N the fraction of naive agents.

Next, turning the optimal pricing equation, (17), can be log linearized to

$$\frac{\phi}{\theta - 1}\pi_t(j) = \hat{P}_t - \hat{P}_t(j) + \hat{mc}_t + \frac{\beta\phi}{\theta\bar{mc}}\tilde{E}_t^j\pi_{t+1}(j), \tag{41}$$

or

$$\hat{\pi}_t(j) = \frac{\theta - 1}{\phi}(\hat{P}_t - \hat{P}_t(j)) + \frac{\theta - 1}{\phi}\hat{mc}_t + \beta\tilde{E}_t^j\pi_{t+1}(j), \tag{42}$$

where we used (24) to substitute for steady state marginal cost. Just as in the case of consumption, it follows from Assumption 1 that $\tilde{E}_t^j[\hat{\pi}_{t+1}(j)] = \tilde{E}_t^j[\hat{\pi}_{t+1}]$. Using this, and using the definition of the aggregate price level ($\hat{P}_t = \int_0^1 \hat{P}_t(j)dj$), we can now integrate the above firms specific inflation equation over all firms to get

$$\pi_t = \int_0^1 \hat{\pi}_t(j)dj = \frac{\theta - 1}{\phi}\hat{mc}_t + \beta\bar{E}_t\pi_{t+1}, \tag{43}$$

Log linearizing (15), (7) and (9), and combining with (38) gives

$$\hat{mc}_t = \hat{w}_t - \hat{A}_t = \eta\hat{H}_t + \sigma\hat{C}_t + \frac{\hat{\tau}_t}{1-\bar{\tau}} - \hat{A}_t = (\sigma+\eta)\hat{Y}_t - \sigma\frac{\hat{g}_t}{1-\bar{g}} + \frac{\hat{\tau}_t}{1-\bar{\tau}} - (1+\eta)\hat{A}_t \quad (44)$$

Inserting this in (43) gives

$$\pi_t = \beta\bar{E}_t[\pi_{t+1}] + \kappa(\sigma+\eta)\hat{Y}_t - \kappa\sigma\frac{\hat{g}_t}{1-\bar{g}} + \kappa\frac{\hat{\tau}_t}{1-\bar{\tau}} - \kappa(1+\eta)\hat{A}_t, \quad (45)$$

where $\kappa = \frac{(\theta-1)}{\phi}$.

Finally, the government budget constraint, (19), can be log linearized to get an equation for the evolution of the government debt to GDP ratio

$$\hat{b}_t = \hat{g}_t - \frac{\theta-1}{\theta}(\hat{\tau}_t + \bar{\tau}\hat{mc}_t) + \frac{1}{\beta}\hat{b}_{t-1} + \frac{\bar{b}}{\beta}(\hat{i}_{t-1} - \hat{\pi}_t - \hat{Y}_t + \hat{Y}_{t-1}), \quad (46)$$

with $\hat{mc}$ given by (44).

We assume the central bank targets only inflation and that the inflation target is zero (which is consistent with the assumption of a zero inflation steady state that was assumed in the log linearizion in the previous section). The log-linearized forward looking Taylor rule is given by

$$i = \phi_1\bar{E}\pi_{t+1} \quad (47)$$

Linearizing (20) and (21) gives

$$\hat{g}_t = -\zeta\gamma_1\max(0, \hat{b}_{t-2} - \hat{DL}_{t-2}), \quad (48)$$

and

$$\hat{\tau}_t = (1-\zeta)\gamma_2\max(0, \hat{b}_{t-2} - \hat{DL}_{t-2}), \quad (49)$$

where are variables are deviations from steady state.

The model is now given by (40), (45), (46), (47), (48) and (49).

3 Heuristics specification

We assume private sector beliefs are formed by two heuristics: fundamentalistic and naive. Naive agents believe future inflation, output gap and government spending to be equal to their last observed values: $\tilde{E}_t^n x_{t+1} = x_{t-1}$, $\tilde{E}_t^n \pi_{t+1} = \pi_{t-1}$, $\tilde{E}_t^n g_{t+1} = g_{t-1}$. Their expectations about future taxes do not show up in the equations of our model, and also do not influence naive agents' predictions about other variables. We therefore do not need to specify these expectations.

Fundamentalists know that future taxes and government spending will depend on the debt level. They therefore base their expectations about these variables on the debt level. We assume they do not know whether the consolidation will be taxed based or spending based, i.e., they do not know the value of ζ. They think it will be spending based with probability α. Furthermore if it is spending based they expect the magnitude of the consolidation to be proportional to the excess debt, with coefficient γ_1. Government spending expectations are therefore given by

$$\tilde{E}_t^f g_{t+1} = -\alpha\gamma_1 \max(0, b_{t-1} - DL_{t-1}) \tag{50}$$

Similarly they expect a tax based consolidation with probability $1 - \alpha$. Taxes are then proportionally increased in response to excess debt with coefficient γ_2

$$\tilde{E}_t^f \tau_{t+1} = (1 - \alpha)\gamma_2 \max(0, b_{t-1} - DL_{t-1}) \tag{51}$$

Fundamentalists take account of these expectations about next period taxes and government spending when they form expectations about the other variables (they do this simultaneously). They do not use long horizon forecast in their decision making process and they do not think about how the fiscal variables (or any other variable) may change after 2 or more periods. Instead they calculate the perfect foresight fixed point values of inflation and output, corresponding to the fiscal regime they expect to be implemented next period, that would arise in the absence of shocks. They know the model equations of inflation and output, and know the specification of the nominal interest rate rule.

It makes sense for fundamentalists to ignore shocks to the economy in their predictions

because shocks are white noise and are not contemporaneously observed by agents. Fundamentalists furthermore know that there are agents in the model that form expectations in a different manner. However, they do not know how many such agents there are in a given period and they also do not know what values other agents will predict. They can therefore not take these other agents into account in there expectation formation process. Instead fundamentalists expect the values that would occur if there were no shocks to the model and all agents made correct (perfect foresight) predictions.

From (47) it follows that in a state of the economy where all variables except debt are constant over time we must have

$$i = \phi_1 \pi \tag{52}$$

Furthermore, (40) reduces in such a fixed point state to

$$Y = Y - \frac{1}{\sigma}(i - \pi) - \frac{1}{1 - \bar{g}}(g - g), \tag{53}$$

from which it follows that

$$\pi = i \tag{54}$$

(52) and (54) can only both hold (assuming $\phi_1 \neq 1$) if

$$\pi = i = 0 \tag{55}$$

Therefore, fundamentalists inflation expectations satisfy $\tilde{E}_t^f \pi_{t+1} = 0$. Using the above, it follows from (43) that in the fixed point $mc = 0$. Plugging this in (44) implies

$$0 = \kappa(\sigma + \eta)Y - \sigma\kappa\frac{g}{1 - \bar{g}} + \kappa\frac{\tau}{1 - \bar{\tau}}, \tag{56}$$

or that

$$Y = \frac{\sigma}{\sigma + \eta}\frac{g}{1 - \bar{g}} - \frac{1}{\sigma + \eta}\frac{\tau}{1 - \bar{\tau}}, \tag{57}$$

Output expectations of fundamentalists therefore satisfy

$$\tilde{E}_t^f Y_{t+1} = \frac{\sigma}{\sigma + \eta}\frac{\tilde{E}_t^f g_{t+1}}{1 - \bar{g}} - \frac{1}{\sigma + \eta}\frac{\tilde{E}_t^f \tau_{t+1}}{1 - \bar{\tau}} \tag{58}$$

Plugging in (50) and (51) gives

$$\tilde{E}_t^f Y_{t+1} = -\frac{\sigma}{\eta + \sigma} \frac{1}{1-\bar{g}} \alpha\gamma_1 \max(0, b_{t-1} - DL) - \frac{1}{\eta + \sigma} \frac{1}{1-\bar{\tau}} (1-\alpha)\gamma_2 \max(0, b_{t-1} - DL),$$

Since fundamentalists make a joint prediction about all variables, the fractions of agents following this heuristic must be based on the performance of all predictions. Since expectations about taxes do not influence agents' decisions directly it should not be included in the fitness measure. The most natural fitness measure seems to be

$$U_{t-1} = -(g_{t-1} - \tilde{E}_{t-2}g_{t-1})^2 - (\pi_{t-1} - \tilde{E}_{t-2}\pi_{t-1})^2 - (y_{t-1} - \tilde{E}_{t-2}y_{t-1})^2, \tag{59}$$

Above we assumed that agents know the model equations and are able to calculate the fixed point values that would arise if all agents made the "correct" forecasts. This does however not mean that these theoretically correct forecast are indeed good predictors of the future. Due to heterogeneity in expectations formations the above fixed point will not necessarily be reached. Instead, it is possible that the presence of naive predictors in combination with shocks to the economy causes completely different dynamics. Naive predictors may then perform better than fundamentalists. This would cause more fundamentalists agents to abandon their models since these are apparently not good enough to make adequate predictions about the actual economy they are in. The fraction of naive agents would then increase.

4 Complete model

Our system is piecewise linear. The equation for inflation and output depend on the last two debt levels, even though these debt levels do not necessarily show up in the equations.

1. When debt is low ($b_{t-2} < DL$ and $b_{t-1} < DL$) we obtain, by plugging in monetary and fiscal policy, the following system for inflation and output

$$\hat{Y}_t = \bar{E}_t[\hat{Y}_{t+1}] - \frac{\phi_1 - 1}{\sigma}\bar{E}_t[\pi_{t+1}] - \frac{1}{1-\bar{g}}\bar{E}_t[\hat{g}_{t+1}] \tag{60}$$

$$\pi_t = \beta\bar{E}_t[\pi_{t+1}] + \kappa(\sigma + \eta)\hat{Y}_t - \kappa(1+\eta)\hat{A}_t, \tag{61}$$

In this region of low debt, fundamentalists expect all variables to be at their steady

state, so taht aggregate expectations are given by

$$\bar{E}_t g_{t+1} = n_t^N g_{t-1} \tag{62}$$

$$\bar{E}_t \pi_{t+1} = n_t^N \pi_{t-1} \tag{63}$$

$$\bar{E}_t Y_{t+1} = n_t^N Y_{t-1} \tag{64}$$

2. When debt has just crossed the critical boundary, but consolidation is not yet implemented ($b_{t-2} < DL$ but $b_{t-1} > DL$), then (60), (61) and (63) still hold, but for aggregate government spending expectations we then have

$$\bar{E}_t g_{t+1} = n_t^N g_{t-1} - (1 - n_t^N)\alpha\gamma_1(b_{t-1} - DL) \tag{65}$$

and for output expectations

$$\bar{E}_t Y_{t+1} = n_t^N Y_{t-1} - (1 - n_t^N)\frac{1}{\sigma + \eta}\left(\frac{\sigma}{1 - \bar{g}}\alpha\gamma_1 + \frac{1}{1 - \bar{\tau}}(1 - \alpha)\gamma_2\right)(b_{t-1} - DL) \tag{66}$$

3. When both $b_{t-2} > DL$ and $b_{t-1} > DL$ expectations are again given by (63), (65) and (66). However, when the consolidation is spending based inflation and output now are equal to

$$\hat{Y}_t = \bar{E}_t[\hat{Y}_{t+1}] - \frac{\phi_1 - 1}{\sigma}\bar{E}_t[\pi_{t+1}] - \frac{1}{1 - \bar{g}}(\bar{E}_t[\hat{g}_{t+1}] + \gamma_1(b_{t-2} - DL)) \tag{67}$$

$$\pi_t = \beta\bar{E}_t[\pi_{t+1}] + \kappa(\sigma + \eta)Y_t + \sigma\kappa\frac{\gamma_1(b_{t-2} - DL)}{1 - \bar{g}} - \kappa(1 + \eta)\hat{A}_t, \tag{68}$$

When the consolidation is tax based output is given by (60) and inflation is given by

$$\pi_t = \beta\bar{E}_t[\pi_{t+1}] + \kappa(\sigma + \eta)Y_t + \kappa\frac{\gamma_2(b_{t-2} - DL)}{1 - \bar{\tau}} - \kappa(1 + \eta)\hat{A}_t, \tag{69}$$

4. At some point the consolidation has worked enough and debt falls again below the critical threshold. One period later consolidation is no longer expected for the future,

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but still implemented in the current period (since $b_{t-2} > DL$ but $b_{t-1} < DL$). In that case, expectations are given by (62), (63) and (64) but output and inflation are given by (67) and (68) (spending based consolidation) or (60) and (69) (tax based consolidation).

Finally, the fraction of naive agents is equal to

$$n_t^N = \frac{e^{\omega U_{t-1}^N}}{e^{\omega U_{t-1}^N} + e^{\omega U_{t-1}^F}} \tag{70}$$

with U_{t-1}^N and U_{t-1}^F given by (59) evaluated at the naive predictions and fundamentalistic predictions respectively. ω is the intensity of choice parameter that determines how sensitive agents are to past performance of heuristics, and how fast they switch between heuristics.

5 Debt above the debt limit

When the debt level is above the debt limit consolidations are expected and implemented. That is, we are first in case 2 and then in case 3 of the previous section. In case 3 the dynamical system is then defined by (65) through (70), (63), and (46). The state vector is

$$\begin{pmatrix} y_t & \pi_t & b_t & y_{t-1} & \pi_{t-1} & b_{t-1} & n_{t+1}^N & n_t^N & \pi_{t-2} \end{pmatrix} \tag{71}$$

Below we analyze the dynamics that arise when consolidations are necessary, and how these dynamics are affected by our key parameters. These parameters are: the magnitude of consolidations (γ_1 and γ_2), the probability fundamentalists place on spending based consolidation (α), and the intensity of choice (ω).

More specifically we look at how these parameters influence the following four quantities: the levels of variables in the period where fundamentalists expect consolidations, but where it is not yet implemented (Section 5.1); the levels of variables in the period of implementations of consolidations (Section 5.2); the existence and stability (largest eigenvalue) of a fixed point above the debt limit (Section 5.3); and finally the level of this fixed point (Section 5.4). The first two quantities determine short run dynamics, while the latter two determine medium to long run dynamics.

Short run dynamics are not only affected by parameters, but also by initial fractions of

fundamentalists and naive agents. The effects of these initial conditions die out in the long run.

5.1 Effects of expected consolidations due to a shock to debt or the debt limit

If in period t debt is above the debt limit for the first time, then in period $t+1$, consolidations are expected by fundamentalists, but not yet implemented. The actual type of consolidation then does not matter yet, but instead dynamics are driven by the type of consolidation that fundamentalists expect. In Proposition 1 it is stated that depending on what type fundamentalists expect, a consolidation can both lead to an expansion or a contraction in output. Furthermore, if the expansion is large enough, expected consolidations lead to a reduction in debt. Proof of Proposition 1 is given in Appendix A.1.

Proposition 1. *Assume that a shock to debt or to the debt limit has increased debt above its threshold in period t ($b_t > DL_t$). This affects next periods output (and thereby also inflation, marginal cost and debt) through fundamentalists' expectations about future governemnt spending and future output. The effect of the debt or debt limit shock on next periods output is given by*

$$\frac{\partial Y_{t+1}}{\partial(b_t - DL_t)} = (1 - n_{t+1}^N)\left(\frac{\eta}{\sigma + \eta}\alpha\frac{\gamma_1}{1 - \bar{g}} - \frac{1}{\sigma + \eta}\frac{1}{1 - \bar{\tau}}(1 - \alpha)\gamma_2\right),$$

This implies that the effect of expected consolidations on output, marginal cost and inflation is positive, if and only if

$$\frac{\alpha\gamma_1}{(1 - \alpha)\gamma_2} > \frac{1 - \bar{g}}{1 - \bar{\tau}}\frac{1}{\eta}, \tag{72}$$

or in terms of the probability they place on a spending based consolidation:

$$\alpha > \frac{\gamma_2(1 - \bar{g})}{\gamma_1(1 - \bar{\tau})\eta + \gamma_2(1 - \bar{g})} \tag{73}$$

When this condition does not hold and the expectations lead to a contraction, then expected consolidation results in an increase in debt. When Condition (72) does hold, the expected consolidation leads to a reduction in debt if and only if the expansion in output is large

enough:

$$\frac{\partial Y_{t+1}}{\partial(b_t - DL_t)} > \frac{1 - \beta}{\beta\left((\bar{\tau}(\sigma + \eta))\frac{\theta-1}{\theta} + \frac{\bar{b}}{\beta}(1 + \kappa(\sigma + \eta))\right)}. \tag{74}$$

It follows from Proposition 1 that it is desirable that agents mainly expect spending based consolidations. The higher the probability that fundamentalists place on spending based consolidation, the larger output, and the lower the debt level in the period before the consolidation is actually implemented. Furthermore, as can be seen in Equation (72), when fundamentalists do mainly expect spending based consolidations, the expansion in output and the reduction in debt are increasing in the fraction of fundamentalists $(1 - n_{t+1}^N)$.

5.2 Effects of implemented consolidations

If after one period debt still is above the debt limit, the expectational effects analyzed above are still present two periods after the shock. However, in addition there are direct effects of implemented consolidations on output, inflation and debt. We now turn to the total of these effects in the period that consolidation is implemented for the first time (two periods after the shock). Proposition 2 states that in this first consolidation period, a tax based consolidation is always more effective than a spending based consolidation. Its proof is provided in Appendix A.2.

Proposition 2. *When we assume tax based based and spending based consolidations that have equal direct impact on the governments budget deficit $(\gamma_1 = \frac{\theta-1}{\theta}\gamma_2)$, then a tax based consolidation always results in lower debt than a spending based consolidation in the period of implementation. Moreover, the difference between $\frac{\partial b_{t+2}}{\partial(b_t - DL_t)}$ for spending based and tax based consolidation is given by*

$$\gamma_1\left((\bar{\tau}\frac{\theta - 1}{\theta} + \frac{\bar{\tau}\frac{\theta-1}{\theta} - \bar{g}}{1 - \beta}\kappa)\left(\frac{\theta}{(\theta - 1)(1 - \bar{\tau})} + \frac{\eta}{1 - \bar{g}}\right) + \frac{\bar{\tau}\frac{\theta-1}{\theta} - \bar{g}}{1 - \beta}\frac{1}{1 - \bar{g}}\right) \tag{75}$$

(75) is always positive, so that spending based consolidation indeed always leads to higher debt than tax based consolidation in the period where it is implemented for the first time. It can furthermore immediately be seen that the difference in this debt level is increasing the magnitude of consolidation $(\gamma_1 = \frac{\theta-1}{\theta}\gamma_2)$ and in steady state taxes $(\bar{\tau})$.

5.3 Existence and stability of high debt fixed point

Above we explicitly analyzed the levels of variables in the first two periods after the debt or debt limit shock. This gives clear insights in the short run dynamics that result form such a shock. Medium to long run dynamics will be determined by the existence and stability of a fixed point in the high debt region. In this section we investigate under what conditions this fixed point exists and how its stability is affected by γ_1, γ_2, α, and ω (intensity of choice).

5.3.1 Existence

We first consider the two benchmark cases where all agents have correct predictions in the fixed point. That is, a spending based consolidation with $\alpha = 1$, and a tax based consolidation with $\alpha = 0$. The same fixed point levels can also be obtained with infinite intensity of choice ($\omega = +\infty$). In this limiting case all agents have correct naive expectations in the fixed point. Stability conditions are then different however, as will be discussed below.

Proposition 3 states when the high debt fixed point lies above the debt limit in case of spending based consolidation. This is also the condition for the fixed point to exist, since below the debt limit variables are governed by the low debt dynamical system and not the high debt system analyzed in this section.

Proposition 3. *When consolidations are spending based, and agents expectations are correct in the high debt fixed point (either because fundamentalists are right, $\alpha = 1$, or because $\omega = +\infty$ and all agents become naive), then the fixed point lies above the debt limit if and only if*

$$\gamma_1 > \frac{1}{\beta} - 1, \tag{76}$$

Proposition 4 states that in case tax based consolidation the condition for existence of the high debt fixed point is the same as in case of spending based consolidation, but with γ_1 replaced by $\gamma_2 \frac{\theta-1}{\theta}$. This is intuitive because if we let $\gamma_1 = \gamma_2 \frac{\theta-1}{\theta}$, tax based and spending based consolidation are of equal magnitude in their effect on the budget deficit.

Proposition 4. *When consolidations are tax based, and agents expectations are correct in the high debt fixed point (either because fundamentalists are right, $\alpha = 0$, or because $\omega = +\infty$*

and all agents become naive), then the fixed point lies above the debt limit if and only if

$$\gamma_2 \frac{\theta - 1}{\theta} > \frac{1}{\beta} - 1, \tag{77}$$

5.3.2 Stability

When the fixed point exists in the high debt region it is of interest whether is locally stable, so that convergence to it can occur. In order to get insight in the stability of the fixed point for different parameter values we numerically calculate the eigenvalues in the fixed point. We therefor first need to calibrate the model. Unless otherwise stated we use the parameter values given in Table 1.

In Figure 1 the largest eigenvalues in the high debt fixed point for spending based (left panels) and tax based (right panels) are plotted in blue, as a function of $\gamma_1 = \frac{\theta - 1}{\theta} \gamma_2$ (the magnitude of consolidation). The lower two panels belong to the case of infinite intensity of choice, where all agents are naive in the fixed point. The top left and top right panel depict the eigenvalues of the benchmark cases of Proposition 3 and 4 respectively. The real parts of the largest eigenvalues are plotted in dotted red. When the solid blue and dotted red line do not coincide, the largest eigenvalue is complex, which implies oscillatory convergence or divergence.

From the previous section we know that for very low values of γ_1 and γ_2, the fixed point does not lie in the high debt region for the four cases plotted in Figure 1. For this reason we do not plot the largest eigenvalue for $\gamma_1 < \frac{1}{\beta} - 1$. It can be seen in the Figure, that in 3 of the four cases the largest eigenvalue is real and equal to unity for the lowest allowed value

Table 1: Parameter values

Parameter	Name	Value
β	Discount factor	0.99
σ	Relative risk aversion	2
$\frac{1}{\eta}$	Frisch elasticity of labor supply	0.5
θ	Elasticity of substitution	6
ϕ_1	Coefficient on inflation in Taylor rule	1.5
ω	Intensity of choice	10,000
ϕ	Price adjustment costs	100
$\bar{g}$	Steady state government spending	0.21
$\bar{\tau}$	Steady state taxes	0.26
$\bar{b}$	Steady state debt	0.66
DL_t	Deviation of debt limit from steady state	0.04
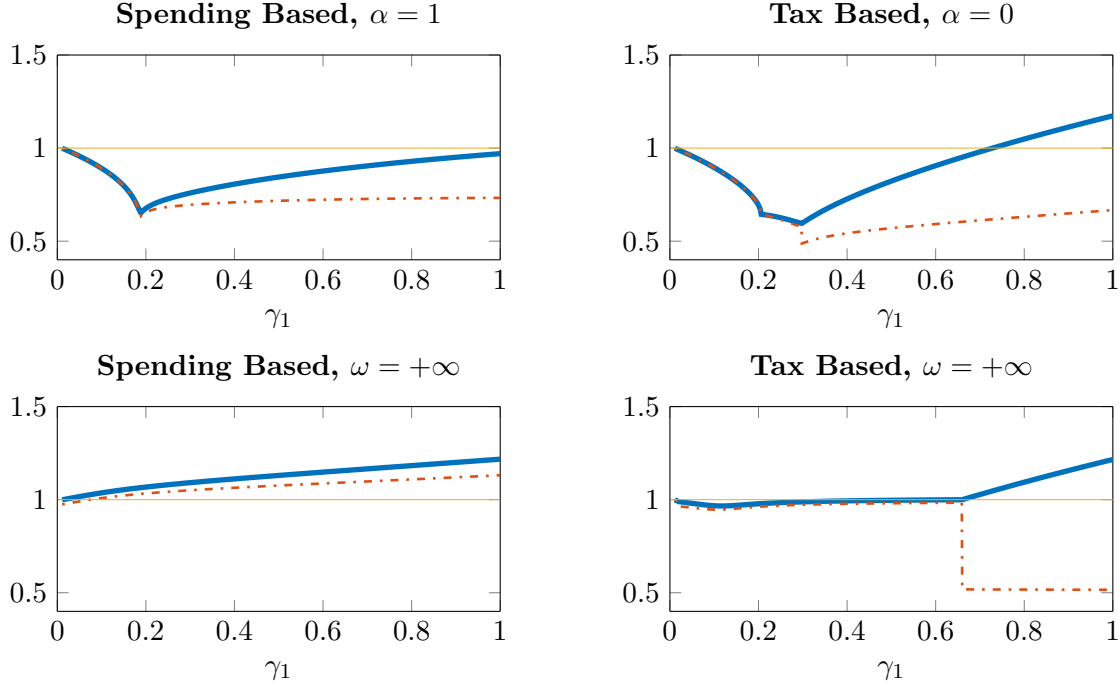
Figure 1: Largest eigenvalue (solid blue) in high debt fixed point for the four benchmark cases of Section 5.3.1. The real part of the largest eigenvalue is plotted in dashed red.

of γ_1. As γ_1 and γ_2 go up, this real eigenvalue decreases and the fixed point becomes more and more stable. In the top two panels with equal fractions of fundamentalists and naive agents the largest eigenvalue decreases to values that are quite low. In contrast, a complex eigenvalue very close to (but lower than 1 almost immediately becomes the largest in case of tax based consolidations with infinite intensity of choice.

When consolidations are spending based this complex eigenvalue is largest for all value of γ_1 when the intensity of choice is infinite. This eigenvalue furthermore always lies outside the unit circle, implying that the fixed point cannot be locally stable in this case. We can conclude that with infinite intensity of choice and aggressive enough fiscal policy (Condition (A.27) is satisfied) there is oscillatory divergence when consolidations are spending based, an very slow oscillatory convergence for tax based consolidations that are not too aggressive.

In the two benchmark cases where fundamentalists are correct about the type of consolidation, complex eigenvalues also become the largest for larger values of γ_1, but this occurs much later. As in the case of infinite intensity of choice, the largest eigenvalue of tax based consolidation is smaller than that of spending based consolidation for a considerable range of fiscal policy parameters. However both are inside the unit circle, so in both cases there is

oscillatory convergence. Furthermore, the eigenvalue of tax based consolidation crosses the unit circle for a lower value of γ than that of spending based consolidation. For aggressive policy there thus is oscillatory divergence when fully expected consolidations are tax based, but oscillatory convergence when they are spending based.

But what can be said about about stability and dynamics when intensity of choice is finite and α is not equal to 0 ore 1? For arbitrary values of α and ω the fraction of naive agents in the fixed point will be between 0.5 (as in the top panels) and 1 (as in the bottom panels). This fraction determines the stability (magnitude of the eigenvalues) of the fixed point. Since the model is linear in fractions, the fixed point levels of other variables do not show up in the Jacobian and do not affect stability. The closer α is to the benchmark value, the more correct fundamentalists are, and the closer the fixed point fraction of naive agents is to 0.5. The largest eigenvalue is then closer to that of the top panels of Figure 1.

This is illustrated in Figure 2, where the largest eigenvalues for spending and tax based consolidation are plotted for different values for α. The bottom left and top right panel correspond to the benchmark cases already depicted in Figure 1. First considering spending based consolidations (left column of Figure 2) it can be seen that as fundamentalists expect more tax based consolidations the largest eigenvalue is increased for any value of γ_1. These eigenvalues therefore indeed come closer to the case of all naive agents, depicted in the bottom left panel of Figure 1. Note that as fundamentalists expect more tax based consolidations, the range of low γ_1 parameters for which the fixed point does not exist becomes larger. The top left two panels therefore start at higher values of γ_1.

Then turning to the right column of tax based consolidations, we see the same pattern. As fundamentalists expect more spending based consolidations (and therefore become more wrong), more agents are naive in the fixed point, which leads to larger eigenvalues. The effect of changing α seems however smaller than in case of spending based consolidation.

When α does not equal the benchmark value the fixed point fraction of naive agents not only depends on the value of α, but also on the intensity of choice. If the intensity of choice is zero, the fractions of naive agents always equal 0.5, no matter the value of α. The higher the intensity of choice, the higher the fixed point fraction of naive agents for any given non benchmark value of α.

This is illustrated in Figure 3. Here α is set to 0.5 and the largest eigenvalue is plotted for
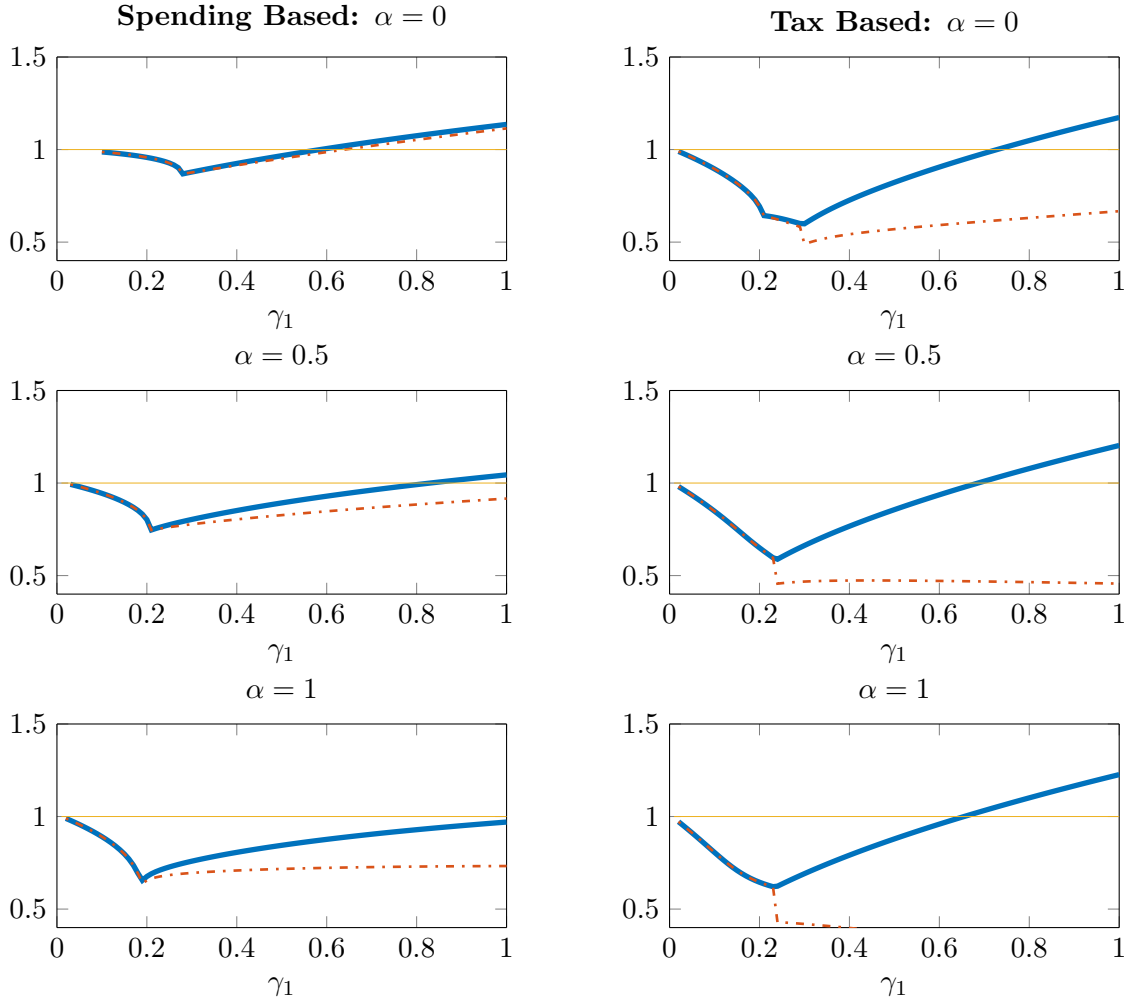
Figure 2: Largest eigenvalue (solid blue) in high debt fixed point for finite intensity of choice and different values of α. The real part of the largest eigenvalue is plotted in dashed red.
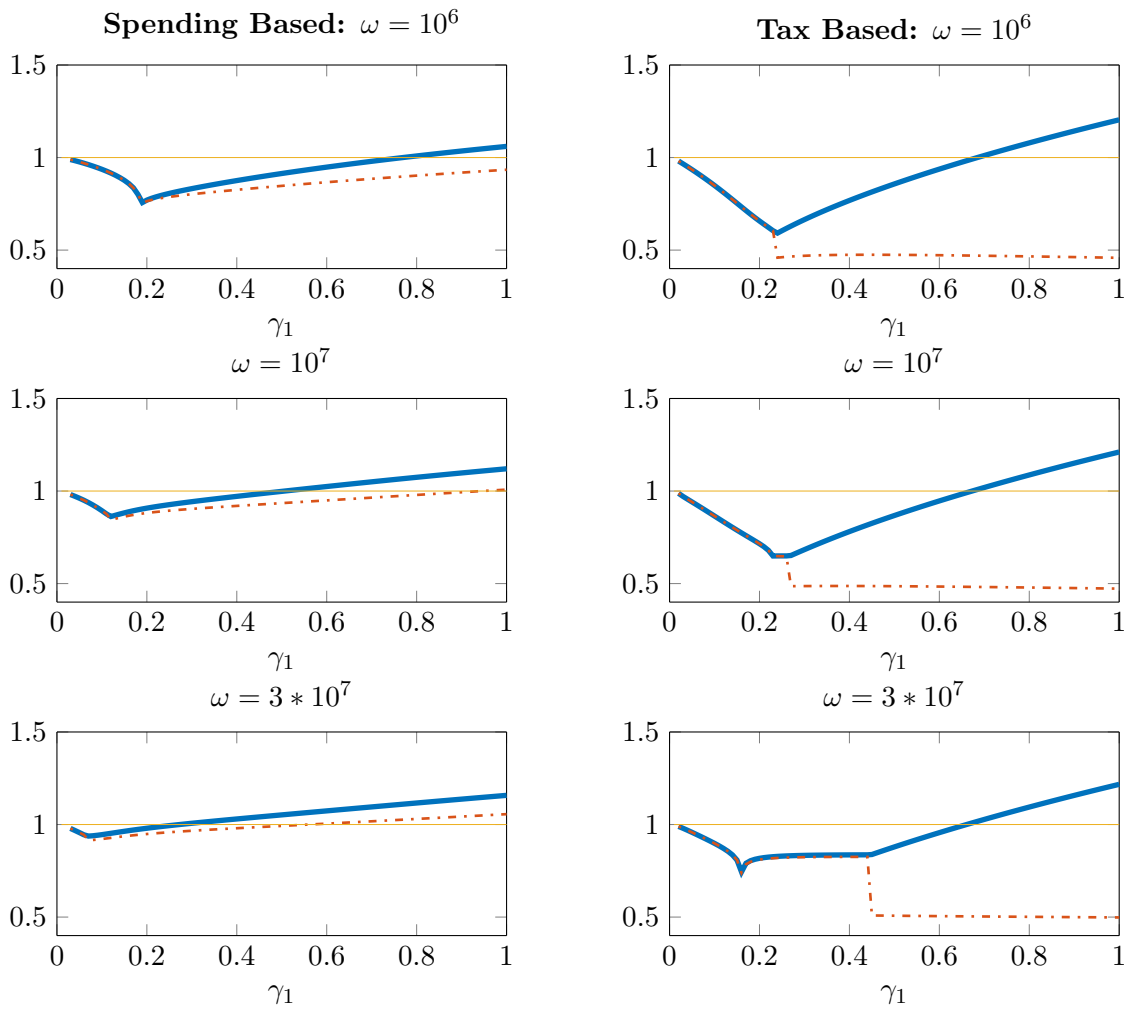
different values of the intensity of choice. In the top two panels the intensity of choice is set to 10^6. Even though this comprises a one hundred time increase compared to the value used in the previous figures, this increase does not effect fixed point fraction of naive agents much. As a consequence the largest eigenvalues of these two cases lie very close to those depicted in the middle two panels of Figure 2. The reason that fixed point fractions are not affected much is that the fixed point debt level lies close to the debt limit, so that, in the fixed point, implemented consolidations are very small. Fundamentalists are therefore not very wrong here.

As we increase the intensity of choice even more, the fixed point fraction of naive agents does increase and the largest eigenvalue does change. This can be seen in the middle panels

of Figure 3, where the intensity of choice is set equal to 10^7. Finally, the bottom two panels of Figure 3, illustrate that that if the intensity of choice is increased even more (to $3*10^7$), the largest eigenvalues move closer and closer to the limiting case of infinite intensity of choice and all naive agents, depicted in the bottom two panels of Figure 1.

We can conclude that when fundamentalists are not fully correct about the type of consolidation a higher intensity of choice leads to less stability and more oscillation around the high debt fixed point.

Figure 3: Largest eigenvalue (solid blue) in high debt fixed point for different intensities of choice (ω), when $\alpha = 0.5$. The real part of the largest eigenvalue is plotted in dashed red.

5.4 Level of the high debt fixed point

When the high debt fixed point is stable, the debt level of this fixed point is of crucial importance. When this debt level lies very close to the debt limit, the government might be content with convergence to this fixed point. However, if the debt level lies considerably above the limit, convergence to the fixed point is not desirable.

The parameters α and ω both affect aggregate expectations in the fixed point. α only shows up in our model through expectations and its effect on the mode variables can therefore easily be analyzed. An increase or decrease in α results (for finite intensity of choice) in an increase or decrease in output.

$$\frac{\partial \bar{E}_t g_{t+1}}{\partial \alpha} = -(1 - n_t^N)\left(\gamma_1(b_{t-1} - DL)\right) < 0 \tag{78}$$

$$\frac{\partial \bar{E}_t Y_{t+1}}{\partial \alpha} = (1 - n_t^N)\frac{1}{\eta + \sigma}\left(-\frac{\sigma}{1 - \bar{g}}\gamma_1(b_{t-1} - DL) + \frac{1}{1 - \bar{\tau}}\gamma_2(b_{t-1} - DL)\right) \tag{79}$$

$$\frac{\partial Y_t}{\partial \alpha} = \left(\frac{\partial \bar{E}_t Y_{t+1}}{\partial \alpha} - \frac{1}{1 - \bar{g}}\frac{\partial \bar{E}_t g_{t+1}}{\partial \alpha}\right) = \frac{\partial \bar{E}_t C_{t+1}}{\partial \alpha} =$$

$$(1 - n_t^N)\left(\frac{\eta}{\eta + \sigma}\frac{\gamma_1}{1 - \bar{g}}(b_{t-1} - DL) + \frac{1}{\eta + \sigma}\frac{\gamma_2}{1 - \bar{\tau}}(b_{t-1} - DL)\right) > 0, \tag{80}$$

Now we know the sign of the effect of a change in α on output we can analyze the effect of a change in α on the high debt fixed point. When intensity of choice is finite and α is not equal to the benchmark values analyzed above, the fixed point values for spending and tax based consolidations are no longer equal to those of the benchmark cases analyzed above.

Because increasing α increases output and does not directly affect any other variable, an increase in α must increase fixed point output, marginal cost and inflation, and decrease fixed point debt.

When α is not equal to the benchmark values of the previous subsection the intensity of choice also affects the fixed point levels. Recall that for infinite intensity of choice the fixed point levels are as in the benchmark cases, no matter what the values of α is. The lower the intensity of choice the more the effect of a non-benchmark α is reinforced. For zero intensity of choice 50% of the agents are fundamentalists and 50% are naive. In this most extreme case the fixed point lies as far from the benchmark as possible for any given α.

Finally we turn to the magnitude of consolidation. From Proposition 3 and 4 it follows about the benchmark cases that, when the high debt fixed point exists and is stable, more aggressive policy leads to a lower debt level in the fixed point. We furthermore numerically find that this result also holds when the intensity of choice is finite and α is not equal to the benchmark values,

5.5   Comparison spending and tax based consolidations

In this Section we analyze the effect of a shock to the debt limit both in the short run and in the long run, using impulse responses. We show the difference between spending based consolidation and tax based consolidation and the difference between strong and weak responses to debt. We also look how these results depend on agents expectations.

Since our model is linear in expectation fractions, impulse responses are only affected by the fractions of naive agents in the initial two periods, and not by other initial conditions (like the level of output when the shock hits). We therefore let the initial fractions of naive agents vary over a two dimensional grid to obtain both a median impulse response, and an upper and lower bound on the impulse responses.

In Figure 4 the impulse responses of output and debt are plotted in case the government implements spending based consolidation, and fundamentalists also expected mostly spending based consolidations ($\alpha = 0.8$). Before period 0 the debt limit was above the debt level, and from period 0 onward the debt limit is lowered to 70% of GDP, which we assume to be below the initial debt level of 75% of GDP. The debt limit is indicated by the blue lines in the right two panels. Black lines indicate median impulse responses, and upper and lower bounds are given by the dotted red curves.

First consider the left column of panels. Here the reaction coefficient to debt is given by $\gamma_1 = 0.3$. In the top left panel we see that in period 1, the period that consolidation is expected but not yet implemented, a large expansion in output can occur. This is in line with Proposition 1, since we are considering $\alpha = 0.8$. As can be seen in Equation 72, the effect of the shock on output in period 1 is scaled by the fraction of fundamentalists. When all agents are naive in period 1, nobody expects consolidations to occur in period two and nothing happens in period 1. When all agents are fundamentalists, everyone expects consolidations and there is a large expansion in output. This explains why the upper and lower bound lie
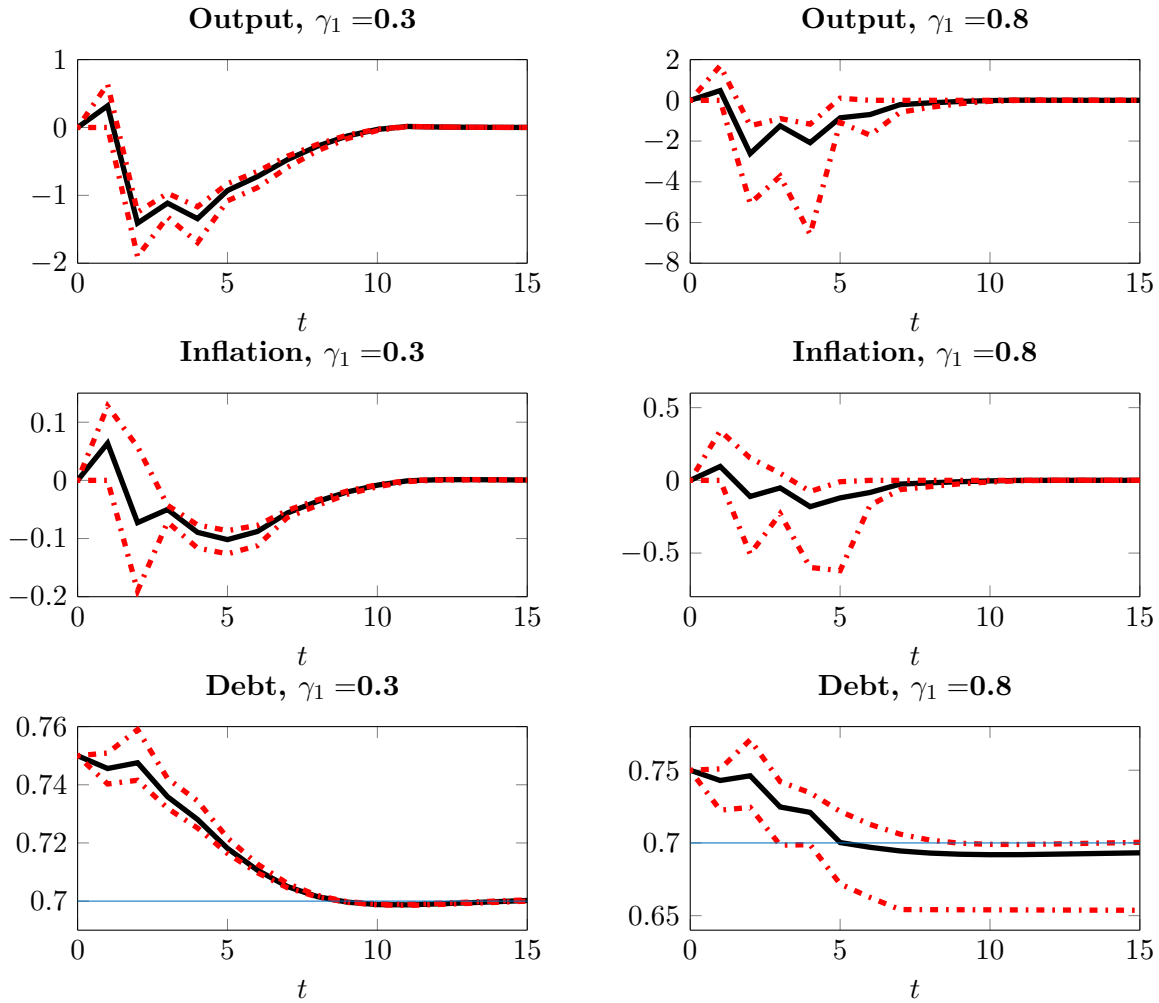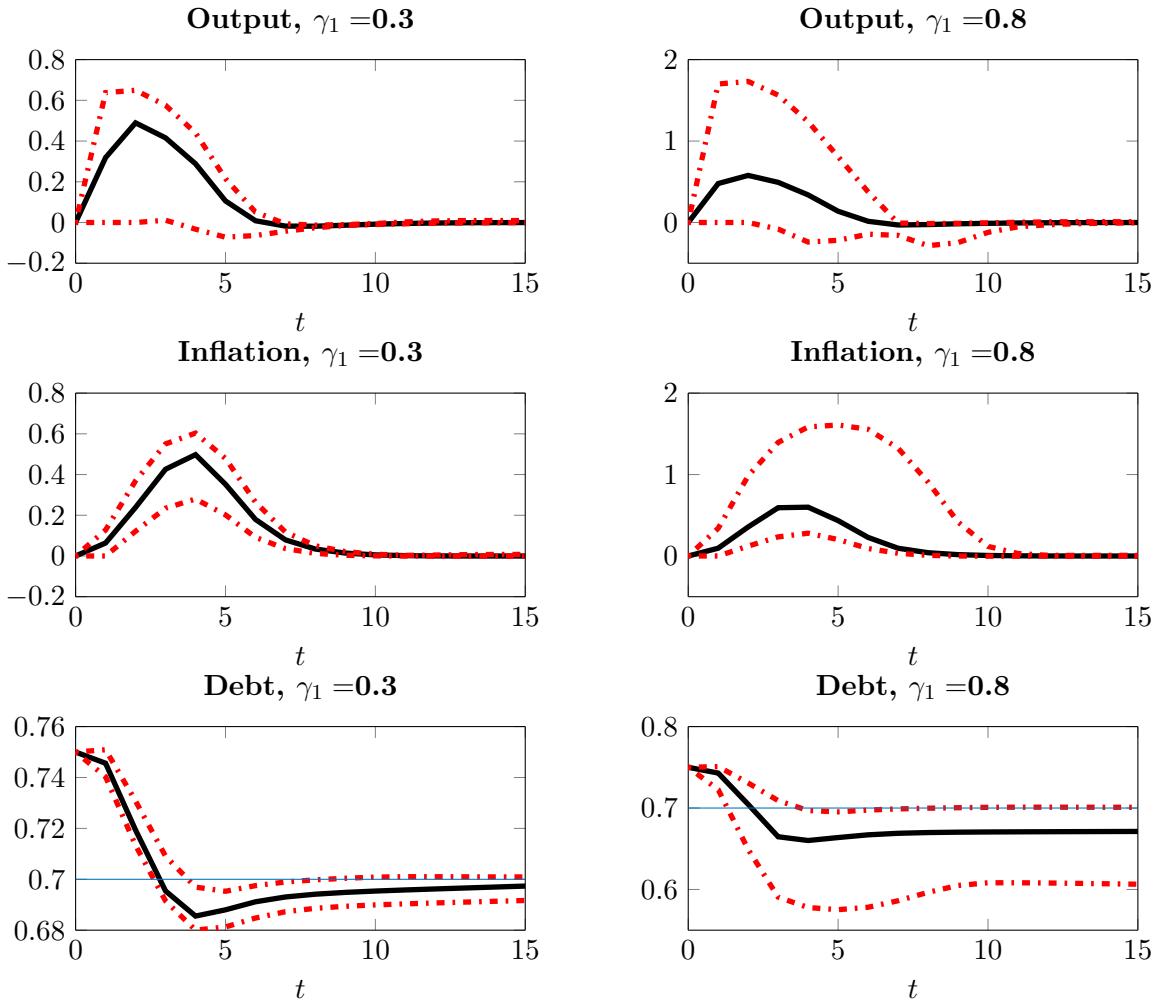
Figure 4: Impulse response to debt limit shock of output, inflation and debt for spending based consolidation when $\alpha = 0.8$. Impulse responses depend on initial fractions of naive agents. The median is plotted in solid black and the maximum and minimum are plotted in dotted red. The debt limit (after the shock) is plotted in blue in the right two panels. The top two panels depict the case of a moderately weak fiscal response and the bottom two panels the case of a strong fiscal response.

relatively far apart here. The middle left panel shows that the effect on inflation is similar to that of output, which also is in line with the theoretical results of Section 5.1. In the bottom left panel it can be seen that the expansion in output and inflation causes a decline in debt. However, when all agents were naive, there is no expansion and debt slightly increases in the first period due to interest rate payments (top red dashed line).

In period 2 government spending is lowered, which leads to a sharp decline in output, and a (less severe) decline in inflation. This initially causes an increase in debt. However, in the long run debt decreases towards the high debt fixed point, which lies slightly above the debt limit. Meanwhile output and inflation converge to their fixed point levels, close to 0. From Section 5.3.2 we know that for almost fully expected spending based consolidation with low γ_1, the largest eigenvalue in the high debt fixed point will be relatively low. In the top right panel of Figure 4 we therefore see that debt decreases to a point below the high debt fixed point, but that it does not overshoot to much. This indicates that the fixed point indeed is fairly stable.

Turning to the right column of panels of Figure 4, where the policy coefficient is increased to $\gamma_1 = 0.8$, we see that there is potentially much more overshooting here. This is in line with the finding of Section 5.3.2 that the largest eigenvalue is larger here. However, from the large difference between the two red dashed lines, it can be seen that the amount of overshooting very much depends on initial fractions of naive agents. Similarly, the qualitative response of output to the shock is the same as in the case of $\gamma_1 = 0.3$, but the upper and lower bound now lie much further apart. We can conclude that in case of mostly expected spending based consolidation, a stronger response will typically work better than a weak response, but it will also bring more uncertainty

In Figure 5 the impulse responses of a shock to the debt limit are plotted when the governemnt implements tax based consolidation, even though agents mostly expected spending based consolidation ($\alpha = 0.8$). In period 1 no consolidation is implemented yet, so dynamics are the same as in Figure 4 (note the difference in scale on te y-axis though). In period 2, when tax based consolidation is implemented, debt is immediately reduced. This is in line with Proposition 2, which says that tax based consolidation always *initially* leads to lower debt than spending based consolidation. Additionally, we see that there now is no contraction in output (neither initially nor in the long run), which is also more desirable. Long run
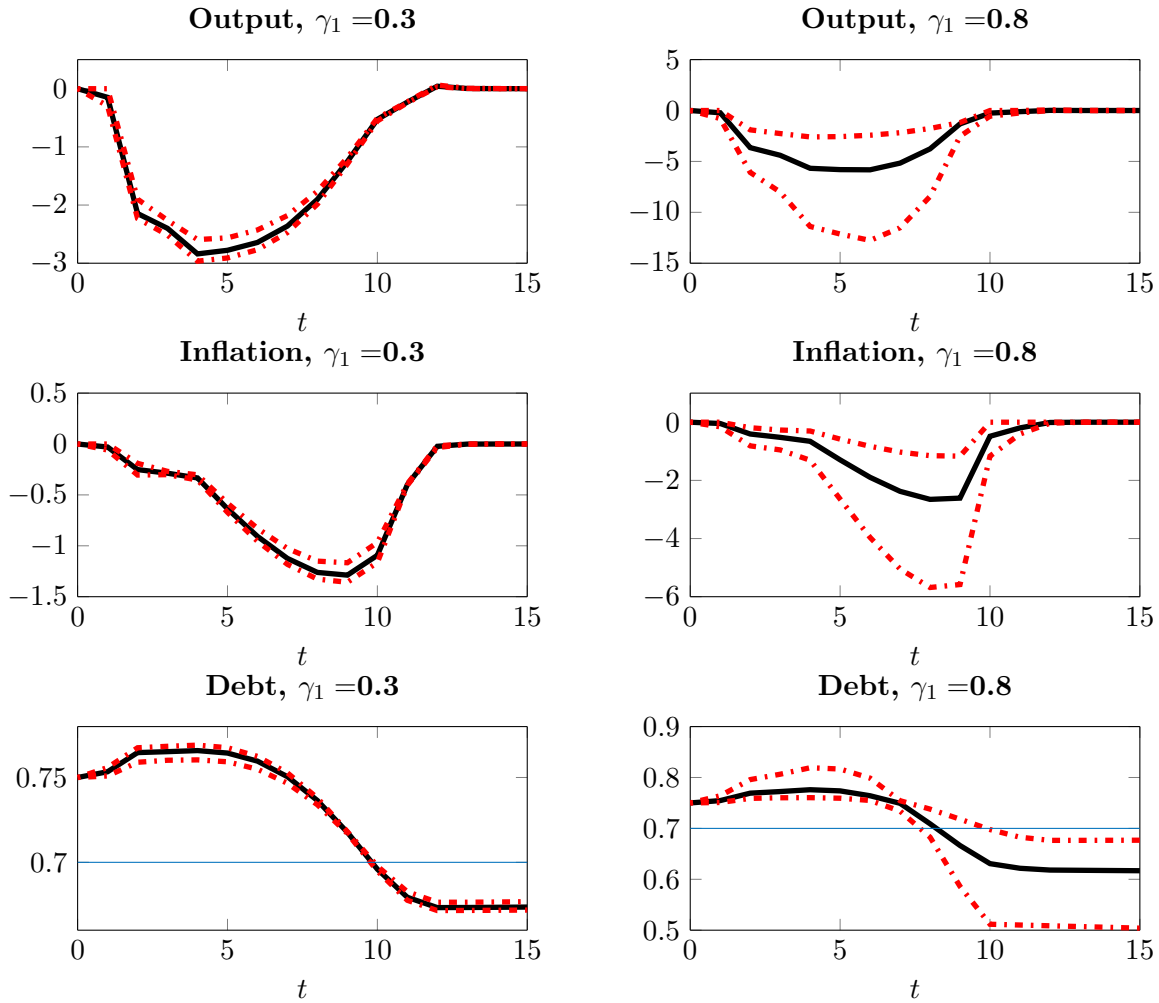
Figure 5: Impulse response to debt limit shock of output and debt for tax based consolidation when $\alpha = 0.8$. Impulse responses depend on initial fractions of naive agents. The median is plotted in solid black and the maximum and minimum are plotted in dotted red. The debt limit (after the shock) is plotted in blue in the right two panels. The top two panels depict the case of a moderately weak fiscal response and the bottom two panels the case of a strong fiscal response.

debt furthermore is as low or lower as in Figure 4. Overall tax based consolidation clearly is preferable when agents expect spending based consolidation.

But what if agents mostly expect tax based consolidation? Figures 6 and 7 plot impulse responses with spending based and tax based consolidation respectively when $\alpha = 0.2$. First of all, as predicted by Proposition 1 there is now a contraction instead of an expansion in output in period 1. Secondly, in Period 2, debt increases in case of spending based consolidation, but decreases in case of tax based consolidation, in line with Proposition 2.

Spending based consolidation now performs worse then in the case of $\alpha = 0.8$, both initially and after a few periods. The cut in government spending leads to a decrease in output that adds to the contraction already in place because of expectations. This combined effect leads to a much deeper and longer recession that is accompanied by a high debt level. Only when output starts to increase again debt decreases towards the debt limit. We furthermore see that for $\gamma_1 = 0.8$ debt decreases more sharply than for $\gamma = 0.3$, but that the reduction in debt does not start earlier. The reason for this is that a stronger response to debt also leads to an even larger contraction in output which prevents policy from effectively controlling debt.

Tax based consolidation on the other hand does not directly depress output. This makes the contraction caused by expectations much milder, as can be seen by comparing the top panels of Figure 7 with those of Figure 6. As a consequence debt dynamics turn out not to be very sensitive to the value of α, as can be seen by comparing the right two panels of Figure 5 and 7. Our conclusion that tax based consolidation is better than spending based consolidation thus holds even more strongly when agents mostly expect tax based consolidation.

Figure 6: Impulse response to debt limit shock of output and debt for spending based consolidation when $\alpha = 0.2$. Impulse responses depend on initial fractions of naive agents. The median is plotted in solid black and the maximum and minimum are plotted in dotted red. The debt limit (after the shock) is plotted in blue in the right two panels. The top two panels depict the case of a moderately weak fiscal response and the bottom two panels the case of a strong fiscal response.
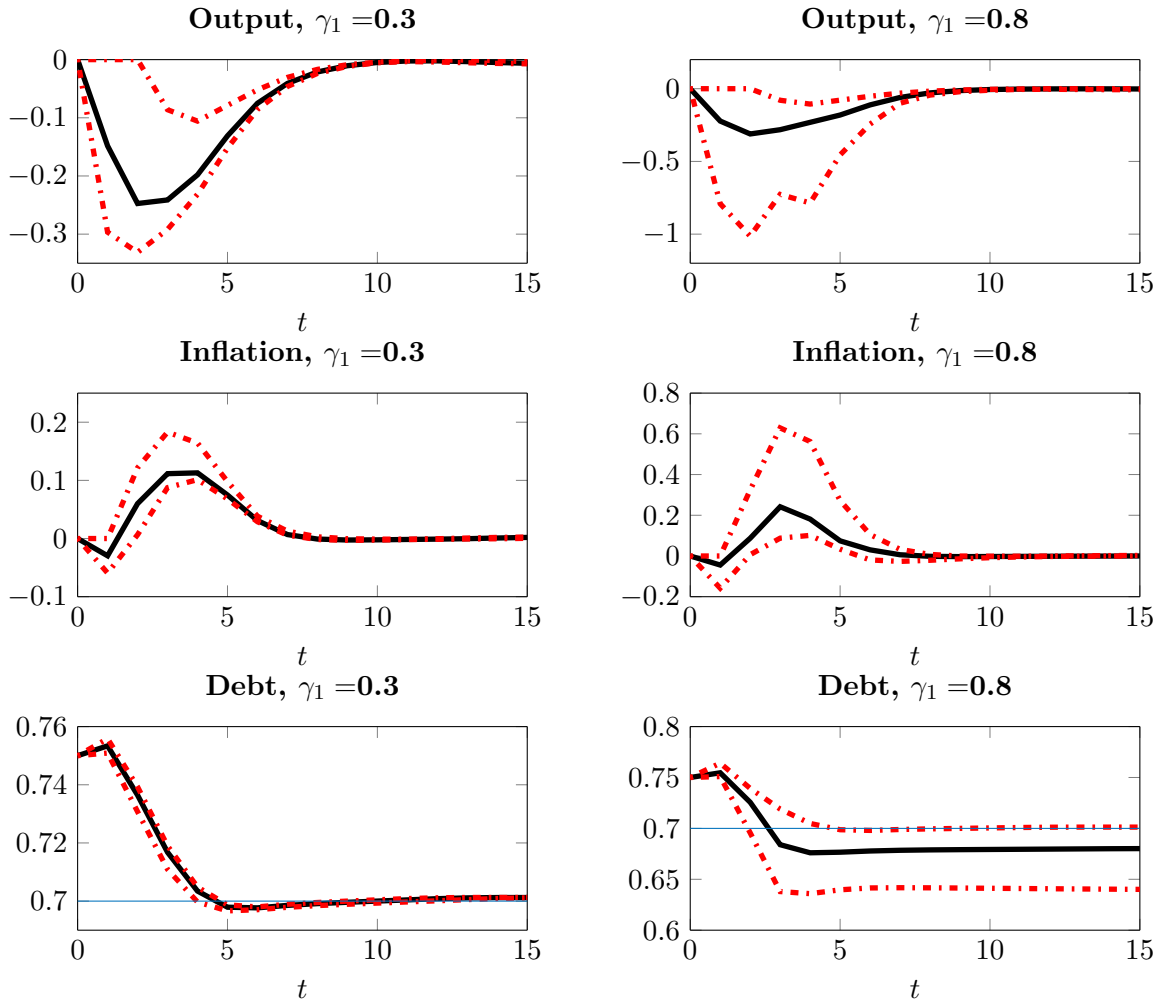
Figure 7: Impulse response to debt limit shock of output and debt for tax based consolidation when $\alpha = 0.2$. Impulse responses depend on initial fractions of naive agents. The median is plotted in solid black and the maximum and minimum are plotted in dotted red. The debt limit (after the shock) is plotted in blue in the right two panels. The top two panels depict the case of a moderately weak fiscal response and the bottom two panels the case of a strong fiscal response.

6 Conclusions

In this paper we have explored the effects of fiscal consolidations when agents are heterogeneous and uncertain about the composition of the consolidations. We assumed agent heterogeneity in the way expectations are formed in the spirit of Brock and Hommes (1997). Agents can switch between two types, namely, the fundamentalist and the naive. The former type consisted of forward looking agents, whereas the latter consisted of backward looking agents.

The Fiscal authority was assumed to engineer a consolidation once debt exceeds an announced debt limit, with a lag. Consolidations were implemented either through spending cuts or tax increases. Prior to the consolidation agents were uncertain about the composition. Given the lag in implementation, such uncertainty affected only the way fundamentalists formed their expectations leaving the expectations of naive agents unaffected.

Our first finding was that tax-based consolidations outperform spending-based ones in the short-run leading to an abrupt fall in the debt-to-GDP ratio. We showed that the type of consolidation anticipated was crucial at determining whether the consolidation would trigger expansions or abrupt contractions in output so long as it lasts. Moreover, whether the type of consolidation anticipated was correct or not, determined its duration. Consolidations last longer as long as agents wrongly anticipate them to be tax-based, but turn out to be spending based. This is due to the persistent contraction in output triggered by the the spending cuts and the rise in real interest rates due to inflationary pressures.

Interestingly, in the medium to long-run it is ambiguous which type of consolidation is more successful in stabilizing the debt ratio. Spending based consolidations of high magnitude tend to outperform tax-based ones, when agents anticipate the latter to occur. The model was complex in its dynamics and we kept the analysis as simple as possible. Cases like the effect of the zero lower bound on the potential of a consolidation to be expansionary and/or successful in stabilizing debt, in a heterogeneous agents model, deserve further research.

A Proofs of propositions

A.1 Proof Proposition 1

$$\frac{\partial \bar{E}_{t+1} g_{t+2}}{\partial (b_t - DL_t)} = -(1 - n_{t+1}^N)\alpha\gamma_1 \tag{A.1}$$

$$\frac{\partial \bar{E}_{t+1} Y_{t+2}}{\partial (b_t - DL_t)} = -(1 - n_{t+1}^N)\frac{1}{\sigma + \eta}\left(\frac{\sigma}{1 - \bar{g}}\alpha\gamma_1 + \frac{1}{1 - \bar{\tau}}(1 - \alpha)\gamma_2\right) < 0 \tag{A.2}$$

$$\frac{\partial Y_{t+1}}{\partial (b_t - DL_t)} = \left(\frac{\partial \bar{E}_{t+1} Y_{t+2}}{\partial (b_t - DL_t)} - \frac{1}{1 - \bar{g}}\frac{\partial \bar{E}_{t+1} g_{t+2}}{\partial (b_t - DL_t)}\right) = \frac{\partial \bar{E}_{t+1} C_{t+2}}{\partial (b_t - DL_t)} =$$
$$(1 - n_{t+1}^N)\left(\frac{\eta}{\sigma + \eta}\alpha\frac{\gamma_1}{1 - \bar{g}} - \frac{1}{\sigma + \eta}\frac{1}{1 - \bar{\tau}}(1 - \alpha)\gamma_2\right), \tag{A.3}$$

It follows that the effect of a debt (or debt limit) shock on next periods output (and marginal cost and inflation) is positive, if and only if

$$\eta\alpha\gamma_1\frac{1}{1 - \bar{g}} > \frac{1}{1 - \bar{\tau}}(1 - \alpha)\gamma_2$$
$$\frac{\alpha\gamma_1}{(1 - \alpha)\gamma_2} > \frac{1 - \bar{g}}{1 - \bar{\tau}}\frac{1}{\eta} \tag{A.4}$$

When $\alpha = 1$ this will always hold and when $\alpha = 0$ it will never hold. Solving for α gives

$$\alpha > \frac{\gamma_2(1 - \bar{g})}{\gamma_1(1 - \bar{\tau})\eta + \gamma_2(1 - \bar{g})}$$

Next we turn the the effect of a debt shock on debt on the other variables of the model. We first of all have

$$\frac{\partial \pi_{t+1}}{\partial (b_t - DL_t)} = \kappa(\sigma + \eta)\frac{\partial Y_{t+1}}{\partial (b_t - DL_t)} \tag{A.5}$$

$$\frac{\partial mc_{t+1}}{\partial (b_t - DL_t)} = (\sigma + \eta)\frac{\partial Y_{t+1}}{\partial (b_t - DL_t)} \tag{A.6}$$

For debt we have

$$\frac{\partial b_{t+1}}{\partial(b_t - DL_t)} = \frac{1}{\beta} - \bar{\tau}\frac{\theta-1}{\theta}\frac{\partial mc_{t+1}}{\partial(b_t - DL_t)} - \frac{\bar{b}}{\beta}\frac{\partial Y_{t+1}}{\partial(b_t - DL_t)} - \frac{\bar{b}}{\beta}\frac{\partial \pi_{t+1}}{\partial(b_t - DL_t)} \qquad (A.7)$$

$$= \frac{1}{\beta} - \left((\bar{\tau}(\sigma+\eta))\frac{\theta-1}{\theta} + \frac{\bar{b}}{\beta}(1+\kappa(\sigma+\eta))\right)\frac{\partial Y_{t+1}}{\partial(b_t - DL_t)} \qquad (A.8)$$

We can conclude that if (A.4) is not satisfied (and a higher debt level leads to a lower output level then an increase in b_{t-1} implies a more than one for one increase in b_t, so that $b_t > b_{t-1}$ and consolidation expectations lead to an increase in debt.

If (A.4) is satisfied, then the expectations of consolidations may reduce period $t+1$ debt compared to period t debt. This happens if and only if

$$\frac{\partial b_{t+1}}{\partial(b_t - DL_t)} = \frac{1}{\beta} - \left((\bar{\tau}(\sigma+\eta))\frac{\theta-1}{\theta} + \frac{\bar{b}}{\beta}(1+\kappa(\sigma+\eta))\right)\frac{\partial Y_{t+1}}{\partial(b_t - DL_t)} < 1 \qquad (A.9)$$

$$\frac{\partial Y_{t+1}}{\partial(b_t - DL_t)} > \frac{1-\beta}{\beta\left((\bar{\tau}(\sigma+\eta))\frac{\theta-1}{\theta} + \frac{\bar{b}}{\beta}(1+\kappa(\sigma+\eta))\right)} \qquad (A.10)$$

A.2 Proof Proposition 2

We first assume spending based consolidation.

Fundamentalists only base their expectations on b_{t-1} and not on b_{t-2} debt, so from them there is only an indirect effect of b_{t-2} on period t variables. However, naive agents base their expectations on lagged variables, that do depend on b_{t-2} in the way analyzed in the previous section.

We therefore have

$$\frac{\partial Y_{t+2}}{\partial(b_t - DL_t)} = \frac{\partial Y_{t+2}}{\partial b_{t+1}}\frac{\partial b_{t+1}}{\partial(b_t - DL_t)} - \frac{1}{1-\bar{g}}\gamma_1 + n_{t+2}^N\frac{\partial Y_{t+1}}{\partial(b_t - DL_t)} - \frac{\phi_1-1}{\sigma}n_{t+2}^N\frac{\partial \pi_{t+1}}{\partial(b_t - DL_t)} \quad (A.11)$$

$$\frac{\partial \pi_{t+2}}{\partial(b_t - DL_t)} = \frac{\partial \pi_{t+2}}{\partial b_{t+1}}\frac{\partial b_{t+1}}{\partial(b_t - DL_t)} + \frac{\sigma\kappa\gamma_1}{1-\bar{g}} + \beta n_{t+2}^N\frac{\partial \pi_{t+1}}{\partial(b_t - DL_t)} + \kappa(\sigma+\eta)\frac{\partial Y_{t+2}}{\partial(b_t - DL_t)} \quad (A.12)$$

$$\frac{\partial mc_{t+2}}{\partial(b_t - DL_t)} = \frac{\partial mc_{t+2}}{\partial b_{t+1}}\frac{\partial b_{t+1}}{\partial(b_t - DL_t)} + \frac{\sigma\gamma_1}{1-\bar{g}} + (\sigma+\eta)\frac{\partial Y_{t+2}}{\partial(b_t - DL_t)} \qquad (A.13)$$

where $\frac{\partial Y_{t+2}}{\partial b_{t+1}}$ is obtained by replacing $n_{t+1}^N t$ by n_{t+2}^N in (A.3). Updating (A.5) and (A.6) accordingly gives the other derivatives.

Finally, we have

$$
\frac{\partial b_{t+2}}{\partial(b_t - DL_t)} = -\gamma_1 + \frac{1}{\beta}\frac{\partial b_{t+1}}{\partial(b_t - DL_t)} - \bar{\tau}\frac{\theta - 1}{\theta}\frac{\partial mc_{t+2}}{\partial(b_t - DL_t)} + \frac{\bar{b}}{\beta}\left(\frac{\partial Y_{t+1}}{\partial(b_t - DL_t)} - \frac{\partial Y_{t+2}}{\partial(b_t - DL_t)} - \frac{\partial \pi_{t+2}}{\partial(b_t - DL_t)}\right)
$$

$$
= -\gamma_1 + \frac{1}{\beta}\frac{\partial b_{t+1}}{\partial(b_t - DL_t)} - \left(\bar{\tau}\frac{\theta - 1}{\theta} + \frac{\bar{b}}{\beta}\kappa\right)\frac{\sigma\gamma_1}{1 - \bar{g}} - \left(\bar{\tau}\frac{\theta - 1}{\theta}(\sigma + \eta) + \frac{\bar{b}}{\beta}\kappa(\sigma + \eta)\right)\frac{\partial Y_{t+2}}{\partial b_{t+1}}\frac{\partial b_{t+1}}{\partial(b_t - DL_t)}
$$

$$
- \left((\bar{\tau}(\sigma + \eta))\frac{\theta - 1}{\theta} + \frac{\bar{b}}{\beta}(1 + \kappa(\sigma + \eta))\right)\frac{\partial Y_{t+2}}{\partial(b_t - DL_t)} + \frac{\bar{b}}{\beta}\left(1 - \beta n_{t+2}^N\kappa(\sigma + \eta)\right)\frac{\partial Y_{t+1}}{\partial(b_t - DL_t)} \quad (\text{A}.)
$$

In case of tax based consolidation we get

$$
\frac{\partial Y_{t+2}}{\partial(b_t - DL_t)} = \frac{\partial Y_{t+2}}{\partial b_{t+1}}\frac{\partial b_{t+1}}{\partial(b_t - DL_t)} + n_{t+2}^N\frac{\partial Y_{t+1}}{\partial(b_t - DL_t)} - \frac{\phi_1 - 1}{\sigma}n_{t+2}^N\frac{\partial \pi_{t+1}}{\partial(b_t - DL_t)} \quad (\text{A.15})
$$

$$
\frac{\partial \pi_{t+2}}{\partial(b_t - DL_t)} = \frac{\partial \pi_{t+2}}{\partial b_{t+1}}\frac{\partial b_{t+1}}{\partial(b_t - DL_t)} + \frac{\kappa\gamma_2}{1 - \bar{\tau}} + \beta n_{t+2}^N\frac{\partial \pi_{t+1}}{\partial(b_t - DL_t)} + \kappa(\sigma + \eta)\frac{\partial Y_{t+2}}{\partial(b_t - DL_t)} \quad (\text{A.16})
$$

$$
\frac{\partial mc_{t+2}}{\partial(b_t - DL_t)} = \frac{\partial mc_{t+2}}{\partial b_{t+1}}\frac{\partial b_{t+1}}{\partial(b_t - DL_t)} + \frac{\gamma_2}{1 - \bar{\tau}} + (\sigma + \eta)\frac{\partial Y_{t+2}}{\partial(b_t - DL_t)} \quad (\text{A.17})
$$

Finally, we have

$$
\frac{\partial b_{t+2}}{\partial(b_t - DL_t)} =
$$

$$
-\frac{\theta - 1}{\theta}\gamma_2 + \frac{1}{\beta}\frac{\partial b_{t+1}}{\partial(b_t - DL_t)} - \bar{\tau}\frac{\theta - 1}{\theta}\frac{\partial mc_{t+2}}{\partial(b_t - DL_t)} + \frac{\bar{b}}{\beta}\left(\frac{\partial Y_{t+1}}{\partial(b_t - DL_t)} - \frac{\partial Y_{t+2}}{\partial(b_t - DL_t)} - \frac{\partial \pi_{t+2}}{\partial(b_t - DL_t)}\right)
$$

$$
= -\frac{\theta - 1}{\theta}\gamma_2 + \frac{1}{\beta}\frac{\partial b_{t+1}}{\partial(b_t - DL_t)} - \left(\bar{\tau}\frac{\theta - 1}{\theta} + \frac{\bar{b}}{\beta}\kappa\right)\frac{\gamma_2}{1 - \bar{\tau}} - \left(\bar{\tau}\frac{\theta - 1}{\theta}(\sigma + \eta) + \frac{\bar{b}}{\beta}\kappa(\sigma + \eta)\right)\frac{\partial Y_{t+2}}{\partial b_{t+1}}\frac{\partial b_{t+1}}{\partial(b_t - DL_t)}
$$

$$
- \left((\bar{\tau}(\sigma + \eta))\frac{\theta - 1}{\theta} + \frac{\bar{b}}{\beta}(1 + \kappa(\sigma + \eta))\right)\frac{\partial Y_{t+2}}{\partial(b_t - DL_t)} + \frac{\bar{b}}{\beta}\left(1 - \beta n_{t+2}^N\kappa(\sigma + \eta)\right)\frac{\partial Y_{t+1}}{\partial(b_t - DL_t)} \quad (\text{A.18})
$$

The difference between spending based and tax based second period debt is

$$
-(\gamma_1 - \frac{\theta - 1}{\theta}\gamma_2) - (\bar{\tau}\frac{\theta - 1}{\theta} + \frac{\bar{b}}{\beta}\kappa)(\frac{\sigma\gamma_1}{1 - \bar{g}} - \frac{\gamma_2}{1 - \bar{\tau}}) + \left((\bar{\tau}(\sigma + \eta))\frac{\theta - 1}{\theta} + \frac{\bar{b}}{\beta}(1 + \kappa(\sigma + \eta))\right)\frac{\gamma_1}{1 - \bar{g}}
$$

assuming consolidations of equal impact on the budget deficit $(\gamma_1 - \frac{\theta - 1}{\theta}\gamma_2)$, this reduces

to

$$\gamma_1 \left((\bar{\tau}\frac{\theta - 1}{\theta} + \frac{\bar{b}}{\beta}\kappa)(\frac{\theta}{(\theta - 1)(1 - \bar{\tau})} - \frac{\sigma}{1 - \bar{g}}) + \left((\bar{\tau}(\sigma + \eta))\frac{\theta - 1}{\theta} + \frac{\bar{b}}{\beta}(1 + \kappa(\sigma + \eta)) \right) \frac{1}{1 - \bar{g}} \right) \quad (A.19)$$

We can rewrite this as

$$\gamma_1 \left((\bar{\tau}\frac{\theta - 1}{\theta} + \frac{\bar{b}}{\beta}\kappa) \left(\frac{\theta}{(\theta - 1)(1 - \bar{\tau})} + \frac{\eta}{1 - \bar{g}} \right) + \frac{\bar{b}}{\beta}\frac{1}{1 - \bar{g}} \right) \quad (A.20)$$

This implies that a spending based consolidation leads to a higher debt then a tax based consolidation after two periods. Substituting for steady state debt we get

$$\gamma_1 \left((\bar{\tau}\frac{\theta - 1}{\theta} + \frac{\bar{\tau}\frac{\theta - 1}{\theta} - \bar{g}}{1 - \beta}\kappa) \left(\frac{\theta}{(\theta - 1)(1 - \bar{\tau})} + \frac{\eta}{1 - \bar{g}} \right) + \frac{\bar{\tau}\frac{\theta - 1}{\theta} - \bar{g}}{1 - \beta}\frac{1}{1 - \bar{g}} \right) \quad (A.21)$$

We can conclude that the difference between spending based and tax based in the second period is increasing in: the magnitude of consolidation (γ_1), steady state taxes ($\bar{\tau}$). The relation with steady state government spending is not immediately clear.

A.3 Proof Proposition 3

We assume steady state levels and we assume a spending based consolidation is implemented. In a fixed point where both fundamentalists and naive agents are correct (because $\alpha = 1$) inflation and output satisfy

$$\pi(1 - \beta n^N) = \kappa(\sigma + \eta)Y + \kappa\sigma\frac{\gamma_1(b - DL)}{1 - \bar{g}} \quad (A.22)$$

$$(1 - n_t^N)Y = -(1 - n_t^N)\frac{1}{\eta + \sigma}\sigma\frac{\gamma_1(b - DL)}{1 - \bar{g}} - \frac{\phi_1 - 1}{\sigma}n_t^N\pi. \quad (A.23)$$

Solving this two equations shows that fixed point inflation and marginal cost are zero and fixed point output is given by the fundamentalists expected value:

$$Y = -\frac{1}{\eta + \sigma}\sigma\frac{\gamma_1(b - DL)}{1 - \bar{g}}, \quad (A.24)$$

When fundamentalists are wrong ($\alpha \neq 1$), but the intensity of choice is infinite ($\omega = +\infty$), then in the fixed point $n_t^N = 1$. In this case the fixed point levels of output, inflation and

marginal cost are the same as above.

For debt we have in the fixed point where marginal cost and inflation are zero:

$$b = -\gamma_1(b - DL) + \frac{1}{\beta}b, \tag{A.25}$$

The fixed point debt level therefore is

$$b = \frac{DL\gamma_1}{1 - \frac{1}{\beta} + \gamma_1} \tag{A.26}$$

This is indeed a fixed point when the fixed point debt level lies above the debt limit. This is the case if and only if

$$\gamma_1 > \frac{1}{\beta} - 1, \tag{A.27}$$

A.4   Proof Proposition 4

In the benchmark case of tax based consolidations that are fully expected by fundamentalists ($\alpha = 0$) we again have a fixed point where marginal cost and inflation are zero. Output is now given by

$$Y = -\frac{1}{\eta + \sigma}\left(\frac{\gamma_2(b - DL)}{1 - \bar{\tau}}\right),$$

For infinite intensity of choice the same fixed point is reached, but with all naive expectations. For debt we now have

$$b = \frac{DL\gamma_2\frac{\theta-1}{\theta}}{1 - \frac{1}{\beta} + \gamma_2\frac{\theta-1}{\theta}} \tag{A.28}$$

This is indeed a fixed point when this debt level lies above the debt limit

The condition now becomes

$$\gamma_2\frac{\theta - 1}{\theta} > \frac{1}{\beta} - 1, \tag{A.29}$$

References

Alesina, Alberto and Roberto Perotti (1995), Fiscal Expansions and Fiscal Adjustments in OECD Countries, NBER Working Papers 5214, National Bureau of Economic Research, Inc.

Alesina, Alberto and Silvia Ardagna (1998), 'Tales of fiscal adjustment', *Economic Policy* **13**(27), 487–545.

Alesina, Alberto and Silvia Ardagna (2010), Large Changes in Fiscal Policy: Taxes versus Spending, *in* 'Tax Policy and the Economy, Volume 24', NBER Chapters, National Bureau of Economic Research, Inc, pp. 35–68.

Ardagna, Silvia (2004), 'Fiscal stabilizations: When do they work and why', *European Economic Review* **48**(5), 1047–1074.

Bi, Huixin, Eric M. Leeper and Campbell Leith (2013), 'Uncertain fiscal consolidations', *The Economic Journal* **123**(566).

Brock, William and Cars Hommes (1997), 'A rational route to randomness', *Econometrica* **65**(5), 1059–1096.

IMF (2011), 'Shifting gears: Tackling challenges on the road to fiscal adjustment', **51**(0), 180 – 203.

Perotti, Roberto (1996), 'Fiscal consolidation in europe: Composition matters', *The American Economic Review* **86**(2), pp. 105–110.