BANK OF JAPAN

July 30, 2007: Introduction to Dynare

Michel Juillard

July 30, 2007

DYNARE

1. computes the solution of deterministic models (arbitrary accuracy)
2. computes first and second order approximation to solution of stochastic models
3. estimates (maximum likelihood or Bayesian approach) parameters of DSGE models
4. computes optimal policy.
5. performs global sensitivity analysis of a model (thanks to Marco Ratto's tools)

Schorfheide (2000)

- ▶ Standard Cash-in-advance model (see also Nason and Cogley, 1994)
- ▶ Portfolio adjustment cost model

- ▶ Main Results:
  - ▶ Standard CIA model outperforms the PAC model in terms of both better in-sample properties and posterior distribution
  - ▶ The PAC model outperforms the CIA model when analyzing the response to a money growth shock.

Standard CIA model

- ▶ Three agents: Household, a firm and a financial intermediary
- ▶ Decisions are made **after** the current period surprise change in money growth and technology.

Firm

The firm chooses desired capital, K_t, labour demand, N_t, dividends F_t and loans L_t

$$\max_{\{K_t, N_t, F_t\}_{t=0}^{\infty}} E_0 \left[\sum_{t=0}^{\infty} \beta^t \frac{F_t}{C_{t+1} P_{t+1}} \right]$$

s.t.

$$
\begin{aligned}
F_t &\leq L_t + P_t Y_t - K_t + (1-\delta)K_{t-1} - W_t N_t - R_{F,t} L_t \\
W_t N_t &\leq L_t \\
Y_t &= K_{t-1}^{\alpha} (A_t N_t)^{1-\alpha}
\end{aligned}
$$

where Y_t is output, A_t, technical change index, P_t, the price index, W_t, the wage rate, and $R_{F,t}$, the gross interest rate on loans.

Financial Intermediary

The following maximisation problem:

$$\max_{\{B_t, L_t, D_t\}_{t=0}^{\infty}} E_0 \left[\sum_{t=0}^{\infty} \beta^t \frac{B_t}{C_{t+1} P_{t+1}} \right]$$

s.t.

$$\begin{aligned}
B_t &= D_t + R_{F,t} L_t - R_{H,t} D_t - L_t + M_t - M_{t-1} \\
L_t &\leq M_t - M_{t-1} + D_t
\end{aligned}$$

where B_t is dividends from the financial intermediaries, D_t, the deposits, and $R_{H,t}$ is the gross deposit interest rate.

Household

The household chooses consumption C_t, hours worked H_t, and (non-negative) deposits D_t so that

$$\max_{\{C_t, H_t, M_t, D_t\}_{t=0}^{\infty}} E_0 \left[\sum_{t=0}^{\infty} \beta^t \left[(1 - \phi) \log C_t + \phi \log(1 - H_t) \right] \right]$$

s.t.

$$
\begin{aligned}
P_t C_t &\leq M_{t-1} - D_t + W_t H_t \\
D_t &\geq 0 \\
M_t &= M_{t-1} - D_t + W_t H_t - P_t C_t + R_{H,t} D_t + F_t + B_t
\end{aligned}
$$

Market clearing conditions

- Labor market

$$H_t = N_t$$

- Money market

$$P_t C_t = M_t$$

- Goods market

$$C_t + (K_t - (1 - \delta)K_{t-1}) = K_{t-1}^{\alpha}(A_t N_t)^{1-\alpha}$$

Shock processes

Technology:

$$g_{At} = \frac{A_t}{A_{t-1}}$$
$$\log g_{At} = \gamma + \varepsilon_{At}$$

Money:

$$g_{Mt} = \frac{M_t}{M_{t-1}}$$
$$\log g_{Mt} = (1 - \rho) \log g_M^\star + \rho \log g_{Mt-1} + \varepsilon_{Mt}$$

Optimality conditions

$$E_t\left\{\frac{P_t}{C_{t+1}P_{t+1}}\right\} = \beta E_t\left\{\frac{P_{t+1}\alpha K_t^{\alpha-1}\left(A_{t+1}N_{t+1}\right)^{1-\alpha} + 1 - \delta}{C_{t+2}P_{t+2}}\right\}$$

$$\frac{\psi}{1-\psi}\frac{C_t P_t}{1-N_t} = W_t$$

$$\frac{1}{C_t P_t} = \beta E_t\left\{\frac{R_t}{C_{t+1}P_{t+1}}\right\}$$

$$R_t = \frac{P_t(1-\alpha)K_{t-1}^{\alpha}A_t^{1-\alpha}N_t^{-\alpha}}{W_t}$$

Other equilibrium conditions

$$W_t = \frac{L_t}{N_t}$$
$$L_t = M_t - M_{t-1} + D_t$$
$$P_t C_t = M_t$$
$$C_t + (K_t - (1-\delta)K_{t-1}) = K_{t-1}^\alpha (A_t N_t)^{1-\alpha}$$
$$g_{At} = \frac{A_t}{A_{t-1}}$$
$$\log g_{At} = \gamma + \varepsilon_{At}$$
$$g_{Mt} = \frac{M_t}{M_{t-1}}$$
$$\log g_{Mt} = (1-\rho)\log g_M^\star + \rho \log g_{Mt-1} + \varepsilon_{Mt}$$

Stationarized model

In order for a local approximation to make sense, the model variables must fluctuate in the neighborhood of the steady state. It is therefor necessary to stationarize the model, before computing the local approximation of the solution.

- ▶ K_t, C_t, are detrended by the productivity A_t,
- ▶ P_t is detrended by M_{t-1}/A_t
- ▶ W_t, L_t, and D_t are detrended by M_{t-1}.
- ▶ R_t and N_t are stationary

First-order conditions

Euler equation:

$$E_t\left\{\frac{\widehat{P}_t\frac{M_{t-1}}{A_t}}{\widehat{C}_{t+1}A_{t+1}\widehat{P}_{t+1}\frac{M_t}{A_{t+1}}}\right\}$$

$$= \beta E_t\left\{P_{t+1}\frac{M_t}{A_{t+1}}\frac{\alpha\left(\widehat{K}_tA_t\right)^{\alpha-1}(A_{t+1}N_{t+1})^{1-\alpha}+1-\delta}{\widehat{C}_{t+2}A_{t+2}\widehat{P}_{t+2}\frac{M_{t+1}}{A_{t+2}}}\right\}$$

after simplification

$$E_t\left\{\frac{\widehat{P}_t}{\widehat{C}_{t+1}\widehat{P}_{t+1}g_{M_t}}\right\} = \beta E_t\left\{P_{t+1}\frac{\alpha g_{A_{t+1}}^{-\alpha}\widehat{K}_t^{\alpha-1}N_{t+1}^{1-\alpha}+\frac{1-\delta}{g_{A_{t+1}}}}{\widehat{C}_{t+2}\widehat{P}_{t+2}g_{M_{t+1}}}\right\}$$

First-order conditions (continued)

Firm's borrowing constraint:

$$\widehat{W}_t M_{t-1} = \frac{\widehat{L}_t M_{t-1}}{N_t}$$

after simplification

$$\widehat{W}_t = \frac{\widehat{L}_t}{N_t}$$

Intertemporal labor market optimality condition:

$$\frac{\psi}{1-\psi} \frac{\widehat{C}_t A_t \widehat{P}_t \frac{M_{t-1}}{A_t}}{1-N_t} = \widehat{W}_t M_{t-1}$$

after simplification

$$\frac{\psi}{1-\psi} \frac{\widehat{C}_t \widehat{P}_t}{1-N_t} = \widehat{W}_t$$

First order conditions (continued)

Equilibrium interest rate:

$$R_t = \frac{(1-\alpha)\widehat{P}_t \frac{M_{t-1}}{A_t} \left(\widehat{K}_{t-1} A_{t-1}\right)^{\alpha-1} (A_t N_t)^{-\alpha}}{\widehat{W}_t M_{t-1}}$$

after simplification

$$R_t = \frac{(1-\alpha)\widehat{P}_t g_{A_t}^{-\alpha} \widehat{K}_{t-1}^{\alpha-1} N_t^{-\alpha}}{\widehat{W}_t}$$

Credit market optimality condition:

$$\frac{1}{\widehat{C}_t A_t \widehat{P}_t \frac{M_{t-1}}{A_t}} = \beta E_t \left\{ \frac{R_t}{\widehat{C}_{t+1} A_{t+1} \widehat{P}_{t+1} \frac{M_t}{A_{t+1}}} \right\}$$

after simplification

$$\frac{1}{\widehat{C}_t \widehat{P}_t} = \beta E_t \left\{ \frac{R_t}{g_{M_t} \widehat{C}_{t+1} \widehat{P}_{t+1}} \right\}$$

Other equilibrium conditions

$$
\begin{aligned}
\widehat{W}_t &= \frac{\widehat{L}_t}{N_t} \\
\widehat{L}_t &= g_{Mt} - 1 + \widehat{D}_t \\
\widehat{P}_t \widehat{C}_t &= g_{Mt} \\
\widehat{C}_t + \widehat{K}_t - (1 - \delta)\frac{\widehat{K}_{t-1}}{g_{At}} &= \left(\frac{\widehat{K}_{t-1}}{g_{At}}\right)^\alpha N_t^{1-\alpha} \\
\log g_{At} &= \gamma + \varepsilon_{At} \\
\log g_{Mt} &= (1 - \rho)\log g_M^\star + \rho \log g_{Mt-1} + \varepsilon_{Mt}
\end{aligned}
$$

fs2000A.mod

```
var P C W R K D N L Y gA gM;
varexo e_a e_m;

parameters alp bet gam gMstar rho psi del;

alp = 0.33;
bet = 0.99;
gam = 0.003;
gMstar = 1.011;
rho = 0.7;
psi = 0.787;
del = 0.02;
```

fs2000A.mod (continued)

```
model;
P/(C(+1)*P(+1)*gM) = bet*P(+1)*(alp*gA(+1)^(-alp)*
         K^(alp-1)*N(+1)^(1-alp)+(1-del)/gA(+1))/
         (C(+2)*P(+2)*gM(+1));
W = L/N;
(psi/(1-psi))*(C*P/(1-N)) = W ;
R = P*(1-alp)*gA^(-alp)*K(-1)^alp*N^(-alp)/W;
1/(C*P) = bet*R/(gM*C(+1)*P(+1));
C+K = Y+(1-del)*K(-1)/gA;
P*C = gM;
gM-1+D = L;
Y = K(-1)^alp*N^(1-alp)*gA^(-alp);
log(gA) = gam + e_a;
log(gM) = (1-rho)*log(gMstar)+rho*log(gM(-1))+e_m;
end;
```

```
initval;
K = 6;
P = 2.25;
C = 0.45;
W = 4;
R = 1.02;
D = 0.85;
N = 0.19;
L = 0.86;
Y = 0.6;
gA = exp(gam);
gM = dMstar;
end;
```

```
shocks;
var e_a; stderr 0.014;
var e_m; stderr 0.005;
end;

steady;
check;

stoch_simul(order=1);
```

Dating variables in Dynare

Dynare will automatically recognize predetermined and non–predetermined variables, but you must observe a few rules:

- ▶ period t variables are set during period t on the basis of the state of the system at period $t-1$ and shocks observed at the beginning of period t.

- ▶ therefor, stock variables must be on an end–of–period basis: investment of period t determines the capital stock at the end of period t.

Log–linearization

- ▶ Taking a log–linear approximation of a model is equivalent to take a linear approximation of a model with respect to the logarithm of the variables.
- ▶ In practice, it is sufficient to replace all occurences of variable X with $exp(LX)$ where $LX = \log X$.
- ▶ It is possible to make the substitution for some variables and not anothers. You wouldn't want to take a log approximation of a variable whose steady state value is negative . . .
- ▶ There is no evidence that log–linearization is more accurate than simple linearization. In a growth model, it is often more natural to do a log–linearization.

Computing the steady state

```
steady;
```

- ▶ initval contains guess values for nonlinear solver
- ▶ initval can alternatively contains the exact solution when it is possible to derive it analytically.
- ▶ It is also possible to write the analytical solution of the steady state as a Matlab function called in this case fs2000A_steadystate.m

Checking unicity of a locally stable solution

```
check;
```

- ▶ Blanchard and Kahn condition for the unicity of a stable solution in a linear rational expectation model: There must be as many explosive roots as there are forward–looking variables in the model (and a particular rank condition must be satisfied).

Computing the approximate solution

```
stoch_simul(order=1);
```

Options:

- In a large model, it is possible to limit results to certain variables. Example:`stoch_simul(order=1) C K R;`
- `order`: order of approximation (1 or 2; default=2)
- `irf`: number of periods for the IRFs (default 40)
- `periods`: runs an actual Monte Carlo simulation

Approximated decision rule

At the first order:

$$K_t = 5.80 + 0.95 \left(K_{t-1} - \overline{K} \right) + 0.16 \left(g_{Mt-1} - \overline{g_M} \right) - 5.49 \varepsilon_{At} + 0.22 \varepsilon_{Mt}$$

At the second order:

$$
\begin{aligned}
K_t &= 5.80 + 4.35 \times 10^{-4} + 0.95 \left(K_{t-1} - \overline{K} \right) + 0.16 \left(g_{Mt-1} - \overline{g_M} \right) \\
&\quad - 5.49 \varepsilon_{At} + 0.22 \varepsilon_{Mt} - 0.0004 \left(K_{t-1} - \overline{K} \right)^2 \\
&\quad - 0.07 \left(g_{Mt-1} - \overline{g_M} \right)^2 + 0.02 \left(K_{t-1} - \overline{K} \right) \left(g_{Mt-1} - \overline{g_M} \right) \\
&\quad + 2.73 \varepsilon_{At}^2 - 0.15 \varepsilon_{At} \varepsilon_{Mt} - 0.14 \varepsilon_{Mt}^2 - 0.79 \varepsilon_{At} \left(K_{t-1} - \overline{K} \right) \\
&\quad + 0.17 \varepsilon_{Mt} \left(K_{t-1} - \overline{K} \right) - 0.69 \varepsilon_{At} \left(g_{Mt-1} - \overline{g_M} \right) \\
&\quad - 0.69 \varepsilon_{Mt} \left(g_{Mt-1} - \overline{g_M} \right)
\end{aligned}
$$

The role of the Dynare parser

- ► the Dynare toolbox solves generic problems
- ► the parser reads your *.mod file and translates it in specific Matlab files
- ► *filename*.m: main Matlab script for your model
- ► *filename*_static.m: static model
- ► *filename*_dynamic.m: dynamic model

Priors in DYNARE

NORMAL_PDF	$N(\mu, \sigma)$	R
GAMMA_PDF	$G_2(\mu, \sigma, p_3)$	$[p_3, +\infty)$
BETA_PDF	$B(\mu, \sigma, p_3, p_4)$	$[p_3, p_4]$
INV_GAMMA_PDF	$IG_1(\mu, \sigma)$	R^+
UNIFORM_PDF	$U(p_3, p_4)$	$[p_3, p_4]$

By default, $p_3 = 0$, $p_4 = 1$.

Schorfheide's priors

```
estimated_params;
alp, beta_pdf, 0.356, 0.02;
bet, beta_pdf, 0.993, 0.002;
gam, normal_pdf, 0.0085, 0.003;
mst, normal_pdf, 1.0002, 0.007;
rho, beta_pdf, 0.129, 0.223;
psi, beta_pdf, 0.65, 0.05;
del, beta_pdf, 0.01, 0.005;
stderr e_a, inv_gamma_pdf, 0.035449, inf;
stderr e_m, inv_gamma_pdf, 0.008862, inf;
end;
```

List of observed variables

```
varobs gp_obs gy_obs;
```

Estimation

```
estimation(datafile=fsdat,nobs=192,
           mh_replic=10000,mh_jscale=0.8);
```

Usefull options

first_obs=n : first observation (default: 1)

nobs=n or nobs=([n1:n2)]: number of observations (default: the entire data file)

mode_file : filename of previous results (default: none)

compute_mode : optimization algorithm

        0 : no optimization

        1 : Maltab's fmincon

        2 : Lester Ingber's adaptive simulated annealing

        3 : Matlab's fminunc

        4 : Chris Sims' csminwel (default)

        5 : Marco Ratto's robust optpimizer

mode_check : draws objective function in each parameter direction.

More options

prefilter : 0, no prefiltering; 1, the data are demeaned before estimation (default: 0).

presample : number of initial periods that don't enter into likelihood computation (default: 0).

like_init : initial covariance matrix of state variable prediction. 1, for stationary models, unconditional variance of state variables; 2, for nonstationary models, diffuse prior, diagonal matrix with 10 on the diagonal.

loglinear : computes a log–linear approximation of the model instead of a linear (default) approximation.

More options

optim=() : changes options for Matlab optimizer (see Matlab `optimset` command).

moments_varendo : computes posterior distribution of moments of endogenous variables.

bayesian_irf : computes posterior distribution of IRF's.

smoother : computes posterior distribution of smoothed variables.

filtered_vars : computes posterior distribution of filtered variables.

forcast=n : computes forecasts for n periods.

Slides and examples

Slides and examples are available at