EABCN Training School: Monetary-Fiscal Policy Interactions

Lecture 3. Policy Interactions with Tax Distortions

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The Messages

- Will study three models with distorting taxes
- First draws on Gordon-Leeper (2005,2006): growth model w/ transactions demand for money
- Second draws on Leeper-Yun (2006): provides micro foundations for FTPL
- Once models completely solved out, can understand price-level determination more deeply
- Emphasizes the role of asset substitution, which is absent from simple models
- Gets us away from the fiscal theory story about wealth effects
- Useful models to keep in your head: “roll your own policies”
- Characterize eqm as function of general sequences of policy variables

First Model

- Growth model w/ capital, money, nominal government debt
  - arbitrages among assets determine their relative demands
  - returns to real balance holdings and after-tax returns to capital determine the relative values of real and nominal assets
  - expected macro policies determine expected returns on real and nominal assets
  - so price level depends on interactions among current and expected future MP & FP
- Quantity theory and fiscal theory emerge as special cases
- QT & FT employ common money demand

$$\frac{M^d}{P} = h(i, y)$$

  - how can this be?

THE MODEL

- We exploit the analytic convenience that comes with log prefs, C-D technology, complete depreciation of capital
  - none of the general points depend on these simplifying assumptions
- Aggregate resource constraint

$$c_t + k_t + g_t = f(k_{t-1})$$

- Goods producing firm rents k at rental rate r and pays taxes levied against sales of goods to solve

$$\max_{k_{t-1}} D_{Gt} = (1 - \tau_t) f(k_{t-1}) - r_t k_{t-1}$$

- Transactions services producing firm hires labor l at wage rate w to solve

$$\max_{l_t} D_{Tt} = P_{Tt} T(l_t) - w_t l_t$$

THE MODEL

- Household owns firms and pays taxes on capital income
- HH has income

$$I_t = r_t k_{t-1} + D_{Gt} + w_t l_t + D_{Tt} + z_t$$

  where $z_t \geq 0$ is lump-sum transfers from the government

- HH's expenditures on c & k must be financed with real money balances, M_{t-1}/P_t, or with transactions services, T_t, to satisfy the constraint

$$\frac{M_{t-1}}{P_t} + T_t(c_t + k_t) \geq c_t + k_t$$

  T_t gives fraction of expenditures financed w/ transactions services

The Model

- HH's problem

$$\max_{\{c_t, l_t, T_t, M_t, B_t, k_t\}} E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, 1 - l_t), \quad 0 < \beta < 1$$

  where $1 - l_t$ is leisure, subject to the finance constraint, the budget constraint

$$c_t + k_t + \frac{M_t + B_t}{P_t} + P_{Tt} T_t \leq I_t + \frac{M_{t-1} + R_{t-1} B_{t-1}}{P_t}$$

  and $0 \leq l_t \leq 1$

- Future government policy is the sole source of uncertainty; the operator E denotes equilibrium expectations of private agents over future policy

THE MODEL

- The government finances expenditures on goods, g_t, and transfer payments, z_t, by levying taxes, issuing new debt, and creating new money to satisfy:

$$\tau_t f(k_{t-1}) + \frac{M_t - M_{t-1}}{P_t} + \frac{B_t + R_{t-1}B_{t-1}}{P_t} = g_t + z_t$$

- Assume the following functional forms:

$$f(k_{t-1}) = k_{t-1}^{\sigma}, \quad 0 < \sigma < 1$$

$$T(l_t) = 1 - (1 - l_t)^{\alpha}, \quad \alpha > 1$$

$$U(c_t, 1 - l_t) = \log(c_t) + \gamma \log(1 - l_t), \quad \gamma > 0$$

Solving the Model

- State at t depends on assets and expectations of macro policies

- denote state by
  $z_t = (k_{t-1}, M_{t-1}, (1 + i_{t-1})B_{t-1}, \{E_t\rho_j, E_t\tau_j, E_t s_j^g\}_{j=t}^{\infty})$

  - $\rho_t = M_t/M_{t-1}, s_t^g = g_t/f(k_{t-1})$
  - emphasizes that a complete specification of policy must allow agents to form expectations over infinite future of policies

Solving the Model

- First-order conditions
  - firms'

$$1 + r_t = \sigma(1-\tau_t)k_{t-1}^{\sigma-1} \qquad w_t = \alpha(1-l_t)^{\alpha-1}P_{Tt}$$

  - household's

$$\begin{aligned}
\varphi_t + \lambda_t &= \frac{1}{c_t} + \lambda_t T_t^d \\
\frac{\gamma}{1-l_t} &= w_t \varphi_t \\
\varphi_t P_{Tt} &= \lambda_t (c_t + k_t) \\
\frac{\varphi_t}{P_t} &= \beta E_t \left[\frac{\varphi_{t+1} + \lambda_{t+1}}{P_{t+1}} \right] \\
\frac{\varphi_t}{P_t} &= \beta(1+i_t) E_t \left[\frac{\varphi_{t+1}}{P_{t+1}} \right] \\
\varphi_t + \lambda_t &= \lambda_t T_t^d + \beta E_t (1 + r_{t+1}) \varphi_{t+1}
\end{aligned}$$

EQUILIBRIUM

- Characterize eqm in terms of policy expectations functions (μ_t, η_t), government claims to goods, s_t^g, and assets, $(k_{t-1}, M_{t-1}, (1+i_{t-1})B_{t-1})$
- Solution maps policy expectations into portfolio choices
  - of course, policy expectations are restricted to policy paths that are consistent with eqm
- η and μ capture portfolio balance effects of policies
  - η measures direct tax distortion on investment & extent to which gov't expends are financed by taxing output
  - μ reflects expected inflation & the expected return on nominal assets
- Assume (similar to "Monetary Doctrines")

$$\rho_{t+j} = \rho_F, \forall j > 0$$
$$\tau_{t+j} = \tau_F, \forall j > 0$$
$$s_{t+j}^g = s_F^g, \forall j > 0$$

EQUILIBRIUM

- Two dynamical equations to solve: real asset & nominal assets
- k Euler equation in terms of $s_t = k_t/(c_t + k_t)$ yields

$$\frac{1}{1-s_t} = \sigma\beta E_t\left[\frac{1-\tau_{t+1}}{1-s^g_{t+1}}\left(\frac{1}{1-s_{t+1}}\right)\right] + E_t\left[1-\sigma\beta\frac{\gamma}{\alpha}\frac{1-\tau_{t+1}}{1-s^g_{t+1}}\right]$$

whose solution is

$$\frac{1}{1-s_t} = \eta_t$$

where

$$\eta_t \equiv E_t\sum_{i=0}^{\infty}(\sigma\beta)^i d^\eta_i\left[1-\sigma\beta\frac{\gamma}{\alpha}\frac{1-\tau_{t+i+1}}{1-s^g_{t+i+1}}\right]$$

$$d^\eta_i = \prod_{j=0}^{i-1}\left(\frac{1-\tau_{t+j+1}}{1-s^g_{t+i+1}}\right), \qquad d^\eta_0 = 1$$

EQUILIBRIUM

- Euler equation for M yields d.q. in velocity, $1 - T_t$

$$(1-T_t)\left[\frac{1}{1-s_t} - \frac{\gamma}{\alpha}\right] = \beta\frac{1}{\rho_t}E_t\left\{(1-T_{t+1})\left[\frac{1}{1-s_{t+1}} - \frac{\gamma}{\alpha}\right] + \frac{\gamma}{\alpha}\right\}$$

whose solution is

$$(1-T_t)\left[\frac{1}{1-s_t} - \frac{\gamma}{\alpha}\right] = \frac{\mu_t}{\rho_t}$$

where

$$\mu_t \equiv \beta\frac{\gamma}{\alpha}E_t\sum_{i=0}^{\infty}\beta^i d_i^{\mu}, \qquad d_i^{\mu} \equiv \prod_{j=0}^{i-1}\frac{1}{\rho_{t+j+1}}, \qquad d_0^{\mu} = 1$$

EQUILIBRIUM

- Imposing the stationary policy assumptions yields the policy expectations functions

$$\eta_t(\overset{(-)}{\tau_F}, \overset{(+)}{s_F^g}) = \frac{1 - \sigma\beta\frac{\gamma}{\alpha}\left(\frac{1-\tau_F}{1-s_F^g}\right)}{1 - \sigma\beta\left(\frac{1-\tau_F}{1-s_F^g}\right)}$$

$$\mu_t(\overset{(-)}{\rho_F}) = \frac{\beta\frac{\gamma}{\alpha}}{1 - \beta/\rho_F}$$

- Eqm capital stock is

$$k_t = \left(1 - \frac{1}{\eta_t})\right)(1 - s_t^g)f(k_{t-1})$$

- Eqm real money balances are

$$\frac{M_t}{P_t} = \left(\frac{\mu_t}{\eta_t - \gamma/\alpha}\right)(1 - s_t^g)f(k_{t-1})$$

Price-Level Determination

- Can think of price level being determined "through eqm real balances"

$$\frac{M_t}{P_t} = \Delta_t (1 - s_t^g) f(k_{t-1})$$

where

$$\Delta_t = \frac{\mu_t}{\eta_t - \gamma/\alpha}$$

with

$$\Delta_t(\overset{(-)}{\rho_F}, \overset{(+)}{\tau_F}, \overset{(-)}{s_F^g}) = \frac{\beta \frac{\gamma}{\alpha}}{1 - \gamma/\alpha} \left[\frac{1 - \sigma\beta \left(\frac{1-\tau_F}{1-s_F^g} \right)}{1 - \beta/\rho_F} \right]$$

- $1/\Delta_t$ is velocity; it gives the value of nominal assets
  - Δ_t depends on expected MP & FP

The Role of Policy Expectations

- μ and η capture 3 distinct influences of expectations on P
  1. μ: the marginal value of real money balances; higher expected money growth lowers μ and induces substitution away from money, raising P
  2. η: direct tax distortion that alters return on investment; higher expected taxes reduce return on investment and induces substitution away from k into c and into M (Tobin effect), raising money demand and lowering P
  3. η summarizes composition of expected fiscal financing; higher η reflects increase in expected nominal liability creation & reduction in relative role of real taxation

To see (3), note terms $(1-\tau)/(1-s^g)$ in η and write gbc as

$$\frac{1-\tau_t}{1-s_t^g} = 1 + \frac{(M_t - M_{t-1} + B_t - (1+i_{t-1})B_{t-1})/P_t}{(1-s_t^g)f(k_{t-1})}$$

Jointly Consistent (Equilibrium) Policies

- Dynamic interactions among policies
  - current policies constrain future policy options
  - expected fiscal financing constrains current policies
  - expected policies affect P_t & real value of gov't liabilities
- How do jointly consistent combinations of current & future policies affect P?
  1. Which policies are consistent with eqm given current expectations $(\mu$ & $\eta)$?
  2. How do current policy changes affect the set of future policies that are consistent with eqm?

Jointly Consistent Policies

1. Which policies are consistent with eqm given current expectations $(\mu$ & $\eta)$?
2. How do current policy changes affect the set of future policies that are consistent with eqm?

- Eqm government b.c. at t

$$\left[\frac{\rho_t - 1}{\rho_t} + \left(\frac{B}{M}\right)_t - \frac{1 + i_{t-1}}{\rho_t}\left(\frac{B}{M}\right)_{t-1}\right]\Delta_t = \frac{s_t^g - \tau_t}{1 - s_t^g}$$

where $(B/M)_s \equiv B_s/M_s$ and Δ_t summarizes given expected policies

- Eqm government b.c. in future

$$\Delta_t = \left(\frac{s_F^g - \tau_F}{1 - s_F^g}\right)\left[\frac{1}{\left(\frac{B}{M}\right)_F - \frac{1}{\beta}\left(\frac{B}{M}\right)_t + \left(\frac{\rho_F - 1}{\rho_F}\right)}\right]$$

Money Demand

$$\frac{M_t}{P_t} = \beta\frac{\gamma}{\alpha}\left(\frac{1+i_t}{i_t}\right)\frac{1}{\eta_t - \gamma/\alpha}(1-s_t^g)f(k_{t-1})$$

- In general, both MP and FP affect P
- When is P determined by MP alone?
- Under policy assumptions that dichotomize real & nominal sides
- Balanced net-of-interest surplus: $\tau_t = s_t^g$ all t
- Now $\eta_t = (1-\sigma\beta\gamma/\alpha)/(1-\sigma\beta)$ and M^d is

$$\frac{M_t}{P_t} = h(i_t, c_t + k_t)$$

- P independent of FP but *not of debt*
  - money growth must finance interest obligations
  - higher $B \Rightarrow$ higher debt service $\Rightarrow$ higher P & π
- In general, cannot rid M/P of η

Unpleasant Monetarist Arithmetic

- Open-market sale of B_t, holding $M_t + B_t$ fixed
- Fix (s_t^g, s_F^g) and τ_t
- B/c $B_t \Uparrow$, some future policy must adjust—either τ_F or ρ_F
  1. suppose $\tau_F \Uparrow$: $\eta_t \Downarrow$, $k_t \Downarrow$ $P_t \Downarrow$ (but future $P \Uparrow$)
  2. suppose $\rho_F \Uparrow$: $\mu_t \Downarrow$, tend to make $P_t \Uparrow$ (but future $P \Uparrow$) But $M_t \Downarrow$, so ultimate effect on P_t can go either way, depending on B/M
- Monetary policy is constrained by the government's fiscal obligations
  - works through seigniorage

Canonical FTPL

- Bond-financed tax cut: $\tau_t \Downarrow$, $B_t \Uparrow$
- Fix (ρ_F, τ_F, s_F^g) and s_t^g
- B/c B_t rises, if M_t unchanged, $(B/M)_t$ rises and some future policy must adjust
- By ass'n no future policy can adjust
- Only eqm policy is for M_t to rise in proportion to the B_t increase so that $(B/M)_t$ unchanged
- Required increase in M_t is exactly enough so increase in future seigniorage (b/c the *level* of money supplied is now higher) suffices to service higher debt
- The fixed policies peg i_t and M_t/P_t, so $P_t \Uparrow$
- Monetary policy is constrained by the government's fiscal obligations
  - works through nominal asset revaluation

Pure Fiscal Effects

- FP can affect P independently of MP
- Consider a debt-financed tax cut to which future taxes adjust
- Fix $(\rho_t, \rho_F, s_t^g, s_F^g)$
- Lower τ_t & higher $(B/M)_t \Rightarrow$ higher τ_F
- Lower return on capital induces substitution away from real assets toward nominal assets
- With M_t fixed, P_t falls
- This Tobin effect gives debt a natural role in determining P
- Quite non-Keynesian: current fiscal expansion reduces nominal demand and price level
- Note that even though money growth is unchanged, because M/P rises, seigniorage revenues rise

Countercyclical Fiscal Policies

- Extend previous models in several ways
  - add human capital, h: $f(k, h)$, f CRS
  - incomplete depreciation of both k and h
  - total investment, $x = x_k + x_h$, and consumption enters finance constraint
  - add lump-sum transfers
  - calibrate to U.S. data
- Need to compute expectations functions, $\{\eta_t, \mu_t\}$
  - assume perfect foresight
  - use data on $\{s_t^g, s_t^z, \tau_t, \rho_t\}$
  - handle “infinite sum” in a couple of ways
- Simulate time paths of investment & velocity

Countercyclical Fiscal Policies

- Basic intuition:
  - economic downturn: $g/y \uparrow$ and $T/y \downarrow$
  - debt-financed deficit
  - if agents expect higher future taxes, return on investment $\downarrow$
  - investment in the downturn declines *more* than in absence of countercyclical policy
  - capital stock *lower* than in absence of countercyclical policy
  - downturn is deeper and more prolonged that in absence of countercyclical policy

Countercyclical Fiscal Policies

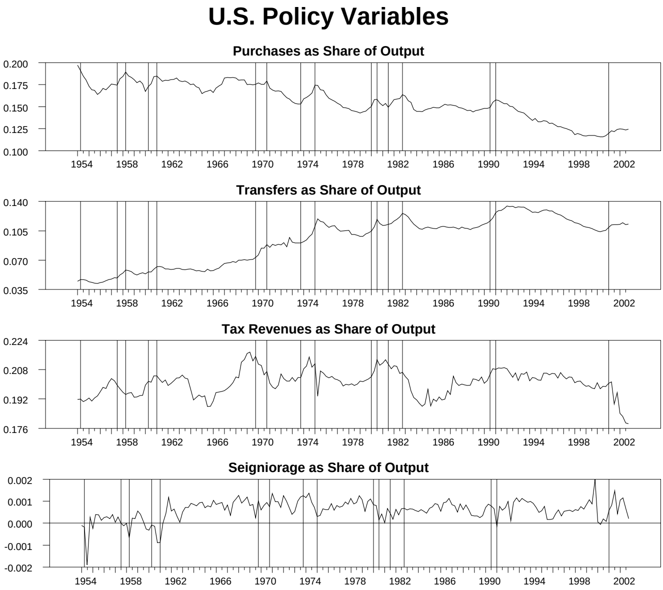
- Capital accumulation

$$k_t + h_t = \left(1 - \frac{1}{\eta_t}\right)(1 - \delta s_t^g) f(k_{t-1}, h_{t-1})$$

  - $s_t^g \uparrow \Rightarrow$ capital $\downarrow$
  - η_t plays two roles
    1. heavy dependence of direct taxation $\Rightarrow \eta$ high
       - elasticity of capital wrt/ s_t^g is high
    2. if future taxes rise, η_t rises
       - further raising elasticity of capital wrt/ s_t^g

- How countercyclical policies are expected to be financed influences their effectiveness

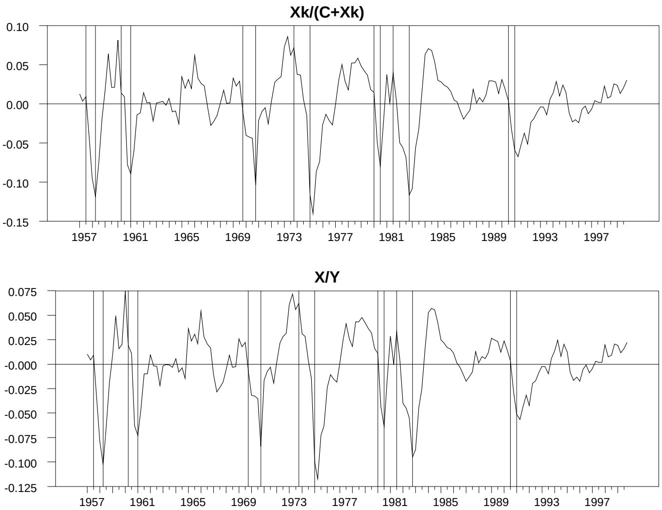
COUNTERCYCLICAL FISCAL POLICIES



COUNTERCYCLICAL FISCAL POLICIES

Portfolio Choices (U.S. Data)

Cyclical



Countercyclical Fiscal Policies

Portfolio Choices (U.S. Data)

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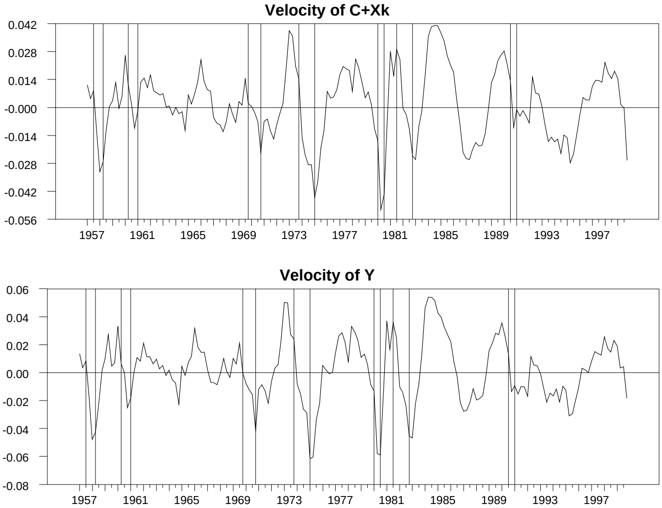
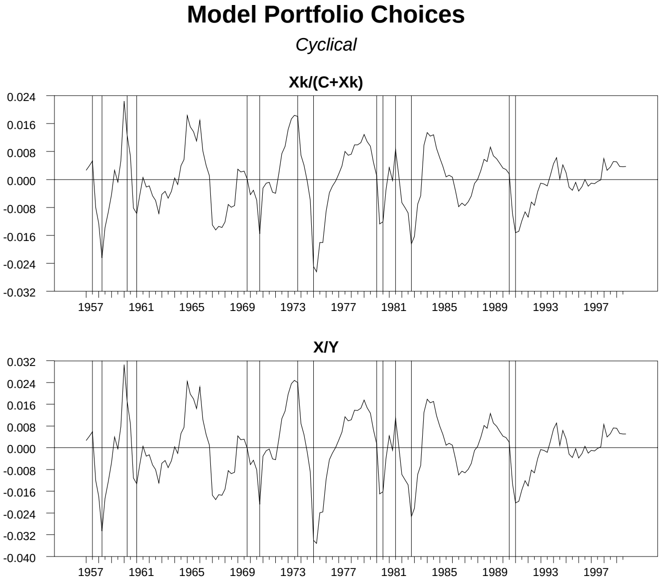


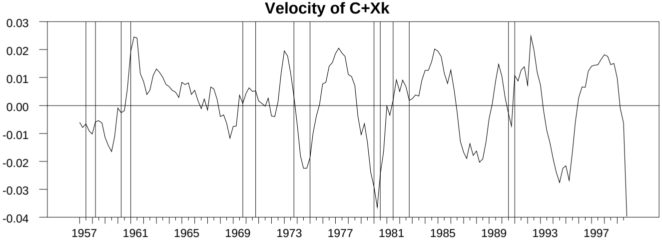
Countercyclical Fiscal Policies

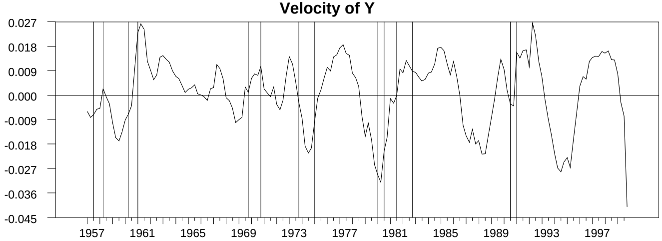


COUNTERCYCLICAL FISCAL POLICIES

Model Portfolio Choices

Cyclical





THIRD MODEL

- Seek to provide micro foundations for the FTPL
- Elastic labor supply; fixed capital stock
- Proportional tax levied against labor income has both "supply" and "demand" effects
- FTPL typically focuses only on "demand" effects
- Complete contingent claims, fiat currency, nominal government debt
- CRS production in labor
- Derive effects of tax policies on balance sheets of HHs

THE MODEL

- Preferences

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t, m_t) + v(1 - h_t) \right]$$

- HH budget constraint

$$c_t + m_t + E_t \left[Q_{t,t+1} \frac{B_{t,t+1}}{P_t} \right] \leq (1 - \tau_t)(w_t h_t + \Phi_t) + \frac{B_{t-1,t} + M_{t-1}}{P_t}$$

$Q_{t,t+1}$ is stochastic discount factor (nominal value at t of \$1 at $t + 1$; Φ_t is real dividends
$E_t \left[Q_{t,t+1} \frac{B_{t,t+1}}{P_t} \right]$ is real value at t of nominal contingent claims

$$1 + i_t = \frac{1}{E_t[Q_{t,t+1}]}, \qquad Q_{t,t+1} = q_{t,t+1} \frac{P_t}{P_{t+1}}$$

The Model

- Rewrite the HH's flow b.c. as

$$c_t + \frac{i_t}{1+i_t} m_t + E_t[q_{t,t+1} a_{t+1}] \leq (1-\tau_t)(w_t h_t + \Phi_t) + a_t$$

$$a_t = \frac{B_{t-1,t} + M_{t-1}}{P_t}, \qquad \text{value of nominal assets}$$

- HH's present-value b.c. is

$$E_0 \sum_{t=0}^{\infty} q_t \left[c_t + \frac{i_t}{1+i_t} m_t - (1-\tau_t)(w_t h_t + \Phi_t) \right] \leq a_0$$

with $\lim_{t \to \infty} E_0[q_t a_t] = 0$

The Model

- First-order conditions

$$\beta^t u_c(c_t, m_t) = \lambda q_t$$

$$\beta^t u_m(c_t, m_t) = \lambda q_t \left(\frac{i_t}{1 + i_t} \right)$$

where $\lambda = u_c(c_0, m_0)$

$$\frac{v'(1 - h_t)}{u_c(c_t, m_t)} = (1 - \tau_t) w_t$$

- Use these in PV b.c.

$$\frac{E_0 \sum_{t=0}^{\infty} \beta^t \left[u_c(c_t, m_t) c_t + u_m(c_t, m_t) m_t - (1 - \tau_t) y_t u_c(c_t, m_t) \right]}{u_c(c_0, m_0)} = a_0$$

- Note: LHS entirely in terms of allocations
- When allocations are unique, have a unique real value of nominal assets, $a_0 = \frac{B_{-1,0} + M_{-1}}{P_0}$

EQUILIBRIUM

$$\frac{E_0 \sum_{t=0}^{\infty} \beta^t \left[u_c(c_t, m_t) c_t + u_m(c_t, m_t) m_t - (1 - \tau_t) y_t u_c(c_t, m_t) \right]}{u_c(c_0, m_0)} = a_0$$

- Under rational expect, HH knows a_0 when it optimizes
- This is an eqm balance sheet relation, where LHS is PV of HH's assets at time 0
- Get cond in policy variables, subst $y_t = c_t + g_t$ in relation

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{u_c(c_t, m_t)}{u_c(c_0, m_0)} \left[(\tau_t y_t - g_t) + \frac{u_m(c_t, m_t)}{u_c(c_t, m_t)} m_t \right] = a_0$$

Noting that $q_t = \beta^t \frac{u_c(c_t, m_t)}{u_c(c_0, m_0)}$

$$E_0 \sum_{t=0}^{\infty} q_t \left[(\tau_t y_t - g_t) + \frac{i_t}{1 + i_t} m_t \right] = a_0$$

An equilibrium condition!

A Fiscal Theory Equilibrium

$$E_0 \sum_{t=0}^{\infty} q_t \left[(\tau_t y_t - g_t) + \frac{i_t}{1+i_t} m_t \right] = a_0$$

- Suppose: $\tau_t y_t$ are lump-sum tax revenues; y & g exogenous, then q given; let MP peg $i_t = \bar{i} \Rightarrow m_t = h(y_t)$ independent of MP & FP; let FP set $\{\tau_t\}$ exogenously
- Under these ass'ns, LHS a number, call it PVS, so

$$P_0 = \frac{B_{-1,0} + M_{-1}}{PVS}$$

- At $t = 0$, $B_{-1,0} + M_{-1}$ given, so this determines P_0
- Can think of $1/PVS$ as price of nominal assets, which plays the same role as $1/\Delta$ in earlier model

Policy Experiments

1. Expect lower $\tau_{t+k} \Rightarrow PVS \Downarrow \Rightarrow P_0 \Uparrow$
2. Reduce current $\tau_0 \Rightarrow PVS \Downarrow \Rightarrow P_0 \Uparrow$
3. Expect lower $\frac{\bar{i}}{1+\bar{i}} m \Rightarrow PVS \Downarrow \Rightarrow P_0 \Uparrow$

- (3) seems perverse relative to standard theory
  - lower expected seigniorage iff lower $\bar{i} \Rightarrow$ lower π^e in most monetary models $\Rightarrow$ higher expected return to $M \Rightarrow M^d \Uparrow \Rightarrow P_0 \Downarrow$
  - what's going on?
  - in standard models, the **ubiquitous eqm condition** is present but it doesn't restrict the nature of the eqm b/c it is assumed that taxes adjust to alter the PVS for any given P_0
  - in FTPL, lower $\frac{\bar{i}}{1+\bar{i}} m \Rightarrow$ less "backing" for nominal assets, so nominal assets are worth less, meaning $1/P_0 \Downarrow$
- Whether the **ubiquitous eqm condition** should be treated as a *constraint* or an *eqm condition* is at the heart of Buiter's critique of the FTPL

A PRICE-THEORETIC VIEW OF THE FTPL

- Follow public finance to extend Slutsky-Hicks decomposition to include a third effect
- FT works through a type of wealth effect that arises when ΔP revalues nominal assets in HH portfolios
- Decompose impacts of tax change as
  - total effect $=$ substitution effect $+$ wealth effect $+$ revaluation effect
- Let y_t^F be Becker's "full income" (dividend income $+$ maximum labor income if HH works entire time endowment—1 unit)

$$y_t^F = (1 - \tau_t) w_t \cdot 1 + \Phi_t$$

- HH takes y_t^F as given and from it purchases consumption, real balances, leisure

A Price-Theoretic View of the FTPL

- HH flow b.c.

$$c_t + \frac{i_t}{1+i_t} m_t + (1-\tau_t) w_t (1-h_t) + E_t[q_{t,t+1} a_{t+1}] \leq y_t^F + a_t$$

- HH present value b.c.

$$E_0 \sum_{t=0}^{\infty} q_t \left[c_t + \frac{i_t}{1+i_t} m_t + (1-\tau_t) w_t (1-h_t) \right] \leq a_0 + v_0$$

with $\lim_{t\to\infty} E_0[q_t a_t] = 0, \lim_{t\to\infty} E_0[q_t y_t^F] = 0$

  - v_0 is expected PV of full income flows, $v_0 = E_0 \sum_{t=0}^{\infty} [q_t y_t^F]$

- HH takes both a_0 and v_0 parametrically

A Price-Theoretic View of the FTPL

- Lagrange multiplier on the PV b.c. is $\lambda = \frac{e_0}{a_0+v_0}$

$$e_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left[u_c(c_t, m_t)c_t + u_m(c_t, m_t)m_t + v'(1-h_t)(1-h_t)\right]$$

  e_0 is expected PV of expenditures (including leisure)
- λ is shadow price of wealth
  - wealth rises ($a_0 + v_0 \Uparrow$) $\Rightarrow \lambda \Downarrow$
  - expenditures rise ($e_0 \Uparrow$) $\Rightarrow \lambda \Uparrow$
- Demand functions

$$\begin{aligned} c_t &= c\left(\frac{q_t}{\beta^t}, \frac{i_t}{1+i_t}, \frac{a_0+v_0}{e_0}\right) \\ m_t &= m\left(\frac{q_t}{\beta^t}, \frac{i_t}{1+i_t}, \frac{a_0+v_0}{e_0}\right) \\ h_t &= h\left((1-\tau_{t)} w_t, \frac{q_t}{\beta^t}, \frac{i_t}{1+i_t}, \frac{a_0+v_0}{e_0}\right) \end{aligned}$$

A Price-Theoretic View of the FTPL

- Conventional wealth effect vs. revaluation effect
  - suppose $B_{-1,0} + M_{-1} = 0$
  - revaluation effect is zero $(a_0 = 0)$
  - conventional wealth effect still operates through v_0 & e_0
- Of course, taxes can affect P_0 even if FTPL not operative
  - suppose $\tau_0 \Uparrow$
  - substitution effect reduces labor supply
  - wealth effect raises labor supply
  - final impact depends on relative sizes
  - but then the resulting ΔP_0 and Δa_0 imposes restrictions on $\{\tau_t\}_{t=1}^{\infty}$ necessary for eqm

Substitution, Wealth & Revaluation

- Suppose $\{\tau_t^*\}_{t=0}^{\infty}$ changes to $\{\tau_t^{\dagger}\}_{t=0}^{\infty}$
- Problem (*)

$$\max_{\{c_t,m_t,h_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t,m_t) + v(1-h_t)\right]$$

s.t. $$E_0 \sum_{t=0}^{\infty} q_t \left[c_t + \frac{i_t}{1+i_t} m_t + (1-\tau_t^*) w_t (1-h_t)\right] \leq a_0 + v_0$$

yields $\{c_t^*, m_t^*, h_t^*, w_t^*, a_t^*, v_t^*, e_t^*, P_t^*, q_t^*, R_t^*, \Phi_t^*\}_{t=0}^{\infty}$

- Problem (†)

$$\max_{\{c_t,m_t,h_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t,m_t) + v(1-h_t)\right]$$

s.t. $$E_0 \sum_{t=0}^{\infty} q_t \left[c_t + \frac{i_t}{1+i_t} m_t + (1-\tau_t^{\dagger}) w_t (1-h_t)\right] \leq a_0 + v_0$$

yields $\{c_t^{\dagger}, m_t^{\dagger}, h_t^{\dagger}, w_t^{\dagger}, a_t^{\dagger}, v_t^{\dagger}, e_t^{\dagger}, P_t^{\dagger}, q_t^{\dagger}, R_t^{\dagger}, \Phi_t^{\dagger}\}_{t=0}^{\infty}$

SUBSTITUTION EFFECT

- Set lump-sum transfers, T_0^s, so HH can achieve same level of utility it would have obtained under the (*) tax even though it optimizes under the (†) tax
- Problem (Substitution)

$$\max_{\{c_t, m_t, h_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t, m_t) + v(1 - h_t)\right]$$

s.t. $$E_0 \sum_{t=0}^{\infty} q_t^{\dagger} \left[c_t + \frac{i_t^{\dagger}}{1 + i_t^{\dagger}} m_t + (1 - \tau_t^{\dagger}) w_t^{\dagger} (1 - h_t)\right] \leq a_0^{\dagger} + v_0^{\dagger} + T_0^s$$

  - constraining prices to be eqm prices under (†) tax $\Rightarrow$ budget line of this problem tangent to HH's indifference surface under (†) tax

A Hicksian Decomposition

- Problem (No Revaluation)

$$\max_{\{c_t, m_t, h_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t, m_t) + v(1 - h_t)\right]$$

s.t.

$$E_0 \sum_{t=0}^{\infty} q_t^{\dagger} \left[c_t + \frac{i_t^{\dagger}}{1 + i_t^{\dagger}} m_t + (1 - \tau_t^{\dagger}) w_t^{\dagger} (1 - h_t)\right] \leq a_0^* + v_0^{\dagger}$$

  - HH assumes revaluation does not result from the tax change, so assets have value $a_0 = a_0^*$ under (†) tax

A Hicksian Decomposition

- Set lump-sum transfers, T_0^w, so HH can achieve the same level of utility it would have obtained under the (†) tax, with and without asset revaluation
- Problem (Revaluation)

$$\max_{\{c_t, m_t, h_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t, m_t) + v(1 - h_t) \right]$$

$$\text{s.t.} \qquad E_0 \sum_{t=0}^{\infty} q_t^{\dagger} \left[c_t + \frac{i_t^{\dagger}}{1 + i_t^{\dagger}} m_t + (1 - \tau_t^{\dagger}) w_t^{\dagger} (1 - h_t) \right] \leq a_0^{\dagger} + v_0^{\dagger} + T_0^w$$

- T_0^w permits Problem (No Revaluation) and Problem (Revaluation) to achieve the same level of utility
- Total Effect: Problem (*) vs. Problem (†)
- Substitution Effect: Problem (*) vs. Problem (Substitution)
- Revaluation Effect: Problem (No Revaluation) vs. Problem (Revaluation)
- Wealth Effect $=$ Total $-$ Substitution $-$ Revaluation

A Hicksian Decomposition

- Solutions to optimization problems are cons demands

$$(*) : c_t^* = c\left(\frac{q_t^*}{\beta^t}, \frac{i_t^*}{1+i_t^*}, \frac{a_0^* + v_0^*}{e_0^*}\right)$$

$$(\dagger) : c_t^\dagger = c\left(\frac{q_t^\dagger}{\beta^t}, \frac{i_t^\dagger}{1+i_t^\dagger}, \frac{a_0^\dagger + v_0^\dagger}{e_0^\dagger}\right)$$

$$(\text{Substitution}) : c_t^\dagger \Big|_{u_0=u_0^*} = c\left(\frac{q_t^\dagger}{\beta^t}, \frac{i_t^\dagger}{1+i_t^\dagger}, \frac{a_0^* + v_0^* + T_0^s}{e_0^\dagger \Big|_{u_0=u_0^*}}\right)$$

$$(\text{Revaluation}) : c_t^\dagger \Big|_{u_0=u_0^\dagger, a_0=a_0^*} = c\left(\frac{q_t^\dagger}{\beta^t}, \frac{i_t^\dagger}{1+i_t^\dagger}, \frac{a_0^\dagger + v_0^\dagger + T_0^w}{e_0^\dagger \Big|_{u_0=u_0^*, a_0=a_0^*}}\right)$$

  - $c_t^\dagger \Big|_{u_0=u_0^*}$: planned consumption under (†) tax, with utility at u_0^*, the level under (*) tax
  - $c_t^\dagger \Big|_{u_0=u_0^\dagger, a_0=a_0^*}$: planned consumption without revaluation under (†) tax with utility at $u_0^\dagger$, the level under (†) tax

A HICKSIAN DECOMPOSITION

- The full decomposition

$$\underbrace{\log\left(\frac{c_t^\dagger}{c_t^*}\right)}_{\text{total effect}} = \underbrace{\log\left(\frac{c_t^\dagger|_{u_0=u_0^*}}{c_t^*}\right)}_{\text{substitution effect}} + \underbrace{\log\left(\frac{c_t^\dagger|_{u_0=u_0^\dagger,a_0=a_0^*}}{c_t^\dagger|_{u_0=u_0^*}}\right)}_{\text{wealth effect}} + \underbrace{\log\left(\frac{c_t^\dagger}{c_t^\dagger|_{u_0=u_0^\dagger,a_0=a_0^*}}\right)}_{\text{revaluation effect}}$$

An Example Economy

- Assume log preferences

$$u(c, m) + v(1 - h) = \log c + \log m + \log(1 - h)$$

Then

$$e_0 = \frac{3}{1 - \beta}$$

$$\begin{aligned} c_t &= \left(\frac{1-\beta}{3}\right)\left(\frac{\beta^t}{q_t}\right)(a_0 + v_0) \\ m_t &= \left(\frac{1-\beta}{3}\right)\left(\frac{\beta^t}{q_t}\right)\left(\frac{1+i_t}{i_t}\right)(a_0 + v_0) \\ h_t &= 1 - \left(\frac{1-\beta}{3}\right)\left(\frac{\beta^t}{(1-\tau_t)w_t q_t}\right)(a_0 + v_0) \end{aligned}$$

- Then can compute all the objects in the decomposition
- Assume MP pegs i_t to satisfy: $\beta(1 + i_t) = 1, \qquad t \geq 0$
- See Leeper-Yun (2006) for details and case of lump-sum

AN EXAMPLE ECONOMY

- Income taxes set $\tau_t > 0$

$$
\begin{aligned}
y_t &= \frac{1-\tau_t}{2-s^g-\tau_t} \\
q_t &= \beta^t \frac{(1-\tau_0)}{(1-\tau_t)} \frac{(2-s^g-\tau_t)}{(2-s^g-\tau_0)}
\end{aligned}
$$

- Present value full income flows is

$$
v_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{(1-\tau_0)(2-s^g-\tau_t)}{2-s^g-\tau_0} \right]
$$

- HH present value b.c.

$$
a_0 + v_0 = \frac{3}{1-\beta} \frac{(1-\tau_0)(1-s^g)}{2-\tau_0-s^g}
$$

- A Laffer curve in $\tau_t y_t$ with revenues maximized at

$$
\bar{\tau} = 2 - s^g - \sqrt{(2-s^g)(1-s^g)}
$$

An Example Economy

- Suppose taxes constant at τ
- y & c constant; i pegged; $q_t = \beta^t$

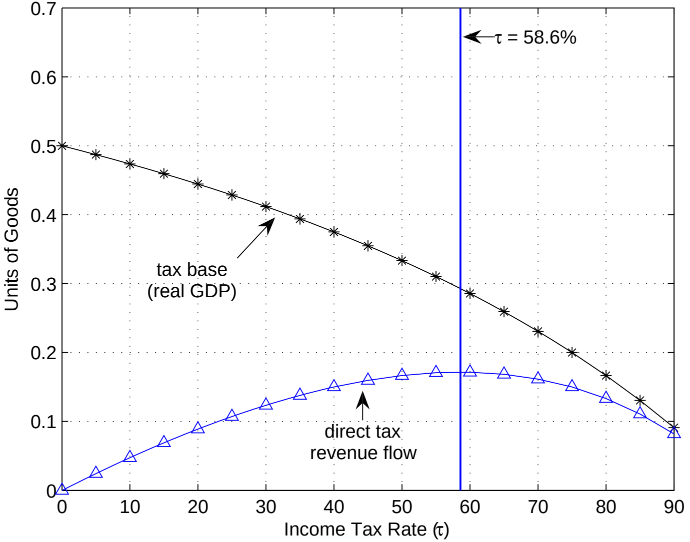
$$v_0 = \frac{1-\tau}{1-\beta}, \qquad a_0 = \frac{1-\beta}{1-\tau}\frac{2-s^g-\tau}{1-2s^g+\tau}$$

- Equilibrium price level

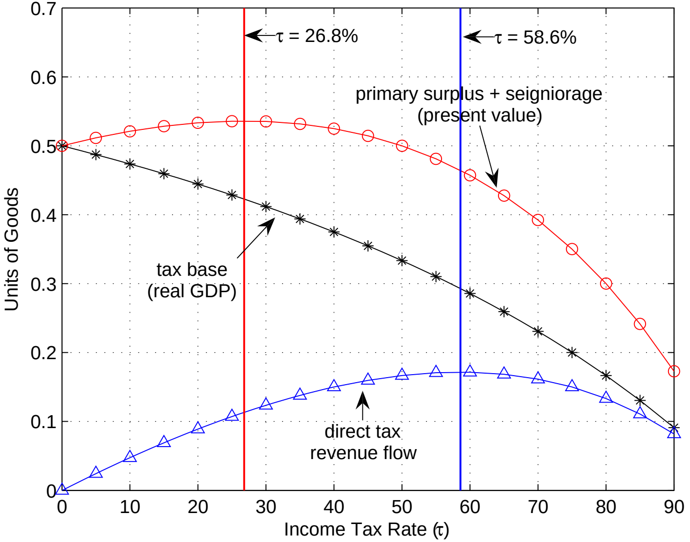
$$P_0 = \frac{1-\beta}{1-\tau}\frac{2-s^g-\tau}{1-2s^g+\tau}(B_{-1}+M_{-1})$$

- Note from $a_0 = \frac{1-\beta}{1-\tau}\frac{2-s^g-\tau}{1-2s^g+\tau}$, quadratic in $\tau \implies$ Laffer curve in sum of PV surpluses $+$ seigniorage
- Laffer curves in $\tau_t y_t$ and in a_0 can look very different

CONVENTIONAL LAFFER CURVE



Fiscal Theory Laffer Curve



TWO LAFFER CURVES

- Why are these different?
- Tax bases differ
  - conventional: $\tau_t y_t$
  - fiscal theory: $PV\left(\tau_t y_t + \frac{i_t}{1+i_t} m_t\right)$
  - changes in conventional tax base, y_t, feed into m_t and the seigniorage tax base
- Should we care about this?
  - presents tradeoffs
  - relevant for inflation-targeting countries to think about the fiscal consequences of MP

Wrap Up

- Fiscal theory has been accused of being "incoherent," "inconsistent with economic theory," and worse
- This shows that with the right kind of price-theoretic analysis, the revaluation effect that lies at the heart of the FTPL can be understood as a natural extension in an environment with nominal assets of standard the Slutsky-Hicks decomposition
- Critics have also accused FTPL of ignoring the government's budget *constraint*
- Here we have shown that you can get an eqm condition that determines P_0 without any reference to government variables
- Introducing distorting taxes to a FTPL analysis reveals a second kind of Laffer curve that has been largely overlooked