# Low-Order Perturbation Analysis of the Complete Markets Case

Robert Kollmann, Sunghyun H. Kim, and Jinill Kim

Paris School of Economics Presentation

September 1, 2007

- Algorithm for Perturbation
  - Split between State and Control Variables
  - Split Not Needed
    * versatility
    * interpretation

- Output of Perturbation: Nonlinear Difference Equation
  - How to use it?
  - This paper is using the pruning approach presented in KKSS.
    - stability of the quadratic solution
    - correct quadratic approximation of $x_{t+2}$ w.r.t. $x_t$

- Results (to be continued)

- **ACCURACY HIGHEST CLOSE TO STEADY STATE**

- **APPROXIMATION ERRORS INCREASING IN VOLATILITY OF PRODUCTIVTY**

- **<u>ACCURACY GAIN FROM QUADRATIC APPROX.</u> (COMPARED TO LINEAR APPROX) LARGER FOR BIG SHOCKS CASE ($\sigma = 0.01$) THAN FOR SMALL SHOCKS CASE ($\sigma = 0.001$).**

**ACROSS ALL SPECIFICATIONS WITH $\sigma = 0.01$, MEAN ACCURACY GAIN FOR $S_{\max}$ STATISTIC ($S_{\max}^{L} - S_{\max}^{Q}$): 0.80%**

**ACROSS CASES WITH** $\sigma = 0.001$ mean $S_{\max}^{L} - S_{\max}^{Q}$: $0.001\%$.

- Approximation errors increasing in number of countries, risk aversion; decreasing in capital adjustment cost

- Simulated den Haan-Marcet (DM) statistics favor very slightly quadratic approximation (over linear approx.), but only when $\sigma = 0.01$.

KKK (2005) found that DM more clearly in favor when $\sigma = 0.1$.

- Computing speed: N=10 model solved in a few seconds

## Table 1. Accuracy tests: results for sets of specifications

| Linear approximation | | | | | 2nd order approximation | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Test 1 | | | Test 2 | | Test 1 | | | Test 2 | |
| $T_{.01}$ | $T_{.1}$ | $T_{.3}$ | $S_{\max}$ | $S_{mean}$ | $T_{.01}$ | $T_{.1}$ | $T_{.3}$ | $S_{\max}$ | $S_{mean}$ |
| *(1)* | *(2)* | *(3)* | *(4)* | *(5)* | *(6)* | *(7)* | *(8)* | *(9)* | *(10)* |
| **All specifications** | | | | | | | | | |
| -2.57 | -1.40 | -0.59 | -1.63 | -2.80 | -4.41 | -2.53 | -1.13 | -2.50 | -4.03 |
| **Small shocks** $(\sigma = 0.001)$ | | | | | | | | | |
| -3.95 | -1.98 | -1.10 | -4.65 | -5.31 | -6.01 | -3.06 | -1.70 | -6.90 | -7.85 |
| **Big shocks** $(\sigma = 0.01)$ | | | | | | | | | |
| -2.57 | -1.40 | -0.59 | -1.63 | -2.50 | -4.41 | -2.53 | -1.13 | -2.50 | -3.73 |

Table shows $\log_{10}$ of test statistics

## Table 2. Relating accuracy to model parameters: regression results

$$\log_{10}(S^{L,i}_{\max}) = \underset{(51.25)}{-4.93} + \underset{(0.60)}{0.005}\, N^i \; \underset{(4.94)}{-0.46}\, \gamma^i \; \underset{(49.16)}{+315.8}\, \sigma^i \; \underset{(4.52)}{-0.02}\, \varphi^i \qquad R^2 = .96;$$

$$\log_{10}(S^{Q,i}_{\max}) = \underset{(49.54)}{-7.94} + \underset{(0.15)}{0.002}\, N^i \; \underset{(4.66)}{-0.67}\, \gamma^i \; \underset{(46.74)}{+455.7}\, \sigma^i \; \underset{(6.96)}{-0.06}\, \varphi^i \qquad R^2 = .96;$$

$$\log_{10}(S^{L,i}_{\max} - S^{Q,i}_{\max}) = \underset{(52.32)}{-4.94} + \underset{(0.57)}{0.005}\, N^i \; \underset{(4.96)}{-0.46}\, \gamma^i \; \underset{(49.61)}{+312.5}\, \sigma^i \; \underset{(4.41)}{-0.02}\, \varphi^i \qquad R^2 = .96;$$

In parentheses: absolute t-stats.

# Table 3. Accuracy tests: results for selected specifications

| | | | | | | Linear approximation | | | | | | | 2nd order approximation | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Mo= | | | | | | **Test 1** | | **Test 2** | | **Test 3** | | | **Test 1** | | **Test 2** | | **Test 3** | | |
| **del** | $N$ | $\gamma$ | $\eta$ | $\chi$ | $\mu$ | $T_{.01}$ | $T_{.3}$ | $S_{\max}$ | $S_{mean}$ | $P_{.05}$ | $P_{.5}$ | $P_{.95}$ | $T_{.01}$ | $T_{.3}$ | $S_{\max}$ | $S_{mean}$ | $P_{.05}$ | $P_{.5}$ | $P_{.95}$ |
| *(1)* | *(2)* | *(3)* | *(4)* | *(5)* | *(6)* | *(7)* | *(8)* | *(9)* | *(10)* | *(11)* | *(12)* | *(13)* | *(14)* | *(15)* | *(16)* | *(17)* | *(19)* | *(20)* | *(21)* |
| | | | | | | ***(a) Small shocks:*** **$\sigma = 0.001$** | | | | | | | | | | | | | |
| A1 | 2 | 1 | -- | -- | -- | -4.18 | -1.33 | -5.36 | -6.01 | .04 | .49 | .96 | -6.33 | -2.01 | -7.61 | -8.32 | .04 | .49 | .96 |
| A1 | 6 | 1 | -- | -- | -- | -4.70 | -1.85 | -5.49 | -5.84 | .99 | .99 | 1.00 | -7.01 | -2.66 | -7.65 | -8.22 | .99 | .99 | 1.00 |
| | | | | | | ***(b) Big shocks*:** **$\sigma = 0.01$** | | | | | | | | | | | | | |
| A1 | 2 | 1 | -- | ---- | | -3.26 | -0.99 | -2.34 | -3.02 | .03 | .43 | .94 | -5.40 | -1.71 | -3.66 | -4.44 | .04 | .45 | .94 |
| A1 | 6 | 1 | -- | -- | -- | -3.15 | -1.51 | -2.35 | -2.80 | .00 | .06 | .63 | -5.41 | -2.19 | -3.69 | -4.23 | .00 | .06 | .63 |

-----------------------------------------------

Table shows $\log_{10}$ of $T_r, S_{\max}, S_{mean}$.