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Christian Matthes
Francesca Rondina

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Two-sided Learning in New Keynesian Models: Dynamics, (Lack of) Convergence and the Value of Information*

Christian Matthes †   Francesca Rondina‡

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Abstract

This paper investigates the role of learning by both private agents and the central bank (*two-sided learning*) in a New Keynesian framework populated by private agents and a central bank that have asymmetric imperfect knowledge about the true data generating process. We assume that all agents employ the data they observe (which can be different for different sets of agents) to form beliefs about the aspects of the true model of the economy that they do not know, use these beliefs to decide on actions, and revise beliefs through a statistical learning algorithm as new information becomes available. We study the short-run dynamics of the model and policy recommendations coming out of our model, in particular concerning central bank communication. Two-sided learning can generate large increases in volatility and persistence, and can alter the behavior of the variables in the model in a significant way. We also show that our model does not converge to a symmetric rational expectations equilibrium and highlight one source that disables the convergence results of Marcet & Sargent (1989). Finally, we identify a novel aspect of central bank communication in models of learning: communication can be harmful if the central bank's model is substantially misspecified.

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†Universitat Pompeu Fabra and Barcelona GSE, email: christian.matthes@upf.edu

‡Institute for Economic Analysis, CSIC and Barcelona GSE, email: francesca.rondina@iae.csic.es

1 Introduction

This paper studies the role of asymmetric information and learning in a New Keynesian framework in which both private agents and the monetary authority have imperfect knowledge about the true model of the economy. In particular, we focus on the short run dynamics (or, more generally, dynamics when beliefs have not yet converged in the sense that agents do not know other agents' policy rules) that can arise when given their respective beliefs and information sets, policymakers optimally choose the policy rule to be implemented and private agents form their expectations rationally. In this environment, a rich and complex variety of interactions between beliefs and actions can potentially arise, with important consequences on the time series patterns of the variables of the model.

In a large number of situations and contexts, it is reasonable to assume that two or more interacting agents have asymmetric information about the environment in which they operate. In this work, we consider a model of the economy in which private agents do not observe a policy shock and the monetary policy rule implemented by the central bank, while the monetary authority does not observe a technology shock and the beliefs private agents have when forming their expectations. Agents employ all available information to estimate the aspects of the true data generating process that they do not know, and use a statistical learning algorithm to revise their beliefs as new data becomes available. In each period, these updated beliefs will be the base for the decisions of both the policy maker and the private agents.

An extensive literature in economics focuses on the analysis of monetary policy in environments characterized by imperfect knowledge and learning. Some recent contributions are Barnett and Ellison (2011), Bullard and Mitra (2002), Cho et al. (2002), Cogley et al. (2011), Evans and Honkapohja (2006), Honkapohja and Mitra (2005), Marcet and Nicolini (2003), Milani (2008), Orphanides and Williams (2005), Bullard and Eusepi (2005) (for additional references, Evans and Honkapohja, 2009, provide an extensive review of this literature). A large part of the research in this area focuses on the conditions under which the economy converges to a determinate rational expectations equilibrium (REE), and on the role of monetary policy in attaining this result.[1] Within this branch of the literature, two-sided learning has previously been studied by Marcet and Sargent (1989), who describe a general framework upon which we build, Bullard and Eusepi (2005), who use a New Keyne-

[1]Bullard and Mitra (2002), for instance, investigate this issue for a variety of alternative policies under the assumption that the central bank adopts Taylor-type instrument rules. On the other hand, Evans and Honkapohja (2003, 2006) focus on expectations-based targeting rules obtained from the optimization of the central bank's objective function (Svensson, 2002, strongly argues that these rules are superior to Taylor-type policy rules because they reflect optimal behavior on the part of the central bank).

sian model, but do not model the central bank's decision problem explicitly and endow both the central bank and private agents with the same information set, and Barnett and Ellison (2011), who build two-sided learning into a version of Sargent (1999). Honkapohja and Mitra (2005) study convergence to rational expectations equilibria under two-sided learning (using different assumptions on the perceived law of motions of agents than we do), but also do not explicitly model the central bank's decision problem. Evans and Honkapohja (2003) study convergence to rational expectations equilibria under two-sided learning (with an optimizing central bank), but using substantially different assumptions on the perceived laws of motion of agents and their respective information sets (in particular, they do not consider structural shocks that are only observed by some agents in the economy). Giannitsarou (2005) studies convergence to rational expectations equilibria under different learning algorithms in a framework similar to Marcet and Sargent (1989). Our paper departs from most of the literature on two-sided learning when it comes to the assumptions made on policymakers and the focus on short and medium-run dynamics.
More specifically, in our framework both private agents and the monetary authority have incomplete knowledge of the true data generating process, and they adopt the same approach to deal with their lack of full information: they form some beliefs using the data that they have available in each period, and they decide optimally based on these beliefs. The substantially different assumptions about the central bank's knowledge and behavior in our environment also implies that the focus of our analysis is not on the ability of the monetary authority to enforce a particular equilibrium of the model, but rather on the short run dynamics that the interactions of beliefs and actions between private agents and policymakers can generate. To our knowledge, the study of optimal policymaking and two-sided learning in an environment characterized by asymmetric information has not received much attention yet, particularly in the context of the business-cycle dynamics of a New Keynesian model.

With respect to learning, we assume that both private agents and the monetary authority use statistical models to estimate and predict the behavior of the variables for which they do not know the true data generating process. Then, we allow them to update their estimates as new data becomes available using a recursive learning algorithm. These assumptions follow the large branch of the learning literature originating from Marcet and Sargent (1989), Cho et al. (2002), and Evans and Honkapohja (1998). However, as in Cogley et al. (2011), the agents' perceived laws of motion in our framework have the important feature of incorporating the cross-equation restrictions that originate from their respective beliefs.

Because of the complex relationships between parameter updates and optimal decisions, the actual law of motion of the variables in the framework under analysis cannot be characterized analytically. For this reason, we study the impact of asymmetric information and learning in a number of simulations that are performed using standard parameter values for this model. Our results show that two-sided learning can significantly alter the dynamics of the model. More specifically, we find that it can generate large departures of the variables of interest from rational expectations equilibrium values, and changes in their behavior in terms of autocorrelations and correlations with the other variables. We find that convergence to an asymmetric rational expectations equilibrium in a frameworks like ours might not occur. Our exercises also suggest that, in this environment, information communication by the central bank does not seem effective in reducing the impact of asymmetric information and learning on the equilibrium dynamics. The fact that central bank communication can be harmful when the central bank has a misspecified model of the economy, is, to the best of our knowledge, a novel result in the literature [2].

The remainder of the paper is organized as follows. Section 2 presents the basic features of the New Keynesian model under analysis. Section 3 discusses the information available to agents, the learning procedure, and the decision-making approach. Section 4 derive agents' perceived laws of motion for the variables of interest, and their implied actual law of motion. Section 5 describes the simulation exercises that we perform, and presents their results. Section 6 concludes.

2 The true model of the economy

The true model of the economy is a standard New Keynesian framework as developed in Gali (2008). We assume perfect indexation of prices that cannot be reset to past inflation, as in Christiano et al. (2001), which makes sure that the pricing equations are unaffected by the presence of positive trend inflation, which leads to the steady state level of output being independent of the steady state level of inflation.[3]

Given these assumptions, private agents' behavior in this economy can be described by the following equations:

$$y_t = E_t^P\left(y_{t+1}\right) - \frac{1}{\sigma}\left(i_t - E_t^P\left(\pi_{t+1}\right) - r_t^n\right) \tag{1}$$

$$\pi_t = \frac{1}{(1+\beta)}\pi_{t-1} + \frac{\beta}{(1+\beta)}E_t^P\left(\pi_{t+1}\right) - \frac{\kappa}{(1+\beta)}(y_t - \overline{y}) \tag{2}$$

$$r_t^n = \overline{r} + u_t \tag{3}$$

[2] Eusepi and Preston (2010) study central bank communication when only the private sector is learning. We compare our results to theirs in section 3.

[3] See Ascari (2004) for a discussion.

$$u_t = \rho_u u_{t-1} + \varepsilon_t^u \tag{4}$$

where y_t is output π_t is the inflation rate, i_t is the nominal interest rate, r_t^n is the natural rate of interest and $\overline{y}$ is the steady state level of output (all other steady variables drop out in the equations above). The variable r_t^n is assumed to be the sum of the steady state real interest rate $\overline{r}$ and a technology shock u_t, or shock to the real side of the economy, which evolves according to the $AR(1)$ process described by (4). All variables are in logs. The parameters σ, β and κ have standard interpretations, and are obtained from the underlying problem of consumers and firms; see Gali (2008) for further details. Equations (1) and (2) have the standard interpretation of a IS equation and Phillips curve equation. Differently from the standard New Keynesian framework, the superscript P in the expectation operator $E_t^P(\cdot)$ in (1) and (2) denotes the fact that at each time t private agents will form these expectations based only the information that they have available, which will generally be different from the information available to policymakers.

In addition to the private sector, the economy is populated by a central bank or public authority, which is assumed to have some control over the nominal interest rate and to use it as its policy instrument. More specifically, the central bank is assumed to be able to set the value of i_t up to a monetary policy shock v_t. Let x_t be the value of the policy instrument chosen by the central bank for time t, then the interest rate will be:

$$i_t = x_t + v_t \tag{5}$$

where v_t is assumed to follow the AR(1) process:

$$v_t = \rho_v v_{t-1} + \varepsilon_t^v \tag{6}$$

Private agents and policymakers do not have full knowledge of the economy. In particular, the central bank does not observe the technology shock and does not know how private agents form expectations about the future values of the variables of interest and decide on prices and output. On the other hand, private agents are not aware of the policy rule that the central bank uses to set x_t, and do not observe the monetary policy shock. A more thorough description of the information set available to each side, and of its changes over time through learning, is given next.

3 Information and decisions

The imperfect and asymmetric information that private agents and the monetary authority employ in the decision making process are the central features that differentiate this work from the previous literature. However, these features require that we make some additional assumptions about the way in which each side will use its specific knowledge to estimate the aspect of the economy that are not known, to take decisions in each period, and finally to update its information based on the new data that can be observed.

The state and noise vectors that include all the information available in the economy are:

$$z_t = \begin{bmatrix} y_t & \pi_t & i_t & u_t & v_t & 1 \end{bmatrix}' \tag{7}$$

$$\varepsilon_t = \begin{bmatrix} \varepsilon_t^u & \varepsilon_t^v \end{bmatrix}' \tag{8}$$

As previously mentioned, the vectors z_t and ε_t are not perfectly observed. In particular, private agents do not observe the policy shock v_t, while the central bank does not observe the technology shock u_t. Thus, the vectors of variables that each side will use in their decision process can be written as:

$$z_t^P = \begin{bmatrix} y_t & \pi_t & i_t & u_t & 1 \end{bmatrix}' \tag{9}$$

$$z_t^{CB} = \begin{bmatrix} y_t & \pi_t & i_t & v_t & 1 \end{bmatrix}' \tag{10}$$

or:

$$z_t^P = M^P z_t$$

$$z_t^{CB} = M^{CB} z_t$$

where M^P and M^{CB} are just selection matrices that pick the relevant variables from the overall state z_t.

Private agents use the vector z_t^P to estimate the policymaker's interest rate rule and to predict future values of the nominal interest rate. In the same way, the monetary authority employs z_t^{CB} to approximate and predict the behavior of output and the inflation rate. We assume that agents make use of reduced-form regressions for this purpose, thus estimating a simple linear relationship between the variables for which they have limited knowledge and the information that they observe[4]. Given this framework, the decision process can be decomposed in two steps. First, private agents and policymakers need to estimate the

[4] In our model agents only estimate regressions for variables that they can not control. This is in contrast to most of the previous literature on learning, in which agents estimate VARs on *all* equilibrium variables, including their own decision variables.

parameters of the model using the available data. Second, they use their perceived model of the economy, together with the estimates of its parameters, to make their respective decisions. These steps are updated in each period according to the new information that is observed over time.

3.1 Estimation and learning

Private agents do not know the interest rate rule that the central bank uses to set the value of the interest rate. However, they know that the nominal interest rate affects output and the inflation rate through equations (1) and (2). For this reason, in order to be able to form their expectations on the future values of these variables, they need to make some conjecture about the relationship between i_t and the variables that they can observe.

Private agents behave like econometricians and estimate the simple linear relationship:

$$i_t = \psi_t z_{t-1}^P + \omega_t^P$$

$$i_t = \psi_{0t} + \psi_{\pi t}\pi_{t-1} + \psi_{yt}y_{t-1} + \psi_{it}i_{t-1} + \psi_{ut}u_{t-1} + \omega_t^P \tag{11}$$

where the error term ω_t^P just captures all the determinants of the nominal interest rate that are orthogonal to the information included in the state vector z_{t-1}^P.

The central bank has imperfect knowledge on the private side of the economy. However, the policy decision process requires the monetary authority to have some beliefs on the way in which the nominal interest rate affects the variables of interest. Akin to the assumptions we made for private agents, we assume that policymakers behave like econometricians and estimate simple reduced-form relationships between y_t and π_t and the state vector z_{t-1}^{CB} which includes the variables that they can observe:

$$y_t = c_{yt} z_{t-1}^{CB} + \omega_{yt}^{CB} \tag{12}$$

$$\pi_t = c_{\pi t} z_{t-1}^{CB} + \omega_{\pi t}^{CB} \tag{13}$$

As new data becomes available, private agents update their estimates of the vector of coefficients ψ, and the central bank updates its estimates of c_{yt} and $c_{\pi t}$. We assume that all agents in the economy use a standard recursive least squares algorithm (see, for instance, Evans and Honkapohja, 2001). We focus on the case of decreasing gains, in which the values of ψ, c_{yt} and $c_{\pi t}$ converge to OLS estimates for appropriate gain sequences. Further details about the learning approach that we adopt in this work are provided in the Appendix; the study of additional learning approaches in one of the directions of our future research.

We augment our learning algorithm with a projection facility, which ensures that parameter estimates remain within a predetermined region of values which we regard as suitable.[5] To be specific, we allow agents to make use of three types of projection facilities. The first type refers to the coefficients on the inflation rate and output in the perceived and actual policy rules, which are restricted to assume only positive values. The second type of facilities allow private agents to disregard estimates of the policy rule coefficients for which the solution of the expectational difference equation that they need to solve in their decision process does not exist or is not unique.[6] Finally, the third projection facility allows policymakers to rule out estimates of (12) and (13) that would cause the perceived law of motion of the variables of interest to be nonstabilizable. The actual projection facility we use os that if an estimate violates one of the restrictions we impose, the agents construct an estimate by averaging over the estimates from the previous two years (8 periods) and use this as their beliefs. A more formal description of the impact of projection facilities on the learning algorithm adopted in this paper is provided in the Appendix.

Projection facilities might rule out some potentially interesting dynamics of the variables of interest. However, in the environment under analysis, which is characterized both by asymmetric imperfect information and two-sided learning, it is important to endow agents with reasonable priors on the behavior of the other agents in the economy because agents' decisions are based on beliefs, which are in turn affected by the other side's decisions. It follows that it is very well possible that unreasonable beliefs over the estimated coefficients keep reinforcing each other instead of being redirected towards more sensible values. In any case, the simulation section of the paper will provide further discussion about the role of projection facilities in our simulations.

Notice that because we allow for the presence of trend inflation in (2), the true long-run level of inflation in this model is not known, and it is not constant. However, private agents can estimate trend inflation in each period as function of the estimated policy rule and the steady state level of the real interest rate. It follows that this framework has all the features

[5] For a more thorough discussion about the use of projection facilities in a number of learning algorithms, see Carceles-Poveda and Giannitsarou (2007).

[6] While it seems reasonable to assume that agents would rule out parameter estimates for which a stable pattern of the variables in z_t^P does not exist, the case in which the solution is indeterminate is a little bit more complex. The analysis of learning in environments in which multiple equilibria can potentially arise requires not only to take a stand on the way in which one of the alternative solutions should be selected, but also to model the way in which agents should account for the indeterminacy of the equilibrium when updating their beliefs. The learning patterns emerging in this environment could potentially be very complicated. For these reasons, we decided to start with a simpler scenario, in which private agents behave conservatively and disregard parameter estimates that would lead to indeterminate equilibria. Nonetheless, we do believe that the study of two-sided learning in the case of indeterminate equilibria is very interesting, and we aim to extend our research in this direction in the future.

to allow us to investigate the impact of the uncertainty in the long-run level of inflation on current decisions, which is a direction we are currently pursuing in parallel work.

Before moving to the description of the decision process, we would like to address one issue related to the structure of the information set and learning approach that we assume in this paper. One possible objection to the framework that we adopt would be that the central bank should know the learning problem of the agents just by introspection, given that the central bank decision makers are private agents after all. However, the model could be rewritten under the assumption that each private agent does not know a priori that all the other agents use the same forecasting scheme. Rather, each household (or firm) i could be endowed with a conditional expectations operator indexed by i, E_t^i. If we assume that all those expectation operators are indeed equal, we can integrate and over i and since work with a linear model we would still get the standard aggregate equilibrium conditions.

3.2 Policy decisions and expectations formation

The actual law of motion of the variables in the model depends on the decisions of private agents and policymakers. More specifically, private agents use their knowledge of the private side of the economy and their beliefs about the interest rate rule to form expectations, which in turn affect the behavior of y_t and π_t through (1) and (2). On the other hand, the central bank uses its beliefs on the processes for y_t and π_t to set the value of the policy instrument x_t.

With respect to the private sector, we assume that decisions follow the same timing as in Cogley et al. (2011). Private agents estimate the parameters of the policy rule (11) using information up to and including time $t-1$. Then, they observe current period shocks and the value of the policy instrument, and use them, together with the previously available information, when making decisions on actions. This approach means that agents enter time t with predetermined parameter estimates, but then use the current period shock realizations to form expectations.

The central bank, on the other hand, has the power to decide the value of x_t in (5). The policy rule for x_t is chosen by minimizing the expected discounted quadratic loss function:

$$E_{t-1}^{CB}\sum_{j=0}^{\infty}\beta^j[(\pi_{t+j})^2+\lambda_y(y_{t+j})^2+\lambda_i(i_{t+j}-i_{t+j-1})^2] \tag{14}$$

given (12) and (13), and the estimated values of c_{yt} and $c_{\pi t}$. The parameters λ_y and λ_i represent the weight attached to the output variable relative to inflation, and the relative cost of changing the nominal interest rate. The subscript in E_{t-1}^{CB} means that expectations are taken with respect to the information set available to the central bank. We do not allow the central bank to react contemporaneously to the monetary policy shock since the central bank could otherwise just undo any effect of the monetary policy shock.

In their decisions, private agents and policymakers are assumed to behave as anticipated utility decision makers (Kreps, 1998), which means that they will treat parameter estimates as true values, thus disregarding parameter uncertainty and the effects of learning. This assumption is common in the literature on learning in macroeconomics (see, for instance, Evans and Honkapohja, 2001).

4 Model solution

Policymakers and private agents base their decisions on their respective perceived law of motion (PLM) for the variables of interest. However, their decisions will affect the true model of the economy, i.e. the actual law of motion (ALM) of these variables. This section provides more details about the agents' PLMs, their decision process, and the resulting ALM.

4.1 PLM for the central bank

The central bank's PLM for output and inflation is defined by equations (12) and (13), and can be rewritten in state space form using the vector z_t^{CB} as:

$$A^{CB} z_t^{CB} = B^{CB} x_t + C_t^{CB} z_{t-1}^{CB} + D^{CB} \varepsilon_t^{CB} \tag{15}$$

The time subscripts in the matrices of parameters emphasize the fact that the estimates of c_{yt} and $c_{\pi t}$ in (12) and (13) are updated over time even if, as previously mentioned, the assumption of anticipated utility implies that policymakers will not take these updates into account in their decision process. The problem of the central bank is then to find the sequence $\{x_t\}$ that minimizes (14) subject to (15) under the assumption of constant parameter values. It is well known that, under standard conditions, the solution to this problem is linear in the state z_{t-1}^{CB}, i.e.:

$$\begin{aligned} x_t &= -F_t z_{t-1}^{CB} \\ &= f_{0t} + f_{\pi t}\pi_{t-1} + f_{yt} y_{t-1} + f_{it} i_{t-1} + f_{vt} v_{t-1} \end{aligned}$$

so that the expression for the nominal interest rate becomes:

$$i_t = f_{0t} + f_{\pi t}\pi_{t-1} + f_{yt} y_{t-1} + f_{it} i_{t-1} + f_{vt} v_{t-1} + v_t \tag{16}$$

If the matrices of parameters in (15) were constant over time, standard results in the optimal control literature would deliver a time-invariant optimal policy vector F. However, because in our setup the optimization problem is repeated in every period given updated values of c_{yt} and $c_{\pi t}$ in (12) and (13), the optimal policy vector will actually depend on the current period estimates of these parameters.

Given estimates of the parameters for time t and the chosen policy rule, the PLM for the central bank can be rewritten as:

$$A^{CB} z_t^{CB} = \left(C_t^{CB} - B^{CB} F_t\right) z_{t-1}^{CB} + D^{CB} \varepsilon_t^{CB}$$

or

$$z_t^{CB} = \Phi_{1,t} z_{t-1}^{CB} + \Phi_2 \varepsilon_t^{CB} \tag{17}$$

where $\Phi_{1,t} = \left(A^{CB}\right)^{-1} \left(C_t^{CB} - B^{CB} F_t\right)$ and $\Phi_2 = \left(A^{CB}\right)^{-1} D^{CB}$.

The central bank implements the policy rule defined by (16), so that the chosen value of the vector of coefficients F_t will in fact have an impact on the ALM of the nominal interest rate and, through it, on the ALM of output and inflation.

4.2 PLM for private agents

The PLM for private agents can be obtained from equations $(1) - (4)$ and the perceived interest rate rule expressed by (11). In matrix form, this PLM can be written as:

$$A^P z_t^P = B^P E_t^p \left(z_{t+1}^P\right) + C_t^P z_{t-1}^P + D^P \varepsilon_t^P \tag{18}$$

where the time subscript in the matrix C_t^P emphasizes the fact that the estimated coefficients of the perceived policy rule are updated over time. The matrices of coefficients A^P, B^P, C_t^P and D^P are specified in the Appendix.

Private agents use (18) as the basis to solve the expectation term $E_t^p \left(z_{t+1}^P\right)$. As previously mentioned, we use the same approach as in Cogley et al. (2011). In more detail, in every period private agents estimate the coefficients of the perceived policy rule, and then solve the vector-valued expectational difference equation that features the equilibrium conditions including the estimated policy rule. This approach has the important consequence that agents do take into account cross-equations restrictions when forming forecasts. In addition, because of our assumptions, the PLM is just the reduced form VAR associated with (18), with reduced form coefficients that are time-varying because they depend on the estimate of the policy rule coefficients. For this reason, we can guess a solution of the form:

$$z_t^P = \Gamma_{1,t} z_{t-1}^P + \Gamma_{2,t} \varepsilon_t^P$$

which we can plug in for the expectation to get:

$$(A^P - B^P \Gamma_{1,t}) z_t^P = C_t^P z_{t-1}^P + D^P \varepsilon_t^P$$

This gives the following equation for the reduced form matrices:

$$\Gamma_{1,t} = (A^P - B^P \Gamma_{1,t})^{-1} C_t^P$$

$$\Gamma_{2,t} = (A^P - B^P \Gamma_{1,t})^{-1} D^P$$

We use Sims'(2001) Gensys program to find the values of $\Gamma_{1,t}$ and $\Gamma_{2,t}$. When z_t^P is determinate, this program delivers the unique nonexplosive solution for these matrices of parameters. When z_t^P is indeterminate, the program delivers one of the many possible non-explosive solutions. As previously mentioned, we will endow private agents with a projection facility allowing them to rule out coefficient estimates for which a stable solution does not exist. In addition, we allow private agents to employ an additional projection facility according to which they will disregard estimates of the parameters in A^P, B^P, C_t^P and D that would lead to an indeterminate outcome for z_t^P. This facility has the consequence of preventing the Gensys program from randomly selecting a solution that private agents should implement.

4.3 ALM

The ALM for the variables in the model involves equations $(1)-(4)$, which describe the true behavior of the private sector, together with the true interest rate rule expressed by (16). In matrix form, this ALM can be written as:

$$Az_t = BE_t^p \left(z_{t+1}^P \right) + C_t z_{t-1} + D\varepsilon_t$$

The matrices A, B, C and D are defined in the Appendix. Notice that the matrix C_t is time-variant because it includes the true policy coefficients, which the central bank will update in each period.

From the PLM for the private sector, we know that:

$$\begin{aligned} E_t^p\left(z_{t+1}^P\right) &= \Gamma_{1,t} z_{t-1}^P \\ &= \Gamma_{1,t} M^P z_t \end{aligned}$$

which implies:

$$Az_t = B\Gamma_{1,t} M^P z_t + C_t z_{t-1} + D\varepsilon_t \tag{19}$$

It follows that the ALM of the model can be written as:

$$z_t = \Psi_{1,t} z_{t-1} + \Psi_{2,t}\varepsilon_t$$

where:

$$\Psi_{1,t} = (A - B\Gamma_{1,t} M^P)^{-1} C_t$$

$$\Psi_{2,t} = (A - B\Gamma_{1,t} M^P)^{-1} D$$

These last two expressions, which define the matrices of coefficients in the ALM of the economy, do not include the matrix $\Gamma_{2,t}$. Hence the inclusion of a perceived policy shock in the PLM of agents is actually unnecessary.

5 Simulation Exercises

The model that we have described in the previous section involves complex interactions of beliefs and actions between private agents and policymakers that can not be solved in closed form. In particular, while the learning procedure that agents use to update their beliefs has a recursive structure, policymakers' optimization approach and private agents' expectation formation process are highly nonlinear functions of their estimated parameters . For this reason, the equilibrium pattern implied by the learning and decision sequence assumed in this paper cannot be characterized analytically. The main goal of this section is thus to offer some insights about the role of asymmetric information and two-sided learning in the context of the New Keynesian framework described in the previous section by performing a Monte Carlo simulation.

We focus on the short run (i.e. non-asymptotic) behavior of the endogenous variables of the model, and we investigate the patterns, magnitude and length of the departures of these variables from their values in a rational expectation equilibrium. We define the REE as the one emerging from an environment in which policymakers set a fixed policy rule for the instrument x_t in (5) and maintain this policy for the entire simulation period, while private agents still learn and compute expectations based on the procedure described in the previous section. The fixed policy rule that we use in this case is a standard Taylor-type rule in the form: $x_t = 0.5\overline{r} + 0.5y_{t-1} + 1.5\pi_{t-1} + 0.5i_{t-1}$. While this specific policy rule is chosen arbitrarily, the lessons that we will draw from comparing the model under learning and the rational expectations version would emerge under virtually any monetary policy rule [7].

In addition to the benchmark scenario incorporating learning and optimal decisions as described in the previous section, we also perform one additional exercise. We allow the central bank to communicate its perceived steady state value of inflation to private agents. More specifically, we assume that at the end of period t, the central bank announces $\overline{\pi}_t$, their time t estimate of the long run value of inflation, which can be calculated using the policy rule computed in period t. Private agents trust this announcement, and, because of anticipated utility, they treat this announced level of the steady state inflation rate as fixed, and use it in their regressions. In more detail, they estimate the policy rule with the time t left hand side variable being $i_t - \overline{i}_t$ and the right hand side variables being deviations from the implied steady state. $\overline{i}_t$ is the sum of the announced steady state inflation rate and $\overline{r}$. When making their decisions next period, the private agents also use their knowledge of $\overline{\pi}_t$. Note that the central bank will update its estimate to $\overline{\pi}_{t+1}$ that period. We believe that this setup is interesting because it allows us to study the impact that a reduction in the asymmetry of the information available to agents has on the process of learning. However, notice that, because the central bank has imperfect information about the economy, it is not necessarily the case that the communication of $\overline{\pi}_t$ will improve economic outcomes. On the contrary, policymakers' incorrect beliefs over the long run value of the inflation rate can potentially have a destabilizing effect on private agents' learning and decision process.

In all the simulations, we set the parameters of the true model of the economy and those in policymakers' loss function as: $\sigma = 1$; $\kappa = 0.2$; $\beta^P = \beta^{CB} = 0.99$; $\overline{r} = 1/\beta^P - 1$; $\lambda_y = 1/16$ and $\lambda_i = 0.5$. For the real shock u_t and the policy shocks v_t, we assume normal distributions with parameters: $\sigma^2_{\varepsilon^u} = 0.008^2$; $\sigma^2_{\varepsilon^v} = 0.008^2$; $\sigma^2_{\varepsilon^v} = 0.008^2$; $\rho_u = 0$; $\rho_v = 0$.[8] In order to initialize the learning and decisions procedure, we need to set an initial value for agents' beliefs. We do this by using population regressions and population moments from the

[7] Ideally, we would like to compare our learning model to its asymmetric information rational expectations counterpart (instead of a rational expectations model with an arbitrary policy rule), but as we will see below our simulations don't allow us to identify this asymmetric information rational expectations counterpart and in our setup the differential equation approach of Marcet and Sargent (1989) does not give us any advantage over running simulations since would have to solve the ODEs numerically as well, which would be cumbersome because of the non-linear mapping from parameter estimates to reduced form dynamics.

[8] We are not advocating that uncorrelated exogenous shocks are necessarily the best option when one wants to fit data, but since we are interested in the relative volatility and persistence of variables in our model (relative to rational expectations) we make that simplifying assumption. Our results are robust to (and even become somewhat stronger when) assuming autocorrelated shocks.

rational expectation solution of the model obtained using the same fixed policy rule adopted in the rational expectations scenario. While the initial transition path in the simulations is affected by this choice, our conclusions are not.[9]

We report the results for the case of a recursive learning algorithm with decreasing gains, as described in the Appendix.[10] The value of t_0 was set equal to 12. In all the exercises, we set the period length to $T = 1000$, and we performed $N = 1000$ simulations. We study the impact of asymmetric information and two-sided learning in the New Keynesian model under analysis by looking at the distributions of the patterns of the variables of interest and of the policy parameters obtained from the different simulations. We show the median, and 15th and 85th percentile bands of these distributions, and we report their relevant statistics[11].

Figures $1-3$ and tables $1-2$ provide clear evidence about the fact that the impact of two-sided learning in this environment is significant. Compared to their distribution in the rational expectations case, all the variables exhibit patterns that are much more volatile when asymmetric information and two-sided learning are included. This increase in volatility happens for all the variables, and in all the exercises that we performed. Moreover, the magnitude of this increase in volatility is very large. Table 1 shows that the median standard deviation of the output variable is more than twice its value in the rational expectations case, and the median standard deviation of the annualized inflation rate is more than 6 times its value in the rational expectations case in both exercises. This table also highlights the fact that reducing asymmetric information through central bank communications does not decrease the volatility originated by two-sided learning. On the contrary, the median standard deviations of log output and of the annualized nominal interest rate are even higher under this assumption than in the benchmark case.

In figure 5, we report the distribution of the autocorrelations of the variables of interest in the 3 alternative scenarios under analysis. This figure shows that, relative to the rational expectations case, two-sided learning can potentially increases the persistence of the variables in the model at all orders, in some cases in a substantial way. This result holds in all the exercises that we performed and is more pronounced in the case in which we allow the central bank to communicate its perceived long run value of the inflation rate. Tables $3-5$ report the correlations between variables for the different cases. The correlations between the output gap and inflation and between the output gap and the nominal interest rate do not seem

[9] Carceles-Poveda and Giannitsarou (2007) provide a discussion of different methods that can be employed to initialize agents' beliefs in frameworks characterized by adaptive learning.

[10] We also experimented with constant gains, $g = 0.015$, and we found much larger volatility of the variables of interest in this case. As previously mentioned, the analysis of the learning patterns under alternative algorithms is one of the extensions that we intend to pursue.

[11] If we reported means instead of medians the difference vis-a-vis rational expectations would be more striking, but would be partly driven by a relatively small fraction of outliers.

to be much affected by the introduction of two-sided learning. The only relationship that changes in a relevant way is the one between the inflation rate and the nominal interest rate, for the case of asymmetric information and learning without central bank communication. More specifically, the negative correlation between these two variables becomes much smaller in this scenario. This result is interesting, because it suggests that in this case the perceived ability of the central bank to affect the inflation rate through the policy instrument would be reduced. In all, we can conclude that two-sided learning has the potential to affect the persistence of the variables in the models, and also some important contemporaneous relationships between them.

In terms of policy parameters, we report the distribution of the actual and estimated coefficient attached to the lagged inflation rate (figure 4).[12] When asymmetric information and learning are introduced in the model, convergence of these two sets of parameters to fixed values is not guaranteed. Since the private agents use different variables in their perceived policy rule than those that enter the actual monetary policy rule, the coefficients on lagged inflation (or other variables that appear in both the perceived and actual policy rules) in the perceived and actual policy rules do not have to converge to the same values. It is worth emphasizing that the distribution of policy rule coefficients is non-degenerate even after 1000 periods. This hints at the possibility of non-convergence to an asymmetric rational expectations equilibrium or the existence of multiple self-confirming-equilibria in our model [13]. We will discuss below why the convergence theorems of Marcet and Sargent (1989) do not apply in our setup (and thus why non-convergence might be an issue).

In addition, we find that the projection facility that requires the optimal policy parameters to assume reasonable values (in this case, a value greater than zero), is invoked in a non-negligible number of the simulations, especially in the first periods of the learning and decision process. It is interesting that communication of the estimated long run value of the inflation rates does not seem to help private agents' learning of the actual policy coefficients and, consequently, does not seem to reduce the volatility of policymakers' beliefs and policy decisions. As previously mentioned, the fact that the perceived long run inflation rate is obtained using a misspecified model has the potential to destabilize, rather than stabilize, the learning and decision process, even if this communication actually reduces the asymmetry of information. This result is confirmed by figure 7, which reports the absolute difference

[12] For the other parameters in the policy rule, we find that they exhibit patterns that are similar to those reported in figure 4.

[13] As mentioned before, unfortunately our set-up, in which private agents basically solve a rational expectations model given their beliefs every period, makes it infeasible for us to get any analytical results about the existence of multiple self-confirming equilibria using the standard ODE approach pioneered by Marcet and Sargent (1989).

in the one-step ahead inflation forecasts by the central bank and private agents in the last period of the simulations [14]. This figure clearly shows that this difference can be much larger in the case in which policymakers communicate their beliefs on long run inflation relative to the baseline asymmetric information environment. The two panels of figure 7 also highlight the fact that two-sided learning has important consequences on the agents' ability to predict future values of the variables of interest, even for the very short run, i.e. for the one-period ahead forecast. Indeed, different knowledge and beliefs over the underlying true coefficients of the model and policy rule, will lead private agents and policymakers to disagree on the predicted patterns of the variables in the future, sometimes even in a substantial way.

Finally, we study the stability properties of the framework under analysis. Figure 6 reports the probability of bad outcomes, which we define as inflation rates higher than 20%, in the three alternative scenarios.[15] In the rational expectations model, in which policymakers set a fixed policy rule and and private agents know that rule, this probability is zero. When asymmetric information and two-sided learning are introduced, the probability of inflation rates higher than 20% becomes positive, and it increases with the length of the sample period under consideration. If in addition the central bank is allowed to communicate its beliefs over the long run inflation rate, this probability becomes quite large. This result confirms our previous observation that "naive" communication of information that is based on incorrect beliefs about the true data generating process can have destabilizing effects on the economy, rather than facilitating learning and convergence of perceived and actual decisions. Figure 8 provides further support to this conclusion. The projection facilities that we implement ensure that private agents and policymakers will rule out parameter estimates which, according to their respective PLMs, would generate explosive patterns of the endogenous variables in the model. However, as previously discussed, this is not sufficient to guarantee the stability of the ALM, because in some circumstances decisions based on the individual PLMs could still lead to a situation in which one or more eigenvalues of $\Psi_{1,t}$ are larger than one in absolute value. Figure 8 shows that the probability of such an event to materialize is nonzero but yet very small in the baseline scenario, while it becomes considerably higher when communication from the central bank to private agents is allowed. To get further insight into the reasons behind our results, it is instructive to compare our setup to Marcet and Sargent (1989). Marcet and Sargent (1989) require the ALM to be stable at all times to guarantee convergence to an asymmetric information rational expectations equilibrium. The existence of periods where the ALM is not stable in our simulations means

[14] We report forecast differences in annualized percentages up to the 80 percentile distribution to control for outliers, similarly to our previous graphs

[15] This probability was computed as the fraction of times in which inflation was above the 20% value in the simulations that we performed.

that the convergence results in Marcet and Sargent(1989) are not applicable in our case, even though we endow agents with perceived laws of motion that are always stable. The result of unstable dynamics coupled with stable perceived dynamics is also present in models with one-sided learning, as discussed in Cogley, Matthes and Sbordone (2011).
Eusepi and Preston (2010) provide conditions under which the central bank's communication of a (time-invariant) inflation target does not lead to convergence to the rational expectations equilibrium. While their result has a similar flavor to the outcomes of our simulations, their result is obtained by either letting (at least) one of the exogenous processes in their model becoming arbitrarily close to a random walk or price stickiness vanishing and having persistent exogenous processes. We, on the other hand, have i.i.d. disturbances and a fixed level of price stickiness (as encoded in the slope of the Phillips curve).[16]

Uur exercises suggest that the impact of asymmetric information and two-sided learning in the context of a New Keynesian model of the economy is significant. We find that the pattern of the variables of the model can change considerably in terms of volatility and autocorrelations, that bad outcomes are much more likely, and that large differences between policymakers' and private agents' forecasts of the future values of the variables of interest can potentially arise. We also show that communication of information from the central bank does not necessarily improve private agents' learning of the true policy coefficients and, as a consequence, might not result in a more stable economic environment. On the contrary, in our simulations the scenario in which policymakers disclose their beliefs over the long run inflation rate is the one that exhibits the highest potential for extreme outcomes to emerge. This increase in volatility happens because, even if communication between the agents reduces the asymmetry of the information employed in the respective decision processes, it will still be based upon each side's perceptions and imperfect knowledge of the true model of the economy.

[16] In the economic literature on information and signal extraction, a number of contributions originating from the seminal work of Morris and Shin (2002) have investigated the impact of transparency and public information on social welfare. In general, this research finds that more transparency on the side of the central bank might be welfare increasing or decreasing depending on the precision of the public signal relative to the private signal, and on the weight that private agents attach to the departures of their individual actions from the aggregate. Our results on central bank communication are related to the conclusions of this literature; however, the focus of our paper is not on the analysis of the relationship between public information and social welfare, but rather on the study of the changes in the dynamics of a standard New Keynesian model implied by the introduction of asymmetric information and learning.

6 Conclusions

This work represents a first attempt at investigating the role of asymmetric information and two-sided learning in a New Keynesian model of the economy. The assumption that both monetary authorities and private agents have imperfect knowledge of the true data generating process, and try to learn over time from the new information that becomes available, seems to reflect fairly well what we observe in the real world scenario. For this reason, we believe that the study of the way in which this learning process can potentially alter the dynamics of a New Keynesian framework, which is often used as the basis for policy analysis, is of primary importance. The results of our simulations support this idea by showing that two-sided learning can cause large departures of beliefs and decisions from their rational expectations values. We also emphasize a novel (at least to our knowledge) aspect of central communication in learning models: if the central bank has a misspecified model of the economy, central bank communication can lead to substantially more volatility, not only for an initial transition period, but for the entire time period considered. This part of our analysis complements Eusepi and Preston (2010), who conduct an analysis of central bank communication under private sector learning.

The analysis in this paper can be extended in a number of directions. First, the impact of alternative assumptions about agents' learning approach, for instance the use of different learning algorithms, could be investigated. Second, we think that it would be interesting to further analyze the effects that communication between the agents might have on their process of learning about the true data generating process. In this work, we have studied the case in which policymakers inform the private agents about their perceived long run value of the inflation rates. Communications of different types of information, or communications from private agents to policymakers, could also be investigated. Third, the framework that we employ in this paper allows us to investigate the impact of the uncertainty about the long run level of inflation on current beliefs and decisions, and this is also a direction that we are interested in pursuing.

Finally, one route that we would like to explore is in the direction of estimating our framework using real world data, which would allow us to employ this model to provide an interpretation of past event and to offer more punctual policy recommendations.

Appendix

Learning algorithm

Let the equations to be estimated by agents be written in general terms as:

$$q_t = p'_{t-1}\phi_t + \eta_t$$

where q_t is the dependent variable or a vector of dependent variables, p_{t-1} a vector or matrix of regressors, η_t the residual(s) and ϕ_t the vector of parameters of interest. In the case of private agents, this equation corresponds to (11), while for policymakers it encompasses (12) and (13). Using this notation, the learning algorithm can be written as:

$$R_t = R_{t-1} + g_t \left(p_{t-1}p'_{t-1} - R_{t-1}\right)$$

$$\phi_t = \phi_{t-1} + g_t R_t^{-1} p_{t-1} \left(q_t - p'_{t-1}\phi_{t-1}\right)$$

where g_t represents the gain. In the simulations, we focus on Recursive Least Squares (RLS) learning, in which $g_t = \frac{1}{t_0+t}$.[17] However, we also perform some comparative exercises using Constant Gain (CG) learning, in which g_t is a constant positive and small number, i.e. $g_t = g$, $0 < g < 1$. For a more thorough description of these learning algorithms and their properties, see Evans and Honkapohja (2001); for a discussion of their performance in a few standard macroeconomic models, see Carceles-Poveda and Giannitsarou (2007).

As mentioned in the main text, the basic learning algorithm will be augmented with a number of projection facilities. We assume that whenever the value of the estimated parameters by private agents and policymakers "hits" the projection facility, i.e. moves outside the predetermined parameter region Q, agents will use an average over past estimates as the current period estimate. Thus, the algorithm can be rewritten as:

$$\widehat{R}_t = \widehat{R}_{t-1} + g_t \left(p_{t-1}p'_{t-1} - \widehat{R}_{t-1}\right)$$

$$\widehat{\phi}_t = \widehat{\phi}_{t-1} + g_t \widehat{R}_t^{-1} p_{t-1} \left(q_t - p'_{t-1}\widehat{\phi}_{t-1}\right)$$

$$(\phi_t, R_t) = \begin{array}{ll} \left(\widehat{\phi}_t, \widehat{R}_t\right) & \text{if } \left(\widehat{\phi}_t, \widehat{R}_t\right) \in Q \\ \left((1/N)\sum_{j=1}^{N} \phi_{t-j}, (1/N)\sum_{j=1}^{N} R_{t-N}\right) & \text{if } (\phi_t, R_t) \notin Q \end{array}$$

The specific restrictions that we impose via projection facilities are discussed in the main text.

[17] As mentioned in the main text, t_0 is set to 12 quarters in our simulations.

Matrices in the PLMs and ALM

The matrices of the PLM for the central bank can easily be obtained using the state space representation (15) and the policy rule (16) emerging as a result of the optimization problem. We have that:

$$A^{CB} z_t^{CB} = \left(C_t^{CB} - B^{CB} F_t \right) z_{t-1}^{CB} + D^{CB} \varepsilon_t^{CB}$$

or more explicitly:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} y_t \\ \pi_t \\ i_t \\ v_t \\ 1 \end{pmatrix} = \begin{pmatrix} c_{1yt} & c_{2yt} & c_{3yt} & c_{4yt} & c_{5yt} \\ c_{1\pi t} & c_{2\pi t} & c_{3\pi t} & c_{4\pi t} & c_{5\pi t} \\ -f_{\pi t} & -f_{yt} & -f_{it} & -f_{vt} & -f_{0t} \\ 0 & 0 & 0 & \rho_v & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} y_{t-1} \\ \pi_{t-1} \\ i_{t-1} \\ v_{t-1} \\ 1 \end{pmatrix} + \\ + \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \omega_{yt}^{CB} \\ \omega_{\pi t}^{CB} \\ \varepsilon_t^{v} \end{pmatrix}$$

The PLM for private agents is given by the true equations $(1) - (4)$ together with the perceived interest rate rule expressed by (11).

$$y_t = E_t^P \left(y_{t+1} \right) - \tfrac{1}{\sigma} \left(i_t - E_t^P \left(\pi_{t+1} \right) - \overline{r} - u_t \right)$$

$$\pi_t = \tfrac{1}{(1+\beta)} \pi_{t-1} + \tfrac{\beta}{(1+\beta)} E_t^P \left(\pi_{t+1} \right) - \tfrac{\kappa}{(1+\beta)} \left(y_t - \overline{y} \right)$$

$$i_t = \psi_{0t} + \psi_{\pi t} \pi_{t-1} + \psi_{yt} y_{t-1} + \psi_{it} i_{t-1} + \psi_{ut} u_{t-1} + \omega_t^P$$

$$u_t = \rho_u u_{t-1} + \varepsilon_t^u$$

These equations can be rewritten in matrix form as:

$$\begin{pmatrix} 1 & 0 & \frac{1}{\sigma} & -\frac{1}{\sigma} & -\frac{1}{\sigma}\overline{r} \\ \frac{\kappa}{(1+\beta)} & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} y_t \\ \pi_t \\ i_t \\ u_t \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & \frac{1}{\sigma} & 0 & 0 & 0 \\ 0 & \frac{\beta}{(1+\beta)} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} E_t^p \left[\begin{pmatrix} y_{t+1} \\ \pi_{t+1} \\ i_{t+1} \\ u_{t+1} \\ 1 \end{pmatrix} \right] +$$

$$+ \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{(1+\beta)} & 0 & 0 & \frac{\kappa}{(1+\beta)}\overline{y} \\ \psi_{yt} & \psi_{\pi t} & \psi_{it} & \psi_{ut} & \psi_{0t} \\ 0 & 0 & 0 & \rho_u & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} y_{t-1} \\ \pi_{t-1} \\ i_{t-1} \\ u_{t-1} \\ 1 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \varepsilon_t^u \\ \omega_t^P \end{pmatrix}$$

or

$$A^P z_t^P = B^P E_t^p \left(z_{t+1}^P \right) + C_t^P z_{t-1}^P + D^P \varepsilon_t^P$$

The ALM for the variables in the model can be obtained from the true equations (1) – (4) together with the true interest rate rule expressed by (16), and can be written in matrix form as:

$$\begin{pmatrix} 1 & 0 & \frac{1}{\sigma} & -\frac{1}{\sigma} & 0 & -\frac{1}{\sigma}\overline{r} \\ \frac{\kappa}{(1+\beta)} & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} y_t \\ \pi_t \\ i_t \\ u_t \\ v_t \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & \frac{1}{\sigma} & 0 & 0 & 0 \\ 0 & \frac{\beta}{(1+\beta)} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} E_t^p \left[\begin{pmatrix} y_{t+1} \\ \pi_{t+1} \\ i_{t+1} \\ u_{t+1} \\ 1 \end{pmatrix} \right]$$

$$+ \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{(1+\beta)} & 0 & 0 & 0 & \frac{\kappa}{(1+\beta)}\overline{y} \\ f_{yt} & f_{\pi t} & f_{it} & 0 & f_{vt} & f_{0t} \\ 0 & 0 & 0 & \rho_u & 0 & 0 \\ 0 & 0 & 0 & 0 & \rho_v & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} z_{t-1} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \varepsilon_t^u \\ \varepsilon_t^v \end{pmatrix}$$

or:

$$Az_t = BE_t^p \left(z_{t+1}^P \right) + C_t z_{t-1} + D\varepsilon_t$$

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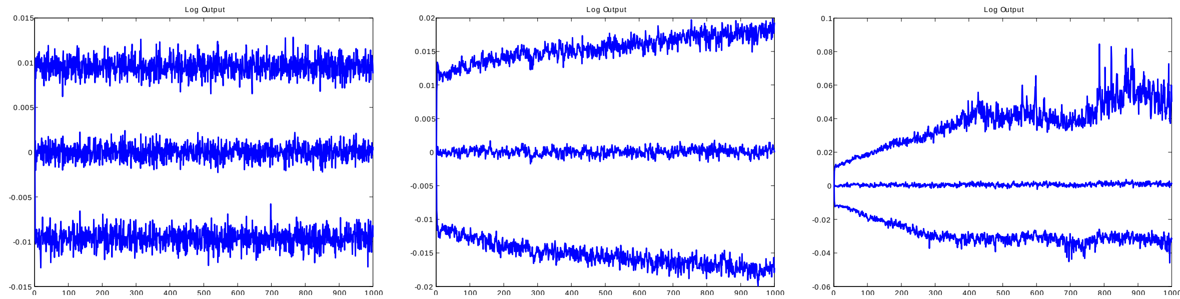
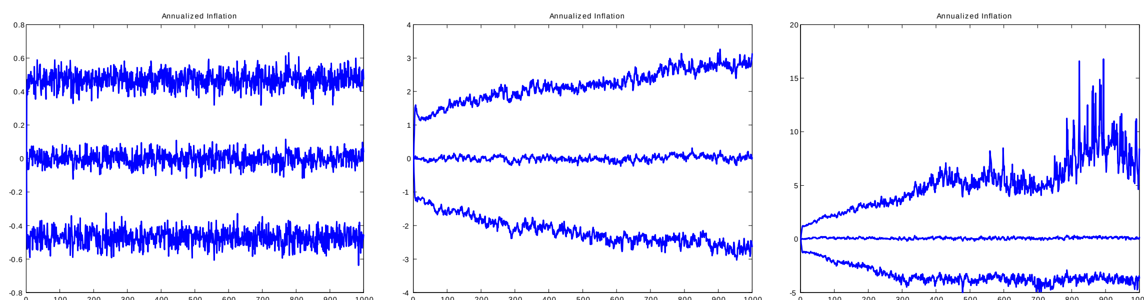
Figures and Tables

Figure 1 - Log output



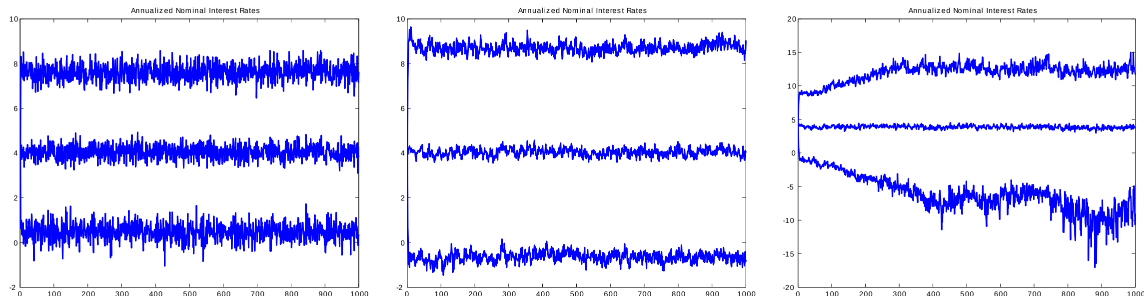
Note: Median value and 15th and 85th percentile bands of the simulated pattern for log output. The panels report the following scenarios: 1) REE; 2) benchmark asymmetric information and learning case; 3) central bank communication.

Figure 2 - Annualized inflation



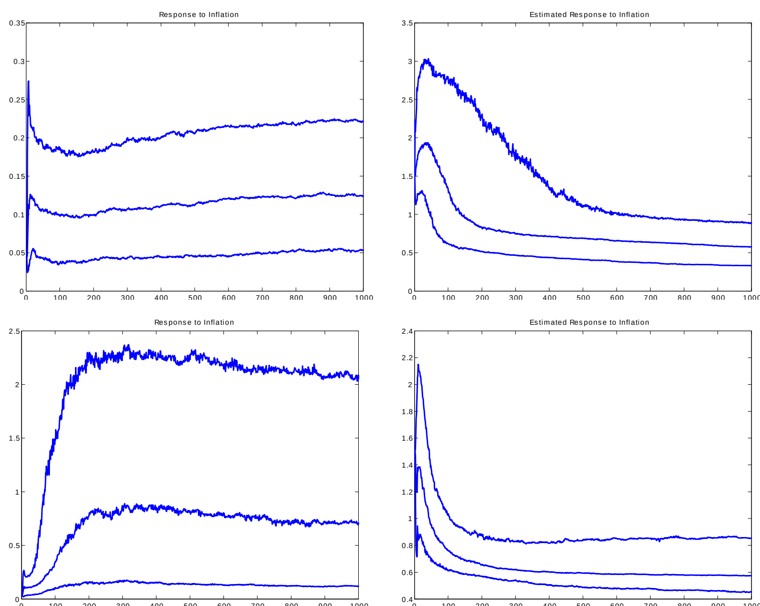
Note: Median value and 15th and 85th percentile bands of the simulated pattern for the annualized inflation rate. The panels report the following scenarios: 1) REE; 2) benchmark asymmetric information and learning case; 3) central bank communication.

Figure 3 - Annualized nominal interest rate



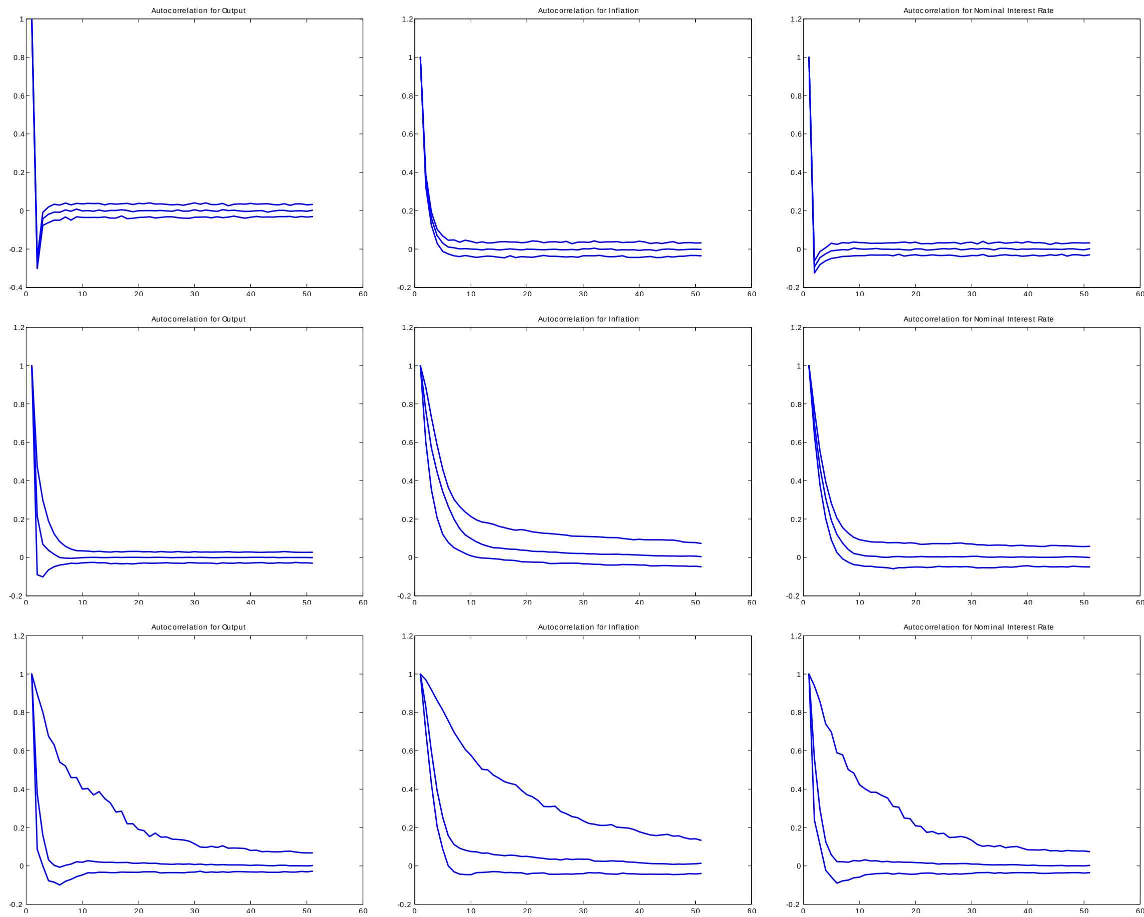
Note: Median value and 15th and 85th percentile bands of the simulated pattern for the annualized interest rate. The panels report the following scenarios: 1) REE; 2) benchmark asymmetric information and learning case; 3) central bank communication.

Figure 4 - Selected policy coefficients - response to the inflation rate



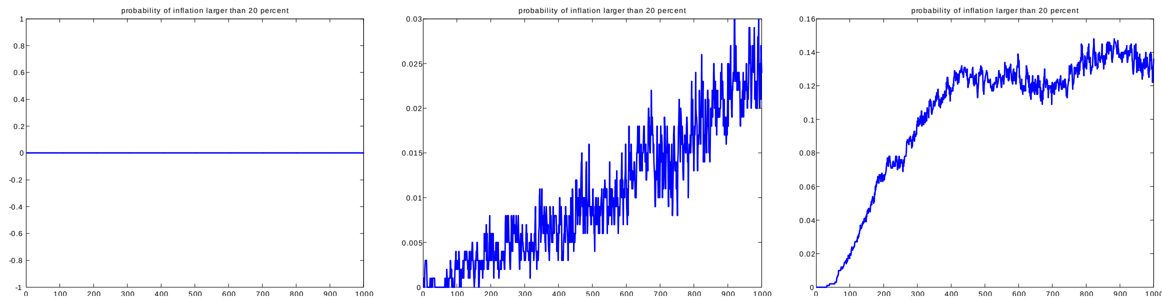
Note: Median value and 15th and 85th percentile bands of the simulated actual and estimated policy response to the inflation rate. Each row reports a different scenario: 1) benchmark asymmetric information and learning case; 2) central bank communication.

Figure 5 - Autocorrelations



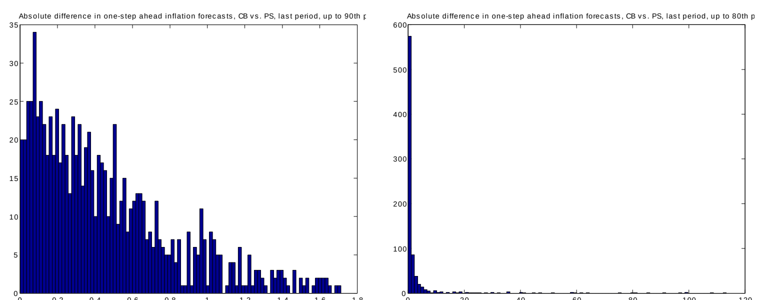
Note: Median value and 15th and 85th percentile bands of the autocorrelations of the variables of interest in the performed simulations. Each row reports a different scenario: 1) RE; 2) benchmark asymmetric information and learning case; 3) central bank communication.

Figure 6 - Probability of inflation larger than 20%



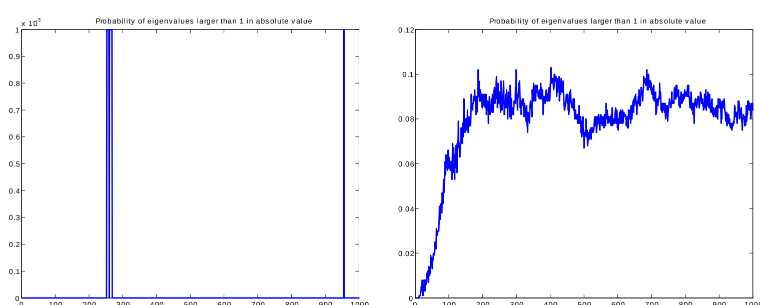
Note: Probability of inflation exceeding 20%. The panels report the following scenarios: 1) RE; 2) benchmark asymmetric information and learning case; 3) central bank communication.

Figure 7 - Absolute difference inflation forecast



Note: Absolute difference in the one-step ahead inflation forecasted by the Central Bank and private agents. The panels report the following scenarios: 1) benchmark asymmetric information and learning case; 2) central bank communication.

Figure 8 - Probability of explosive ALM



Note: Probability of eigenvalues larger than 1 in absolute value in the ALM. The panels report the following scenarios: 1) benchmark asymmetric information and learning case; 2) central bank communication.

Table 1 - Median standard deviations relative to the RE case

variable	**benchmark**	**CB comm.**
output	2.3539	2.6698
annualized inflation	7.9535	6.1471
annualized interest rate	1.3287	2.0812

Table 2 - Standard deviations, RE case

variable	**standard deviation**
output	0.0092
annualized inflation	0.4537
annualized interest rate	3.4487

Table 3 - Correlations in the RE case

correlations - RE case

	y_t	π_t	i_t
y_t	1	0.7874	-0.8461
π_t	0.7874	1	-0.7342
i_t	-0.8461	-0.7342	1

Table 4 - Correlations in the benchmark case

	y_t	π_t	i_t
y_t	1	0.6474	-0.6524
π_t	0.6474	1	-0.2876
i_t	-0.6524	-0.2876	1

Table 5 - Correlations in the case of central bank communication

	y_t	π_t	i_t
y_t	1	0.7614	-0.9621
π_t	0.7614	1	-0.8350
i_t	-0.9621	-0.8350	1