Dynare Working Papers Series
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Working Paper no. 29

December 2013

CEPREMAP
CENTRE POUR LA RECHERCHE ECONOMIQUE ET SES APPLICATIONS
142, rue du Chevaleret — 75013 Paris — France
http://www.cepremap.fr

Productivity shocks and monetary policy in a two-country model*

Tae-Seok Jang† and Eiji Okano‡

November 10, 2013

Abstract

In this paper, we examine the effects of foreign productivity shocks on monetary policy in a symmetric open economy. Our two-country model incorporates the New Keynesian features of price stickiness and monopolistic competition based on the cost channel of Ravenna and Walsh (2006). In particular, in response to asymmetric productivity shocks, firms in one country achieve a more efficient level of production than those in another economy. Because the terms of trade are directly affected by changes in both economies' output levels, international trade creates a transmission channel for inflation dynamics in which a deflationary spiral in foreign producer prices reduces domestic output. When there is a decline in economic activity, the monetary authority should react to this adverse situation by lowering the key interest rate. The impulse response function from the model shows that a productivity shock can cause a real depreciation of the exchange rate when economies are closely integrated through international trade.

JEL Classification: E24, E31, J3

Keywords: cost channel; New Keynesian model; productivity shocks; terms of trade; two-country model

*A preliminary version of this paper was presented at the Asia Pacific Econophysics Conference in 2013 at Pohang University of Science and Technology, Korea, and at the 9th Dynare Conference at Shanghai University of Finance and Economics, China.

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1 Introduction

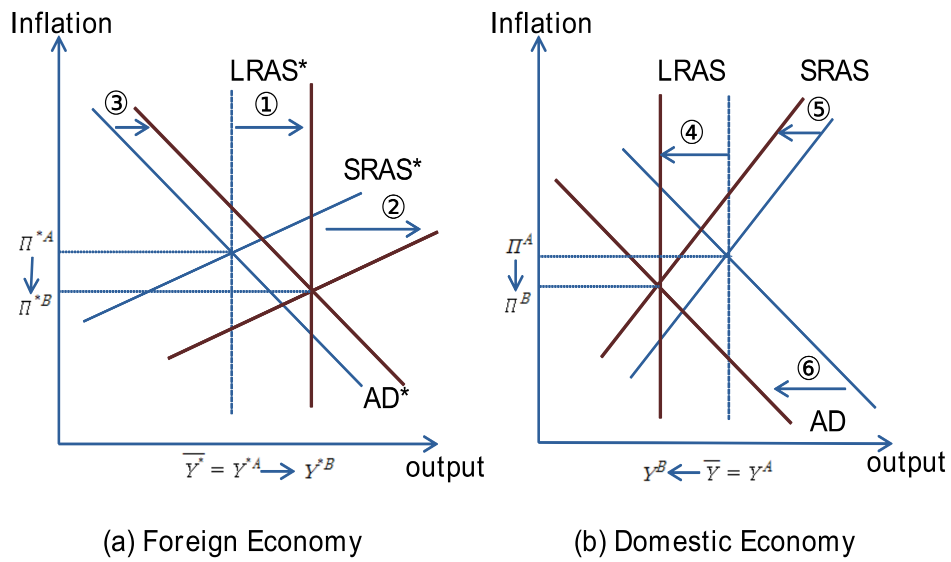
Free trade often provides a litmus test for global economic growth and the success of the world economy; see, for example, Baldwin (2003) and Billmeier and Nannicini (2007). Indeed, a well-developed industry can benefit from international trade, especially when the economy is in a sufficiently good position to meet foreign demand.[1] On the other hand, globalization and trade openness can put pressure on local industry by exposing it to a loss of international competitiveness. Thus, small open economies remain fragile and vulnerable to overseas shocks; one country can easily lose its competitive edge against another country that is booming. Hence, policy makers have a vested interest in world economic development.

In this paper, we consider how economic productivity in one country affects another country. Our objective is to understand the shock transmission between the two countries. In real business cycle models, economic agents efficiently allocate consumption and investment in response to productivity shocks. However, in a Keynesian framework, because of nominal rigidities, the ways in which agents adjust to economic changes can distort shock transmission channels in an open economy. To show this, we examine the endogenous dynamic economic relationships between nominal adjustment and procyclical fluctuations in open economies. Our model predicts that monetary policy is strongly affected by changes in foreign output and price levels, especially when economies are integrated through international trade; good news in one country can act much like bad news in another country.

We apply the open-economy framework of Gali and Monacelli (2005) and Okano et al. (2012). In particular, our model incorporates the New Keynesian features of price stickiness and monopolistic competition based on the cost channel of Ravenna and Walsh (2006). The real and financial sectors are linked to incorporate the effects of productivity shocks on market demand. For example, suppose that a productivity shock occurs in a foreign economy; see Fig. 1. The positive shock causes marginal cost to fall and the natural level of output to increase. The shift of the aggregate supply curve to the right reduces the price level in the short run. A monetary authority that follows a standard Taylor rule lowers the nominal interest rate. Because of the cost channel, through which firms borrow money from financial intermediaries, a fall in the foreign interest rate increases aggregate demand. However, the shock causes a deterioration in the domestic economy's terms of trade. The domestic economy suffers a contraction in aggregate supply and falls behind the booming foreign economy. Moreover, consumers spend more money on imported goods and less on domestically produced goods. The monetary authority should react to this adverse situation by

[1] Better terms of trade may reflect an economy's improved global position; see Cheptea et al. (2005).

lowering the key interest rate.



Note: * asterisk denotes foreign economy. AD, SRAS, and LRAS mean aggregate demand, short-run and long-run aggregate supply, respectively.

Figure 1: The effects of a foreign productivity shock on the domestic economy

The transmission of shocks between two countries has become an important aspect of open-economy macroeconomics; see Erceg, Gust, and Salido (2007). Many researchers focus on the time series behavior of key economic variables by estimating structural vector autoregressive models; see Cushman and Zha (1997), Jacobson (1999) and Jacobson *et al.* (2001). Other researchers use a New Keynesian framework to focus on theoretical and empirical aspects of open economies; see Giordani (2004), Gali and Monacelli (2005) and De Paoli (2009). Although there is much research on the shock transmission mechanism between countries, less attention has been paid to endogenous economic dynamics in New Keynesian models. In this respect, we aim to examine the effects of price stickiness and differentiated goods on the adjustment process in two economies.

First, we focus on a symmetric two-country model in which purchasing power parity (PPP) does not necessarily hold. In general, deviations from PPP can be used to enhance the model's ability to capture short-run price movements, and can enable an examination of the effects of trade openness on the exchange rate. Second, it is assumed in the model that households in the two countries can be separated; that is, domestic households are located on the interval (0, 1) and foreign households are on the interval (1, 2). Thus, although we do not incorporate the ratio of the countries' GDP levels into the model, relative GDP is taken into consideration by consumers when they buy goods. This simplification facilitates the analysis of connectivity between the two economies.

Because of the model's analytical tractability, we can generate impulse response functions to convey the effect of a positive productivity shock. We also vary the degree of home bias in the model, and examine the effect of economic fluctuations on monetary policy. Several key results emerge from our analysis. First, depending on the degree of price stickiness incorporated into the model, the duration of output and inflation responses to foreign productivity shocks is strongly affected by trade openness: that is, the higher the degree of openness, the longer the effects of the shock last. The strength of this correlation depends on the degrees of price stickiness and substitutability between domestic and imported goods. Second, because the terms of trade are directly affected by output levels in both economies, international trade can create a transmission channel for inflation dynamics in which a deflationary spiral in the producer price index (PPI) in one country reduces economic activity in another country. In particular, our model predicts that a foreign productivity shock can cause a real depreciation (appreciation) of the exchange rate when the domestic economy is relatively open (closed) to international trade; appreciation occurs because the domestic price level is hardly affected by changes in the relative competitiveness of the two countries. This means that the monetary authority must react to changes in foreign productivity.

This paper is organized as follows. In Section 2, we explain how demand and supply in the New Keynesian model generates symmetry between the two economies. In Section 3, using calibrated values for the model's parameters, we simulate impulse responses to a productivity shock based on different degrees of openness to trade. Section 4 concludes the paper. Technical details are relegated to the Appendix.

2 A model of two symmetric economies

Following Gali and Monacelli (2005), we model a world economy comprising two countries. Each country is populated with a continuum of unit mass; the population in the segment $h \in [0,1]$ belongs to country H and the population in the segment $f \in [1,2]$ belongs to country F.

2.1 Households

The representative households' preferences in the open economy are given by:

$$\mathcal{U} \equiv \mathrm{E}_0 \left(\sum_{t=0}^{\infty} \beta^t U_t \right) \; ; \; \mathcal{U}^* \equiv \mathrm{E}_0 \left(\sum_{t=0}^{\infty} \beta^t U_t^* \right), \tag{1}$$

where $U_t \equiv \frac{1}{1-\sigma} C_t^{1-\sigma} - \frac{1}{1+\varphi} N_t^{1+\varphi}$ and $U_t^* \equiv \frac{1}{1-\sigma} \left(C_t^*\right)^{1-\sigma} - \frac{1}{1+\varphi} \left(N_t^*\right)^{1+\varphi}$ denote utility levels in period t in countries H and F, respectively. E_t denotes the expectation conditional on the information set at period t, and $\beta \in (0,1)$ is the subjective discount factor. C_t and C_t^* denote the consumption index in countries H and F, respectively. $N_t \equiv \int_0^1 N_t(h)\, dh$ and $N_t^* \equiv \int_1^2 N_t^*(f)\, df$ are hours of labor in countries H and F, respectively. Quantities and prices in country F are denoted by asterisks, and quantities and prices without asterisks are those in country H.

The consumption indices are defined as follows:

$$\begin{aligned} C_t &\equiv \left[(1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} ; \\ C_t^* &\equiv \left[(1-\alpha)^{\frac{1}{\eta}} \left(C_{F,t}^*\right)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} \left(C_{H,t}^*\right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \end{aligned} \tag{2}$$

where $C_{H,t} \equiv \left[\int_0^1 C_t(h)^{\frac{\varepsilon-1}{\varepsilon}} dh \right]^{\frac{\varepsilon}{\varepsilon-1}}$ is an index of the consumption of goods produced in country H. Similarly, $C_{F,t} \equiv \left[\int_1^2 C_t(f)^{\frac{\varepsilon-1}{\varepsilon}} df \right]^{\frac{\varepsilon}{\varepsilon-1}}$ is an index of the consumption of goods produced in country F. $\varepsilon > 1$ is the elasticity of substitution between the differentiated goods produced in each country. $\alpha \in [0,1]$ is a measure of trade openness. $\eta > 0$ is the elasticity of substitution between domestic and foreign goods from the viewpoint of the domestic consumer.

The maximization of Eq. (1) is subject to a sequence of budget constraints of the following form:

$$\begin{aligned} \int_0^1 P_t(h)\, C_t(h)\, dh + \int_0^1 P_t(f)\, C_t(f)\, df + \mathrm{E}_t\left(Q_{t,t+1} D_{t+1}^n\right) &\leq D_t^n + W_t N_t + TR_t; \\ \int_1^2 P_t^*(h)\, C_t^*(h)\, dh + \int_1^2 P_t^*(f)\, C_t^*(f)\, df + \mathrm{E}_t\left(Q_{t,t+1}^* D_{t+1}^{n*}\right) &\leq D_t^{n*} + W_t^* N_t^* + TR_t^*, \end{aligned} \tag{3}$$

where $P_t(h)$ and $P_t(f)$ are the prices of generic goods produced in country H and country F in terms of country H's currency, respectively. D_{t+1}^n denotes the nominal payoff in period $t+1$ of the portfolio held at the end of period t in terms of country H's currency. $Q_{t,t+1}$ denotes the stochastic discount factor for the one-period-ahead nominal payoffs relevant to domestic households. W_t and TR_t are the nominal wage and lump-sum transfers (taxes), respectively.

Optimally allocating any given expenditure within each category of goods yields the following demand functions:

$$\begin{aligned} C_t(h) &= \left(\frac{P_t(h)}{P_{H,t}}\right)^{-\varepsilon} C_{H,t}\ , \qquad C_t(f) = \left(\frac{P_t(f)}{P_{F,t}}\right)^{-\varepsilon} C_{F,t}; \\ C_t^*(h) &= \left(\frac{P_t^*(h)}{P_{H,t}^*}\right)^{-\varepsilon} C_{H,t}^*\ , \quad C_t^*(f) = \left(\frac{P_t^*(f)}{P_{F,t}^*}\right)^{-\varepsilon} C_{F,t}^*, \end{aligned} \tag{4}$$

where $P_{H,t} \equiv \left(\int_0^1 P_t(h)^{1-\varepsilon} dh\right)^{\frac{1}{1-\varepsilon}}$ and $P_{F,t}^* \equiv \left(\int_1^2 P_t(f)^{1-\varepsilon} df\right)^{\frac{1}{1-\varepsilon}}$ denote the PPI. The price indices of imported goods, $P_{H,t}^*$ and $P_{F,t}$, are defined analogously to $P_{H,t}$ and $P_{F,t}^*$.

The optimal allocations of expenditures between domestic and imported goods are:

$$\begin{aligned} C_{H,t} &= (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t\ \ ,\ \ C_{F,t} = \alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t; \\ C_{H,t}^* &= \alpha\left(\frac{P_{H,t}^*}{P_t^*}\right)^{-\eta} C_t^*\quad ,\ \ C_{F,t}^* = (1-\alpha)\left(\frac{P_{F,t}^*}{P_t^*}\right)^{-\eta} C_t^*. \end{aligned} \tag{5}$$

Note that the consumer price indices (CPIs) are given by:

$$P_t \equiv \left[(1-\alpha) P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta}\right]^{\frac{1}{1-\eta}}; \tag{6}$$

$$P_t^* \equiv \left[(1-\alpha)^* \left(P_{F,t}^*\right)^{1-\eta} + (1-(1-\alpha)^*)\left(P_{H,t}^*\right)^{1-\eta}\right]^{\frac{1}{1-\eta}}. \tag{7}$$

Eq. (4) implies that total amount of expenditure in country H is the sum of $\int_0^1 P_t(h)\,C_t(h)\,dh = P_{H,t}C_{H,t}$ and $\int_0^1 P_t(f)\,C_t(f)\,df = P_{F,t}C_{F,t}$. Similarly, total expenditure in country F is the sum of $\int_1^2 P_t^*(h)\,C_t^*(h)\,dh = P_{H,t}^*C_{H,t}^*$ and $\int_1^2 P_t^*(f)\,C_t^*(f)\,df = P_{F,t}^*C_{F,t}^*$. Then, from Eq. (5), the following relations hold: $P_{H,t}C_{H,t} + P_{F,t}C_{F,t} = P_tC_t$ and $P_{H,t}^*C_{H,t}^* + P_{F,t}^*C_{F,t}^* = P_t^*F_t^*$. These expressions can be used to rewrite Eq. (3) as:

$$P_tC_t + \mathrm{E}_t\left(Q_{t,t+1}D_{t+1}^n\right) \leq D_t^n + W_tN_t + TR_t; \tag{8}$$

$$P_t^*C_t^* + \mathrm{E}_t\left(Q_{t,t+1}^*D_{t+1}^{n*}\right) \leq D_t^{*n} + W_t^*N_t^* + TR_t^*. \tag{9}$$

Representative households maximize Eq. (1) subject to Eq. (9). The intertemporal optimality conditions are given by:

$$\begin{aligned} \beta\mathrm{E}_t\left(\frac{C_{t+1}^{-\sigma}P_t}{C_t^{-\sigma}P_{t+1}}\right) &= \frac{1}{R_t}; \\ \beta\mathrm{E}_t\left(\frac{\left(C_{t+1}^*\right)^{-\sigma}P_t^*}{\left(C_t^*\right)^{-\sigma}P_{t+1}^*}\right) &= \frac{1}{R_t^*}, \end{aligned} \tag{10}$$

where $R_t \equiv 1+r_t$ and $R_t^* \equiv 1+r_t^*$ denote the gross nominal interest rates that satisfy $\frac{1}{R_t} = \mathrm{E}_t\left(Q_{t,t+1}\right)$ and $\frac{1}{R_t^*} = \mathrm{E}_t\left(Q_{t,t+1}^*\right)$, respectively, where r_t and r_t^* are the real interest rates for countries H and F, respectively. The intratemporal optimality conditions are then given by:

$$C_t^{\sigma} N_t^{\varphi} = \frac{W_t}{P_t} \; ; \; \left(C_t^*\right)^{\sigma} \left(N_t^*\right)^{\varphi} = \frac{W_t^*}{P_t^*}. \tag{11}$$

For future reference, note the following log-linearized version of Eq. (10):

$$\begin{aligned} c_t = & \quad \mathrm{E}_t\left(c_{t+1}\right) - \frac{1}{\sigma}\{\hat{r}_t - \mathrm{E}_t\left(\pi_{t+1}\right)\}; \\ c_t^* = & \quad \mathrm{E}_t\left(c_{t+1}^*\right) - \frac{1}{\sigma}\{\hat{r}_t^* - \mathrm{E}_t\left(\pi_{t+1}^*\right)\}, \end{aligned} \tag{12}$$

where lowercase letters denote the percentage deviation from the steady state of variables denoted by the corresponding uppercase letters.[2] Thus, the real interest rate is $\hat{r}_t \equiv \frac{dR_t}{R}$. Hence, CPI inflation is $\pi_t \equiv p_t - p_{t-1}$.

To determine the link between CPI inflation and PPI inflation, we consider the price of goods produced in country F in terms of the price of goods produced in country H. Note that the terms of trade can be expressed as $\mathcal{S}_t \equiv \frac{P_{F,t}}{P_{H,t}}$. Log-linearizing Eq. (7) yields:

$$\pi_t = \pi_{H,t} + \alpha\left(s_t - s_{t-1}\right) \; ; \; \pi_t^* = \pi_{F,t}^* - \alpha\left(s_t - s_{t-1}\right), \tag{13}$$

where $\pi_{H,t} \equiv p_{H,t} - p_{H,t-1}$ and $\pi_{F,t}^* \equiv p_{F,t}^* - p_{F,t-1}^*$ denote PPI inflation in countries H and F, respectively. Eq. (13) shows that CPI inflation depends on PPI inflation and changes in the terms of trade.

In the context of our open-economy model, we assume complete financial markets at both domestic and international levels. The equilibrium price of a risk-free bond denominated in country H's currency is given by $\mathrm{E}_t\left(Q_{t,t+1}^*\right)\mathcal{E}_t = \mathrm{E}_t\left(Q_{t,t+1}\mathcal{E}_{t+1}\right)$, where $\mathcal{E}_t$ denotes the price of country F's currency in terms of country H's currency; that is, the nominal exchange rate. Hence, we obtain the following version of the uncovered interest rate parity (UIP) condition:

$$\frac{R_t}{R_t^*} = \mathrm{E}_t\left(\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\right).$$

This expression shows that the difference in the nominal interest rates on risk-free bonds of countries H and F is equal to expected changes in the nominal exchange rate. The UIP condition can be

[2] For example, $v_t \equiv \ln\left(\frac{V_t}{V}\right)$, for some arbitrary variable V_t. Unless stated otherwise, V denotes the steady-state value of V_t.

log-linearized as follows:

$$\hat{r}_t - \hat{r}_t^* = \mathrm{E}_t\left(e_{t+1}\right) - e_t,$$

for which we have used $e_t \equiv \ln \mathcal{E}_t$.

Combining the first and second equalities in Eq. (10), under the assumption of complete securities markets, yields the following international risk-sharing condition:

$$C_t = \vartheta C_t^* \mathcal{Q}_t^{\frac{1}{\sigma}}, \tag{14}$$

where $\mathcal{Q}_t \equiv \frac{\mathcal{E}_t P_t^*}{P_t}$ denotes the real exchange rate. The risk-sharing condition includes a constant (ϑ) that depends on initial conditions.

Note that we assume that the law of one price holds for individual goods at all times, both for import and export prices, which implies that $P_t\left(f\right) = \mathcal{E}_t P_t^*\left(f\right)$ and $P_t\left(h\right) = \mathcal{E}_t P_t^*\left(h\right)$. Substituting these conditions into the price indices for the two countries yields $P_{F,t} = \mathcal{E}_t P_{F,t}^*$ and $P_{H,t} = \mathcal{E}_t P_{H,t}^*$. However, PPP only holds for intermediate degrees of trade openness; that is, $P_t \neq \mathcal{E}_t P_t^*$. Log-linearizing the real exchange rate yields:

$$q_t = \left(1 - 2\alpha\right) s_t, \tag{15}$$

with $q_t \equiv \ln \mathcal{Q}_t$ and $s_t \equiv \ln \mathcal{S}_t$. Eq. (15) shows that $q_t = 0$ applies only if $\alpha = \frac{1}{2}$; that is, PPP applies only if half of the goods on which the CPI is based are domestic goods. Generally, PPP applies in a simple symmetric two-country setting. Although our model, like other two-country models, is symmetric, the marginal utility of consumption is the same in both countries only if half of the goods consumed are produced domestically.

Substituting Eq. (15) into the log-linearized Eq. (14) yields:

$$c_t = c_t^* + \frac{1 - 2\alpha}{\sigma} s_t, \tag{16}$$

where $c_t = c_t^*$ only if the marginal utility of consumption is the same in both countries; that is, $\alpha = \frac{1}{2}$. However, if $\alpha \neq \frac{1}{2}$, the marginal utility of consumption differs between countries and PPP does not apply. The incorporation of deviations from PPP constitutes a major difference between our model and other symmetric two-country models.

2.2 Firms

A typical firm in each country produces a differentiated good according to the following production function incorporating linear technology:

$$Y_t(h) = A_t N_t(h) \quad ; \quad Y_t^*(f) = A_t^* N_t^*(f),$$

where $Y_t(h)$ and $Y_t^*(f)$ denote the output of a generic good in countries H and F, respectively. A_t and A_t^* represent the productivity levels in countries H and F, respectively; N_t and N_t^* denote the levels of labor input used to produce output for both countries.

Analogous to the consumption indices, the constant elasticity of substitution production functions for both economies are defined as $Y_t \equiv \left[\int_0^1 Y_t(h)^{\frac{\varepsilon-1}{\varepsilon}} dh\right]^{\frac{\varepsilon}{\varepsilon-1}}$ and $Y_t^* \equiv \left[\int_1^2 Y_t^*(f)^{\frac{\varepsilon-1}{\varepsilon}} df\right]^{\frac{\varepsilon}{\varepsilon-1}}$. Combining these expressions with the PPIs yields:

$$Y_t(h) = \left(\frac{P_t(h)}{P_{H,t}}\right)^{-\varepsilon} Y_t \; ; \; Y_t^*(f) = \left(\frac{P_t^*(f)}{P_{F,t}^*}\right)^{-\varepsilon} Y_t^*. \tag{17}$$

Eq. (17) shows that firms are subject to the market demand for good h. Given that, in a currency union, the demand function is based on the production technology, we can relate aggregate employment to the production function as follows:

$$N_t = \frac{Y_t D_t}{A_t} \; ; \; N_t^* = \frac{Y_t D_t^*}{A_t^*}, \tag{18}$$

where $D_t \equiv \int_0^1 \left(\frac{P_t(h)}{P_{H,t}}\right)^{-\varepsilon} dh$ and $D_t^* \equiv \int_1^2 \left(\frac{P_t^*(f)}{P_{F,t}^*}\right)^{-\varepsilon} df$ denote the levels of price dispersion in countries H and F, respectively. As shown by Gali and Monacelli (2005), there are second-order equilibrium variations in d_t and d_t^* around the deterministic steady state. Hence, the log-linearized version of Eq. (18) is given by:

$$y_t = a_t + n_t \; ; \; y_t^* = a_t^* + n_t^*. \tag{19}$$

When a monopolistic firm produces differentiated goods, markets can be considered imperfectly competitive. In this case, each firm sets the prices $P_t(h)$, $P_t(f)$ and $P_t^*(f)$ taking as given P_t, $P_{H,t}$, $P_{F,t}$ and C_t. In addition, following Calvo–Yun, we assume that firms set prices in a staggered fashion, so that each seller can change its price with a given probability of $1-\theta$. Thus, an individual firm's probability of re-optimizing in any given period is independent of the time elapsed since it last reset its price. When a new price is set in period t, the firm seeks to maximize the expected discounted value of its net profits. The first-order necessary conditions (FONCs) for firms are given by:

$$\begin{aligned}\tilde{P}_{H,t} &= \mathrm{E}_t\left(\frac{\sum_{k=0}^{\infty}\theta^k Q_{t,t+k}Y_{t+k}\frac{\varepsilon}{\varepsilon-1}P_{H,t+k}MC_{H,t+k}}{\sum_{k=0}^{\infty}\theta^k Q_{t,t+k}Y_{t+k}}\right);\\ \tilde{P}^*_{F,t} &= \mathrm{E}_t\left(\frac{\sum_{k=0}^{\infty}\theta^k Q^*_{t,t+k}Y_{F,t+k}\frac{\varepsilon}{\varepsilon-1}P^*_{F,t+k}MC^*_{F,t+k}}{\sum_{k=0}^{\infty}\theta^k Q^*_{t,t+k}Y_{F,t+k}}\right),\end{aligned}$$

where $MC_{H,t} \equiv \frac{(1-\tau)W_{H,t}R_t}{P_{H,t}A_{H,t}}$ and $MC^*_{F,t} \equiv \frac{(1-\tau)W^*_{F,t}R^*_t}{P^*_{F,t}A_{F,t}}$ denote the real marginal costs in countries H and F, respectively. $\tilde{P}_{H,t}$ and $\tilde{P}^*_{F,t}$ are the updated prices set in countries H and F, respectively. The employment subsidiary given to firms is τ. In an efficient state ($\tau = \frac{1}{\varepsilon}$), no distortion arises from monopolistic competition.

Note that firms borrow W_tN_t from households via the financial markets at the gross nominal interest rate R_t and W_tN_t; this creates a channel between the real and financial sectors. Thus, the nominal wage corresponds to the discounted value of the nominal payoff in period $t+1$ generated by the portfolio held by households; see Ravenna and Walsh (2006).

Although the FONCs are functions of many parameters, in the steady state, they simplify to:

$$\begin{aligned}\tilde{P}_{H,t} &= \frac{\varepsilon}{\varepsilon-1}P_{H,t}MC_{H,t};\\ \tilde{P}^*_{F,t} &= \frac{\varepsilon}{\varepsilon-1}P^*_{F,t}MC^*_{F,t}.\end{aligned}$$

If there is no price stickiness in the model ($\theta = 0$), prices reach their flexible limit. That is, firms set prices to be a constant markup, $\frac{\varepsilon}{\varepsilon-1}$, of their nominal marginal costs. Log-linearizing firms' FONCs gives:

$$\begin{aligned}\tilde{p}_{H,t} &= (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k\,\mathrm{E}_t\left(p_{H,t+k}+mc_{t+k}\right);\\ \tilde{p}^*_{F,t} &= (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k\,\mathrm{E}_t\left(p^*_{F,t+k}+mc^*_{t+k}\right).\end{aligned} \quad (20)$$

According to Eq. (20), given price stickiness, the model includes firms' FONCs. This implies that firms set their prices to be the sum of the discounted value of nominal marginal costs.

2.3 Equilibrium

2.3.1 Aggregate demand

The market-clearing conditions are given by:

$$Y_t(h) = C_t(h) + C_t^*(h);$$
$$Y_t^*(f) = C_t(f) + C_t^*(f).$$

Substituting Eqs. (4), (5), (14) and (17) into these equalities yields:

$$\begin{aligned} Y_t &= \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t \left[(1-\alpha) + \alpha \mathcal{Q}_t^{\eta - \frac{1}{\sigma}}\right]; \\ Y_t^* &= \left(\frac{P_{F,t}^*}{P_t^*}\right)^{-\eta} C_t^* \left[(1-\alpha) + \alpha \mathcal{Q}_t^{-\left(\eta - \frac{1}{\sigma}\right)}\right], \end{aligned}$$

which can be log-linearized as follows:

$$\begin{aligned} y_t = &\ c_t + \frac{\alpha\left[2(1-\alpha)(\sigma\eta - 1) + 1\right]}{\sigma} s_t; \\ y_t^* = &\ c_t^* - \frac{\alpha\left[2(1-\alpha)(\sigma\eta - 1) + 1\right]}{\sigma} s_t. \end{aligned} \tag{21}$$

Eq. (21) shows that the terms of trade positively affect output in country H but negatively affect output in country F. If each country behaves like a closed economy (that is, $\alpha \to 0$), Eq. (21) reduces to $y_t = c_t$ and $y_t^* = c_t^*$. This means that each country's output equals its domestic consumption.

Substituting Eqs. (13) and (21) into (12) yields:

$$\begin{aligned} y_t &= \mathrm{E}_t(y_{t+1}) - \frac{1}{\sigma}\{\hat{r}_t - \mathrm{E}_t(\pi_{H,t+1})\} - \frac{\omega_2}{\sigma}\mathrm{E}_t(s_{t+1} - s_t); \\ y_t^* &= \mathrm{E}_t\left(y_{t+1}^*\right) - \frac{1}{\sigma}\{\hat{r}_t^* - \mathrm{E}_t\left(\pi_{F,t+1}^*\right)\} + \frac{\omega_2}{\sigma}\mathrm{E}_t(s_{t+1} - s_t), \end{aligned} \tag{22}$$

with $\omega_2 \equiv 2\alpha(1-\alpha)(\sigma\eta - 1)$. Because of international trade, Eq. (22) indicates that output depends on the terms of trade. However, when each country is closed, or when both the coefficient of relative risk aversion and the elasticity of substitution between goods produced in countries H and F are unity (that is, $\sigma = \eta = 1$), the terms of trade disappears from Eq. (22). This gives the New Keynesian IS curve in a closed economy.

Hence, by combining Eqs. (16) and (21), we obtain the following expression:

$$s_t = \frac{\sigma}{\omega_4 + 1}(y_t - y_t^*), \tag{23}$$

with $\omega_4 \equiv 4\alpha(1-\alpha)(\sigma\eta-1)$. The terms of trade depend on the difference in the output of countries H and F.

Substituting Eq. (23) into Eq. (22) yields:

$$\begin{aligned} y_t = & \ \mathrm{E}_t(y_{t+1}) - \frac{1}{\sigma_\omega}\{\hat{r}_t - \mathrm{E}_t(\pi_{H,t+1})\} + \frac{\omega_2}{\omega_2+1}\mathrm{E}_t\left(y^*_{t+1} - y^*_t\right); \\ y^*_t = & \ \mathrm{E}_t\left(y^*_{t+1}\right) - \frac{1}{\sigma_\omega}\{\hat{r}^*_t - \mathrm{E}_t\left(\pi^*_{F,t+1}\right)\} + \frac{\omega_2}{\omega_2+1}\mathrm{E}_t(y_{t+1} - y_t), \end{aligned} \tag{24}$$

with $\sigma_\omega \equiv \frac{(\omega_2+1)\sigma}{\omega_4+1}$.

2.3.2 Aggregate supply

By rearranging Eq. (20) after tedious calculation, we obtain:

$$\begin{aligned} \pi_{H,t} &= \beta \mathrm{E}_t(\pi_{H,t+1}) + \kappa \cdot mc_{H,t}; \\ \pi^*_{F,t} &= \beta \mathrm{E}_t\left(\pi^*_{F,t+1}\right) + \kappa \cdot mc^*_{F,t}, \end{aligned} \tag{25}$$

with $\kappa \equiv \frac{(1-\theta)(1-\theta\beta)}{\theta}$. Eq. (25) shows that current inflation is increasing in both current marginal cost and, because expected inflation appears on the right-hand side of Eq. (21), future marginal cost. This result is consistent with what is implied by Eq. (20).

Substituting Eq. (11) into the expression for real marginal cost yields:

$$MC_{H,t} = \frac{P_t}{P_{H,t}} \frac{(1-\tau) C^\sigma_t N^\varphi_t R_t}{A_t} \ ; \ MC^*_{F,t} = \frac{P^*_t}{P^*_{F,t}} \frac{(1-\tau)(C^*_t)^\sigma (N^*_t)^\varphi R^*_t}{A^*_t}.$$

As mentioned above, $\tau = \frac{1}{\varepsilon}$ characterizes the efficient steady state. The FONCs of firms imply that real marginal cost is the inverse of the constant markup; that is, $MC_H = MC^*_F = \frac{\varepsilon-1}{\varepsilon} < 1$. Hence, in the steady state, the marginal utility of consumption equals the marginal product of labor; that is, $C^{-\sigma} = N^\varphi$, which implies that the steady state is efficient and not distorted. Log-linearizing the above equalities yields:

$$\begin{aligned} mc_{H,t} = & \ \frac{\varsigma}{\omega_4+1} y_t + \frac{\omega_2\sigma}{\omega_4+1} y^*_t + r_t - (1+\varphi) a_t; \\ mc_{F,t} = & \ \frac{\varsigma}{\omega_4+1} y^*_t + \frac{\omega_2\sigma}{\omega_4+1} y_t + r^*_t - (1+\varphi) a^*_t, \end{aligned} \tag{26}$$

where $\varsigma \equiv (\omega_2+1)\sigma + (\omega_4+1)\varphi$, $\omega_2 \equiv 2\alpha(1-\alpha)(\sigma\eta-1)$ and $\omega_4 \equiv 4\alpha(1-\alpha)(\sigma\eta-1)$. Note that Eqs. (19), (21) and (23) are used to derive the above expression. Clearly, a_t and a^*_t denote the percentage deviations of productivity from their steady-state values. We assume that productivity shocks follow exogenous AR(1) processes with coefficients of ρ and ρ^*.

2.3.3 Dynamics

Following Gali and Monacelli (2005), we measure the output gap (the difference between output and its natural level) as follows:

$$x_t \equiv y_t - \bar{y}_t \ ; \ x_t^* \equiv y_t^* - \bar{y}_t^*, \tag{27}$$

where $\bar{y}_t$ and $\bar{y}_t^*$ are the natural levels of output for countries H and F, respectively.

Note that the natural level of output is consistent with $mc_{H,t} = mc_{F,t}^* = 0$. Hence, at the natural level of output, real marginal cost is constant and corresponds to the inverse of the constant markup in the flexible-price equilibrium. Thus, the derivation of the output gap in the steady state is based on instantaneous adjustment. Using these conditions to solve Eq. (26) yields:

$$\bar{y}_t = \frac{\varsigma\psi}{\delta} a_t - \frac{\omega_2\sigma\psi}{\delta} a_t^* \ ; \ \bar{y}_t^* = \frac{\varsigma\psi}{\delta} a_t^* - \frac{\omega_2\sigma\psi}{\delta} a_t, \tag{28}$$

with $\psi \equiv (\omega_4 + 1)(1 + \varphi)$ and $\delta \equiv \sigma^2 (2\omega_2 + 1) + 2\sigma\varphi (\omega_2 + 1)(\omega_4 + 1) + \varphi^2 (\omega_4 + 1)^2$. When all goods are domestically produced, or when the coefficient of relative risk aversion and the elasticity of substitution between goods produced in countries H and F are both unity, Eq. (28) reduces to $\bar{y}_t = \frac{1+\varphi}{\sigma+\varphi} a_t \ ; \ \bar{y}_t^* = \frac{1+\varphi}{\sigma+\varphi} a_t^*$. This implies that the natural rate of output depends only on domestic productivity. The same result emerges when domestic consumers buy only foreign goods ($\alpha = 1$). In these limiting cases, foreign productivity shocks do not affect the domestic economy.

By substituting Eqs. (27) and (28) into Eq. (24), we obtain the New Keynesian IS curve for the open economy:

$$\begin{aligned} x_t = &\ \mathrm{E}_t (x_{t+1}) - \frac{1}{\sigma_\omega}\{\hat{r}_t - \mathrm{E}_t (\pi_{H,t+1})\} + \frac{\omega_2}{\omega_2 + 1}\{\mathrm{E}_t (x_{t+1}^*) - x_t^*\} + \frac{1}{\sigma_\omega}\bar{r}_t; \\ x_t^* = &\ \mathrm{E}_t (x_{t+1}^*) - \frac{1}{\sigma_\omega}\{\hat{r}_t^* - \mathrm{E}_t (\pi_{F,t+1}^*)\} + \frac{\omega_2}{\omega_2 + 1}\{\mathrm{E}_t (x_{t+1}) - x_t\} + \frac{1}{\sigma_\omega}\bar{r}_t^*, \end{aligned} \tag{29}$$

where $\bar{r}_t \equiv -\Theta a_t - \Omega_1 a_t^*$ and $\bar{r}_t^* \equiv -\Theta a_t^* - \Omega_1 a_t$ denote the real natural interest rates, in which $\Theta \equiv \frac{\sigma(1-\rho)\psi[(\omega_2+1)\varsigma - \omega_2^2\sigma]}{(\omega_4+1)\delta}$ and $\Omega_1 \equiv \frac{\sigma(1-\rho)\omega_2\psi[\varsigma - \sigma(\omega_2+1)]}{(\omega_4+1)\delta}$. The parameters ρ and ρ^* are the AR(1) coefficients for the productivity shocks.

Next, we combine Eq. (27) and the condition on real marginal costs under the flexible-price equilibrium, $mc_{H,t} = mc_{F,t}^* = 0$, under which real marginal cost is constant and corresponds to the inverse of the constant markup. By inserting this relationship into Eq. (26), we obtain the following equations:

$$mc_{H,t} = \frac{\varsigma}{\omega_4 + 1} x_t + \frac{\omega_2 \sigma}{\omega_4 + 1} x_t^* + r_t \ ; \ mc_{F,t}^* = \frac{\varsigma}{\omega_4 + 1} x_t^* + \frac{\omega_2 \sigma}{\omega_4 + 1} x_t + r_t^*. \tag{30}$$

These expressions imply that fluctuations in real marginal costs depend on the output gap and the cost channel. If we assume that each country is closed ($\alpha \to 0$), Eq. (30) reduces to $mc_{H,t} = (\sigma + \varphi) x_t + r_t$ and $mc_{F,t}^* = (\sigma + \varphi) x_t^* + r_t^*$. This shows that fluctuations in the real marginal costs are mainly driven by domestic productivity.

Substituting Eq. (30) into Eq. (25) yields the New Keynesian Philips curve for an open economy:

$$\begin{aligned} \pi_{H,t} = & \quad \beta \mathrm{E}_t (\pi_{H,t+1}) + \kappa_\omega (x_t + x_t^*) + r_t; \\ \pi_{F,t}^* = & \quad \beta \mathrm{E}_t (\pi_{F,t+1}) + \kappa_\omega (x_t^* + x_t) + r_t^*, \end{aligned} \tag{31}$$

with $\kappa_\omega \equiv \frac{\kappa \varsigma}{\omega_4 + 1}$.

To establish a link between CPI inflation and PPI inflation, we substitute Eqs. (23), (27) and (28) into Eq. (13) to obtain:

$$\begin{aligned} \pi_t &= \pi_{H,t} + \frac{\alpha \sigma}{\omega_4 + 1} (x_t - x_{t-1}) - \frac{\alpha \sigma}{\omega_4 + 1} (x_t^* - x_{t-1}^*) + r_t + \Omega_2 (a_t - a_{t-1}) - \Omega_2 (a_t^* - a_{t-1}^*); \\ \pi_t^* &= \pi_{F,t}^* + \frac{\alpha \sigma}{\omega_4 + 1} (x_t^* - x_{t-1}^*) - \frac{\alpha \sigma}{\omega_4 + 1} (x_t - x_{t-1}) + r_t^* + \Omega_2 (a_t^* - a_{t-1}^*) - \Omega_2 (a_t - a_{t-1}), \end{aligned} \tag{32}$$

with $\Omega_2 \equiv \frac{\alpha \sigma (1 + \varphi)(\varsigma + \omega_2 \sigma)}{\delta}$.

2.4 Monetary policy

To complete our dynamic open-economy model, we assume that the central bank in each country adopts an *ad hoc* Taylor rule as follows:

$$\begin{aligned} \hat{r}_t = & \quad \varrho \hat{r}_{t-1} + (1 - \varrho) (\phi_\pi \pi_t + \phi_x x_t) + m_t; \\ \hat{r}_t^* = & \quad \varrho^* \hat{r}_{t-1}^* + (1 - \varrho^*) (\phi_\pi^* \pi_t^* + \phi_x^* x_t^*) + m_t^*, \end{aligned} \tag{33}$$

where ϕ_x and ϕ_π are the central bank's reaction coefficients to the output gap and CPI inflation in country H, respectively. Similarly, ϕ_x^* and ϕ_π^* are the foreign central bank's reaction coefficients to the output gap and CPI inflation, respectively. Monetary policy shocks are represented by m_t and m_t^*, which are independent and identically distributed in both economies.

3 Calibration: impulse responses

In this section, we examine the effect of a positive foreign productivity shock on the domestic economy. Before examining the transmission of a shock between the two economies, we simulate the case in which there is no trade. In this case, changes in the exchange rate and the terms of trade are analyzed for symmetric economies. Second, we evaluate the model's dynamics when the domestic economy engages in international trade. The trajectory of macroeconomic variables is simulated based on different degrees of international trade (none, low, high and intermediate). This enables us to examine the effects of trade openness on the domestic economy. Dynare version 4 is used to conduct all simulations; see Adjemian et al. (2011).

3.1 Two economies that do not trade ($\alpha = 0.0$)

In this analysis, nominal rigidity is such that domestic prices are more sticky than foreign prices ($\theta_H > \theta_F$). The discount factor β is set to 0.99. The parameter governing the persistence of the foreign productivity shock (ρ^*) is set to 0.55.

Table 1: Calibrated values for a two-country model

Label	Value	Label	Value
σ	4.5	η	2.5
θ_H	0.9	θ_F	0.75
φ	3	ρ_*	0.55
ϕ_π	1.5	ϕ_π^*	1.5
ϕ_x	0.5	ϕ_x^*	0.5
ϱ	0.4	ϱ^*	0.4

Note: The discount factor β is set to 0.99. The simulations are based on different values for trade openness ($\alpha = 0.0, 0.1, 0.6, 0.9$).

In response to a positive productivity shock, both foreign output and its natural level increase; see Appendix B.1. Because the domestic economy behaves like a closed economy, the productivity shock has no direct influence on the domestic economy. In other words, good news in one country is not transmitted directly to the other country. Note that because actual output increases by less than the natural level of output increases, the output gap widens. Thus, following the Taylor rule, the monetary authority lowers the nominal interest rate to close the output gap. Through the cost channel, the fall in the nominal interest lowers PPI inflation; that is, firms can borrow money from households at a lower interest rate. In this period, the New Keynesian Philips curve is relatively flat because of sticky prices; the short-run reaction of supply is small relative to the fall in the output gap. In particular, the fall in interest rates boosts current consumption. This is because the (gross) nominal interest rate is less than the inverse of the subjective discount factor from the Euler equation for consumption. This implies that households prefer current consumption to future consumption. Thus, in this model, the productivity shock acts much like a demand shock.

Although international trade is not incorporated into this simulation, the productivity shock is expected to affect countries' relative competitiveness. This effect is reflected in the terms of trade and the exchange rate. For example, a foreign positive productivity shock may increase the relative price of domestic goods and thus cause a real appreciation. However, because, over time, the price level increases by more than the terms of trade decline, the nominal exchange rate begins to depreciate after two years. The effects of the productivity shock disappear after 10 years, and the natural level of output returns to its steady-state level. In the steady state, the real exchange rate is constant ($\hat{s}_t = 0$) because the law of one price holds in the steady state; see Appendix A for details.

3.2 Two economies that trade

To examine the transmission mechanism between the two economies, we incorporate international trade into the simulations. Initially, we assume (realistically) that the domestic economy engages in an intermediate level of trade; that is, $\alpha = 0.6$. Subsequently, we examine the impulse response functions for the shock based on low and high degrees of trade openness ($\alpha = 0.1$ and $\alpha = 0.9$, respectively).

Case I: intermediate level of trade ($\alpha = 0.6$)

In this simulation, we investigate the response of the macroeconomic variables to a foreign productivity shock predicted by the model when the degree of trade openness is intermediate; see Table 1. From the impulse response function, we can determine how the shock is transmitted between the two economies; see Appendix B.2. First, the foreign positive productivity shock increases the natural level of foreign output. Because output increases by less than natural output, the foreign economy's output gap widens. The Taylor principle obliges the monetary authority to lower the nominal interest rate to relieve deflationary pressure in the economy. Subsequently, the natural level of domestic output falls, which causes a deterioration in the terms of trade. This affects the domestic economy's international competitiveness. Because of the deflationary decline in output, the foreign PPI changes to cause a gradual depreciation of the real exchange rate. This leads the monetary authority to reduce the key interest rate so that the negative effect on the domestic economy of the foreign shock is partially offset. (The international risk-sharing condition is activated.)

According to our simulations, given an intermediate degree of trade openness, the macroeconomic variables reach their steady-state levels in three to four years. This suggests that the time taken to reach the steady state depends on the degree of openness to trade.

Case II: low level of trade ($\alpha = 0.1$)

We investigate the adjustment process based on a low degree of openness to trade. Given a small value of α, the responses of domestic output and inflation to the shock persist for less than 10 years. There are a number of interim effects. A positive foreign productivity shock affects the domestic natural rate of output through international risk sharing. Because the international risk-sharing conditions are gradually relaxed by a deterioration in the terms of trade, the shock has a relatively moderate effect on the output gap. Because of the domestic economy's low exposure to trade, the domestic price level is hardly affected by the change in its relative international competitiveness. This leads to an appreciation of the real exchange rate. Nevertheless, by exporting less and importing more, domestic output contracts.

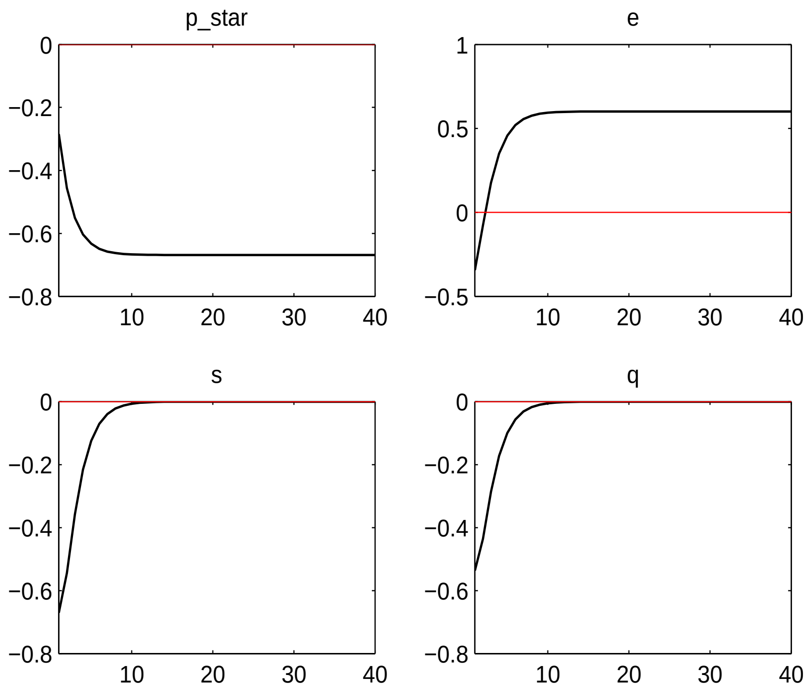


Figure 2: Case II (low trade openness): responses of foreign price level (upper-left panel), nominal exchange rate (upper-right panel), terms of trade (lower-left panel) and real exchange rate (lower-right panel)

Over time, as the effect of the shock dissipates, output returns to its original level. The impulse response for domestic inflation has a humped shape, and implies positive inflation after three or four years; the response is delayed because of the high degree of price stickiness assumed in the open economy. This coincides with a narrowing of the foreign output gap.

Case III: high level of trade ($\alpha = 0.9$)

We investigate adjustment to a productivity shock when open economies are closely integrated through international trade. Given a high value for the trade openness parameter, the responses of output and inflation to a foreign productivity shock persist for a long time. Compared with the case of little trade, the output fall due to international risk sharing greatly exceeds the fall in current output. Thus, similarly to the case of intermediate trade openness, the domestic output gap narrows immediately after the shock. Subsequently, the domestic economy's imports increase, which generates deflationary pressure. This causes current output to fall. The resulting output gap must be closed by monetary policy.

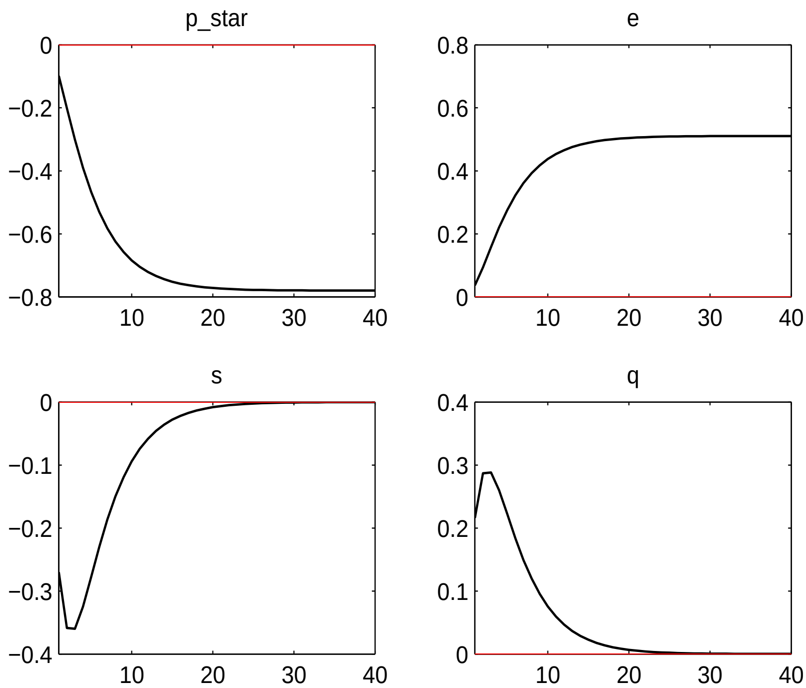


Figure 3: Case III (high trade openness): responses of foreign price level (upper-left panel), nominal exchange rate (upper-right panel), terms of trade (lower-left panel) and real exchange rate (lower-right panel)

Another feature of the model in which two countries are highly integrated through international trade is that the real exchange rate depreciates following a productivity shock. The effect on the exchange rate is greater than when there is an intermediate level of trade. This implies that the domestic economy's increased expenditure on cheap imports generates deflationary pressure. However, as mentioned above, the behavior of the exchange rate is the opposite, particularly when the economy is relatively closed to trade. That is, the domestic economy experiences currency appreciation following a positive foreign productivity shock; see Figs. 2 and 3. Eq. (15) indicates that an improvement in the terms of trade corresponds to a real appreciation of the currency when α is below 0.5.

4 Conclusion

In this paper, we studied the effects of an asymmetric shock in an open economy; our two-country model is based on a Keynesian framework of monopolistic competition and price stickiness that incorporates a cost channel. In addition, the shock could influence the degree to which the real and financial sectors are connected. Simulations were used to investigate the effects of trade openness on the domestic economy.

Our main findings can be summarized as follows. The duration of output and inflation responses to changes in the foreign productivity level is strongly affected by trade openness: that is, the higher the degree of openness, the longer the effects of the shock last. When there is minimal trade, a positive foreign productivity shock causes a real appreciation of the exchange rate. However, our open-economy model predicts that consumers will spend more on cheap imports. This means that the domestic currency is likely to depreciate in the immediate aftermath of the shock before returning to its original level as the effect of the shock dissipates over time. This implies that the behavior of the exchange rate is mainly driven by consumer reactions to the shock.

We conclude from these findings that a monetary authority should respond to changes in foreign productivity levels by lowering its key interest rate, particularly if the economy is highly exposed to international trade. Moreover, open economies should coordinate their policy responses to asymmetric shocks. Policy responses should also consider financial frictions in real economies.

Our analysis would have been more convincing had we used real data to estimate the effects of shocks and trade openness on the cost channel. However, more research is needed to estimate the deep parameters of open-economy macroeconomic models. This is because the complexity of the microfoundations of such models would create identification problems, such as the emergence of multiple local minima in the parameter space during optimization.[3] Future research should be conducted to examine the empirical importance of the cost channel in open economies.

[3]For example, although the moment-matching approaches of Franke *et al.* (2011) and Jang (2012) have been used to estimate the structural parameters of an elementary New Keynesian model, it would not be easy to use classical estimation methodology to alleviate the problem of overparametrization in open-economy macroeconomic models.

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Appendices

A The deterministic steady state

In this section, we report the trajectory of the state variables in the deterministic stationary equilibrium. In the steady state, PPI inflation is zero; that is, $\Pi_{H,t} = \Pi_{F,t} = 1$. Note that $\Pi_{H,t} \equiv \frac{P_{H,t}}{P_{H,t-1}}$ and $\Pi_{F,t} \equiv \frac{P_{F,t}}{P_{F,t-1}}$; a variable without a time subscript takes its nonstochastic steady-state value. [4] Because the steady state is not stochastic, we consider constant productivity growth over time; that is, $A_H = A_N = A_F = A_N^* = 1$.

In the steady state, the gross nominal interest rate is equal to the inverse of the subjective discount factor:

$$R = R^* = \beta^{-1}. \tag{A.1}$$

Further, the nominal exchange rate is constant in the steady state; $\mathcal{E}_t = \mathcal{E}$. The FONCs for firms can be rewritten as:

$$MC_H = MC_F = \frac{\epsilon - 1}{\epsilon}. \tag{A.2}$$

Because the marginal utility of consumption between two countries is identical, the total amount of consumption in the two countries is the same:

$$C = C^*. \tag{A.3}$$

Thus, the international risk-sharing condition reduces to:

$$Q = 1, \tag{A.4}$$

where we have assumed $\vartheta = 1$ for simplicity. This implies that PPP holds in the steady state. In the steady state, the price level is the same in the two economies ($P_H = P_F$). Hence, the terms of trade are constant; that is, $S = 1$.

The market-clearing conditions imply the following:

$$Y = C = Y^* = C^*. \tag{A.5}$$

[4] For example, $\hat{X}_H = X_H = \hat{X}_F = 1$ in the steady state, when $\hat{X}_{H,t} \equiv \frac{\hat{P}_{H,t}}{P_{H,t}}$ and $\hat{X}_{F,t} \equiv \frac{\hat{P}_{F,t}}{P_{F,t}}$.

B Impulse response: transmission of a foreign productivity shock

B.1 The case of no trade

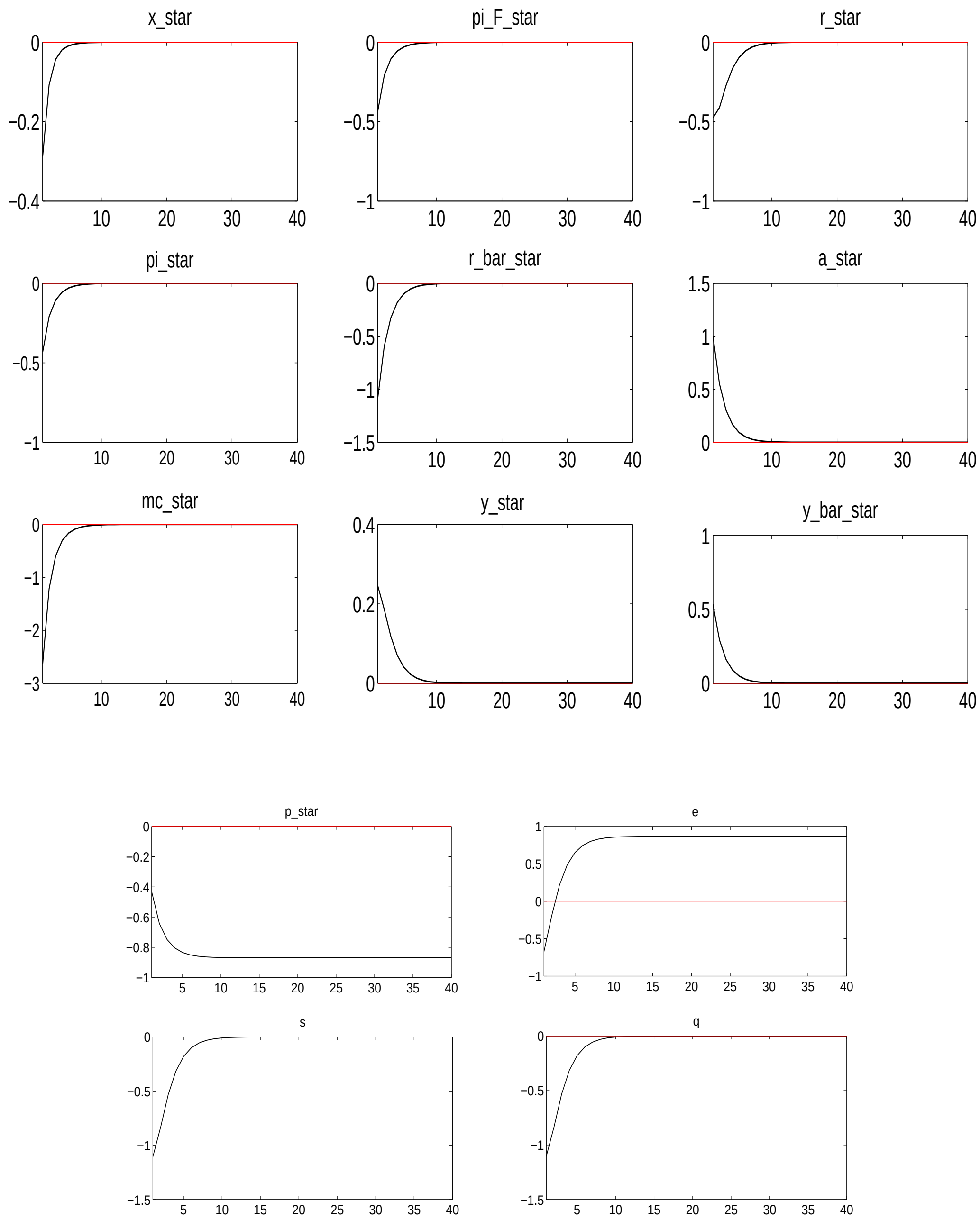
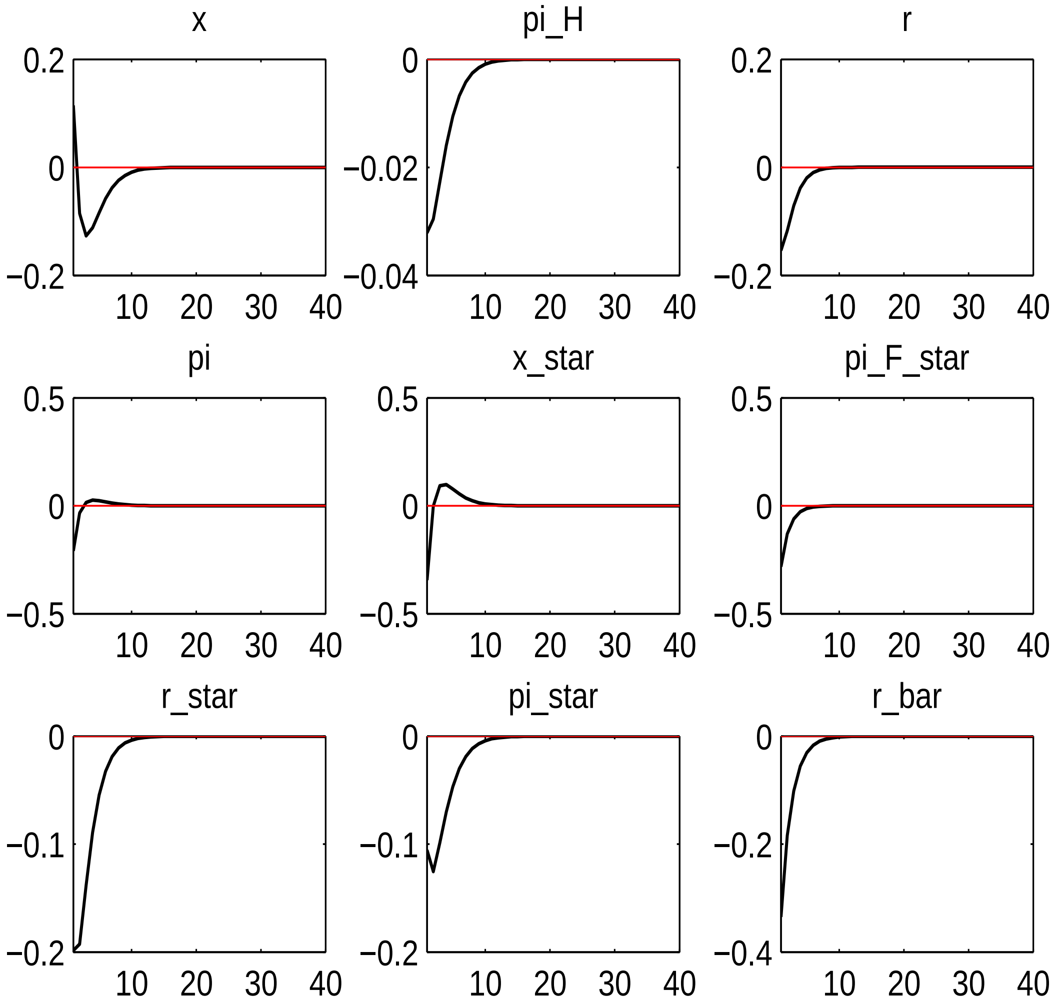
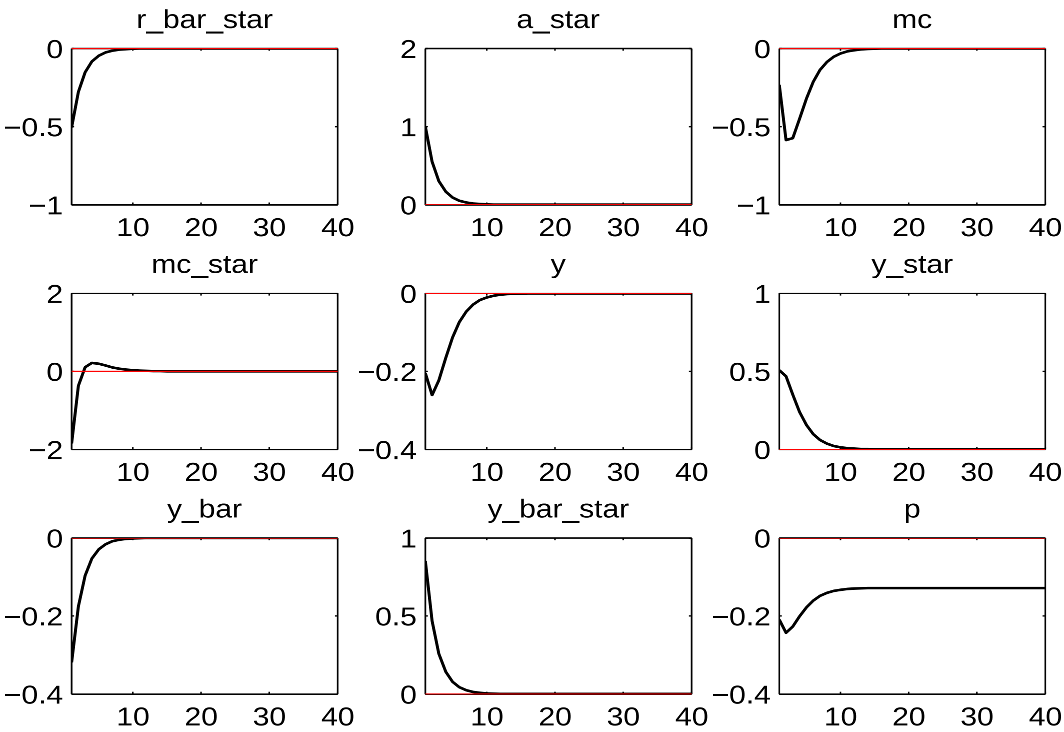


Figure 4: Two economies that do not trade

B.2 The case of the open economy

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a_star
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y
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−0.2
−0.4
y_star
1
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p
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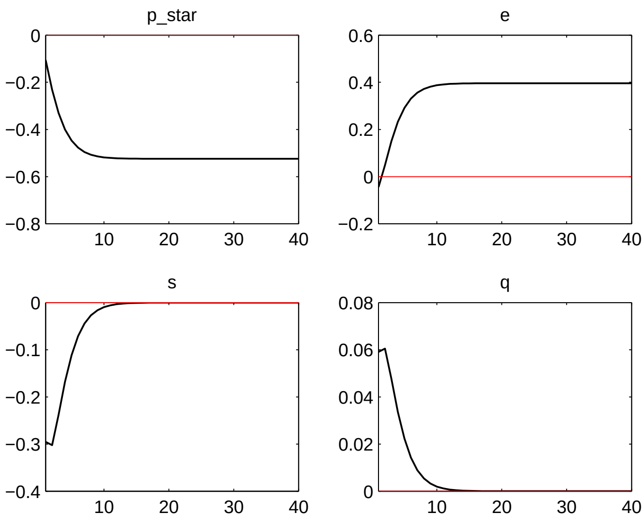
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−0.4
−0.6
−0.8
e
0.6
0.4
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0
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s
0
−0.1
−0.2
−0.3
−0.4
q
0.08
0.06
0.04
0.02
0
10 20 30 40

C The six equations for a two-country model in matrix form

The model developed in this paper is based on the following six equations:

$$
\begin{aligned}
x_t &= E_t(x_{t+1}) - \frac{1}{(\omega_4+1-\alpha\omega_0)\sigma}\{r_t - E_t(\pi_{t+1})\} + \frac{\alpha\omega_0}{\omega_4+1-\alpha\omega_0}\{E_t(x^*_{t+1}) - x^*_t\};\\
\pi_t &= \beta E_t(\pi_{t+1}) - \frac{\beta\alpha\sigma}{\omega_4+1}\{E_t(x_{t+1}) - E_t(x^*_{t+1})\} + \frac{\alpha\sigma(1+\beta)+\kappa_H\varsigma}{\omega_4+1}x_t - \frac{\sigma[\alpha(1+\beta)-\kappa_H\omega_2]}{\omega_4+1}x^*_t\\
&\quad - \frac{\alpha\sigma}{\omega_4+1}\{x_{t-1} - x^*_{t-1}\} + \kappa_H r_t;\\
r_t &= \phi_r r_{t-1} + (1-\phi_r)(\phi_\pi \pi_t + \phi_x x_t);\\
x^*_t &= E_t(x^*_{t+1}) - \frac{1}{(\omega_4+1-\alpha\omega_0)\sigma}\{r^*_t - E_t(\pi^*_{t+1})\} + \frac{\alpha\omega_0}{\omega_4+1-\alpha\omega_0}\{E_t(x_{t+1}) - x_t\};\\
\pi^*_t &= \beta E_t(\pi^*_{t+1}) - \frac{\beta\alpha\sigma}{\omega_4+1}\{E_t(x^*_{t+1}) - E_t(x_{t+1})\} + \frac{\alpha\sigma(1+\beta)+\kappa_F\varsigma}{\omega_4+1}x^*_t - \frac{\sigma[\alpha(1+\beta)-\kappa_F\omega_2]}{\omega_4+1}x_t\\
&\quad - \frac{\alpha\sigma}{\omega_4+1}\{x^*_{t-1} - x_{t-1}\} + \kappa_F r^*_t;\\
r^*_t &= \phi^*_r r^*_{t-1} + (1-\phi^*_r)(\phi^*_\pi \pi^*_t + \phi^*_x x^*_t),
\end{aligned}
$$

where $\omega_0 = 2(1-\alpha)(\sigma\eta-1)$, $\omega_2 = 2\alpha(1-\alpha)(\sigma\eta-1)$ and $\omega_4 = 4\alpha(1-\alpha)(\sigma\eta-1)$. ς, κ_H, and κ_F are defined as $(\omega_2+1)\sigma + (\omega_4+1)\varphi$, $\frac{(1-\theta_H)(1-\theta_H\beta)}{\theta_H}$, and $\frac{(1-\theta_F)(1-\theta_F\beta)}{\theta_F}$, respectively.

We denote by y_t the state vector of $[\ x_t\ \pi_t\ r_t\ x^*_t\ \pi^*_t\ r^*_t\ ]'$. Then, the structural model of a symmetric open economy can be rewritten in canonical form:

$$A\mathrm{E}_t y_{t+1} + By_t + Cy_{t-1} = 0, \tag{C.6}$$

where:

$$
A = \begin{bmatrix}
1 & \frac{1}{(\omega_4+1-\alpha\omega_0)\sigma} & 0 & \frac{\alpha\omega_0}{(\omega_4+1-\alpha\omega_0)} & 0 & 0\\
-\frac{\beta\alpha\sigma}{\omega_4+1} & \beta & 0 & \frac{\beta\alpha\sigma}{\omega_4+1} & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0\\
\frac{\alpha\omega_0}{(\omega_4+1-\alpha\omega_0)} & 0 & 0 & 1 & \frac{1}{(\omega_4+1-\alpha\omega_0)\sigma} & 0\\
\frac{\beta\alpha\sigma}{\omega_4+1} & 0 & 0 & -\frac{\beta\alpha\sigma}{\omega_4+1} & \beta & 0\\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix},
$$

$$B = \begin{bmatrix} -1 & 0 & -\frac{1}{(\omega_4+1-\alpha\omega_0)\sigma} & -\frac{\alpha\omega_0}{(\omega_4+1-\alpha\omega_0)} & 0 & 0 \\ \frac{\beta\alpha\sigma}{\omega_4+1} & -1 & \kappa_H & \frac{\sigma[\alpha(1+\beta)-\kappa_H\omega_2]}{\omega_4+1} & 0 & 0 \\ (1-\phi_r)\phi_x & (1-\phi_r)\phi_\pi & -1 & 0 & 0 & 0 \\ -\frac{\alpha\omega_0}{(\omega_4+1-\alpha\omega_0)} & 0 & 0 & -1 & 0 & -\frac{1}{(\omega_4+1-\alpha\omega_0)\sigma} \\ -\frac{\sigma[\alpha(1+\beta)-\kappa_F\omega_2]}{\omega_4+1} & 0 & 0 & \frac{\alpha\sigma(1+\beta)+\kappa_F\varsigma}{\omega_4+1} & -1 & \kappa_F \\ 0 & 0 & 0 & (1-\phi_r^*)\phi_x^* & (1-\phi_r^*)\phi_\pi^* & -1 \end{bmatrix},$$

$$C = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{\alpha\sigma}{\omega_4+1} & 0 & 0 & \frac{\alpha\sigma}{\omega_4+1} & 0 & 0 \\ 0 & 0 & \phi_r & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{\alpha\sigma}{\omega_4+1} & 0 & 0 & -\frac{\alpha\sigma}{\omega_4+1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \phi_r^* \end{bmatrix}.$$

The method of undetermined coefficients and iterative methods can be used to solve the system of equations. This solution indicates the equilibrium values of the observable variables in the system.

D Dynare code

The following Dynare mod file was used to generate the dynamics of the macroeconomic variables in our two-country New Keynesian model; see Adjemian et al. (2011).

```
var x pi_H r pi x_star pi_F_star r_star pi_star r_bar r_bar_star a a_star mc mc_star y y_star y_bar y_bar_star
p p_star e s q;

varexo m m_star xi xi_star;

parameters sigma eta beta theta_H theta_F alpha varphi kappa_H kappa_F omega_2 omega_4 psi varsigma d
elta sigma_omega oomega_2 rho rho_star phi_pi phi_pi_star phi_x phi_x_star varrho varrho_star;

sigma = 4.5;
eta = 2.5;
beta = 0.99;
theta_H = 0.9;
theta_F = 0.75;
alpha = 0.6;
kappa_H = (1-theta_H)*(1-theta_H*beta)/theta_H;
kappa_F = (1-theta_F)*(1-theta_F*beta)/theta_F;
varphi = 3;
omega_2 = alpha*2*(1-alpha)*(sigma*eta-1);
omega_4 = alpha*4*(1-alpha)*(sigma*eta-1);
psi = (omega_4+1)*(1+varphi);
varsigma = (omega_2+1)*sigma+(omega_4+1)*varphi;
delta = sigma^2*(2*omega_2+1)+2*sigma*varphi*(omega_2+1)*(omega_4+1)+(omega_4+1)^2*varphi^2;
sigma_omega = (omega_2+1)*sigma/(omega_4+1);
oomega_2 = alpha*sigma*(1+varphi)*(varsigma+omega_2*sigma)/delta;
rho_star = 0.55;
phi_pi = 1.5;
phi_pi_star = 1.5;
phi_x = 0.5;
phi_x_star = 0.5;
varrho = 0.4;
varrho_star = 0.4;

model (linear);

x = x(+1)-(omega_4+1)/((omega_2+1)*sigma)*r+(omega_4+1)/((omega_2+1)*sigma)*pi_H(+1)+omega_2/
(omega_2+1)*x_star(+1)-omega_2/(omega_2+1)*x_star+(omega_4+1)/((omega_2+1)*sigma)*r_bar;
```

```
pi_H = beta*pi_H(+1)+kappa_H*varsigma/(omega_4+1)*x+kappa_H*omega_2*sigma/(omega_4+1)*x_star+kappa_H*r;

x_star = x_star(+1)-(omega_4+1)/((omega_2+1)*sigma)*r_star+(omega_4+1)/((omega_2+1)*sigma)*pi_F_star(+1)+omega_2/(omega_2+1)*x(+1)-omega_2/(omega_2+1)*x+(omega_4+1)/((omega_2+1)*sigma)*r_bar_star;

pi_F_star = beta*pi_star(+1)+kappa_F*varsigma/(omega_4+1)*x_star+kappa_F*omega_2*sigma/(omega_4+1)*x+kappa_F*r_star;

r = varrho*r(-1) + (1-varrho)*phi_pi*pi + (1-varrho)*phi_x*x + m;

r_star = varrho_star*r_star(-1)+(1-varrho_star)*phi_pi_star*pi_star+(1-varrho_star)*phi_x_star*x_star+m_star;

r_bar = -sigma*(1-rho)*psi*((omega_2+1)*varsigma-omega_2^2*sigma)/((omega_4+1)*delta)*a-sigma*(1-rho)*omega_2*psi*(varsigma-sigma*(omega_2+1))/((omega_4+1)*delta)*a_star;

r_bar_star = -sigma*(1-rho)*psi*((omega_2+1)*varsigma-omega_2^2*sigma)/((omega_4+1)*delta)*a_star-sigma*(1-rho)*omega_2*psi*(varsigma-sigma*(omega_2+1))/((omega_4+1)*delta)*a;

pi = pi_H+alpha*sigma/(omega_4+1)*x-alpha*sigma/(omega_4+1)*x(-1)-alpha*sigma/(omega_4+1)*x_star+alpha*sigma/(omega_4+1)*x_star(-1)+oomega_2*a-oomega_2*a(-1)-oomega_2*a_star+oomega_2*a_star(-1);

pi_star = pi_F_star+alpha*sigma/(omega_4+1)*x_star-alpha*sigma/(omega_4+1)*x_star(-1)-alpha*sigma/(omega_4+1)*x+alpha*sigma/(omega_4+1)*x(-1)+oomega_2*a_star-oomega_2*a_star(-1)-oomega_2*a+oomega_2*a(-1);

mc = varsigma/(omega_4+1)*x + omega_2*sigma/(omega_4+1)*x_star + r;

mc_star = varsigma/(omega_4+1)*x_star + omega_2*sigma/(omega_4+1)*x + r_star;

y_bar = varsigma*psi/delta*a - omega_2*sigma*psi/delta*a_star;

y_bar_star = varsigma*psi/delta*a_star - omega_2*sigma*psi/delta*a;

y = x + y_bar;

y_star = x_star + y_bar_star;

p = pi + p(-1);

p_star = pi_star+ p_star(-1);

s = sigma/(omega_4+1)*y - sigma/(omega_4+1)*y_star;
```

```
q = (1-2*alpha)*s;

e = q - p_star + p;

a = rho*a(-1) + xi;

a_star = rho_star*a_star(-1) + xi_star;

end;


initval;
x = 0;
pi_H = 0;
r = 0;
pi = 0;
x_star = 0;
pi_F_star = 0;
r_star = 0;
pi_star = 0;
xi = 0;
xi_star = 0;
end;


steady;
check;
shocks;
var xi_star;
stderr 1;
end;

stoch_simul(periods=2100);
```