Animal Spirits in Open Economy: An Interaction-Based Approach to Bounded Rationality

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Abstract

In this paper, we explore the behavior of investors under uncertainty in an open-economy New Keynesian model. The model is extended to include the dynamics of inflation and output generated by the heterogeneous bounded rational agents according to De Grauwe (2011). In particular, we incorporate the waves of optimists and pessimists – the so-called ‘animal spirits’ – into a two-country model. As a result, the model is able to describe the herding behavior of investors and its effect on market volatility. The simulation results suggest that the business cycle goes through periods of high volatility when the large number of optimists or pessimists in one country strongly affects a foreign country.

JEL Classification: C63, E31, F41

Keywords: animal spirits; bounded rationality; New Keynesian; two-country model

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1 Introduction

Recent developments in the world economy coincide with massive amounts of international trade and capital flows. Global market integration means that a sudden disturbance of equilibrium in one economy can generate spillover effects across countries. It is well known that globalization has increased the synchronicity of international business cycles over the last two decades. Indeed, there is empirical evidence that demand shocks are closely associated with global spillovers where complex financial markets strengthen the synchronization of business cycles between countries, especially during the financial turmoil; see also Kose et al. (2003), Kollmann (2013), Berge (2013), Grilli et al. (2014), among others.

![Figure 1: Output gap in US and Euro Area from 1970 to 2013](image)

Note: The Hodrick-Prescott filter is used to compute the output gap with the smoothing parameter of 1600 for quarterly data.

Figure 1 shows the historical output gap in US and Euro Area from 1970 to 2013. There has been a high degree of synchronization in business cycles between the economies during the whole period; the output correlation is 0.549. We can split the data into several sub-periods. Some changes in the synchronization of business cycles are caused by the distinction between these sub-periods. First, the business cycle has become more closely synchronized for the period from 2001 to 2013 (i.e., \( \text{corr}(x_{t}^{us}, x_{t}^{euro}) = 0.736 \)) , while the period from 1991 to 2000 shows relatively a low degree of comovements.

However, economic theory is not clear about whether the synchronization of business cycle is determined by trade and financial integration. The correlation of output between countries can be increased by demand shocks and intra-industry trade, but industry-specific shocks may not contribute to economic comovements due to specialization. Furthermore, productivity shocks and portfolio diversification effects have an ambiguous effect on business cycle. See Berge (2013) for details.
(i.e., \( \text{corr}(x_t^{\text{us}}, x_t^{\text{euro}}) = 0.555 \)). The difference reflects a recent change on global market integration through trade and investment. Second, the market turbulence plays an important role in increasing the synchronization of international business cycles. Indeed, the output correlation in the 1970s was as high as the correlation in the 2000s (i.e., \( \text{corr}(x_t^{\text{us}}, x_t^{\text{euro}}) = 0.706 \)). But the US output is moderately correlated with the Euro area during the Great Moderation period of the 1980s (i.e., \( \text{corr}(x_t^{\text{us}}, x_t^{\text{euro}}) = 0.420 \)).

With a focus on the complex coordination of economic activities, researchers in open macroeconomics have incorporated aspects of optimizing behavior into general equilibrium, i.e. the so-called Redux model by Obstfeld and Rogoff (1995). In particular, dynamic stochastic general equilibrium (DSGE) models include the forward-looking behavior of agents which are based on microfoundations under rational expectations. However, the models of optimizing behavior have been criticized on empirical grounds. In the DSGE models, the persistence in the joint behavior of inflation and aggregate activity is induced by serially correlated markup shocks rather than generated by endogenous dynamics between countries. This suggests that theoretical New Keynesian models are agnostic about bounded rationality in consumption and production (or backward-looking behavior for a narrow sense of expectation formation processes).

To bridge the gap between the model and empirical data, researchers have put efforts in developing endogenous persistence in an optimizing framework. Well-known examples are habit formation and price indexation in the behavior of households and firms; see Galí and Gertler (1999), Fuhrer (2000), Amato and Laubach (2003), among others. However, inertia captured by behavioral anomalies cannot be translated in a tractable manner into the structural dynamics in open economy models due to complicated analytical solutions. Furthermore, the complexity of open DSGE models necessitates the need for computationally intensive empirical methods, where, for example, simulation-based methods are often used to estimate the model parameters. These include Markov Chain Monte Carlo (MCMC) method and Simulated Method of Moments (SMM); see Fernández-Villaverde and Rubio-Ramírez (2011), Flury and Shephard (2011), as well as Ruge-Murcia (2012), among others.

In particular, market incompleteness has been taken into account in DSGE models, which has led to an increase in model complexity. For example, the DSGE models can be extended to include aspects of financial market frictions along the lines of transmission channels in the economy. In other words, policy analysis and economic forecasting need to be structured in terms of relevant transmission mechanisms across countries, as well as within the economy; see Tovar (2009), as well as Driscoll and Holden (2014). By the same token, we claim in this paper that behavioral

\footnote{In addition, Adolfson et. al (2005) find that price stickiness plays an important role in an open-economy transmission channel even if markup shocks are allowed to be autocorrelated.}
uncertainty and coordination issues among investors are one of the most important mechanisms for the global economy (Bryant (1987), Middleton (1996), Akerlof (2002), Pech and Milan (2009), De Grauwe (2012), among others). To show this, we develop endogenous persistence arising from social interactions in an open economy. In other words, the model includes the dynamics of inflation and aggregate economic activities generated by the heterogeneous bounded rational agents according to De Grauwe (2011). The complex nature of interactions between heterogeneous agents can serve as a proxy measure for bounded rational behavior in a two-country model. In particular, we incorporate the waves of optimists and pessimists – the so-called ‘animal spirits’ – into a two-country model.

One of main goals in this study is to examine the importance of behavioral biases affecting expectation formation processes in investment. For example, investors can have ease access to recently developed financial markets; the interactions of bounded rational agents in a two-country model amplify swings and persistence in the business cycle where the economy is subject to the problem of group behavior arising from large uncertainties. Hence, the model is used to establish a link between the investors’ risk-averse behavior (or pessimism/optimism) and the real economy with respect to the economic exposure arising from bounded rationality. The simulation results suggest that the business cycle goes through periods of high volatility when the optimists or pessimists are dominant in the economy.

The paper is organized as follows. Section 2 is devoted to explain the basic framework in an open economy and discuss how demand and supply in the New Keynesian model is influenced by collective behavior of investors emerging from social interactions. Section 3 simulates persistent behavior of inflation and output with respect to behavioral uncertainties. Section 4 examines the importance of animal spirits in open economy and discusses its implication for the global economy in comparison with previous studies. Finally, the paper concludes in Section 5. Technical details are relegated to the Appendix.

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9The micro-founded DSGE models with heterogeneous agents may suffer from schizophrenia. On the one hand, the behavioral heterogeneity cannot avoid the complicated parameterization of structural models. On the other hand, bounded rationality is governed by backward-looking expectations in which simple behavioral rules are often criticized for being an ad hoc analysis from a microeconomic perspective. Refer to Branch and McGough (2009) for the details of indeterminacy of a New Keynesian model with heterogeneous expectations.
2 Model

2.1 Symmetric two-country model in canonical form

A world economy in the model comprises two countries. The model incorporates the features of a standard aggregate demand and aggregate supply following Gali and Monacelli (2005). The monetary policy follows an ad hoc Taylor rule. In contrast with a hybrid New Keynesian model, the persistence and inertia of the model dynamics are mainly driven by the backward-looking expectations arising from agents’ group behavior. The baseline New-Keynesian Model (NKM) in open economy reads as follows:

\[
x_t = E_t(x_{t+1}) - a_1\{r_t - E_t(\pi_{t+1})\} + a_2\{E_t(x^*_t) - x^*_t\} + \varepsilon_{x,t} \\
\pi_t = \beta E_t(\pi_{t+1}) - b_1\{E_t(x_{t+1}) - E_t(\pi^*_{t+1})\} + b_2 x_t - b_3 x^*_t - b_4\{x_{t-1} - x^*_t\} + \varepsilon_{\pi,t} \\
r_t = c_1 r_{t-1} + (1 - c_1)(c_2 \pi_t + c_3 x_t) + \varepsilon_{r,t} \\
x^*_t = E_t(x^*_{t+1}) - a_1\{r^*_t - E_t(\pi^*_{t+1})\} + a_2\{E_t(x^*_t) - x^*_t\} + \varepsilon^*_{x,t} \\
\pi^*_t = \beta E_t(\pi^*_t) - b_1\{E_t(x^*_{t+1}) - E_t(\pi^*_{t+1})\} + b^*_2 x^*_t - b^*_3 x_t - b^*_4\{x^*_t - x^*_t\} + \varepsilon^*_{\pi,t} \\
r^*_t = c^*_1 r^*_{t-1} + (1 - c^*_1)(c^*_2 \pi^*_t + c^*_3 x^*_t) + \varepsilon^*_{r,t}
\]

where \(\beta\) is discount factor. The parameters \(a, b,\) and \(c\) are coefficients of the IS curve, Phillips curve, and Taylor rule, respectively. The asterisk is used to distinguish foreign economy from domestic economy. Note that two economies are symmetric except for some parameters in the Phillips curve.

In other words, we assume that there is no asymmetry in deep parameters between the two countries except for the behavior of central banks and price stickiness. For example, the difference in \(b_2\) (or \(b_3\)) and \(b^*_2\) (or \(b^*_3\)) stems from differences in nominal rigidity between the two countries. In particular, the magnitude of transmission channel between countries is controlled by \(a_2, b_1, b_3, b_4\) and \(b^*_3\). These parameters become zero for the model which does not include spillover effects (i.e., no international trade).

If interest rate rises more than expected in inflation, the economy experiences a decrease in aggregate demand (‘Fisher equation’). The IS relation includes the effects of foreign economy on the domestic country via trade linkage. For example, an increase in the current output gap in the foreign country has a negative impact on the economy due to the deterioration of the terms of trade.

\(^4\)The complete derivation of deep parameters can be found in the literature on standard two-country New Keynesian models. The details of coefficients are given in the Appendix. See also Gali and Monacelli (2005), da Silva (2006), Jang and Okano (2015), among others.
(ToT). However, an expected increase of the foreign output gap in the next period has a positive impact on the domestic economy. The effect is positive, because an open economy’s consumption is closely tied to intertemporal trade. Agents who expect a deterioration in ToT in the future will try to increase current consumption.

The aggregate supply in the domestic economy is derived from profit maximization of individual firms with nominal rigidities and monopolistic competition. The producer price index (PPI) inflation depends on its expected future inflation and the output gap in two countries due to the international trade. However, the consumer producer index (CPI) inflation depends on PPI and ToT. Hence, in contrast with a closed economy, the Phillips curve in an open economy is shifted by some changes in the output gap of the foreign economy. Note here that some changes in the output of the foreign economy have opposing effects on the domestic inflation. On the one hand, the deterioration of ToT in the domestic economy contributes to a decrease in CPI (i.e., \( \pi_t^{CPI} = \pi_t^{PPI} + \alpha \cdot \Delta s_t \), where \( \alpha \) measures the degree of trade openness). However, an increase in the output of foreign economy provides a pressure on the marginal cost in the domestic economy, leading to an increase in PPI (i.e., \( \pi_t^{PPI} = \beta \cdot E_t \pi_{t+1}^{PPI} + k \cdot mc_t \), where \( \kappa \) is the slope of Phillips curve). Since CPI depends on both PPI and ToT, we arrive at somewhat complicated expression for the Phillips curve in open economy. Inflation dynamics are influenced by the leading and lagging differences in the output gap between countries.

The monetary policy rule is governed by Taylor principle with its reaction coefficients of \( c_1, c_2, \) and \( c_3 \) (0 < \( c_1 \) < 1, \( c_2 > 1, c_3 > 0 \)). The foreign economy includes the same type of policy rule, but the corresponding coefficients are indexed by asterisk.

The main feature of bounded rationality modelled in this paper is switching dynamics based on performance in forecasting future output and inflation. We follow expectation formation process based on simple heuristic rules in De Grauwe (2011).

### 2.2 Interaction-based approach in expectation formation process

To make the description of the expectation formation processes more explicit, we assume that agents adopt either an optimistic or pessimistic attitude towards movements in the future output gap (in
the following indicated by the superscripts $O$ and $P$, respectively:

\[ E^O_t y_{t+1} = d_t, \quad E^P_t y_{t+1} = -d_t \]  \hspace{1cm} (1)  
\[ E^O_t y^*_t + 1 = d^*_t, \quad E^P_t y^*_t + 1 = -d^*_t \]  \hspace{1cm} (2)

where

\[ d_t = \frac{1}{2} \cdot [\nu + \delta y_{y,t}] \]  
\[ d^*_t = \frac{1}{2} \cdot [\nu^* + \delta^* y^*_{y,t}] \]

Following De Grauwe (2011), we use the terms of $d_t$ and $d^*_t$ to specify the divergence in beliefs among agents about the output gap in domestic and foreign economies, respectively. The bounded rational agents are uncertain about the future dynamics of the output gap and therefore predict a fixed value of $y_{t+1}$ and $y^*_t + 1$ measured by $\nu$ and $\nu^*$, respectively. The latter can be regarded as the *predicted subjective mean value* of $y_t$. However, the subjective forecast is generally biased and therefore depends on the volatility in the output gap, i.e. given by the unconditional standard deviations $\lambda_{y,t}$ and $\lambda^*_{y,t}$. In this respect, the parameters $\delta$ and $\delta^*$ measure the *degree of divergence* in the movement of economic activity in the two economies. Due to the symmetry in the divergence in beliefs, optimists expect that the output gap will differ positively from the steady state value (which for consistency is set to zero), while pessimists will expect a negative deviation by the same amount. The symmetric two-country model suggests that the foreign economy includes the same underlying structure in bounded expectations formation process but its dynamics are based on different parameter values.

The expression for the *market forecast* regarding the output gap in an open economy is given by

\[ \tilde{E}^R_t y_{t+1} = \alpha^O_{y,t} \cdot E^O_t y_{t+1} + \alpha^P_{y,t} \cdot E^P_t y_{t+1} = (\alpha^O_{y,t} - \alpha^P_{y,t}) \cdot d_t \]  \hspace{1cm} (3)  
\[ \tilde{E}^R_t y^*_t + 1 = \alpha^O_{y,t} \cdot E^O_t y^*_t + \alpha^P_{y,t} \cdot E^P_t y^*_t = (\alpha^O_{y,t} - \alpha^P_{y,t}) \cdot d^*_t, \]  \hspace{1cm} (4)

where $\alpha^O_y + \alpha^P_y = 1$ and $\alpha^O_{y^*} + \alpha^P_{y^*} = 1$ hold. The probabilities $(\alpha^O_y, \alpha^P_y)$ and $(\alpha^O_{y^*}, \alpha^P_{y^*})$ are based on stochastic behavior of the agents who adopt a particular forecasting rule. $\alpha^O_y$ (or $\alpha^P_y$) is regarded as the probability of being an optimist (or pessimist) in the domestic economy. The probabilities for the foreign economy are denoted by $\alpha^O_{y^*}$ (or $\alpha^P_{y^*}$). In the following, we give an explicit description

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5The main goal of this paper is to extend De Grauwe’s model to an open economy. The structure of foreign economy is almost identical to the domestic economy except for parameter values arising from the micro-level behavior of agents. Again, they are indicated by asterisk.
of these probabilities.

First, the selection of the forecasting rules depends on the forecast performances of optimists and pessimists $U^k_t$ (with $k = O, P$) given by the mean squared error of the forecast performance. The foreign economy is governed by the same behavioral rules with different parameters where their values are indexed by asterisk. The utility for the forecast performances can be simply updated in every period as (cf. Brock and Hommes (1997)):

$$U^k_{t} = \rho U^k_{t-1} - (1-\rho)(E^k_{t-1}y_t - y_t)^2; \quad (5)$$

where the parameters $\rho$ and $\rho^*$ are used to measure the memory of agents ($0 \leq \rho, \rho^* \leq 1$). Here $\rho = 0$ suggests that agents have no memory of past observations, while $\rho = 1$ means that they have infinite memory instead. Second, agents can revise their expectations by considering the forecast performances (i.e., the discrete choice theory). The different types of performance measures can be utilized for $(\alpha^O_{y,t}, \alpha^P_{y,t})$ and $(\alpha^O_{ys,t}, \alpha^P_{ys,t})$ as follows:

$$\alpha^O_{y,t} = \frac{\exp(\gamma U^O_t)}{\exp(\gamma U^O_t) + \exp(\gamma U^P_t)}; \quad \alpha^P_{y,t} = \frac{\exp(\gamma U^P_t)}{\exp(\gamma U^O_t) + \exp(\gamma U^P_t)} = 1 - \alpha^O_{y,t}; \quad (7)$$

$$\alpha^O_{ys,t} = \frac{\exp(\gamma^* U^{O*}_t)}{\exp(\gamma^* U^{O*}_t) + \exp(\gamma^* U^{P*}_t)}; \quad \alpha^P_{ys,t} = \frac{\exp(\gamma^* U^{P*}_t)}{\exp(\gamma^* U^{O*}_t) + \exp(\gamma^* U^{P*}_t)} = 1 - \alpha^O_{ys,t} \quad (8)$$

where the parameters $\gamma, \gamma^* \geq 0$ denote the intensity of choice for domestic and foreign economies respectively. The self-selecting mechanism is purely stochastic with $\gamma = 0$ (i.e. $\alpha^O_{y,t} = \alpha^P_{y,t} = 1/2$), whereas it is fully deterministic with $\gamma = \infty$ (i.e. $\alpha^O_{y,t} = 0, \alpha^P_{y,t} = 1$ or vice versa).

Given the past value of the forecast performance ($U^k_{t-1}$), we see that the lower the difference between the expected value of the output gap (taken from the previous period, i.e. $E^k_{t-1}y_t = |d_{t-1}|$) and its realization in period $t$, the higher the corresponding forecast performance $U^k_t$ will be. More precisely, if, for example, the forecast made by the optimists is more accurate than the one made by the pessimists, this will lead to a higher level of utility for the optimistic agents, i.e. $U^O_t > U^P_t$ holds. Hence, the pessimists have the incentive to adopt the forecasting rule used by the optimists (i.e., $E^O_t y_{t+1} = d_t$). Finally, this forecasting rule prevails and the share of pessimists in the market decreases. The same updating mechanism in expectation formation process is applied to the foreign economy. Again, the parameter values for the foreign economy are indicated by asterisk.

Again following De Grauwe (2011), we assume that agents will be either so called inflation (gap) targeters (tar) or extrapolators (ext). In the former case, the central bank anchors expectations by
announcing a target for the inflation gap $\bar{\pi}$. From the viewpoint of the inflation targeters, we consider this pre-commitment strategy to be fully credible in both economies. Hence, the corresponding forecasting rule becomes

\begin{align}
E_{t}^{\text{tar}} \hat{\pi}_{t+1} &= \bar{\pi}; \\
E_{t}^{\text{tar}} \hat{\pi}^{*}_{t+1} &= \bar{\pi}^{*} \\
\end{align}

with $\bar{\pi} = \bar{\pi}^{*} = 0$. The extrapolators instead expect that the future value of the inflation gap is given by its past value:

\begin{align}
E_{t}^{\text{ext}} \hat{\pi}_{t+1} &= \hat{\pi}_{t-1}; \\
E_{t}^{\text{ext}} \hat{\pi}^{*}_{t+1} &= \hat{\pi}^{*}_{t-1}.
\end{align}

Note that the market forecast for the inflation gap is similar to the forecast for the output gap in equations (3) and (4):

\begin{align}
\tilde{E}_{t}^{\text{BR}} \hat{\pi}_{t+1} &= \alpha_{t}^{\text{tar}} E_{t}^{\text{tar}} \hat{\pi}_{t+1} + \alpha_{t}^{\text{ext}} E_{t}^{\text{ext}} \hat{\pi}_{t+1} = \alpha_{t}^{\text{tar}} \bar{\pi} + \alpha_{t}^{\text{ext}} \hat{\pi}_{t-1}; \\
\tilde{E}_{t}^{\text{BR}} \hat{\pi}^{*}_{t+1} &= \alpha_{t}^{\text{tar}} E_{t}^{\text{tar}} \hat{\pi}^{*}_{t+1} + \alpha_{t}^{\text{ext}} E_{t}^{\text{ext}} \hat{\pi}^{*}_{t+1} = \alpha_{t}^{\text{tar}} \bar{\pi}^{*} + \alpha_{t}^{\text{ext}} \hat{\pi}^{*}_{t-1}.
\end{align}

The forecast performances of inflation targeters and extrapolators follow the mean squared forecasting error in domestic and foreign economies:

\begin{align}
U_{t}^{s} &= \rho U_{t-1}^{s} - (1 - \rho)(E_{t-1}^{s} \bar{\pi}_{t} - \hat{\pi}_{t})^{2}; \\
U_{t}^{*s} &= \rho^{*} U_{t-1}^{*s} - (1 - \rho^{*})(E_{t-1}^{s} \bar{\pi}^{*}_{t} - \hat{\pi}^{*}_{t})^{2},
\end{align}

where $s = (\text{tar}, \text{ext})$ holds. Finally, we can write:

\begin{align}
\alpha_{t}^{\text{tar}} &= \frac{\exp(\gamma U_{t}^{\text{tar}})}{\exp(\gamma U_{t}^{\text{tar}}) + \exp(\gamma U_{t}^{\text{ext}})}, \\
\alpha_{t}^{\text{ext}} &= \frac{\exp(\gamma U_{t}^{\text{ext}})}{\exp(\gamma U_{t}^{\text{tar}}) + \exp(\gamma U_{t}^{\text{ext}})} = 1 - \alpha_{t}^{\text{tar}}, \\
\alpha_{t}^{\text{tar}} &= \frac{\exp(\gamma^{*} U_{t}^{\text{tar}})}{\exp(\gamma^{*} U_{t}^{\text{tar}}) + \exp(\gamma^{*} U_{t}^{\text{ext}})}, \\
\alpha_{t}^{\text{ext}} &= \frac{\exp(\gamma^{*} U_{t}^{\text{ext}})}{\exp(\gamma^{*} U_{t}^{\text{tar}}) + \exp(\gamma^{*} U_{t}^{\text{ext}})} = 1 - \alpha_{t}^{\text{tar}}.
\end{align}

\[8\] In this respect (based on an optimal monetary policy strategy), an inflation gap target of zero percent implies that the European Central Bank seeks to minimize the deviation of its (realized) target rate of inflation from the corresponding time-varying steady state value. Thus the deviation should be zero in the optimum.
tations and vice versa. Note here that the memory ($\rho$), as well as the intensive of choice ($\gamma$), do not differ across the expectation formation processes in terms of the output and inflation gap. In the end, the BR model exhibits purely backward-looking behavior from which we can arrive at the solution to the system numerically by backward-induction, as well as the method of undetermined coefficients, together with the brute force iteration procedure (Binder and Pesaran (1995)).

2.3 Bounded rationality and the business cycle

Now we examine the relation between structural parameters and the model dynamics based on animal spirits. In the model, the important role is placed on the synthesis of structural parameters with agents’ behavior. For example, the model predicts that high uncertainty in expectation formation process can increase the persistence in the output and inflation dynamics (see Table 1). We examine the effect of expectation formation process on structural dynamics where a different degree of uncertainty and price stickiness are assumed across the countries.

<table>
<thead>
<tr>
<th>Table 1: Relation between model dynamics and structural parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Persistence in $H$</td>
</tr>
<tr>
<td>---------------------</td>
</tr>
<tr>
<td>High U. in $H$ &amp; low U. in $F$</td>
</tr>
<tr>
<td>High price stickiness in $F$</td>
</tr>
<tr>
<td>Low trade openness ($\alpha$=0.1)</td>
</tr>
<tr>
<td>High trade openness ($\alpha$=0.9)</td>
</tr>
</tbody>
</table>

Note: ‘U.’ indicates the degree of uncertainty in expectation formation process. Trade openness is set to be moderate ($\alpha = 0.6$) for first two cases.

On the other hand, an increase in price stickiness can be translated into strong persistence in the macroeconomic dynamics but does not have an effect on the behavior of agents. Another important aspect is the role of trade openness in the transmission mechanism between countries. Indeed, the model predicts that a low degree of trade linkage hinders the uncertainty from spilling over to another country. Hence, if two economies are strongly integrated, the agents tend to behave in an extreme way leading to more persistence in the dynamics of inflation and output.

However, the relation between agents’ behavior and the business cycle is not unambiguous. First, the analytic autocorrelation function does not exist due to non-linear group behavior. Second, if you consider the negative influence of foreign output gap on the IS relation, there exists countercyclical relation between behavioral uncertainty and the business cycle. But the complexity
of the model dynamics can be strengthened by structural adjustment process in demand and supply.

Table 2: Simulated output correlation

<table>
<thead>
<tr>
<th></th>
<th>$\alpha = 0.1$</th>
<th>$\alpha = 0.6$</th>
<th>$\alpha = 0.9$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu = 0.5$</td>
<td>-0.193</td>
<td>-0.040</td>
<td>0.315</td>
</tr>
<tr>
<td>$\nu = 1.0$</td>
<td>-0.221</td>
<td>-0.037</td>
<td>0.314</td>
</tr>
<tr>
<td>$\nu = 2.0$</td>
<td>-0.261</td>
<td>-0.046</td>
<td>0.104</td>
</tr>
</tbody>
</table>

Note: $\nu^*$ is set to 0.5. Other deep parameters are set to the values given in the Appendix B.

Table 2 shows the output correlation between country $H$ and country $F$ where we assume different values for divergence in belief. For this exercise, divergence of belief for country $F$ is set to 0.5, while we allow for difference in divergence of belief for country $H$ (i.e., $\nu = 0.5, 1.0, 2.0$). The model is simulated 100 times using different stochastic noise. After considering simulation errors, we take the average of the simulated values for the output correlation from the model.

The simulation results support countercyclical behavior when the spillover effects are weak across countries (i.e., weak trade linkage). The business cycles are less synchronized compared to the case where countries are strongly tied to each other. As two countries increase the amount of international trade, the business cycles of two economies tend to be synchronized. However, the model generates countercyclical output dynamics induced by behavioral uncertainty. This suggests that country $H$ is subject to strong behavioral uncertainty regardless of international trade. Nevertheless, its effect on the business cycles is not exactly due to above mentioned reasons. For example, an increase in trade openness does not improve the synchronization of business cycles between countries when large behavioral uncertainty is assumed ($\nu = 2.0$, $\alpha = 0.9$). To see more details, we investigate the model prediction via numerical simulations in the next section.

3 Numerical Simulations

In this section, the model is simulated using a set of different parameter values. In particular, we examine some changes in behavioral parameters and their effects on the dynamics in an open economy. The benchmark values are shown in Table 3. The monetary reaction coefficients remain the same for both countries. Also, the memory parameter is set to 0.05 which suggests almost no memory. This assumption is supported by an evidence that memory decays exponentially (Ander-

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7See the Appendix for the relation between the model coefficients and the deep parameters in open economy.
son (2001)). Note here that the domestic and foreign economies are denoted by $H$ and $F$ respectively.

Table 3: Calibrated values for animal spirits in a two-country model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1$</td>
<td>2</td>
<td>$b_2$</td>
<td>0.8194</td>
<td>$c_1$, $c_1^*$</td>
<td>0.5</td>
<td>$\nu$, $\nu^*$</td>
<td>2</td>
</tr>
<tr>
<td>$a_2$</td>
<td>1</td>
<td>$b_3$</td>
<td>0.4760</td>
<td>$c_2$, $c_2^*$</td>
<td>1.5</td>
<td>$\delta$, $\delta^*$</td>
<td>0.5</td>
</tr>
<tr>
<td>$b_1$</td>
<td>0.2475</td>
<td>$b_4$</td>
<td>0.25</td>
<td>$c_3$, $c_3^*$</td>
<td>1.0</td>
<td>$\rho$, $\rho^*$</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Note: The discount factor $\beta$ is set to 0.99. The simulations are based on different values for divergence in belief, price stickiness, and trade openness ($\alpha = 0.1, 0.9$).

3.1 Case I: high uncertainty in $H$ & low uncertainty in $F$

As discussed in the previous section, animal spirits in open economy are reflected in the group behavior. The behavioral parameters for bounded rational agents provide an approximation to uncertainty about expectation formation processes. For example, the degree of divergence in belief controls the persistence in extreme opinions with respect to the dynamics of output and inflation.

Figure 2: Dynamics of output and inflation gap in a two-country model

Note: The divergence in belief $\nu$ is set to 2.0 in country $H$, while it is set to 0.5 for country $F$.

Note here that one of the most important behavioral parameters is the degree of divergence in belief. To see the effects of behavioral uncertainty on the model dynamics, we use same values for
other parameters in simulations. In addition, we consider a moderate degree of trade openness where
the two economies engage in same amount of trade activities. In other words, domestic residents
buy half of goods from the foreign economy ($\alpha = 0.5$).

The simulated trajectories of output and inflation gap in the two countries are shown in Figure 2.
The parameters $\nu$ and $\nu^*$ are set to 2.0 and 0.5 for countries $H$ and $F$, respectively. The dynamics
of output gap in country $H$ are more persistent than country $F$. This suggests that a large degree
of divergence in belief influences the dynamics of output and inflation, in particular leading to high
persistence. The two countries do not differ in terms of underlying economic structure (i.e., the
same values for the deep parameters), but the differences in the output and inflation dynamics
are caused by the expectation formation process on market uncertainty. The extreme opinions in
country $H$ persist much longer than in country $F$, as the perceived uncertainty plays an important
role in strengthening their strategy instead of changing their opinions on the economy.

Indeed, the simulated data match a high correlation between two countries. For example, high
uncertainty in country $H$ increases the correlation for simulated data (i.e., $\text{corr}(x^H_t, x^F_t) = 0.419$).
Also, autocorrelations at lag one are 0.427 and 0.468 for countries $H$ and $F$ respectively. Those
values are higher than the case where behavioral uncertainty is set to 0.5 for country $H$ (i.e., $\text{corr}(x^H_t, x^F_t) = 0.177$, $\text{autocorr}(x^H_t, x^H_{t-1}) = 0.358$, $\text{autocorr}(x^F_t, x^F_{t-1}) = 0.336$). Accordingly, an increase
in behavioral uncertainty has contributed to the synchronization of international business cycles, as
well as the persistence of output gap for both countries.

3.2 Case II: difference in price stickiness

In this simulation, we investigate the effects of price stickiness on the model dynamics. The price
stickiness controls the inherited persistence from the output gap in the Phillips curve. Note here
that country $F$ keeps a higher degree of price stickiness than country $H$ ($\theta_H=0.75$, $\theta_F=0.9$). This
produces much flatter slope of the Phillips curve in country $F$. Hence, it is shown in Figure 3 that
the dynamics of the output gap in country $F$ remains persistent, where its trajectory is strongly
influenced by country $H$.

This finding is noteworthy because a standard DSGE model with staggered price is often used to
generate endogenous dynamics. According to our simulation results, the persistence of output and
inflation gap in country $F$ stem from nominal rigidities. Note that the persistent dynamics are also
strengthened by the high uncertainty on the future output gap in country $H$.

The simulation results are evaluated by considering the output correlation of the countries. For
3.3 Case III: low and high degree of trade openness

Now we investigate the effect of trade openness on the dynamics of the output gap and inflation gap. As it is shown in the Appendix, the degree of trade openness has an effect on the inherited and intrinsic persistence in the dynamics of inflation and output. In the simulation, we consider different values for trade openness, while keeping behavioral uncertainty the same across countries.

(i) low degree of trade openness \((\alpha = 0.1)\)

The spillover effects across countries are now constrained by low trade openness. For example,
Table 4: Calibrated values for animal spirits in a two-country model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1$</td>
<td>1.2593</td>
<td>$b_2$</td>
<td>0.4783</td>
<td>$b^*_2$</td>
<td>0.1932</td>
</tr>
<tr>
<td>$a_2$</td>
<td>0.2593</td>
<td>$b_3$</td>
<td>0.1350</td>
<td>$b^*_3$</td>
<td>0.1447</td>
</tr>
<tr>
<td>$b_1$</td>
<td>0.0728</td>
<td>$b_4$</td>
<td>0.0735</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: The discount factor $\beta$ is set to 0.99. We assume that there is no change in the monetary reaction coefficients.

The model produces the endogenous persistence of business cycles through international trade, and the high degree of economic uncertainty and price stickiness can increase persistence in the output and inflation dynamics. However, low trade openness hinders the uncertainty in country $H$ from spilling over to country $F$. Accordingly, the output dynamics display a low level of inertia.

Table 4 shows calibrated values for the model based on weak trade linkage. The coefficients of the IS relation and Phillips curve are relatively small when the low degree of trade integration is assumed. The internal propagation plays an important role in the dynamics, because the spillover effects are not strong in this case. The simulated trajectories are shown in Figure 4.

Figure 4: Dynamics of output and inflation gap in a two-country model

Note: We assume the same degree of divergence in belief for countries $H$ and $F$ (i.e., $\nu = \nu^* = 2$). Trade openness controls a degree of spillover effects across countries. The parameter $\alpha$ is set to 0.1.

The simulated data are evaluated by considering the output correlation of countries over time. For example, the correlation for simulated data is 0.379 when we assume low trade openness. Also,
autocorrelations at lag one are 0.557 and 0.577 for countries $H$ and $F$ respectively. They are much higher than the case based on intermediate trade openness ($\alpha = 0.6$). For the latter the model generates a high degree of business cycle synchronization, but the output dynamics behave more regularly than the case where two economies do not trade. Accordingly, an increase in trade integration has contributed to the synchronization of international business cycles, while their effects on the persistence of output gap are ambiguous.

(ii) **high degree of trade openness** ($\alpha = 0.9$)

Now we consider a high degree of trade openness across countries. The dynamics of inflation output are more persistent relative to the model in which the economies are loosely integrated. The simulation results in Figure 4 show that the underlying interactions play an important role in the model dynamics when a high degree of trade openness is assumed. The result is not surprising: a high degree of trade openness puts pressure on the IS relation and Phillips curve, in particular leading to an increase in their slopes (see Table 5).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1$</td>
<td>4.8571</td>
<td>$b_2$</td>
<td>1.6489</td>
<td>$b_2^r$</td>
<td>1.3638</td>
</tr>
<tr>
<td>$a_2$</td>
<td>3.8571</td>
<td>$b_3$</td>
<td>1.3056</td>
<td>$b_3^r$</td>
<td>1.3153</td>
</tr>
<tr>
<td>$b_1$</td>
<td>0.6551</td>
<td>$b_4$</td>
<td>0.6618</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: The discount factor $\beta$ is set to 0.99. The monetary reaction coefficients do not change.

However, high trade openness has an ambiguous effect on the correlation in a two-country model. For example, the correlation for simulated data is 0.364. Also, autocorrelations at lag one are 0.472 and 0.602 for countries $H$ and $F$ respectively. They do not differ from the case where low degree of trade openness is assumed. The rational behind the ambiguous effect is that trade integration is strongly tied to persistence in the output dynamics, which are intermingled with large behavioral uncertainty. Hence, high trade openness does not have direct effects on an increase in the synchronization of international business cycles. But the model predicts a positive correlation between the output gap and trade openness.
4 Comparison with other studies

Recent advances in research on behavioral economics and its application to macroeconomic models reflect an existing gap between theoretical models and data. In other words, the inclusion of financial market frictions in DSGE models is a high priority for further research in macroeconomics. However, most frameworks are limited to simple transmission mechanism, in particular within sectors in the economy. As a result, the misspecification of DSGE models are considered a main drawback when providing appropriate advice to policy makers (see also Tovar (2009)).

Indeed, only a few aspects of financial market distortions have been examined until recently. Examples include balance sheet effects, cost channel, portfolio choice, term premium, among others. For example, firms’ balance sheet effects on investment can serve endogenously as the financial accelerator in DSGE models (Bernanke et al. (1999)). Also, the cost channel between countries can be used to generate propagation mechanism in the global economy (Ravenna and Walsh (2006)). Differences in home and foreign portfolio may have additional effects on equity portfolios under

---

Note: We assume the same degree of divergence in belief for countries $H$ and $F$ (i.e., $\nu = \nu^* = 2$). Trade openness controls a degree of spillover effects across countries. The parameter $\alpha$ is set to 0.9.
nominal price stickiness (Engel and Matsumoto (2005)). Furthermore, the relation between term premium and the economy can be investigated within this context (Rudebusch et al. (2006)).

This study’s aim is to explain financial market frictions much like the above studies with respect to the structure of DSGE models, but the approach is based on social interaction effects where bounded rational agents act as a main driver of market incompleteness in an open economy. As shown in the Section 2, the baseline open DSGE model does not include lagged terms in the IS and Phillips curve. As a result, the expectation formation process in the model is purely forward-looking. By introducing performance measures of optimists and pessimists (or the bounded rational behavior of agents), the model can be used to establish a link between backward-looking expectations and macroeconomic dynamics. In other words, a behavioral channel between countries can be strengthened by risk-averse investors who may over- or under-react to the state of the economy.

5 Conclusion

The global economy is complicated by many aspects of international trade and investment across countries. Distortions might stem from the existence of incomplete markets, cost channel or trade openness. Put differently, distortions and economic complexity can be associated with multi-dimensional aspects of economic variables in which a single exponent is not enough to describe the economic system over time. Hence, a complicated dependency between time series contributes to the economic complexity where economic data are recorded in different time intervals but we observe aggregated ones for economic analysis.

In addition, the complexity of economic dynamics is attributed to a non-rational group behavior which influences economic system in a non-linear way. This paper contributes to the macroeconomic system in this respect. In other words, this study puts an emphasis on behavioral perspective and the perceived uncertainty of heterogeneous agents on the underlying economy. As a result, large uncertainties in economic activities can be generated by the interactions arising from differences in forecast performance by heterogeneous agents.

To show this, we construct an artificial open economy where two countries are constrained by social interactions between heterogeneous investors. Our simulation results show that social interaction effects serve as an transmission channel behind market integration in a global economy. Indeed, economic uncertainty affects the investment decisions. To show this, we examine the effect of the social interactions between boundedly rational agents on the joint behavior of inflation and output
within two countries.

In addition, future work of our current study may lead to interesting results. For example, the estimation of the behavioral parameters from real data can be considered a high priority for current research. For example, Jang and Sacht (2015) make an estimation of De Grauwe model by using SMM from Euro Area data, and found that the backward-looking behavior and bounded rationality play an important role in the approximation of the model to empirical data. Similarly, simulation-based inference can be used to bridge the gap between the model dynamics and real data. Another research direction includes the extension of the expectation formation process in which forecast performance in one country depends on the mean squared forecast errors from foreign country. This kind of extension can be used to analyse the effects of behavioral complexity on the global economy.
Appendixes

A: Canonical form of open-economy model

We denote by $z_t$ the state vector of $[x_t \ \pi_t \ r_t \ x_t^* \ \pi_t^* \ r_t^*]'$. Then, the structural model of a symmetric open economy can be rewritten in canonical form:

$$A \epsilon_t z_{t+1} + B z_t + C z_{t-1} = 0, \quad (19)$$

where:

$$A = \begin{bmatrix} 1 & a_1 & 0 & a_2 & 0 & 0 \\ -b_1 & \beta & 0 & b_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ a_2 & 0 & 0 & 1 & a_1 & 0 \\ b_1 & 0 & 0 & -b_1 & \beta & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} -1 & 0 & -a_1 & -a_2 & 0 & 0 \\ b_2 & -1 & 0 & -b_3 & 0 & 0 \\ (1 - c_1) \cdot c_3 & (1 - c_1) \cdot c_2 & -1 & 0 & 0 & 0 \\ -a_2 & 0 & 0 & -1 & 0 & -a^* \\ -b_3^* & 0 & 0 & b_2^* & -1 & 0 \\ 0 & 0 & 0 & (1 - c_1^*) \cdot c_3^* & (1 - c_1^*) \cdot c_2^* & -1 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ -b_4 & 0 & 0 & b_4 & 0 & 0 \\ 0 & 0 & c_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ b_4 & 0 & 0 & -b_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & c_1^* & 0 \end{bmatrix}$$
The method of undetermined coefficients and iterative methods can be used to solve the system of equations. This solution indicates the equilibrium values of the observable variables in the system.

B: Calibrated values for deep parameters

Table 6: Calibrated values for animal spirits in a two-country model

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>σ</td>
<td>Risk aversion</td>
<td>1.0</td>
</tr>
<tr>
<td>η</td>
<td>Elasticity of substitution between goods</td>
<td>2.0</td>
</tr>
<tr>
<td>ϕ</td>
<td>Labor disutility</td>
<td>3.0</td>
</tr>
<tr>
<td>θ_H</td>
<td>Calvo lotteries in domestic price</td>
<td>0.75</td>
</tr>
<tr>
<td>θ_F</td>
<td>Calvo lotteries in foreign price</td>
<td>0.9</td>
</tr>
<tr>
<td>φπ = φπ*</td>
<td>Taylor rule inflation</td>
<td>1.5</td>
</tr>
<tr>
<td>φy = φy*</td>
<td>Taylor rule output growth</td>
<td>1.0</td>
</tr>
<tr>
<td>φr = φr*</td>
<td>Interest rate smoothing</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Note: The discount factor β is set to 0.99. The simulations are based on different values for trade openness (α = 0.1, 0.9).

Table 5 shows the calibrated values for deep parameters in the model. The parameters are connected with the model coefficients in canonical form as follows:

\[
\begin{align*}
\kappa_H &= (1 - \theta_H) \cdot (1 - \theta_H \cdot \beta)/\theta_H; \\
\kappa_F &= (1 - \theta_F) \cdot (1 - \theta_F \cdot \beta)/\theta_F; \\
\omega_2 &= 2 \cdot \alpha \cdot (1 - \alpha) \cdot (\sigma \cdot \eta - 1); \\
\omega_4 &= 4 \cdot \alpha \cdot (1 - \alpha) \cdot (\sigma \cdot \eta - 1); \\
\zeta &= (\omega_2 + 1) \cdot \sigma + (\omega_4 + 1) \cdot \varphi; \\
a_1 &= (\omega_4 + 1)/(\omega_2 + 1 - \alpha) \cdot \sigma; \\
a_2 &= (\alpha + \omega_2)/(\omega_2 + 1 - \alpha); \\
b_1 &= (\beta \cdot \alpha \cdot \sigma)/(\omega_4 + 1); \\
b_2 &= \{\alpha \cdot \sigma \cdot (1 + \beta) + \kappa_H \cdot \zeta\}/(\omega_4 + 1); \\
b_3 &= \sigma \cdot \{\alpha \cdot (1 + \beta) - \kappa_H \cdot \omega_2\}/(\omega_4 + 1); \\
b_4 &= (\alpha \cdot \sigma)/(\omega_4 + 1); \\
b_2^* &= \{\alpha \cdot \sigma \cdot (1 + \beta) + \kappa_F \cdot \zeta\}/(\omega_4 + 1); \\
b_3^* &= \sigma \cdot \{\alpha \cdot (1 + \beta) - \kappa_F \cdot \omega_2\}/(\omega_4 + 1);
\end{align*}
\]
C: Computer code

The following matlab file was used to generate the behavior of agents and dynamic macroeconomic variables in a two-country New Keynesian model.

```matlab
load data\us\euro;

output_{\text{gap}_\text{euro}} = realdat(:,1);
output_{\text{gap}_\text{us}} = realdat(:,2);

beta = 0.99; sigma = 1.0; eta = 2.0; varphi = 3;
theta_H = 0.75; theta_F = 0.75; alpha = 0.5;

kappa_H = (1-theta_H)*(1-theta_H*beta)/theta_H;
kappa_F = (1-theta_F)*(1-theta_F*beta)/theta_F;

omega_2 = alpha*2*(1-alpha)*(sigma*eta-1);
omega_4 = alpha*4*(1-alpha)*(sigma*eta-1);
varsigma = (omega_2+1)*sigma+(omega_4+1)*varphi;
a_1 = (omega_4+1)/((omega_2+1-alpha)*sigma);
a_2 = (alpha+omega_2)/(omega_2+1-alpha);

b_1 = (beta*alpha*sigma)/(omega_4+1);
b_2 = (alpha*sigma*(1+beta)+kappa_H*varsigma)/(omega_4+1);
b_3 = sigma*(alpha*(1+beta)-kappa_H*omega_2)/(omega_4+1);
b_4 = (alpha*sigma)/(omega_4+1);

b_2_star = (alpha*sigma*(1+beta)+kappa_F*varsigma)/(omega_4+1);
b_3_star = sigma*(alpha*(1+beta)-kappa_F*omega_2)/(omega_4+1);

phi_p = 1.5; phi_p_star = 1.5; phi_x = 1.0;
phi_x_star = 1.0; phi_r = 0.5; phi_r_star = 0.5;
```
sig_{pp} = 0.5; \quad \text{sig}_{xx} = 0.5; \quad \text{sig}_{rr} = 0.5;

sig_{pp/star} = 0.5; \quad \text{sig}_{xx/star} = 0.5; \quad \text{sig}_{rr/star} = 0.5;

drift_H = 2.0; \quad \text{delta}_H = 0.5; \quad \rho_H = 0.05;

mm = 1.0; \quad \text{drift}_F = 6; \quad \text{delta}_F = 0.5; \quad \rho_F = 0.05;

A = \begin{bmatrix}
1 & -(\alpha*\sigma^2*(1+\beta)+\kappa_H \varsigma)/(\omega^4+1) & 0 \\
\sigma^2*(\alpha*(1+\beta)-\kappa_H \omega^2)/(\omega^4+1) & (\omega^4+1)^2*(1-\phi_r)*\phi_p/(\omega^4+1) & 1 \\
(\omega^4+1)^2*(1-\phi_r)*\phi_p/(\omega^4+1) & 0 & (\omega^4+1)^2*(1-\phi_r)*\phi_p/(\omega^4+1) \\
\end{bmatrix};

B = \begin{bmatrix}
\beta & -\beta*\alpha*\sigma^2/(\omega^4+1) & 0 \\
\beta*\alpha^2/(\omega^4+1) & \omega^4+1/(\omega^4+1) & 1 \\
0 & \beta*\alpha^2/(\omega^4+1) & \omega^4+1/(\omega^4+1) \\
\end{bmatrix};

C = \begin{bmatrix}
0 & -\alpha*\sigma^2/(\omega^4+1) & 0 \\
0 & 0 & 0 \\
0 & \alpha*\sigma^2/(\omega^4+1) & 0 \\
0 & 0 & 0 \\
\end{bmatrix};

\text{smooth} = \begin{bmatrix}
0 & 0 \\
-(\omega^4+1)/(\omega^4+1) & 0 \\
0 & 0 \\
0 & -(\omega^4+1)/(\omega^4+1) \\
\end{bmatrix};

BB = 20;

T = 129-BB;

\text{eps}_{pp}(1,1) = 0; \quad \text{eps}_{xx}(1,1) = 0; \quad \text{eps}_{rr}(1,1) = 0;

\text{eps}_{pp/star}(1,1) = 0; \quad \text{eps}_{xx/star}(1,1) = 0; \quad \text{eps}_{rr/star}(1,1) = 0;

\beta_{tar} = 0.5; \quad \text{alfay} = 0.5;

\beta_{tar/star} = 0.5; \quad \text{alfay}_star = 0.5;

\beta_{tart}(1,1) = \beta_{tar}; \quad \text{alfayt}(1,1) = \text{alfay};
\[
\begin{align*}
\text{beta}_{\text{tar}_\ast}(1,1) &= \text{beta}_{\text{tar}_\ast}; \\
\text{alfay}_{\text{tar}_\ast}(1,1) &= \text{alfay}_{\ast}; \\
\text{TAR}_{pp} &= \text{zeros}(T+BB,1); \quad \text{EXT}_{pp} = \text{zeros}(T+BB,1); \\
\text{OP}_{xx} &= \text{zeros}(T+BB,1); \quad \text{PE}_{xx} = \text{zeros}(T+BB,1); \\
pp &= \text{zeros}(T+BB,1); \quad xx = \text{zeros}(T+BB,1); \quad rr = \text{zeros}(T+BB,1); \\
\text{TAR}_{pp \ast} &= \text{zeros}(T+BB,1); \quad \text{EXT}_{pp \ast} = \text{zeros}(T+BB,1); \\
\text{OP}_{xx \ast} &= \text{zeros}(T+BB,1); \quad \text{PE}_{xx \ast} = \text{zeros}(T+BB,1); \\
\text{pp \ast} &= \text{zeros}(T+BB,1); \quad xx \ast = \text{zeros}(T+BB,1); \quad rr \ast = \text{zeros}(T+BB,1); \\
SS &= 100; \\
\text{for } jj = 1:SS \\
\text{randn('state', } jj+100); \\
\text{for } ii = 2:T+BB \\
\text{eps}_{pp}(ii,1) &= \text{sig}_{pp} \cdot \text{randn}; \quad \text{eps}_{xx}(ii,1) = \text{sig}_{xx} \cdot \text{randn}; \\
\text{eps}_{rr}(ii,1) &= \text{sig}_{rr} \cdot \text{randn}; \quad \text{eps}_{pp \ast}(ii,1) = \text{sig}_{pp \ast} \cdot \text{randn}; \\
\text{eps}_{xx \ast}(ii,1) &= \text{sig}_{xx \ast} \cdot \text{randn}; \quad \text{eps}_{rr \ast}(ii,1) = \text{sig}_{rr \ast} \cdot \text{randn}; \\
\text{temp} &= -(\omega_{4}+1)/((\omega_{2}+1-\alpha) \cdot \sigma); \\
\text{shocks} &= [ \text{eps}_{pp}(ii,1); \quad \text{temp} \cdot \text{eps}_{rr}(ii,1)+\text{eps}_{xx}(ii,1); \\
&\quad \text{eps}_{pp \ast}(ii,1); \quad \text{temp} \cdot \text{eps}_{rr \ast}(ii,1)+\text{eps}_{xx \ast}(ii,1) ]; \\
\text{epcs} &= \text{pp}(ii-1,1); \quad \text{epcs}\ast = \text{pp}\ast(ii-1,1); \\
\text{epfs} &= 0; \quad \text{epfs}\ast = 0; \\
\text{eps} &= (1-\text{beta}_{\text{tar}}) \cdot \text{epcs} + \text{beta}_{\text{tar}} \cdot \text{epfs}; \\
\text{if } (ii<20) \\
\text{eyoptim} &= 0.5 \cdot (\text{drift}_H + \delta_{H} \cdot \text{std(output\_gap\_euro}(ii:BB,1))); \\
\text{else} \\
\text{eyoptim} &= 0.5 \cdot (\text{drift}_H + \delta_{H} \cdot \text{std(output\_gap\_euro}(ii-BB:ii-1,1))); \\
\text{end} \\
\end{align*}
\]


\[ eypess = -1 \times eyoptim; \]

\[ eys = alfay \times eyoptim + (1-alfay) \times eypess; \]

\[ eps_{star} = (1-beta_{tar_{star}}) \times epcs_{star} + beta_{tar_{star}} \times epfs_{star}; \]

\[ \text{if } \text{(ii} \leq 20) \]
\[ \quad eyoptim_{\star} = 0.5 \times (\text{drift}_{F} + \text{delta}_{F} \times \text{std(output-gap_us(1:BB,1))}); \]
\[ \quad \text{else} \]
\[ \quad eyoptim_{\star} = 0.5 \times (\text{drift}_{F} + \text{delta}_{F} \times \text{std(output-gap_us(ii-BB:ii-1,1))}); \]
\[ \text{end} \]

\[ eypess_{\star} = -1 \times eyoptim_{\star}; \]

\[ eys_{\star} = alfay_{\star} \times eyoptim_{\star} + (1-alfay_{\star}) \times eypess_{\star}; \]

\[ \text{forecast} = [ eps; eys; eps_{\star}; eys_{\star} ]; \]

\[ pplag = pp(ii-1,1); \quad xxlag = xx(ii-1,1); \quad rrlag = rr(ii-1,1); \]

\[ pplag_{\star} = pp_{\star}(ii-1,1); \quad xxlag_{\star} = xx_{\star}(ii-1,1); \quad rrlag_{\star} = rr_{\star}(ii-1,1); \]

\[ \text{lag} = [ pplag; xxlag; pplag_{\star}; xxlag_{\star} ]; \]

\[ rr_{\star} = [ rrlag; rrlag_{\star} ]; \]

\[ D = B \times \text{forecast} + C \times \text{lag} + \text{smooth} \times rr_{\star} + \text{shocks}; \]

\[ XS = A \times D; \]

\[ pp(ii,1) = XS(1,1); \quad xx(ii,1) = XS(2,1); \]

\[ pp_{\star}(ii,1) = XS(3,1); \quad xx_{\star}(ii,1) = XS(4,1); \]

\[ rr(ii,1) = phi_{r} \times rr(ii-1) + (1-phi_{r}) \times (\text{phi}_{\pi_{pp}} \times pp(ii,1) + \text{phi}_{\pi_{xx}} \times xx(ii,1)) + eps_{r} \times rr(ii,1); \]

\[ TAR_{pp}(ii,1) = \text{rho}_{H} \times TAR_{pp}(ii-1,1) - (1-\text{rho}_{H}) \times (epfs - pp(ii,1)).^{\wedge}2; \]

\[ \text{EXT}_{pp}(ii,1) = \text{rho}_{H} \times EXT_{pp}(ii-1,1) - (1-\text{rho}_{H}) \times (epcs - pp(ii,1)).^{\wedge}2; \]

\[ \text{OP}_{xx}(ii,1) = \text{rho}_{H} \times OP_{xx}(ii-1,1) - (1-\text{rho}_{H}) \times (eyoptim - xx(ii,1)).^{\wedge}2; \]

\[ \text{PE}_{xx}(ii,1) = \text{rho}_{H} \times PE_{xx}(ii-1,1) - (1-\text{rho}_{H}) \times (eypess - xx(ii,1)).^{\wedge}2; \]

\[ \text{beta}_{tar} = \exp(mm \times TAR_{pp}(ii,1)) / (\exp(mm \times TAR_{pp}(ii,1)) + \exp(mm \times EXT_{pp}(ii,1)) ); \]

\[ \text{alfay} = \exp(mm \times OP_{xx}(ii,1)) / (\exp(mm \times OP_{xx}(ii,1)) + \exp(mm \times PE_{xx}(ii,1)) ); \]

\[ beta_{tart}(ii,1) = beta_{tar}; \]
\begin{align*}
\text{alfayt}(i,1) &= \text{alfay}; \\
\text{rr\_star}(i,1) &= \phi_{rr\_star}\text{rr\_star}(i-1,1) + (1-\phi_{rr\_star})\phi_{\pi\_star}\text{pp\_star}(i,1) + \\
&\quad \phi_{x\_star}\text{xx}(i,1) + \epsilon_{rr\_star}(i,1); \\
\text{TAR\_pp\_star}(i,1) &= \rho_{F} \ast \text{TAR\_pp\_star}(i-1,1) - (1-\rho_{F}) \ast (\epsilon_{f\_star} - \text{pp\_star}(i,1))^{\wedge}2; \\
\text{EXT\_pp\_star}(i,1) &= \rho_{F} \ast \text{EXT\_pp\_star}(i-1,1) - (1-\rho_{F}) \ast (\epsilon_{c\_star} - \text{pp\_star}(i,1))^{\wedge}2; \\
\text{OP\_xx\_star}(i,1) &= \rho_{F} \ast \text{OP\_xx\_star}(i-1,1) - (1-\rho_{F}) \ast (\epsilon_{\text{optim\_star}} - \text{xx\_star}(i,1))^{\wedge}2; \\
\text{PE\_xx\_star}(i,1) &= \rho_{F} \ast \text{PE\_xx\_star}(i-1,1) - (1-\rho_{F}) \ast (\epsilon_{\text{pess\_star}} - \text{xx\_star}(i,1))^{\wedge}2; \\
\beta_{\text{tar\_star}} &= \exp(m\text{m}\ast\text{TAR\_pp\_star}(i,1)) / (\exp(m\text{m}\ast\text{TAR\_pp\_star}(i,1)) + \exp(m\text{m}\ast\text{EXT\_pp\_star}(i,1))); \\
\text{alfay\_star} &= \exp(m\text{m}\ast\text{OP\_xx\_star}(i,1)) / (\exp(m\text{m}\ast\text{OP\_xx\_star}(i,1)) + \exp(m\text{m}\ast\text{PE\_xx\_star}(i,1))); \\
\beta_{\text{tar\_star}}(i,1) &= \beta_{\text{tar\_star}}; \quad \text{alfayt\_star}(i,1) = \text{alfay\_star}; \\
\text{end;}
\end{align*}
Reference


