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The New Keynesian Wage Phillips Curve: Calvo vs. Rotemberg*

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Abstract

We systematically evaluate how to translate a Calvo wage duration into an implied Rotemberg wage adjustment cost parameter in medium-scale New Keynesian DSGE models by making use of the well-known equivalence of the two setups at first order. We consider a wide range of felicity functions and show that the assumed household insurance scheme and the presence of labor taxation greatly matter for this mapping, giving rise to differences of up to one order of magnitude. Our results account for the inclusion of wage indexing, habit formation in consumption, and the presence of fixed costs in production.

JEL-Classification: E10, E30

Keywords: Wage Phillips Curve; Wage stickiness; Rotemberg; Calvo.

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1 Introduction

Studying the Great Recession, economists have increasingly come to rely on nonlinear macroeconomic models, be it to study the effects of uncertainty shocks as drivers of business cycles (e.g. Fernández-Villaverde, Guerrón-Quintana, Rubio-Ramírez, and Uribe 2011; Born and Pfeifer 2014; Fernández-Villaverde, Guerrón-Quintana, Kuester, and Rubio-Ramírez 2015) or to model the zero lower bound for the nominal interest rate (e.g. Johannsen 2014; Plante, Richter, and Throckmorton 2015). However, the use of nonlinear solution techniques often makes it impractical to use Calvo (1983)-Yun (1996)-type nominal rigidities. First, Calvo rigidities introduce an additional state variable in the form of price/wage dispersion. Second, they give rise to meaningful heterogeneity when not embedded in the right setup (more on this below) and would require tracking distributions in the model. Rotemberg (1982)-type adjustment costs are therefore currently experiencing a renaissance.[1]

However, it is quite difficult to attach a structural interpretation to the Rotemberg adjustment cost parameter, because there is no natural equivalent in the data. In contrast, for the Calvo approach various papers have computed average price durations, e.g. Bils and Klenow (2004) and Nakamura and Steinsson (2008). The literature on *price* rigidities has therefore regularly made use of the first-order equivalence of Rotemberg- and Calvo-type adjustment frictions[2] by translating the Rotemberg adjustment costs to an implied Calvo price duration via the slope of the New Keynesian Price Phillips Curve.[3] However, such guidance for Rotemberg *wage* adjustment costs is still missing, despite good estimates for wage durations being available for both the US (Taylor 1999; Barattieri, Basu, and

[1]Examples include Basu and Bundick (2012), Jermann and Quadrini (2012), Mumtaz and Zanetti (2013), Plante et al. (2015), and Fernández-Villaverde et al. (2015). Richter and Throckmorton (2016) have recently argued for using Rotemberg-type price adjustment costs to improve the model fit, not just for computational convenience.

[2]This approach can also be justified when using nonlinear methods, because the first-order approximation is only used to generate one restriction required to pin down one parameter. The equivalence, however, does not hold in case of trend inflation and incomplete indexing (see Ascari and Sbordone 2014, for a review).

[3]Early works include Roberts (1995), Keen and Wang (2007), and Nisticó (2007). This literature has also shown that the same value of the Rotemberg adjustment cost parameter can have very different economic effects, depending on the value of other structural parameters like the discount factor or the substitution elasticity.

Gottschalk 2014) and the euro area (Le Bihan, Montornès, and Heckel 2012). This is unfortunate, as there has recently been a renewed focus on the importance of wage rigidities (e.g. Galí 2011; Barattieri et al. 2014; Born and Pfeifer 2015).

The present study closes this gap by systematically assessing the mapping between Calvo and Rotemberg wage rigidities in a prototypical medium-scale New Keynesian model including fiscal policy. A particular goal is to provide guidance for researchers working on nonlinear New Keynesian DSGE models with wage rigidities. We focus especially on how i) the other deep parameters of the model and ii) the assumed labor market structure and insurance scheme in the model affect this mapping. For example, it greatly matters whether households supply idiosyncratic labor services and insurance is conducted via state-contingent securities as in Erceg, Henderson, and Levin (2000) (EHL henceforth) or whether insurance takes place inside of a large family and a labor union supplies distinct labor services as in Schmitt-Grohé and Uribe (2006b) (SGU henceforth).[4] We also consolidate the results in the literature by providing a systematic overview of analytic expressions for the slope of the New Keynesian Wage Phillips Curve arising in the EHL setup when using different utility functions with and without consumption habits.[5]

The study probably most closely related to ours is unpublished work by Schmitt-Grohé and Uribe (2006a), who compare the slope of the New Keynesian Wage Phillips Curve arising under the EHL and the SGU setup. However, they do not analyze the relationship to implied Calvo price durations in both setups and do not consider the role of fiscal policy or fixed costs in this mapping.

The paper proceeds as follows. Sections 2 and 3 consider the EHL and SGU setups, respectively. Section 4 provides a numerical comparison. Section 5 concludes. And appendix with detailed derivations and accompanying computer codes are available online.

[4]While the former is more prominent, the latter has been used e.g. in Trigari (2009), Pariés, Sørensen, and Rodriguez-Palenzuela (2011), and Born, Peter, and Pfeifer (2013).

[5]An early precursor of this work is Sbordone (2006).

2 New Keynesian Phillips Curve in the EHL-setup

In this section we lay out the respective prototypical household setups used in EHL and then derive the slope of the New Keynesian Wage Phillips Curve under Calvo and Rotemberg pricing. In the background, but not of interest here, there are a continuum of firms producing differentiated intermediate goods and a final good firm bundling intermediate goods to a final good. In addition, there is a fiscal authority that finances government spending with distortionary labor and consumption taxation and transfers and a monetary authority conducting monetary policy, e.g. according to a Taylor-type interest rate rule.

2.1 Setup

Following EHL, we assume that the economy is populated by a continuum of monopolistically competitive, infinitely-lived households $j, j \in [0,1]$, supplying differentiated labor services N_t^j at wage W_t^j to intermediate goods producers who aggregate them into a composite labor input. Formally, the aggregation technology for the aggregate labor bundle N_t^d follows Dixit and Stiglitz (1977):

$$N_t^d \equiv \left[\int_0^1 (N_t^j)^{\frac{\varepsilon_w - 1}{\varepsilon_w}} \, dj \right]^{\frac{\varepsilon_w}{\varepsilon_w - 1}} , \tag{2.1}$$

where $\varepsilon_w > 0$ is the elasticity of substitution. The cost of the bundle N_t^d is given by

$$W_t \equiv \left[\int_0^1 (W_t^j)^{1-\varepsilon_w} \, dj \right]^{\frac{1}{1-\varepsilon_w}} . \tag{2.2}$$

Taking wages as given, expenditure minimization yields the familiar downward-sloping demand curve for household j's labor

$$N_t^j = \left(\frac{W_t^j}{W_t} \right)^{-\varepsilon_w} N_t^d . \tag{2.3}$$

2.2 Calvo pricing

In case of Calvo pricing, the household is not able to readjust its wage in any given period with probability θ_w. Therefore, it chooses today's optimal wage W_t^* to maximize the expected utility over the states of the world where this wage is operative:

$$\max_{W_t^*} V_t = E_t \sum_{k=0}^{\infty} (\beta\theta_w)^k U\left(C_{t+k|t}^j, N_{t+k|t}^j, \cdot\right) , \tag{2.4}$$

where V is the utility function and U is the felicity function with partial derivatives $U_C > 0$ and $U_N < 0$. The dot denotes additional variables potentially entering the felicity function (e.g. lagged consumption in the case of habits), and where $0 < \beta < 1$ is the (growth-adjusted) discount factor. The subscript $t + k|t$ indicates a variable in period $t + k$ conditional on having last reset the wage at time t. When choosing the optimal wage W_t^*, the household does so taking into account the demand for its labor services

$$N_{t+k|t}^j = \left(\frac{W_{t+k|t}^j}{W_{t+k}}\right)^{-\varepsilon_w} N_{t+k}^d , \tag{2.5}$$

where the wage operative in period $t + k$, $W_{t+k|t}^j$, is given by the originally chosen wage W_t^* times a term $\Gamma_{t,t+k}^{ind}$ that reflects the indexing of wages to (past) inflation:

$$W_{t+k|t}^j = \Gamma_{t,t+k}^{ind} W_t^* . \tag{2.6}$$

We keep this term generic to encompass the varying indexing schemes in the literature and only require that there is full indexing in steady state, i.e. $\Gamma_k^{ind} = \Pi^k$.[6] Note that $\Gamma_{t,t}^{ind} = 1$. The final constraint of this problem is the budget constraint

$$(1 + \tau_{t+k}^c)P_{t+k}C_{t+k|t}^j = (1 - \tau_{t+k}^n)W_{t+k|t}^j N_{t+k|t}^j , \tag{2.7}$$

where the household earns income from supplying differentiated labor N_t^j at the nominal wage rate W_t^j, which is taxed at rate τ_t^n, and spends its income on consumption C_t^j, priced at the price of the final good P_t and taxed at rate τ_t^c. In this budget constraint all additive terms that drop from the current optimization problem when taking the derivative with respect to W_t^* (e.g. capital income or transfers) have been omitted.

[6] Our formulation encompasses, e.g., the partial indexation scheme of Smets and Wouters (2007), which is of the form $\Gamma_{t,t+k}^{ind} = \prod_{s=1}^{k} \Pi_{t+s-1}^{\iota} \Pi^{1-\iota}$, where $\iota \in [0, 1]$ denotes the degree of indexing to past inflation and Π without subscript denotes steady state inflation. Another indexing scheme nested is the one by Christiano, Eichenbaum, and Evans (2005), who use full indexation to past inflation with $\iota = 1$. The absence of indexing is characterized by $\iota = 0$ and $\Pi = 1$ so that $\Gamma_{t,t+k}^{ind} = 1 \,\forall\, k$.

Define the after-tax marginal rate of substitution as

$$MRS_{t+k|t} = -\frac{\left(1+\tau^c_{t+k}\right)}{\left(1-\tau^n_{t+k}\right)} \frac{U_{N,t+k|t}}{V_{C,t+k|t}} \ , \tag{2.8}$$

where subscripts C and N denote partial derivatives and the index j has been suppressed.[7] The first order condition for the optimal wage W_t^* can then be written as

$$0 = \sum_{k=0}^{\infty} (\beta\theta_w)^k E_t \left[N_{t+k|t} \frac{V_{C,t+k|t}(1-\tau^n_{t+k})}{(1+\tau^c_{t+k})} \left(\frac{\varepsilon_w}{\varepsilon_w - 1} MRS_{t+k|t} - \frac{\Gamma^{ind}_{t,t+k} W_t^*}{P_{t+k}} \right) \right] \ , \tag{2.9}$$

which shows that the household chooses the optimal wage to achieve a desired markup over the weighted average of expected future marginal rates of substitution.

Performing a log-linearization around the deterministic steady state[8] yields

$$\hat{W}_t^* = (1-\beta\theta_w) \sum_{k=0}^{\infty} (\beta\theta_w)^k E_t \left[\widehat{MRS}_{t+k|t} + \hat{P}_{t+k} - \hat{\Gamma}^{ind}_{t,t+k} \right] \ , \tag{2.10}$$

where hats denote percentage deviations from the steady state. In order to derive the New Keynesian Wage Phillips Curve, one needs to express the previous equation recursively and aggregate over households j. Aggregation in particular implies replacing the idiosyncratic marginal rate of substitution $\widehat{MRS}_{t+k|t}$ by an expression not depending on the initial period in which household j last reset the wage.

Log-linearizing (2.8) around the deterministic steady state (denoted with omitted time indices), and combining it with the assumption of complete markets and equal initial wealth, yields[9]

$$\widehat{MRS}_{t+k|t} = \widehat{MRS}_{t+k} + \underbrace{\left[-\frac{V_{CN} \times N}{V_{CC} \times C} \varepsilon_c^{mrs} + \varepsilon_n^{mrs} \right]}_{\equiv \varepsilon_{tot}^{mrs}} \left(\hat{N}_{t+k|t} - \hat{N}_{t+k} \right) \ , \tag{2.11}$$

where ε_n^{mrs} and ε_c^{mrs} denote the steady state elasticities of the marginal rate of substitution with respect to labor and consumption, respectively, and ε_{tot}^{mrs} is the total elasticity of the

[7]This formulation allows for non-time separable utility in consumption as introduced by habits, but excludes habits in leisure (e.g. Uhlig 2007).

[8]Depending on the exact conduct of monetary policy, e.g., in case of an interest rate rule, the steady state of nominal variables like P_t and W_t may not be well-defined (see e.g. Galí 2008). Linearization in this case can be interpreted as being done around the long-run trend of the nominal variables. Linearization around a proper steady state would involve rewriting the problem in terms of stationary variables like the real wage W_t/P_t and inflation rates, but would yield the same results as trend changes only appear as ratios and therefore cancel out.

[9]The computational steps here follow Sbordone (2006).

MRS. The latter simplifies to ε_n^{mrs} in the case of additively separable preferences as in EHL, because $V_{CN} = 0$. $\widehat{MRS}_{t+k}$ is the average MRS in the economy.

Equation (2.11) together with the log-linearized demand function for labor of variety j, (2.5), and the log-linearized indexing equation, (2.6), can be used to yield the recursive representation

$$\hat{W}_t^* = (1 - \beta\theta_w) \left\{ \hat{W}_t + \frac{\widehat{MRS}_t - \left(\hat{W}_t - \hat{P}_t\right)}{1 + \varepsilon_w \varepsilon_{tot}^{mrs}} \right\} + \beta\theta_w E_t \left(\hat{W}_{t+1}^* - \hat{\Gamma}_{t,t+1}^{ind}\right) \ , \tag{2.12}$$

where we have made use of $\hat{\Gamma}_{t,t+k}^{ind} = \hat{\Gamma}_{t,t+1}^{ind} + \hat{\Gamma}_{t+1,t+k}^{ind}$ and $\hat{\Gamma}_{t,t}^{ind} = 0$.

Using the linearized law of motion for the aggregate wage level

$$\hat{W}_t^* = \frac{1}{1 - \theta_w} \hat{W}_t - \frac{\theta_w}{1 - \theta_w} (\hat{\Gamma}_{t-1,t}^{ind} + \hat{W}_{t-1}) \tag{2.13}$$

and defining wage inflation $\Pi_{w,t} = \frac{W_t}{W_{t-1}}$, the New Keynesian Wage Phillips Curve follows after some tedious algebra as

$$\hat{\Pi}_t^w = \beta E_t \hat{\Pi}_{w,t+1} - \frac{(1 - \theta_w)(1 - \beta\theta_w)}{\theta_w (1 + \varepsilon_w \varepsilon_{tot}^{mrs})} \hat{\mu}_t^w - \frac{\beta\theta_w}{1 - \theta_w} E_t \hat{\Gamma}_{t,t+1}^{ind} + \frac{\theta_w}{1 - \theta_w} \hat{\Gamma}_{t-1,t}^{ind} \ , \tag{2.14}$$

where $\hat{\mu}_t^w$ defines the deviation of the wedge between the average marginal rate of substitution and the real wage from its long-run value, i.e. the steady state markup:

$$\hat{\mu}_t^w \equiv \left(\hat{W}_t - \hat{P}_t\right) - \widehat{MRS}_t \ . \tag{2.15}$$

Equation (2.14) has the familiar intuition that if $\hat{\mu}_t^w < 0$, the wage markup is below its long-run value, inducing wage setters ceteris paribus to adjust wages upwards, leading to wage inflation.

2.3 Rotemberg pricing

In case of Rotemberg pricing the problem of household j is choosing W_t^j to maximize

$$V_t = E_t \sum_{k=0}^{\infty} \beta^k U\left(C_{t+k}^j, N_{t+k}^j, \cdot\right) \ , \tag{2.16}$$

taking into account the demand for its labor variety

$$N_{t+k}^{j} = \left(\frac{W_{t+k}^{j}}{W_{t+k}} \right)^{-\varepsilon_w} N_{t+k}^{d} \tag{2.17}$$

and subject to the relevant parts of the budget constraint

$$(1 + \tau_{t+k}^{c}) P_{t+k} C_{t+k}^{j} = (1 - \tau_{t+k}^{n}) W_{t+k}^{j} N_{t+k}^{j} - \frac{\phi_w}{2} \left(\Pi^{-1} \frac{W_{t+k}^{j}}{W_{t+k-1}^{j}} - 1 \right)^{2} \Xi_{t+k} \, . \tag{2.18}$$

Here, the last term represents the quadratic Rotemberg costs of adjusting the wage, with ϕ_w being the Rotemberg wage adjustment cost parameter. The costs are proportional to the nominal adjustment cost base Ξ_t and arises whenever wage changes differ from the steady state inflation rate Π. After imposing symmetry and making use of the definition of the after-tax MRS, equation (2.8), the resulting FOC can be written as

$$\begin{aligned} 0 = \varepsilon_w \frac{MRS_t}{\frac{W_t}{P_t}} (1 - \tau_t^n) + & \left\{ (1 - \varepsilon_w)(1 - \tau_t^n) - \phi_w \left(\Pi^{-1} \Pi_{w,t} - 1 \right) \Pi_t \frac{1}{N_t} \Pi^{-1} \frac{\frac{\Xi_t}{P_t}}{\frac{W_{t-1}}{P_{t-1}}} \right\} \\ + E_t \beta \frac{V_{C,t+1}}{V_{C,t}} \frac{(1 + \tau_t^c)}{(1 + \tau_{t+1}^c)} \frac{1}{N_t} \frac{1}{\frac{W_t}{P_t}} & \left\{ \phi_w \left(\Pi^{-1} \Pi_{w,t+1} - 1 \right) \Pi^{-1} \Pi_{w,t+1} \frac{\Xi_{t+1}}{P_{t+1}} \right\} \, . \end{aligned} \tag{2.19}$$

Linearizing (2.19) around the steady state, ignoring inconsequential tax changes, and making use of the definition of $\hat{\mu}_t^w$, (2.15), yields

$$\hat{\Pi}_{w,t} = \beta E_t \hat{\Pi}_{w,t+1} - \frac{(\varepsilon_w - 1)(1 - \tau^n) \aleph}{\phi_w} \hat{\mu}_t^w \, , \tag{2.20}$$

where $\aleph \equiv \frac{N \times W}{\Xi}$ denotes the steady state share of the wage bill in the adjustment cost base. Most papers assume that wage adjustment costs are proportional to either current or steady state output.[10] Thus, the real steady state adjustment costs base Ξ/P is equal to output Y, which is produced via a production function of the type $Y = F(K, N) - \Phi$, where F is a constant returns to scale production function and $\Phi \geq 0$ denotes the fixed cost in production. The literature typically either abstracts from fixed costs, i.e. $\Phi = 0$, or sets them to the value of monopolistic pure profits so that there is no incentive for entry or exit in steady state. In that case, $\Phi = \varepsilon_p^{-1} Y$, where $\varepsilon_p > 0$ is the elasticity of substitution between monopolistically competitive intermediate goods firms. Steady state

[10]For the purpose of this paper it is only important that this term is exogenous from the perspective of the wage setting household so that the effects of household decisions on it are not internalized.

output then is $Y = \frac{\varepsilon_p - 1}{\varepsilon_p} F(K, N)$.

With firms choosing a gross markup of $\varepsilon_p/(\varepsilon_p - 1)$ over marginal cost, $\aleph$ is given by

$$\aleph = \frac{WN}{\Xi} = \frac{\frac{\varepsilon_p - 1}{\varepsilon_p} F_N N}{Y} = \begin{cases} \frac{\varepsilon_p - 1}{\varepsilon_p} (1 - \alpha) , & \text{if } \Phi = 0 , \\ (1 - \alpha) , & \text{if } \Phi = \varepsilon_p^{-1} Y . \end{cases} \tag{2.21}$$

Here, $1 - \alpha$ denotes the steady state elasticity of the production function with respect to labor, e.g. the labor exponent in a Cobb-Douglas production function. Expression (2.21) shows that the relevant steady state labor share $\aleph$ is bigger in case of fixed costs, because net output Y in the denominator only includes capital and labor payments, while in case of no fixed costs, it also includes pure profits. Hence, the slope of the Wage Phillips Curve is ceteris paribus flatter in the absence of fixed costs.

2.4 Comparison

Comparing the slopes of the two Wage Phillips Curves, equations (2.14) and (2.20), yields

$$\frac{(1 - \theta_w)(1 - \beta\theta_w)}{\theta_w (1 + \varepsilon_w \varepsilon_{tot}^{mrs})} = \frac{(\varepsilon_w - 1)(1 - \tau^n)\aleph}{\phi_w} , \tag{2.22}$$

from which the Rotemberg wage adjustment cost parameter ϕ_w implied by a particular Calvo wage duration θ_w can be inferred as

$$\phi_w = \frac{(\varepsilon_w - 1)(1 - \tau^n)\aleph}{(1 - \theta_w)(1 - \beta\theta_w)} \theta_w (1 + \varepsilon_w \varepsilon_{tot}^{mrs}) . \tag{2.23}$$

The left-hand side of equation (2.22) shows that, similar to the case of the New Keynesian Price Phillips curve, the discount factor β and the Calvo wage duration θ_w determine the slope of Wage Phillips Curve in the Calvo case. But there is an additional correction factor in the denominator depending on the elasticity of substitution ε_w and the total elasticity of the marginal rate of substitution, ε_{tot}^{mrs}. This correction factor arises from the EHL setup due to the idiosyncratic marginal rate of substitution being used to evaluate the labor-leisure tradeoff. Table 1 displays the respective expressions for ε_{tot}^{mrs} for different felicity functions (see Appendix C for details). In case of standard additively separable preferences and for Greenwood, Hercowitz, and Huffman (1988)-preferences, ε_{tot}^{mrs} simply corresponds

Table 1: Elasticity ε_{tot}^{mrs} for different felicity functions

	$U(C,N)$	ε_{tot}^{mrs}	Habits
Add. separable	$\frac{C^{1-\sigma}-1}{1-\sigma}-\psi\frac{N^{1+\varphi}}{1+\varphi}$	φ	✓
GHH (1988)	$\frac{\left(C-\psi N^{1+\varphi}\right)^{1-\sigma}-1}{1-\sigma}$	φ	✓
Add. sep., log leisure	$\frac{C^{1-\sigma}-1}{1-\sigma}+\psi\log\left(1-N\right)$	$\frac{N}{1-N}$	✓
Multipl. separable	$\frac{\left(C^{\eta}\left(1-N\right)^{1-\eta}\right)^{1-\sigma}-1}{1-\sigma}$	$\left[1-\frac{\left(1-\eta\right)\left(\sigma-1\right)}{\eta(1-\sigma)-1}\right]\times\frac{N}{1-N}$	✓$^{(*)}$

Notes: Total elasticity of the after-tax marginal rate of substitution, ε_{tot}^{mrs}, for additively separable preferences in consumption and hours worked (first row), for Greenwood, Hercowitz, and Huffman (1988)-type preferences (second row), additively separable preferences in consumption and log leisure (third row), and multiplicative preferences (fourth row). The last column indicates whether the computed elasticity is robust to the inclusion of internal or external habits in consumption of the form $C_t-\phi_c C_{t-1}$. (*) For multiplicatively separable preferences, the resulting expression becomes somewhat more complex, see Appendix C.1.2.

to the inverse Frisch-elasticity parameter φ. For additively separable preferences with log leisure, the total elasticity is pinned down by the ratio of hours worked to leisure. For multiplicatively separable Cobb-Douglas-type preferences, ε_{tot}^{mrs} depends on the degree of risk aversion, the weight of leisure in the utility function, and the ratio of hours worked to leisure.

With Frisch elasticity estimates ranging from 0.75 using micro data (Chetty, Guren, Manoli, and Weber 2011) to 2-4 using macro data (e.g. Smets and Wouters 2007; King and Rebelo 1999) as well as a share of hours worked in total time of 0.2 to 0.33, plausible values for the elasticity range between 0.25 and 1.5. With multiplicative preferences, realistic calibrations are in the same range as those obtained for separable preferences. For example, Backus, Kehoe, and Kydland (1992) use $\sigma=2$, $\eta=0.34$, and $N/(1-N)=0.5$ so that $\varepsilon_{tot}^{mrs}\approx 0.75$.[11]

For the Rotemberg case on the right-hand side of (2.22), the slope depends on the

[11]The lower bound is obtained with $\varepsilon_{tot}^{mrs}=0$ for $\sigma=0$, reaches $\varepsilon_{tot}^{mrs}=N/(1-N)=0.5$ for $\sigma=1$ (i.e. the additively separable case) and then keeps increasing.

elasticity of substitution, the Rotemberg adjustment cost parameter ϕ_w, and on the share of the wage bill in the adjustment cost tax base $\aleph$. In contrast to the previously considered Calvo case, the slope of the Wage Phillips Curve with Rotemberg pricing is decreasing in the labor tax rate τ^n. The reason is that the labor tax rate drives a wedge between the real wage and the marginal rate of substitution. In the limit case of $\tau^n \to 1$, it does not pay off for the household to invest any resources into changing the nominal wage. Wage inflation then becomes completely decoupled from $\hat{\mu}_t$.[12]

Two remarks are in order. The first, technical one, is that Rotemberg wage adjustment cost estimates from papers abstracting from labor taxes cannot be simply used in models with such taxes, because they will correspond to a flatter Phillips curve than intended. The second point is an economic one. If one believes that the Rotemberg price adjustment cost parameter is structural, then equation (2.20) implies that permanent increases in labor taxes can flatten the Wage Phillips Curve. Therefore, if presumed permanent, the gradual increase of labor taxes in the U.S. from below 15% before 1960 to its new plateau of about 23% is, ceteris paribus, associated with a flattening of the Wage Phillips Curve of 8 percentage points in this framework.

3 New Keynesian Phillips Curve in the SGU-setup

In this section we first derive the slope of the New Keynesian Wage Phillips curve in the Schmitt-Grohé and Uribe (2006b)-setup under Calvo pricing and under Rotemberg pricing.

3.1 Setup

Schmitt-Grohé and Uribe (2006b) assume that the economy is populated by a household with a continuum of members that supply the same homogenous labor service N_t, have the same consumption level due to insurance within the household, and work the same

[12] The same does not hold true for the time-dependent Calvo wage adjustment. Whenever the household is allowed to reset its wage, it can do so costlessly. For that reason, as shown in (2.23), the wage adjustment cost parameter ϕ_w implied by a particular Calvo duration, which appears in the denominator of (2.20), is decreasing at rate $(1 - \tau^n)$, canceling the overall effect of τ^n.

amount of hours. This contrasts with EHL, where households supply differentiated labor services and insurance takes place via complete markets.[13] The homogenous labor input in the SGU setup is supplied to a labor union that takes its members utility into account and acts as a monopoly supplier of a continuum of j differentiated labor services N_t^j. These differentiated labor services are bundled into a composite labor input by intermediate goods producers exactly as in the EHL setup in section 2.

The household has lifetime utility function

$$V_t = E_t \sum_{k=0}^{\infty} \beta^k U(C_{t+k}, N_{t+k}, \cdot) \ , \tag{3.1}$$

where $N_{t+k} = \int_0^1 N_{t+k}^j \, dj$ is the market clearing condition assuring that total hours worked across all markets equal the supply by households. The relevant part of the household's nominal budget constraint is

$$(1+\tau_t^c) P_t C_t \leq (1-\tau_t^n) \int_0^1 W_t^j N_t^j \, dj \ , \tag{3.2}$$

where the household earns income from differentiated labor N_t^j at the nominal wage rate W_t^j through the labor services supplied by the union.

3.2 Calvo pricing

The labor union chooses the optimal wage W_t^* in all labor markets where it is able to reoptimize in order to maximize its members' utility, equation (3.1). It takes into account the demand for labor variety j, equation (2.5), and the relevant part of the budget constraint (3.2):

$$(1+\tau_{t+k}^c) P_{t+k} C_{t+k} = \left(1-\tau_{t+k}^n\right) W_{t+k}^{\varepsilon_w} N_{t+k}^d \theta_w^k \left(\Gamma_{t,t+k}^{ind} W_t^*\right)^{1-\varepsilon_w} \ . \tag{3.3}$$

The latter makes use of the fact that, at each point in time $t+k$, the union is able to reset the wage in a fraction $1-\theta_w$ of labor markets, which therefore become irrelevant for the wage setting decision at time t. This leaves a fraction θ_w^k of labor markets where the

[13] Galí (2015) provides a different microfoundation of the EHL setup. He assumes a household with j members, each supplying a differentiated labor service, who are perfectly insured within the family. He then pairs this with j labor unions responsible for the wage setting in market j. Because unions only take the utility of their members into account, i.e. use the idiosyncratic MRS, this setup isomorphic to EHL.

time t optimal wage W_t^* is still active. The FOC of the problem is given by

$$0 = E_t \sum_{k=0}^{\infty} (\beta\theta_w)^k V_{C,t+k} N_{t+k}^d \left(\frac{W_{t+k}}{\Gamma_{t,t+k}^{ind}}\right)^{\varepsilon_w} \frac{1-\tau_{t+k}^n}{1+\tau_{t+k}^c} \left[MRS_{t+k} \frac{\varepsilon_w}{\varepsilon_w - 1} - \Gamma_{t,t+k}^{ind} \frac{W_t^*}{P_{t+k}}\right] . \quad (3.4)$$

After some tedious algebra the New Keynesian Wage Phillips Curve follows as

$$\hat{\Pi}_t^w = \beta E_t \hat{\Pi}_{t+1}^w - \frac{(1-\beta\theta_w)(1-\theta_w)}{\theta_w} \hat{\mu}_t^w - \frac{\beta\theta_w}{1-\theta_w} E_t \hat{\Gamma}_{t,t+1}^{ind} + \frac{\theta_w}{1-\theta_w} \hat{\Gamma}_{t-1,t}^{ind} . \quad (3.5)$$

Comparing the slope of the Wage Phillips Curve in (3.5) to the one of EHL in (2.14), the EHL slope is smaller by a factor of $(1 + \varepsilon_w \varepsilon_n^{mrs})^{-1}$.

3.3 Rotemberg pricing

The Rotemberg problem of the labor union is similar to the household wage setting problem in the EHL case. The relevant part of the budget constraint is given by

$$(1+\tau_t^c) P_t C_t = (1-\tau_t^n) \int_0^1 W_t^j N_t^j \, dj - \frac{\phi_w}{2} \int_0^1 \left(\Pi^{-1} \frac{W_t^j}{W_{t-1}^j} - 1\right)^2 dj \; \Xi_t . \quad (3.6)$$

Following the steps outlined in section 2.3, it can be verified that this leads to the same Wage Phillips Curve as in the EHL case

$$\hat{\Pi}_{w,t} = \beta E_t \hat{\Pi}_{w,t+1} - \frac{(\varepsilon_w - 1)(1-\tau^n)\aleph}{\phi_w} \hat{\mu}_t^w \quad (2.20)$$

3.4 Comparison

Comparison of the slopes of the two Wage Phillips Curves, equations (3.5) and (2.20), yields an expression for the Rotemberg parameter ϕ_w implied by a Calvo wage duration θ_w in the SGU setup:

$$\phi_w = \frac{(\varepsilon_w - 1)(1-\tau^n)\aleph}{(1-\theta_w)(1-\beta\theta_w)} \theta_w \; , \quad (3.7)$$

which differs from the EHL case, equation (2.23). The latter has an additional term $(1 + \varepsilon_w \varepsilon_n^{mrs})$ arising from the different insurance scheme.

Table 2: Implied Rotemberg adjustment cost parameters ϕ_w (quarterly model)

	$\varepsilon_{tot}^{mrs} = 0.25$	$\varepsilon_{tot}^{mrs} = 1$	$\varepsilon_{tot}^{mrs} = 1.5$	$\beta = 0.985$	$\beta = 0.99$	$\beta = 0.995$
SGU	61.36	61.36	61.36	60.48	61.36	62.27
EHL	230.10	736.31	1073.79	725.74	736.31	747.19
	$\varepsilon_w = 6$	$\varepsilon_w = 11$	$\varepsilon_w = 21$	$\tau^n = 0$	$\tau^n = 0.21$	$\tau^n = 0.4$
SGU	30.68	61.36	122.72	77.67	61.36	46.60
EHL	214.76	736.31	2699.81	932.04	736.31	559.22
	$\Phi = \varepsilon_w^{-1} Y$	$\Phi = 0$				
SGU	61.36	55.78				
EHL	736.31	669.37				

Notes: Implied Rotemberg wage adjustment cost parameter ϕ_w that corresponds to an implied Calvo wage duration of 4 quarters ($\theta_w = 0.75$) for different parameter values in the SGU and EHL framework. All other parameters are kept at their baseline value: $\beta = 0.99$, $\tau^n = 0.21$, $\varepsilon_w = \varepsilon_p = 11$, $\alpha = 0.3$, $\varepsilon_{tot}^{mrs} = 1$, $\Phi = \varepsilon_w^{-1} Y$.

4 Numerical example

Table 2 shows the implied Rotemberg wage adjustment cost parameter corresponding to an implied Calvo wage duration of 4 quarters ($\theta_w = 0.75$) for different parameter values in the SGU and EHL frameworks at quarterly frequency. All parameters except for the one under consideration are kept at their baseline values. For the baseline calibration we choose a discount factor of $\beta = 0.99$, corresponding to a 4% real interest rate. The labor tax rate is set to 0.21, which is the mean U.S. effective tax rate over the sample 1960Q1:2015Q4, computed following Jones (2002). The substitution elasticities are set to $\varepsilon_w = \varepsilon_p = 11$, implying a steady state markup of 10%. $\aleph$ is set to 2/3, corresponding to an exponent of capital in a Cobb-Douglas production function of $\alpha = 0.3$ and the presence of fixed costs that make steady state firm profits 0. The total elasticity of the marginal rate of substitution, ε_{tot}^{mrs}, is set to 1 as is the case with additively separable preferences and an inverse Frisch elasticity of $\varphi = 1$.

As can be seen in the rows labeled SGU and EHL, the particular household setup assumed makes a big difference due to the multiplicative $(1 + \varepsilon_w \varepsilon_{tot}^{mrs})$ factor appearing in the EHL-setup. For our baseline parameterization, this factor amounts to $1 + 11 \times 1 = 12$. This factor is also what makes the slope of the Wage Phillips Curve increase (almost) proportionally with the total elasticity of the marginal rate of substitution, ε_{tot}^{mrs}, in the EHL-setup (second row, left panel). In contrast, ε_{tot}^{mrs} does not affect the slope in the SGU case (first row, left panel). The implied Rotemberg parameter increases proportionally in the elasticity of substitution between goods ε_w for the SGU setup (third row, left panel). However, it increases overproportionally in the EHL setup (fourth row, left panel). Increasing ε_w by a factor of 3.5 from 6 to 21 results in an increase of the implied ϕ_w by a factor of 12.6. Assuming the absence of fixed costs, $\Phi = 0$, hardly changes the implied cost parameter in both setups for plausible calibrations (fifth and sixth rows, left panel). The first two rows of the right panel of Table 2 show that the effect of varying the discount factor β is relatively minor in both setups. Finally, the third and fourth rows of the right panel show that the steady state labor tax rate τ^n significantly impacts the implied Rotemberg costs parameter as already discussed in section 2.4.

5 Conclusion

We have provided applied researchers with guidance on how to translate a Calvo wage duration into an implied Rotemberg wage adjustment cost parameter by using the equivalence of their setups at first order. In doing so, we have shown that both the presence of labor taxation and the assumed household insurance scheme matter greatly for this mapping, giving rise to differences of up to one order of magnitude. Our results account for the inclusion of wage indexing, habit formation in consumption, and the presence of fixed costs in production, features commonly used in medium-scale New Keynesian DSGE models.

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A EHL algebra

A.1 Calvo

The Lagrangian for the EHL Calvo setup is given by

$$L = \sum_{k=0}^{\infty} (\beta\theta_w)^k E_t \Bigg[U \left(C_{t+k|t} \ , \left(\frac{\Gamma^{ind}_{t,t+k} W^*_t}{W_{t+k}} \right)^{-\varepsilon_w} N^d_{t+k}, \cdot \right)$$

$$- \lambda_{t+k|t} \left\{ (1+\tau^c_{t+k}) P_{t+k} C_{t+k|t} - (1-\tau^n_{t+k}) \Gamma^{ind}_{t,t+k} W^*_t \left(\frac{\Gamma^{ind}_{t,t+k} W^*_t}{W_{t+k}} \right)^{-\varepsilon_w} N^d_{t+k} \right\} \Bigg] \ , \quad \text{(A.1)}$$

where $\lambda_{t+k|t}$ is the Lagrange multiplier and the j index has been suppressed. The FOC for consumption is given by

$$(1+\tau^c_{t+k}) \lambda_{t+k|t} P_{t+k} = V_{C,t+k|t} \ . \quad \text{(A.2)}$$

The FOC for W^*_t is given by

$$0 = \sum_{k=0}^{\infty} (\beta\theta_w)^k E_t \Bigg[U_N \left(C_{t+k|t}, \left(\frac{\Gamma^{ind}_{t,t+k} W^*_t}{W_{t+k}} \right)^{-\varepsilon_w} N^d_{t+k}, \cdot \right) (-\varepsilon_w) \left(\frac{\Gamma^{ind}_{t,t+k} W^*_t}{W_{t+k}} \right)^{-\varepsilon_w} \frac{N^d_{t+k}}{W^*_t}$$

$$+ \lambda_{t+k|t} \left\{ (1-\varepsilon_w)(1-\tau^n_{t+k}) \Gamma^{ind}_{t,t+k} \left(\frac{\Gamma^{ind}_{t,t+k} W^*_t}{W_{t+k}} \right)^{-\varepsilon_w} N^d_{t+k} \right\} \Bigg] \ , \quad \text{(A.3)}$$

where U_N denotes the partial derivative of the felicity function with respect to N. Using (2.5) and suppressing the arguments of the felicity function this can be rewritten as:

$$0 = \sum_{k=0}^{\infty} (\beta\theta_w)^k E_t \left[N_{t+k|t} \lambda_{t+k|t} \left(\frac{\varepsilon_w}{\varepsilon_w - 1} \frac{U_{N,t+k|t}}{\lambda_{t+k|t}} + (1-\tau^n_{t+k}) \Gamma^{ind}_{t,t+k} W^*_t \right) \right] \ . \quad \text{(A.4)}$$

Replacing $\lambda_{t+k|t}$ using (A.2) yields

$$0 = \sum_{k=0}^{\infty} (\beta\theta_w)^k E_t \left[N_{t+k|t} \frac{V_{C,t+k|t}(1-\tau^n_{t+k})}{(1+\tau^c_{t+k})} \left(\frac{\varepsilon_w}{\varepsilon_w - 1} \frac{U_{N,t+k|t}(1+\tau^c_{t+k})}{V_{C,t+k|t}(1-\tau^n_{t+k})} + \frac{\Gamma^{ind}_{t,t+k} W^*_t}{P_{t+k}} \right) \right] \ . \quad \text{(A.5)}$$

Making use of the definition of the after-tax marginal rate of substitution

$$MRS_{t+k|t} = - \frac{\left(1+\tau^c_{t+k}\right)}{\left(1-\tau^n_{t+k}\right)} \frac{U_{N,t+k|t}}{V_{C,t+k|t}} \quad \text{(2.8)}$$

this yields

$$0 = \sum_{k=0}^{\infty} (\beta\theta_w)^k E_t \left[N_{t+k|t} \frac{V_{C,t+k|t}(1-\tau^n_{t+k})}{(1+\tau^c_{t+k})} \left(\frac{\varepsilon_w}{\varepsilon_w - 1} MRS_{t+k|t} - \frac{\Gamma^{ind}_{t,t+k} W^*_t}{P_{t+k}} \right) \right] \ . \quad \text{(A.6)}$$

Performing a log-linearization around the deterministic steady state yields

$$0 = \sum_{k=0}^{\infty} (\beta\theta_w)^k \, E_t \left[\frac{\varepsilon_w}{\varepsilon_w - 1} MRS \times \widehat{MRS}_{t+k|t} - \Gamma_k^{ind} \frac{W^*}{P} \left(\hat{W}_t^* - \hat{P}_{t+k} + \hat{\Gamma}_{t,t+k}^{ind} \right) \right] \tag{A.7}$$

or

$$\hat{W}_t^* = (1 - \beta\theta_w) \sum_{k=0}^{\infty} (\beta\theta_w)^k \, E_t \left[\widehat{MRS}_{t+k|t} + \hat{P}_{t+k} - \hat{\Gamma}_{t,t+k}^{ind} \right] \, . \tag{2.10}$$

Expand $MRS_{t+k|t}(C_{t+k|t}, N_{t+k|t})$ by the average MRS in the economy

$$MRS_{t+k|t} = \frac{MRS_{t+k|t}}{MRS_{t+k}} MRS_{t+k} \tag{A.8}$$

and log-linearize around the deterministic steady state:[14]

$$\widehat{MRS}_{t+k|t} = \varepsilon_c^{mrs} \left(\hat{C}_{t+k|t} - \hat{C}_{t+k} \right) + \varepsilon_n^{mrs} \left(\hat{N}_{t+k|t} - \hat{N}_{t+k} \right) + \widehat{MRS}_{t+k} \, , \tag{A.9}$$

where $\varepsilon_c^{mrs} \equiv (MRS_C \times C)/MRS$ and $\varepsilon_n^{mrs} \equiv (MRS_N \times N)/MRS$ denote the elasticities of the MRS with respect to C and N, respectively. Due to the required assumption of complete markets and equal initial wealth, marginal utilities are equal across households. Therefore

$$V_{C,t+k} = V_{C,t+k|t} \tag{A.10}$$

and log-linearized

$$V_{CC} C \hat{C}_{t+k} + V_{CN} N \hat{N}_{t+k} = V_{CC} C \hat{C}_{t+k|t} + V_{CN} N \hat{N}_{t+k|t} \, . \tag{A.11}$$

Rearranging

$$V_{CC} C \left(\hat{C}_{t+k|t} - \hat{C}_{t+k} \right) = -V_{CN} N \left(\hat{N}_{t+k|t} - \hat{N}_{t+k} \right) \tag{A.12}$$

and plugging into (A.9) yields

$$\widehat{MRS}_{t+k|t} = \widehat{MRS}_{t+k} + \underbrace{\left[-\frac{V_{CN} N}{V_{CC} C} \varepsilon_c^{mrs} + \varepsilon_n^{mrs} \right]}_{\equiv \varepsilon_{tot}^{mrs}} \left(\hat{N}_{t+k|t} - \hat{N}_{t+k} \right) \, . \tag{2.11}$$

This together with the linearized labor demand

$$\hat{N}_{t+k|t} = -\varepsilon_w \left(\hat{\Gamma}_{t,t+k}^{ind} + \hat{W}_t^* - \hat{W}_{t+k} \right) + \hat{N}_{t+k}^d \tag{A.13}$$

and the fact that up to first order wage dispersion is zero and therefore $N_{t+k}^d = N_{t+k}$ can be used to express the idiosyncratic MRS as

$$\widehat{MRS}_{t+k|t} = \widehat{MRS}_{t+k} - \varepsilon_w \varepsilon_{tot}^{mrs} \left(\hat{\Gamma}_{t,t+k}^{ind} + \hat{W}_t^* - \hat{W}_{t+k} \right) \, . \tag{A.14}$$

[14] If the MRS depends on additional variables like housing or durables, the same approach as in the following can be followed to replace the idiosyncratic MRS by the aggregate one.

Plug into (2.10) to get

$$\hat{W}_t^* = (1-\beta\theta_w)\left(\hat{W}_t + \frac{1}{1+\varepsilon_w\varepsilon_{tot}^{mrs}}\left(\widehat{MRS}_t - \left(\hat{W}_t - \hat{P}_t\right)\right)\right) + \beta\theta_w E_t\left(\hat{W}_{t+1}^* - \hat{\Gamma}_{t,t+1}^{ind}\right) . \tag{2.12}$$

Next, plug in from the linearized LOM for wages in the economy

$$\hat{W}_t^* = \frac{1}{1-\theta_w}\hat{W}_t - \frac{\theta_w}{1-\theta_w}(\hat{\Gamma}_{t-1,t}^{ind} + \hat{W}_{t-1}) \tag{2.13}$$

to get

$$\begin{aligned}\frac{1}{1-\theta_w}\hat{W}_t - \frac{\theta_w}{1-\theta_w}\left(\hat{\Gamma}_{t-1,t}^{ind} + \hat{W}_{t-1}\right) &= (1-\beta\theta_w)\left(\hat{W}_t - \frac{1}{1+\varepsilon_w\varepsilon_{tot}^{mrs}}\hat{\mu}_t^w\right) \\ &+ \beta\theta_w E_t\left(-\hat{\Gamma}_{t,t+1}^{ind} + \frac{1}{1-\theta_w}\hat{W}_{t+1} - \frac{\theta_w}{1-\theta_w}\left(\hat{\Gamma}_{t,t+1}^{ind} + \hat{W}_t\right)\right) .\end{aligned} \tag{A.15}$$

Now add 0 to the left-hand side and expand the right-hand side:

$$\begin{aligned}&\frac{1}{1-\theta_w}\hat{W}_t - \frac{\theta_w}{1-\theta_w}\left(\hat{\Gamma}_{t-1,t}^{ind} + \hat{W}_{t-1}\right) + \left(\frac{1}{1-\theta_w}\hat{W}_{t-1} - \frac{1}{1-\theta_w}\hat{W}_{t-1}\right) \\ &= (1-\beta\theta_w)\hat{W}_t - \beta\theta_w\left(\frac{\theta_w}{1-\theta_w}\left(E_t\hat{\Gamma}_{t,t+1}^{ind} + \hat{W}_t\right) + \frac{1-\theta_w}{1-\theta_w}E_t\hat{\Gamma}_{t,t+1}^{ind}\right) \\ &+ \frac{\beta\theta_w}{1-\theta_w}E_t\left(\hat{W}_{t+1}\right) - \frac{(1-\beta\theta_w)}{1+\varepsilon_w\varepsilon_{tot}^{mrs}}\hat{\mu}_t^w .\end{aligned} \tag{A.16}$$

Factor the left-hand side and collect terms related to W_t on the right-hand side

$$\begin{aligned}&\frac{1}{1-\theta_w}\left(\hat{W}_t - \hat{W}_{t-1}\right) + \hat{W}_{t-1} - \frac{\theta_w}{1-\theta_w}\hat{\Gamma}_{t-1,t}^{ind} \\ &= \underbrace{\left(\frac{1-\beta\theta_w - \theta_w(1-\beta\theta_w) - \beta\theta_w\theta_w}{1-\theta_w}\right)}_{1-\frac{\beta\theta_w}{1-\theta_w}}\hat{W}_t \\ &- \frac{\beta\theta_w}{1-\theta_w}E_t\hat{\Gamma}_{t,t+1}^{ind} + \frac{\beta\theta_w}{1-\theta_w}E_t\left(\hat{W}_{t+1}\right) - \frac{(1-\beta\theta_w)}{1+\varepsilon_w\varepsilon_{tot}^{mrs}}\hat{\mu}_t^w .\end{aligned} \tag{A.17}$$

Subtract W_t from both sides

$$\begin{aligned}&\frac{1}{1-\theta_w}\left(\hat{W}_t - \hat{W}_{t-1}\right) - \frac{\theta_w}{1-\theta_w}\hat{\Gamma}_{t-1,t}^{ind} - \left(\hat{W}_t - \hat{W}_{t-1}\right) \\ &= \frac{\beta\theta_w}{1-\theta_w}E_t\left(\hat{W}_{t+1} - \hat{W}_t\right) - \frac{(1-\beta\theta_w)}{1+\varepsilon_w\varepsilon_{tot}^{mrs}}\hat{\mu}_t^w - \frac{\beta\theta_w}{1-\theta_w}E_t\hat{\Gamma}_{t,t+1}^{ind} .\end{aligned} \tag{A.18}$$

Collecting terms:

$$\begin{aligned}&\frac{\theta_w}{1-\theta_w}\left(\hat{W}_t - \hat{W}_{t-1}\right) - \frac{\theta_w}{1-\theta_w}\hat{\Gamma}_{t-1,t}^{ind} \\ &= \frac{\beta\theta_w}{1-\theta_w}E_t\left(\hat{W}_{t+1} - \hat{W}_t\right) - \frac{(1-\beta\theta_w)}{1+\varepsilon_w\varepsilon_{tot}^{mrs}}\hat{\mu}_t^w - \frac{\beta\theta_w}{1-\theta_w}E_t\hat{\Gamma}_{t,t+1}^{ind} .\end{aligned} \tag{A.19}$$

Solve for wage inflation:

$$\hat{\Pi}_t^w = \beta E_t \hat{\Pi}_{w,t+1} - \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w (1+\varepsilon_w \varepsilon_{tot}^{mrs})} \hat{\mu}_t^w - \frac{\beta\theta_w}{1-\theta_w} E_t \hat{\Gamma}_{t,t+1}^{ind} + \frac{\theta_w}{1-\theta_w} \hat{\Gamma}_{t-1,t}^{ind} \ . \tag{2.14}$$

A.2 Rotemberg

The Lagrangian for the EHL Rotemberg setup is given by

$$L = \sum_{k=0}^{\infty} \beta^k E_t \left[\begin{array}{l} U\left(C_{t+k}, \left(\frac{W_{t+k}^j}{W_{t+k}}\right)^{-\varepsilon_w} N_{t+k}^d \right) \\ -\lambda_{t+k} \left\{ \begin{array}{l} \left(1+\tau_{t+k}^c\right) P_{t+k} C_{t+k}^j - \left(1-\tau_{t+k}^n\right) W_{t+k}^j \left(\frac{W_{t+k}^j}{W_{t+k}}\right)^{-\varepsilon_w} N_{t+k}^d \\ + \frac{\phi_w}{2} \left(\Pi^{-1} \frac{W_{t+k}^j}{W_{t+k-1}^j} - 1 \right)^2 \Xi_{t+k} \end{array} \right\} \end{array} \right] . \tag{A.20}$$

The FOC for consumption is given by

$$(1+\tau_{t+k}^c)\lambda_{t+k}^j P_{t+k} = V_{C,t+k} \ . \tag{A.21}$$

The corresponding FOC for the optimal wage is given by

$$\begin{aligned} 0 \ &= U_N \left(C_t^j, \left(\frac{W_t^j}{W_t}\right)^{-\varepsilon_w} N_t^d, \cdot \right) (-\varepsilon_w) \left(\frac{W_t^j}{W_t}\right)^{-\varepsilon_w} \frac{N_t^d}{W_t^j} \\ &+ \lambda_t^j \left\{ (1-\varepsilon_w)(1-\tau_t^n) \left(\frac{W_t^j}{W_t}\right)^{-\varepsilon_w} N_t^d - \phi_w \left(\Pi^{-1} \frac{W_t^j}{W_{t-1}^j} - 1 \right) \frac{\Pi^{-1}\Xi_t}{W_{t-1}^j} \right\} \\ &- E_t \lambda_{t+1}^j \left\{ \phi_w \left(\Pi^{-1} \frac{W_{t+1}^j}{W_t^j} - 1 \right) (-1) \frac{W_{t+1}^j}{\left(W_t^j\right)^2} \Pi^{-1} \Xi_{t+1} \right\} \ . \end{aligned} \tag{A.22}$$

As there is no wage dispersion in the Rotemberg case, imposing symmetry means that $N_t^j = N_t^d = N_t$. Additionally substituting for λ_t from (A.21) and dividing by $V_{C,t}/(1+\tau_t^c)$, the above equation can be written as

$$\begin{aligned} 0 \ &= \frac{U_{N,t}}{V_{C,t}} (1+\tau_t^c)(-\varepsilon_w) \frac{N_t}{W_t} + \frac{1}{P_t} \left\{ (1-\varepsilon_w)(1-\tau_t^n) N_t - \phi_w \left(\Pi^{-1} \frac{W_t}{W_{t-1}} - 1 \right) \frac{\Pi^{-1}\Xi_t}{W_{t-1}} \right\} \\ &+ E_t \beta \frac{V_{C,t+1}}{V_{C,t}} \frac{(1+\tau_t^c)}{(1+\tau_{t+1}^c)} \frac{1}{W_t} \left\{ \phi_w \left(\Pi^{-1} \frac{W_{t+1}}{W_t} - 1 \right) \frac{W_{t+1}}{W_t} \frac{\Pi^{-1}\Xi_{t+1}}{P_{t+1}} \right\} \ , \end{aligned} \tag{A.23}$$

or, dividing by N_t, multiplying by P_t, and making use of the definition of the after-tax MRS (2.8), as

$$\begin{aligned}
0 =& \varepsilon_w \frac{MRS_t}{\frac{W_t}{P_t}}(1-\tau_t^n) + \left\{ (1-\varepsilon_w)(1-\tau_t^n) - \phi_w \left(\Pi^{-1}\Pi_{w,t} - 1\right) \Pi_t \frac{1}{N_t} \Pi^{-1} \frac{\frac{\Xi_t}{P_t}}{\frac{W_{t-1}}{P_{t-1}}} \right\} \\
&+ E_t \beta \frac{V_{C,t+1}}{V_{C,t}} \frac{(1+\tau_t^c)}{(1+\tau_{t+1}^c)} \frac{1}{N_t} \frac{1}{\frac{W_t}{P_t}} \left\{ \phi_w \left(\Pi^{-1}\Pi_{w,t+1} - 1\right) \Pi^{-1}\Pi_{w,t+1} \frac{\Xi_{t+1}}{P_{t+1}} \right\} \qquad (2.19)
\end{aligned}$$

Linearizing (2.19) around the steady state, ignoring inconsequential tax changes, and making use of (2.15), yields

$$\begin{aligned}
0 \ =& \ \underbrace{\varepsilon_w \frac{MRS}{\frac{W}{P}}}_{\frac{\varepsilon_w - 1}{\varepsilon_w}} (1-\tau_t^n)(-1)\hat{\mu}_t^w \\
&- \phi_w \underbrace{\left(\Pi^{-1}\Pi_w - 1\right)}_{0} \Pi^{-1}\Pi \frac{1}{N} \frac{\frac{\Xi}{P}}{\frac{W}{P}} \left(\hat{\Pi}_t - \hat{N}_t + \hat{\Xi}_t - \hat{W}_t\right) \\
&- \phi_w \Pi^{-1}\Pi \frac{1}{N} \Pi^{-1} \frac{\frac{\Xi}{P}}{\frac{W}{P}} \Pi^{-1}\Pi_w \hat{\Pi}_{w,t} \\
&+ E_t \beta \frac{1}{N} \frac{1}{\frac{W}{P}} \phi_w \underbrace{\left(\Pi^{-1}\Pi_w - 1\right)}_{0} \Pi^{-1}\Pi_w \frac{\Xi}{P} \left(\hat{V}_{C,t+1} - \hat{V}_{C,t} - \hat{N}_t - \hat{W}_t - \hat{\Xi}_{t+1}\right) \\
&+ E_t \beta \frac{1}{N} \frac{1}{\frac{W}{P}} \phi_w \Pi^{-1} \frac{\Xi}{P} \left(2\Pi^{-1}\Pi_w^2 \hat{\Pi}_{w,t+1} - \Pi_w \hat{\Pi}_{w,t+1}\right) \ . \qquad (A.24)
\end{aligned}$$

Simplifying and using the steady state relation $\Pi = \Pi_w$ yields

$$0 = (-1)\varepsilon_w \frac{\varepsilon_w - 1}{\varepsilon_w}(1-\tau^n)\hat{\mu}_t^w - \phi_w \underbrace{\frac{\frac{\Xi}{P}}{N\frac{W}{P}}}_{\frac{1}{\aleph}} \hat{\Pi}_{w,t} + E_t \beta \frac{\frac{\Xi}{P}}{N\frac{W}{P}} \phi_w \hat{\Pi}_{w,t+1} \qquad (A.25)$$

and thus

$$\hat{\Pi}_{w,t} = \beta E_t \hat{\Pi}_{w,t+1} - \frac{(\varepsilon_w - 1)(1-\tau^n)\aleph}{\phi_w} \hat{\mu}_t^w \ . \qquad (2.20)$$

B SGU algebra

B.1 Calvo

The associated Lagrangian is given by

$$L = \sum_{k=0}^{\infty} \beta^k E_t \Bigg[U\left(C_{t+k}\ , N_{t+k}, \cdot\right) \\ - \lambda_{t+k} \left\{ (1+\tau^c_{t+k}) P_{t+k} C_{t+k} - \left(1-\tau^n_{t+k}\right) W^{\varepsilon_w}_{t+k} N^d_{t+k} \theta^k_w \left(\Gamma^{ind}_{t,t+k} W^*_t\right)^{1-\varepsilon_w} \right\} \Bigg] , \tag{B.1}$$

where in the budget constraint we have made use of

$$\int_0^1 W^j_{t+k} N^j_{t+k} dj = \int_0^1 W^j_{t+k} \left(\frac{W^j_{t+k}}{W_{t+k}}\right)^{-\varepsilon_w} N^d_{t+k} dj \\ = W^{\varepsilon_w}_{t+k} N^d_{t+k} \int_0^1 \left(W^j_{t+k}\right)^{1-\varepsilon_w} dj = W^{\varepsilon_w}_{t+k} N^d_{t+k} \left(\theta^k_w \left(\Gamma^{ind}_{t,t+k} W^*_t\right)^{1-\varepsilon_w} + \left(1-\theta^k_w\right) X_{1,t+k}\right) \tag{B.2}$$

The last term, $X_{1,t+k}$, captures the wage level in the other labor markets where price resetting has taken place. Hence, it is independent of W^*_t and can be omitted as it drops out when taking the derivative.

The FOC for consumption is given by

$$(1+\tau^c_{t+k}) \lambda_{t+k} P_{t+k} = V_{C,t+k}\ , \tag{B.3}$$

while the FOC for W^*_t is given by

$$0 = \sum_{k=0}^{\infty} \beta^k E_t \Bigg[U_{N,t+k} \frac{\partial N_{t+k}}{\partial W^*_t} \\ + \lambda_{t+k} \left\{ (1-\varepsilon_w)\left(1-\tau^n_{t+k}\right) W^{\varepsilon_w}_{t+k} N^d_{t+k} \theta^k_w \left(\Gamma^{ind}_{t,t+k}\right)^{1-\varepsilon_w} \left(W^*_t\right)^{-\varepsilon_w} \right\} \Bigg] . \tag{B.4}$$

Making use of

$$N_{t+k} \equiv \int_0^1 N^j_{t+k} dj = \int_0^1 \left(\frac{W^j_{t+k}}{W_{t+k}}\right)^{-\varepsilon_w} N^d_{t+k}\ dj = W^{\varepsilon_w}_{t+k} N^d_{t+k} \int_0^1 \left(W^j_{t+k}\right)^{-\varepsilon_w} dj \\ = W^{\varepsilon_w}_{t+k} N^d_{t+k} \left(\theta^k_w \left(\Gamma^{ind}_{t,t+k} W^*_t\right)^{-\varepsilon_w} + \left(1-\theta^k_w\right) X_{2,t+k}\right)\ , \tag{B.5}$$

we can evaluate the inner derivative in the first line of (B.4) to get

$$0 = \sum_{k=0}^{\infty} \beta^k E_t \Bigg[U_{N,t+k} \left(-\varepsilon^w\right) \frac{N^d_{t+k}}{W^{-\varepsilon^w}_{t+k}} \theta^k_w \left(\Gamma^{ind}_{t,t+k}\right)^{-\varepsilon_w} \left(W^*_t\right)^{-\varepsilon_w - 1}$$
$$+ \lambda_{t+k} \left\{ (1-\varepsilon_w)\,(1-\tau^n_{t+k}) W^{\varepsilon_w}_{t+k} N^d_{t+k} \theta^k_w \left(\Gamma^{ind}_{t,t+k}\right)^{1-\varepsilon_w} \left(W^*_t\right)^{-\varepsilon_w} \right\} \Bigg] . \tag{B.6}$$

Factoring out, and multiplying by $\left(W^*_t\right)^{-\varepsilon_w - 1}$ yields

$$0 = \sum_{k=0}^{\infty} \left(\beta\theta_w\right)^k E_t \lambda_{t+k} N^d_{t+k} W^{\varepsilon_w}_{t+k} \left(\Gamma^{ind}_{t,t+k}\right)^{-\varepsilon_w}$$
$$\times \left[\frac{U_{N,t+k}}{\lambda_{t+k}} \left(-\varepsilon_w\right) + \left(1-\tau^n_{t+k}\right) \left(1-\varepsilon_w\right) \Gamma^{ind}_{t,t+k} W^*_t \right] \tag{B.7}$$

or

$$0 = \sum_{k=0}^{\infty} \left(\beta\theta_w\right)^k E_t \lambda_{t+k} N^d_{t+k} W^{\varepsilon_w}_{t+k} \left(1-\tau^n_{t+k}\right) \left(\Gamma^{ind}_{t,t+k}\right)^{-\varepsilon_w}$$
$$\times \left[\frac{U_{N,t+k}\left(1+\tau^c_{t+k}\right) P_{t+k}}{V_{C,t+k}\left(1-\tau^n_{t+k}\right)} \left(-\varepsilon_w\right) + \left(1-\varepsilon_w\right) \Gamma^{ind}_{t,t+k} W^*_t \right] . \tag{B.8}$$

Using the after-tax MRS definition, this is equal to

$$0 = E_t \sum_{k=0}^{\infty} (\beta\theta_w)^k V_{C,t+k} N^d_{t+k} W^{\varepsilon_w}_{t+k} \frac{1-\tau^n_{t+k}}{1+\tau^c_{t+k}} \left(\Gamma^{ind}_{t,t+k}\right)^{-\varepsilon_w} \left[MRS_{t+k} \frac{\varepsilon_w}{\varepsilon_w - 1} - \Gamma^{ind}_{t,t+k} \frac{W^*_t}{P_{t+k}} \right] . \tag{3.4}$$

Performing a log-linearization around the deterministic steady state yields

$$0 = \sum_{k=0}^{\infty} \left(\beta\theta_w\right)^k E_t \left[\frac{\varepsilon_w}{\varepsilon_w - 1} \times MRS\widehat{MRS}_{t+k} - \Gamma^{ind}_k \frac{W^*}{P} \left(\hat{W}^*_t - \hat{P}_{t+k} + \hat{\Gamma}^{ind}_{t,t+k} \right) \right] \tag{B.9}$$

or

$$\hat{W}^*_t = \left(1-\beta\theta_w\right) \sum_{k=0}^{\infty} \left(\beta\theta_w\right)^k E_t \left[\widehat{MRS}_{t+k} + \hat{P}_{t+k} - \hat{\Gamma}^{ind}_{t,t+k} \right] . \tag{B.10}$$

Note that compared to the EHL case, it is the economy-wide MRS that shows up here, not the individual one. Subtracting $\hat{W}_t$ from both sides and using (2.15), we can write this recursively as

$$\hat{W}^*_t - \hat{W}_t = -\beta\theta_w \hat{W}_t + \left(1-\beta\theta_w\right) \hat{\mu}^w_t \left(-1\right) + \beta\theta_w E_t \left(\hat{W}^*_{t+1} - \hat{\Gamma}^{ind}_{t,t+1} \right) . \tag{B.11}$$

Using (2.13) we obtain

$$\left(\frac{1}{1-\theta_w} \hat{W}_t - \frac{\theta_w}{1-\theta_w} \left(\hat{\Gamma}^{ind}_{t-1,t} + \hat{W}_{t-1} \right) \right) - \hat{W}_t \;=\; -\beta\theta_w \hat{W}_t + \left(1-\beta\theta_w\right) \hat{\mu}^w_t \left(-1\right)$$
$$+ \beta\theta_w E_t \left(\frac{1}{1-\theta_w} \hat{W}_{t+1} - \frac{\theta_w}{1-\theta_w} \left(\hat{\Gamma}^{ind}_{t,t+1} + \hat{W}_t \right) - \hat{\Gamma}^{ind}_{t,t+1} \right) \tag{B.12}$$

from which the New Keynesian Wage Phillips Curve follows as

$$\hat{\Pi}_t^w = \beta E_t \hat{\Pi}_{t+1}^w - \frac{(1-\beta\theta_w)(1-\theta_w)}{\theta_w}\hat{\mu}_t^w - \frac{\beta\theta_w}{1-\theta_w} E_t \hat{\Gamma}_{t,t+1}^{ind} + \frac{\theta_w}{1-\theta_w}\hat{\Gamma}_{t-1,t}^{ind} \ . \tag{3.5}$$

B.2 Rotemberg

The Lagrangian is

$$L = \sum_{k=0}^{\infty} \beta^k E_t \left[\begin{array}{l} U(C_{t+k}, N_{t+k}, \cdot) \\ -\lambda_{t+k} \left\{ \begin{array}{l} \left(1+\tau_{t+k}^c\right) P_{t+k} C_{t+k} - \left(1-\tau_{t+k}^n\right) N_{t+k}^d \int_0^1 W_{t+k}^j \left(\frac{W_{t+k}^j}{W_{t+k}}\right)^{-\varepsilon_w} dj \\ +\frac{\phi_w}{2}\int_0^1 \left(\Pi^{-1}\frac{W_{t+k}^j}{W_{t+k-1}^j} - 1\right)^2 dj \Xi_{t+k} \end{array} \right\} \end{array} \right] . \tag{B.13}$$

The corresponding first order condition for the optimal wage is given by

$$\begin{aligned} 0 \ &= U_{N,t}(-\varepsilon_w)\int_0^1 \left(\frac{W_t^j}{W_t}\right)^{-\varepsilon_w} \frac{N_t^d}{W_t^j}\ dj \\ &+ \lambda_t \left\{ (1-\varepsilon_w)(1-\tau_t^n) N_t^d \int_0^1 \left(\frac{W_t^j}{W_t}\right)^{-\varepsilon_w} dj - \phi_w \int_0^1 \left(\Pi^{-1}\frac{W_t^j}{W_{t-1}^j} - 1\right)\left(\frac{\Pi^{-1}}{W_{t-1}^j}\right) dj\ \Xi_t \right\} \\ &- E_t\lambda_{t+1}\left\{ \phi_w \int_0^1 \left(\Pi^{-1}\frac{W_{t+1}^j}{W_t^j} - 1\right)(-1)\frac{W_{t+1}^j}{\left(W_t^j\right)^2}\Pi^{-1}\ dj\ \Xi_{t+1} \right\} . \end{aligned} \tag{B.14}$$

Imposing symmetry

$$\begin{aligned} 0 = U_N(C_t, N_t, \cdot)(-\varepsilon_w)\frac{N_t^d}{W_t} + \lambda_t \left\{ (1-\varepsilon_w)(1-\tau_t^n) N_t^d - \phi_w \left(\Pi^{-1}\frac{W_t}{W_{t-1}} - 1\right)\left(\frac{\Pi^{-1}}{W_{t-1}}\right)\Xi_t \right\} \\ - E_t\lambda_{t+1}\left\{ \phi_w \left(\Pi^{-1}\frac{W_{t+1}}{W_t} - 1\right)(-1)\frac{W_{t+1}}{(W_t)^2}\Pi^{-1}\Xi_{t+1} \right\} , \end{aligned} \tag{B.15}$$

which is identical to equation (A.23).

C Elasticities of the after-tax MRS

C.1 Habits

C.1.1 Additively separable

First consider additively separable preferences with habits of the form

$$\frac{(C_t - \phi_c C_{t-1})^{1-\sigma} - 1}{1-\sigma} - \psi \frac{N^{1+\varphi}}{1+\varphi} \ , \tag{C.1}$$

where $0 \leq \phi_c \leq 1$ measures the degree of habits, $\varphi \geq 0$ is the inverse of the Frisch elasticity, $\sigma \geq 0$ determines the intertemporal elasticity of substitution, and $\psi > 0$ determines the weight of the disutility of labor.

If habits are internal, we get

$$V_{C_t} = (C_t - \phi_c C_{t-1})^{-\sigma} - \beta\phi_c \left(C_{t+1} - \phi_c C_t\right)^{-\sigma}$$

and in steady state

$$V_C = (1 - \beta\phi_c)\left((1-\phi_c)\, C\right)^{-\sigma} \ .$$

Similarly, the other partial derivatives are given by

$$\begin{aligned}
U_{N_t} &= -\psi N_t^{\varphi} \\
U_N &= -\psi N^{\varphi} \\
V_{C_t C_t} &= -\sigma (C_t - \phi_c C_{t-1})^{-\sigma-1} + \beta\phi_c^2 \left(-\sigma\right)\left(C_{t+1} - \phi_c C_t\right)^{-\sigma-1} \\
V_{CC} &= \left(1 + \beta\phi_c^2\right)(-\sigma)\left((1-\phi_c)\, C\right)^{-\sigma-1} \\
V_{CN} &= 0
\end{aligned}$$

The marginal rate of substitution and its derivatives follow as

$$\begin{aligned}
MRS &= \frac{1+\tau^c}{1-\tau^n} \frac{\psi N^{\varphi}}{(1-\beta\phi_c)\left((1-\phi_c)\, C\right)^{-\sigma}} \\
MRS_N &= \varphi \frac{1+\tau^c}{1-\tau^n} \frac{\psi N^{\varphi-1}}{(1-\beta\phi_c)\left((1-\phi_c)\, C\right)^{-\sigma}} \\
MRS_C &= \frac{1+\tau^c}{1-\tau^n} \psi N^{\varphi}\,(-1)\,\frac{1}{(V_C)^2} V_{CC}
\end{aligned}$$

Therefore,

$$\varepsilon_n^{mrs} = \varphi \tag{C.2}$$

Table 3: Elasticities ε_n^{mrs},ε_c^{mrs}, and ε_{tot}^{mrs} for different felicity functions

	Add. separable	Add. sep. log leisure	Mult. separable	GHH
U	$\frac{C^{1-\sigma}-1}{1-\sigma}-\psi\frac{N^{1+\varphi}}{1+\varphi}$	$\frac{C^{1-\sigma}-1}{1-\sigma}+\psi\log(1-N_t)$	$\frac{\left(C^{\eta}(1-N)^{1-\eta}\right)^{1-\sigma}}{1-\sigma}$	$\frac{\left(C-\psi N^{1+\varphi}\right)^{1-\sigma}-1}{1-\sigma}$
V_N	$-\psi N^{\varphi}$	$-\psi\frac{1}{1-N}$	$-(1-\eta)(1-\sigma)\frac{U}{1-N}$	$\left(C-\psi N^{1+\varphi}\right)^{-\sigma}$ $\times(-\psi)(1+\varphi)N^{\varphi}$
V_C	$C^{-\sigma}$	$C^{-\sigma}$	$\eta(1-\sigma)\frac{U}{C}$	$\left(C-\psi N^{1+\varphi}\right)^{-\sigma}$
V_{CC}	$-\sigma C^{-\sigma-1}$	$-\sigma C^{-\sigma-1}$	$\eta(1-\sigma)(\eta(1-\sigma)-1)\frac{U}{C^2}$	$-\sigma\left(C-\psi N^{1+\varphi}\right)^{-\sigma-1}$
V_{CN}	0	0	$\eta(1-\sigma)(1-\eta)(\sigma-1)$ $\times\frac{U}{C(1-N)}$	$\sigma\left(C-\psi N^{1+\varphi}\right)^{-\sigma-1}$ $\times\psi(1+\varphi)N^{\varphi}$
MRS	$\frac{1+\tau^c}{1-\tau^n}\frac{\psi N^{\varphi}}{C^{-\sigma}}$	$\frac{1+\tau^c}{1-\tau^n}\frac{\psi(1-N_t)^{-1}}{C^{-\sigma}}$	$\frac{1+\tau^c}{1-\tau^n}\frac{1-\eta}{\eta}\frac{C}{1-N}$	$\frac{1+\tau^c}{1-\tau^n}\psi(1+\varphi)N^{\varphi}$
MRS_N	$\varphi\frac{1+\tau^c}{1-\tau^n}\frac{\psi N^{\varphi-1}}{C^{-\sigma}}$	$\frac{1+\tau^c}{1-\tau^n}\frac{\psi(1-N_t)^{-2}}{C^{-\sigma}}$	$\frac{1+\tau^c}{1-\tau^n}\frac{1-\eta}{\eta}\frac{C}{(1-N)^2}$	$\frac{1+\tau^c}{1-\tau^n}\psi(1+\varphi)\varphi N^{\varphi-1}$
MRS_C	$\frac{1+\tau^c}{1-\tau^n}\sigma\frac{\psi N^{\varphi}}{C^{1-\sigma}}$	$\frac{1+\tau^c}{1-\tau^n}\sigma\frac{\psi(1-N)^{-1}}{C^{1-\sigma}}$	$\frac{1+\tau^c}{1-\tau^n}\frac{1-\eta}{\eta}\frac{(1-\phi_c)}{(1-N)}$	0
ε_n^{mrs}	φ	$\frac{N}{1-N}$	$\frac{N}{1-N}$	φ
ε_c^{mrs}	σ	σ	1	0
ε_{tot}^{mrs}	φ	$\frac{N}{1-N}$	$\left[1-\frac{(1-\eta)(\sigma-1)}{\eta(1-\sigma)-1}\right]\frac{N}{1-N}$	φ
ε_{tot}^{mrs} (int. habits)	φ	$\frac{N}{1-N}$	$\left[1-\frac{(1-\eta)(\sigma-1)}{\eta(1-\sigma)-1}\frac{(1-\phi_c)}{(1+\phi_c^2\beta)}\right]\frac{N}{1-N}$	φ
ε_{tot}^{mrs} (ext. habits)	φ	$\frac{N}{1-N}$	$\left[1-\frac{(1-\eta)(\sigma-1)}{\eta(1-\sigma)-1}(1-\phi_c)\right]\frac{N}{1-N}$	φ

Notes: Elasticities of the after-tax marginal rate of substitution with respect to hours worked, ε_n^{mrs}, with respect to consumption, ε_c^{mrs}, and the total elasticity, ε_{tot}^{mrs}, for additively separable preferences in consumption and hours worked (first column), additively separable preferences in consumption and log leisure (second column), for multiplicative preferences (third column), and Greenwood, Hercowitz, and Huffman (1988)-type preferences (fourth column). The last two rows display the total elasticity when internal or external habits in consumption of the form $C_t-\phi_c C_{t-1}$ are assumed.

and

$$\varepsilon_c^{mrs} = \frac{\frac{1+\tau^c}{1-\tau^n}(-U_N)\frac{-1}{(V_C)^2}V_{CC}}{\frac{1+\tau^c}{1-\tau^n}\frac{U_N}{V_C}}C = (-1)\frac{V_{CC}C}{V_C} = (-1)\frac{(1+\beta\phi_c^2)(-\sigma)\left((1-\phi_c)C\right)^{-\sigma-1}C}{(1-\beta\phi_c)\left((1-\phi_c)C\right)^{-\sigma}}$$
$$= \frac{1+\beta\phi_c^2}{1-\beta\phi_c}\frac{\sigma}{(1-\phi_c)} , \tag{C.3}$$

Because of $V_{CN} = 0$, we also have[15]

$$\varepsilon_{tot}^{mrs} = \varepsilon_n^{mrs} , \tag{C.6}$$

If habits are external, we get the partial derivatives

$$\begin{aligned}
V_{N_t} &= -\psi N_t^{\varphi} \\
V_N &= -\psi N^{\varphi} \\
V_{C_t} &= (C_t - \phi_c C_{t-1})^{-\sigma} \\
V_C &= ((1-\phi_c)C)^{-\sigma} \\
V_{C_tC_t} &= (-\sigma)(C_t - \phi_c C_{t-1})^{-\sigma-1} \\
V_{CC} &= (-\sigma)((1-\phi_c)C)^{-\sigma-1} \\
V_{CN} &= 0
\end{aligned}$$

and the marginal rate of substitution

$$\begin{aligned}
MRS &= \frac{1+\tau^c}{1-\tau^n}\frac{\psi N^{\varphi}}{((1-\phi_c)C)^{-\sigma}} \\
MRS_N &= \varphi\frac{1+\tau^c}{1-\tau^n}\frac{\psi N^{\phi-1}}{((1-\phi_c)C)^{-\sigma}} \\
MRS_C &= \frac{1+\tau^c}{1-\tau^n}\frac{\psi N^{\varphi}}{((1-\phi_c)C)^{-\sigma+1}}
\end{aligned}$$

and therefore

$$\varepsilon_c^{mrs} = (-1)\frac{V_{CC}C}{V_C} = (-1)\frac{(-\sigma)((1-\phi_c)C)^{-\sigma-1}C}{((1-\phi_c)C)^{-\sigma}} = \frac{\sigma}{(1-\phi_c)} \tag{C.7}$$

with similar expressions for log leisure. As a consequence, ε_n^{mrs} is the same as in the case

[15] A related functional form with unitary Frisch elasticity considers log utility in leisure:

$$\frac{(C_t - \phi_c C_{t-1})^{1-\sigma} - 1}{1-\sigma} - \psi\log(1-N) . \tag{C.4}$$

and yields

$$\varepsilon_{tot}^{mrs} = \varepsilon_n^{mrs} = N/(1-N) \tag{C.5}$$

.

of internal habits and, because of $V_{CN} = 0$, we also have

$$\varepsilon_{tot}^{mrs} = \varepsilon_{n}^{mrs} \,. \tag{C.8}$$

C.1.2 Multiplicatively separable

Consider a multiplicative felicity function[16] with habits

$$U_t = \frac{\left((C_t - \phi_c C_{t-1})^{\eta} (1-N)^{1-\eta} \right)^{1-\sigma}}{1-\sigma} = \frac{(C_t - \phi_c C_{t-1})^{\eta(1-\sigma)} (1-N)^{(1-\eta)(1-\sigma)}}{1-\sigma} \,, \tag{C.9}$$

where $0 \leq \phi_c \leq 1$ measures the degree of habits, $0 \leq \eta \leq 1$ determines the weight of leisure, and $\sigma \geq 0$ determines the intertemporal elasticity of substitution.

If habits are internal, we have

$$\begin{aligned}
V_{N_t} &= (1-\eta)\,(C_t - \phi_c C_{t-1})^{\eta(1-\sigma)}\,(-1)\,(1-N_t)^{(1-\eta)(1-\sigma)-1} \\
&= -\,(1-\eta)\,(1-\sigma)\,\frac{U_t}{(1-N_t)} \\
V_N &= -\,(1-\eta)\,((1-\phi_c)\,C)^{\eta(1-\sigma)}(1-N)^{(1-\eta)(1-\sigma)-1} \\
&= -\,(1-\eta)\,(1-\sigma)\,\frac{U}{(1-N)} \\
V_{C_t} &= \eta\,(C_t - \phi_c C_{t-1})^{\eta(1-\sigma)-1}\,(1-N_t)^{(1-\eta)(1-\sigma)} \\
&\quad - \phi_c \beta \eta\,(C_{t+1} - \phi_c C_t)^{\eta(1-\sigma)-1}\,(1-N_{t+1})^{(1-\eta)(1-\sigma)} \\
&= \eta\,(1-\sigma)\left(\frac{U_t}{C_t - \phi_c C_{t-1}} - \beta\phi_c \frac{U_{t+1}}{C_{t+1} - \phi_c C_t}\right) \\
V_C &= \eta\,(1-\phi_c\beta)\,((1-\phi_c)\,C)^{\eta(1-\sigma)-1}\,(1-N_t)^{(1-\eta)(1-\sigma)} \\
&= \eta\,(1-\sigma)\,(1-\phi_c\beta)\,\frac{U}{(1-\phi_c)C} \\
V_{C_tC_t} &= \eta\,(\eta\,(1-\sigma)-1)\,(C_t - \phi_c C_{t-1})^{\eta(1-\sigma)-2}(1-N_t)^{(1-\eta)(1-\sigma)} \\
&\quad - \phi_c\beta\eta\,(\eta\,(1-\sigma)-1)\,(-\phi_c)\,(C_{t+1} - \phi_c C_t)^{\eta(1-\sigma)-2}(1-N_{t+1})^{(1-\eta)(1-\sigma)} \\
V_{CC} &= \eta\,(\eta\,(1-\sigma)-1)\left(1+\phi_c^2\beta\right)((1-\phi_c)\,C)^{\eta(1-\sigma)-2}(1-N_t)^{(1-\eta)(1-\sigma)} \\
&= \frac{(\eta\,(1-\sigma))\,(\eta\,(1-\sigma)-1)\,(1+\phi_c^2\beta)\,U}{((1-\phi_c)C)^2} \\
V_{C_tN_t} &= \eta\,(C_t - \phi_c C_{t-1})^{\eta(1-\sigma)-1}\,(1-\eta)\,(1-\sigma)\,(-1)\,(1-N_t)^{(1-\eta)(1-\sigma)-1} \\
V_{CN} &= \eta\,((1-\phi_c)\,C)^{\eta(1-\sigma)-1}\,(1-\eta)\,(\sigma-1)\,(1-N)^{(1-\eta)(1-\sigma)-1} \\
&= (\eta\,(1-\sigma))\,(1-\eta)\,(\sigma-1)\,\frac{U}{(1-\phi_c)C\,(1-N)}
\end{aligned}$$

[16] It has e.g. been used by Backus et al. (1992).

Therefore,

$$
\begin{aligned}
MRS &= \frac{1+\tau^c}{1-\tau^n}\frac{1-\eta}{\eta}\frac{(1-\phi_c)C}{(1-\phi_c\beta)(1-N)} \\
MRS_N &= \frac{1+\tau^c}{1-\tau^n}\frac{1-\eta}{\eta}\frac{(1-\phi_c)C}{(1-\phi_c\beta)(1-N)^2} \\
MRS_C &= \frac{1+\tau^c}{1-\tau^n}\frac{1-\eta}{\eta}\frac{1-\phi_c}{(1-\phi_c\beta)(1-N)}
\end{aligned}
$$

and

$$
\varepsilon_n^{mrs} = \frac{\frac{1+\tau^c}{1-\tau^n}\frac{1-\eta}{\eta}\frac{(1-\phi_c)C}{(1-\phi_c\beta)(1-N)^2}N}{\frac{1+\tau^c}{1-\tau^n}\frac{1-\eta}{\eta}\frac{(1-\phi_c)C}{(1-\phi_c\beta)(1-N)}} = \frac{N}{1-N} \tag{C.10}
$$

$$
\varepsilon_c^{mrs} = \frac{\frac{1+\tau^c}{1-\tau^n}\frac{1-\eta}{\eta}\frac{1-\phi_c}{1-\phi_c\beta}\frac{1}{1-N}}{\frac{1+\tau^c}{1-\tau^n}\frac{1-\eta}{\eta}\frac{1-\phi_c}{1-\phi_c\beta}\frac{C}{1-N}}C = 1 \tag{C.11}
$$

Finally

$$
\begin{aligned}
\frac{V_{CN}}{V_{CC}} &= \frac{(\eta(1-\sigma))(1-\eta)(\sigma-1)\frac{U}{(1-\phi_c)C(1-N)}}{\frac{(\eta(1-\sigma))(\eta(1-\sigma)-1)(1+\phi_c^2\beta)U}{((1-\phi_c)C)^2}} \\
&= \frac{(1-\eta)(\sigma-1)}{\eta(1-\sigma)-1}\frac{(1-\phi_c)}{(1+\phi_c^2\beta)}\frac{C}{1-N}
\end{aligned}
$$

and

$$
\begin{aligned}
\varepsilon_{tot}^{mrs} &= -\frac{V_{CN}N}{V_{CC}C}\varepsilon_c^{mrs} + \varepsilon_n^{mrs} \\
&= -\frac{(1-\eta)(\sigma-1)}{\eta(1-\sigma)-1}\frac{(1-\phi_c)}{(1+\phi_c^2\beta)}\frac{CN}{(1-N)C} \times 1 + \frac{N}{1-N} \\
&= \left[1 - \frac{(1-\eta)(\sigma-1)}{\eta(1-\sigma)-1}\frac{(1-\phi_c)}{(1+\phi_c^2\beta)}\right]\frac{N}{1-N} \qquad \text{(C.12)}
\end{aligned}
$$

In case of $\sigma = 1$, i.e. log utility, utility becomes separable again and (C.12) reduces to (C.5).

With external habits,

$$
\begin{aligned}
V_{C_t} &= \eta \left(C_t - \phi_c C_{t-1}\right)^{\eta(1-\sigma)-1} \left(1 - N_t\right)^{(1-\eta)(1-\sigma)} \\
&= \eta \left(1-\sigma\right) \frac{U_t}{C_t - \phi_c C_{t-1}} \\
V_C &= \eta \left(\left(1-\phi_c\right) C\right)^{\eta(1-\sigma)-1} \left(1 - N_t\right)^{(1-\eta)(1-\sigma)} \\
&= \eta \left(1-\sigma\right) \frac{U}{(1-\phi_c)C} \\
V_{C_tC_t} &= \eta \left(\eta \left(1-\sigma\right) - 1\right) \left(C_t - \phi_c C_{t-1}\right)^{\eta(1-\sigma)-2} \left(1 - N_t\right)^{(1-\eta)(1-\sigma)} \\
V_{CC} &= \eta \left(\eta \left(1-\sigma\right) - 1\right) \left(\left(1-\phi_c\right) C\right)^{\eta(1-\sigma)-2} (1-N)^{(1-\eta)(1-\sigma)} \\
&= \frac{\left(\eta \left(1-\sigma\right)\right) \left(\eta \left(1-\sigma\right) - 1\right) U}{((1-\phi_c)C)^2} \\
V_{C_tN_t} &= \eta \left(C_t - \phi_c C_{t-1}\right)^{\eta(1-\sigma)-1} \left(1-\eta\right) \left(1-\sigma\right) \left(-1\right) \left(1 - N_t\right)^{(1-\eta)(1-\sigma)-1} \\
V_{CN} &= \eta \left(\left(1-\phi_c\right) C\right)^{\eta(1-\sigma)-1} \left(1-\eta\right) \left(\sigma-1\right) \left(1 - N\right)^{(1-\eta)(1-\sigma)-1} \\
&= \left(\eta \left(1-\sigma\right)\right) \left(1-\eta\right) \left(\sigma-1\right) \frac{U}{(1-\phi_c)C\left(1-N\right)} \ .
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
MRS &= \frac{1+\tau^c}{1-\tau^n} \frac{1-\eta}{\eta} \frac{(1-\phi_c)C}{1-N} \\
MRS_N &= \frac{1+\tau^c}{1-\tau^n} \frac{1-\eta}{\eta} \frac{(1-\phi_c)C}{1-N} \\
MRS_C &= \frac{1+\tau^c}{1-\tau^n} \frac{1-\eta}{\eta} \frac{1-\phi_c}{(1-N)}
\end{aligned}
$$

and

$$
\varepsilon_n^{mrs} = \frac{\frac{1+\tau^c}{1-\tau^n} \frac{1-\eta}{\eta} \frac{(1-\phi_c)C}{(1-N)^2} N}{\frac{1+\tau^c}{1-\tau^n} \frac{1-\eta}{\eta} \frac{(1-\phi_c)C}{(1-N)}} = \frac{N}{1-N} \tag{C.13}
$$

$$
\varepsilon_c^{mrs} = \frac{\frac{1+\tau^c}{1-\tau^n} \frac{1-\eta}{\eta} \frac{1-\phi_c}{1-N}}{\frac{1+\tau^c}{1-\tau^n} \frac{1-\eta}{\eta} \frac{(1-\phi_c)C}{1-N}} C = 1 \tag{C.14}
$$

Finally

$$
\begin{aligned}
\frac{V_{CN}}{V_{CC}} &= \frac{\left(\eta \left(1-\sigma\right)\right) \left(1-\eta\right) \left(\sigma-1\right) \frac{U}{(1-\phi_c)C(1-N)}}{\frac{(\eta(1-\sigma))(\eta(1-\sigma)-1)U}{((1-\phi_c)C)^2}} \\
&= \frac{\left(1-\eta\right) \left(\sigma-1\right)}{\eta \left(1-\sigma\right) - 1} \frac{(1-\phi_c)C}{1-N}
\end{aligned}
$$

$$
\begin{aligned}
\varepsilon_{tot}^{mrs} &= -\frac{V_{CN}N}{V_{CC}C}\varepsilon_c^{mrs} + \varepsilon_n^{mrs} \\
&= -\frac{(1-\eta)\,(\sigma-1)}{\eta(1-\sigma)-1}(1-\phi_c)\frac{CN}{(1-N)C} \times 1 + \frac{N}{1-N} \\
&= \left[1 - \frac{(1-\eta)\,(\sigma-1)}{\eta(1-\sigma)-1}(1-\phi_c)\right]\frac{N}{1-N}
\end{aligned}
\tag{C.15}
$$

C.1.3 GHH

Consider GHH preferences with habits of the form

$$
U = \frac{\left(C_t - \phi_c C_{t-1} - \psi N_t^{1+\varphi}\right)^{1-\sigma} - 1}{1-\sigma} - 1\ ,
\tag{C.16}
$$

where $0 \leq \phi_c \leq 1$ measures the degree of habits, $\varphi \geq 0$ is related to the Frisch elasticity, $\sigma \geq 0$ determines the intertemporal elasticity of substitution ($\sigma = 1$ corresponds to log utility), and $\psi > 0$ determines weight of the disutility of labor. In case of internal habits we get

$$
\begin{aligned}
V_{N_t} &= \left(C_t - \phi_c C_{t-1} - \psi N_t^{1+\varphi}\right)^{-\sigma}(-\psi)\,(1+\varphi)N_t^{\varphi} \\
V_N &= \left((1-\phi_c)C - \psi N^{1+\varphi}\right)^{-\sigma}(-\psi)\,(1+\varphi)N^{\varphi} \\
V_{C_t} &= \left(C_t - \phi_c C_{t-1} - \psi N_t^{1+\varphi}\right)^{-\sigma} - \beta\phi_c\left(C_{t+1} - \phi_c C_t - \psi N_{t+1}^{1+\varphi}\right)^{-\sigma} \\
V_C &= (1-\beta\phi_c)\left((1-\phi_c)C - \psi N^{1+\varphi}\right)^{-\sigma} \\
V_{C_tC_t} &= -\sigma\Big(C_t - \phi_c C_{t-1} - \psi N_t^{1+\varphi}\Big)^{-\sigma-1} - \beta\phi_c\,(-\sigma)\,(-\phi_c)\left(C_{t+1} - \phi_c C_t - \psi N_{t+1}^{1+\varphi}\right)^{-\sigma-1} \\
V_{CC} &= -\sigma\left(1+\beta\phi_c^2\right)\left((1-\phi_c)C - \psi N^{1+\varphi}\right)^{-\sigma-1} \\
V_{C_tN_t} &= -\sigma\Big(C_t - \phi_c C_{t-1} - \psi N_t^{1+\varphi}\Big)^{-\sigma-1}(-\psi)\,(1+\varphi)\,N_t^{\varphi} \\
V_{CN} &= \sigma\Big((1-\phi_c)\,C - \psi N^{1+\varphi}\Big)^{-\sigma-1}\psi\,(1+\varphi)\,N^{\varphi}
\end{aligned}
$$

and therefore

$$
\begin{aligned}
MRS &= \frac{1+\tau^c}{1-\tau^n}\frac{\left((1-\phi_c)C - \psi N^{1+\varphi}\right)^{-\sigma}\psi(1+\varphi)N^{\varphi}}{(1-\beta\phi_c)\left((1-\phi_c)C - \psi N^{1+\varphi}\right)^{-\sigma}} = \frac{1+\tau^c}{1-\tau^n}\frac{\psi(1+\varphi)N^{\varphi}}{(1-\beta\phi_c)} \\
MRS_N &= \frac{1+\tau^c}{1-\tau^n}\psi(1+\varphi)\varphi\frac{N^{\varphi-1}}{(1-\beta\phi_c)} \\
MRS_C &= 0
\end{aligned}
$$

and

$$\varepsilon_n^{mrs} = \frac{\frac{1+\tau^c}{1-\tau^n}\frac{\psi(1+\varphi)\varphi}{(1-\beta\phi_c)}N^{\varphi-1}N}{\frac{1+\tau^c}{1-\tau^n}\frac{\psi(1+\varphi)}{(1-\beta\phi_c)}N^{\varphi}} = \varphi \tag{C.17}$$

$$\varepsilon_c^{mrs} = 0 \tag{C.18}$$

and therefore

$$\varepsilon_{tot}^{mrs} = \varepsilon_n^{mrs}\,. \tag{C.19}$$

For external habits

$$\begin{aligned}
V_{N_t} &= \Big(C_t - \phi_c C_{t-1} - \psi N_t^{1+\varphi}\Big)^{-\sigma}(-\psi)(1+\varphi)N_t^{\varphi}\\
V_N &= \Big((1-\phi_c)C - \psi N^{1+\varphi}\Big)^{-\sigma}(-\psi)(1+\varphi)N^{\varphi}\\
V_{C_t} &= \Big(C_t - \phi_c C_{t-1} - \psi N_t^{1+\varphi}\Big)^{-\sigma}\\
V_C &= \Big((1-\phi_c)C - \psi N^{1+\varphi}\Big)^{-\sigma}\\
V_{C_tC_t} &= -\sigma\Big(C_t - \phi_c C_{t-1} - \psi N_t^{1+\varphi}\Big)^{-\sigma-1}\\
V_{CC} &= -\sigma\left((1-\phi_c)C - \psi N^{1+\varphi}\right)^{-\sigma-1}\\
V_{C_tN_t} &= -\sigma\Big(C_t - \phi_c C_{t-1} - \psi N_t^{1+\varphi}\Big)^{-\sigma-1}(-\psi)(1+\varphi)N_t^{\varphi}\\
V_{CN} &= \sigma\Big((1-\phi_c)C - \psi N^{1+\varphi}\Big)^{-\sigma-1}\psi(1+\varphi)N^{\varphi}
\end{aligned}$$

and therefore

$$\begin{aligned}
MRS &= \frac{1+\tau^c}{1-\tau^n}\frac{\left((1-\phi_c)C - \psi N^{1+\varphi}\right)^{-\sigma}\psi(1+\varphi)N^{\varphi}}{\left((1-\phi_c)C - \psi N^{1+\varphi}\right)^{-\sigma}} = \frac{1+\tau^c}{1-\tau^n}\psi(1+\varphi)N^{\varphi}\\
MRS_N &= \frac{1+\tau^c}{1-\tau^n}\psi(1+\varphi)\varphi N^{\varphi-1}\\
MRS_C &= 0
\end{aligned}$$

and

$$\varepsilon_n^{mrs} = \frac{\frac{1+\tau^c}{1-\tau^n}\psi(1+\varphi)\varphi N^{\varphi-1}N}{\frac{1+\tau^c}{1-\tau^n}\psi(1+\varphi)N^{\varphi}} = \varphi \tag{C.20}$$

$$\varepsilon_c^{mrs} = 0 \tag{C.21}$$

and therefore

$$\varepsilon_{tot}^{mrs} = \varepsilon_n^{mrs}\,. \tag{C.22}$$