

Notes on estimating models with unit-roots

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It is now possible to estimate with DYNARE models with unit-roots. These models must satisfy the following restriction: the dynamics of stochastic trends must be (log)–linear and the dynamics of the stationary part around the stochastic trends may be non-linear.

1 Steady state

For stationay models, DYNARE linearizes around the steady state, using the internal function `dynare_solve.m`. When the dynamics of the stochastic trends doesn't contain a deterministic trend (when it is a pure random walk without displacement), there is an infinity of value of \bar{y} satisfying $f(\bar{y}) = 0$ and it is possible to specify one of them in `INITVAL`.

When the stochastic trends contain a deterministic trend, there is no solution to the equation $f(\bar{y}) = 0$. It is then necessary to provide DYNARE with a hand written function called *fname_steadystate.m* that returns the steady state values of the stationary variables and 0 for nonstationary variables when the model is linearized or 1 for nonstationary variables when the model is log–linearized. The variables must be ordered in alphabetical order. See `/examples/fs2000a_steadystate.m` for an example. This procedure is cumbersome and we will be trying to automatize it in the future.

2 Estimation

For initializing the Kalman filter and for computing the smoother, DYNARE uses now the algorithm described in chapter 5 of J. Durbin and S.J. Koopman (2001) and in S.J. Koopman and J. Durbin (2003). Following the advice of these authors, we prefer it to the augmented filter advocated by De Jong. Note that the results are identical.

In DYNARE, it is necessary to declare the stochastic trends thru the following statement:

```
options__unit_root_vars = 'P_obs'; 'Y_obs';
```

This syntax is temporary and will be made more DYNARE-like.

It is also necessary to set the option `lik_init=2` in the estimation statement. In the future, this should be triggered automatically from the declaration of a stochastic trend.

It is also necessary to declare the deterministic components of the stochastic trends with the COEFF_TREND instruction (see /examples/fs2000/fs2000a.mod)

The new functions are

- DgesLikelihood.m: replaces mj_optnumlik for filtering
- DiffuseLikelihood1.m
- DiffuseLikelihood2.m
- DiffuseLikelihoodH1.m
- DiffuseLikelihoodH2.m
- DiffuseLikelihoodH3.m
- DgseSmoother.m: replaces mj_optnumlik for smoothing
- DiffuseKalmanSmoother.m
- DiffuseKalmanSmootherH.m

They still need thorough testing.

3 Examples

DYNARE is now able to estimate purely backward models. The first two examples are such to isolate the problematic of estimating nonstationary models from the difficulties of estimating forward-looking rational expectation models.

3.1 Two unrelated random walks

Artificial data (10000 observations) are generated by /examples/arima/mod1.mod for

$$\begin{aligned}dx_t &= 0.5dx_{t-1} + e_{x_t} \\ dy_t &= -0.3dy_{t-1} + e_{y_t} \\ x_t &= x_{t-1} + dx_t \\ y_t &= y_{t-1} + dy_t\end{aligned}$$

mod1a.mod estimates this model when dx_t and dy_t are observed, and mod1b.mod when the level of the data, x_t and y_t , are observed. As expected, the results are the same.

3.2 A cointegrated system

Artificial data (10000 observations) are generated by /examples/arima/mod2.mod for

$$\begin{aligned}dx_t &= 0.5dx_{t-1} - 0.1(x_{t-1} - y_{t-1}) + e_{x_t} \\dy_t &= -0.3dy_{t-1} + 0.2(x_{t-1} - y_{t-1}) + e_{y_t} \\x_t &= x_{t-1} + dx_t \\y_t &= y_{t-1} + dy_t\end{aligned}$$

mod2a.mod estimates this model when dx_t and dy_t are observed, and x and y are treated as unobservable variables. The estimation procedure fails as the data is first difference don't contain any information about the initial level of x and y .

mod2c.mod again estimates this model when dx_t and dy_t are observed, but the model is reparametrized so as to have the cointegration error as stationary unobservable variable.

mod2b.mod estimates the model with data in level: x_t and y_t , are observed. In this case, and on moderate samples (100 observations), data in level provide additional information on the cointegrating relationship.

3.3 Schorfheide, 2000

fs2000.mod estimates the CIA model of Schorfheide (2000) with observation of the rate of growth of prices and output. fs2000a.mod estimates with data in level. The posterior mode of parameters is somewhat different.

References

- J. Durbin and S.J. Koopman (2001) *Time Series Analysis by State Space Methods*. Oxford: Oxford University Press.
- S.J. Koopman and J. Durbin (2003) "Filtering and Smoothing of State Vector for Diffuse State Space Models" in *Journal of Time Series Analysis*, 24, pp. 85-98.