Tax Reduction Policies of the Productive Sector and Its Impacts on Brazilian Economy

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Abstract

There is a widespread feeling in Brazilian society that tax reform has become necessary. Analysts seek to mitigate the perverse impact of taxation on economic efficiency and competitiveness of the productive sector. In view of this, the objective of this work is to contribute to the discussion about tax reduction in the productive sector through a dynamic stochastic general equilibrium (DSGE) model. To achieve this purpose, two stochastic shocks will be analyzed in the tax rates changes on labor income and capital income. The results suggest that the tax reduction in the first tax is greater than the same effect in the second. In this first shock, there were increases in output, consumption and investment and decreases in public debt and government spending. In the second shock, the poor performance was related to low growth in the capital stock. The results of the tax revenues were similar for the two tax reductions. They showed alignment with the major tax reform proposals for Brazil, a decrease in direct taxes and an increase in indirect taxes.

Key-words: DSGE Models, Tax Reduction, Simulation.
JEL: C63, E37, E62

1 Introduction

Few topics were more discussed by Brazilian economists that the tax reforms of this country. The general feeling is that the Brazilian Constitution of 1988 initially created a system of insufficient funding for the size of the state. Because of this, the government had to create a series of taxes to supplement state funding without much concern about economic rules of taxation. The main result of this policy was a tax system that adversely affects the competitiveness of the productive sector, among other factors.

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The tax literature shows significant differences between the tax reforms in many countries in recent decades. Sandford (1993) contributed a summary listing common elements including reducing the number of tax rates and their maximum marginal value in the income tax of individuals; reduction in the aliquots of corporations; and an increased share of consumption taxes rather than income taxes.

Following this trend, this work aims to contribute to the discussion on tax reforms analyzing tax reductions in the productive sector through a DSGE model. To achieve this purpose, two stochastic shocks will be analysed: the shock from the tax rate on labor income; and the shock from the tax rate on capital income.

There is extensive literature about the possible impact of tax reforms in Brazil: Cavalcanti and Silva (2010); Santana, Cavalcanti and Paes (2012); Paes and Bugarin (2006); Pereira and Ferreira (2010); Araújo and Ferreira (1999); Lledo (2005); and Fochezatto and Salami (2009) evaluated the impacts of proposed reforms in the national tax system. Menezes and Barreto (1999) and Teles and Andrade (2006) simulated the combined effects of tax and pension reforms. The literature cited was basically built using models of overlapping generation (OLG). Instead, this work seeks to contribute to the discussion using a DSGE model.

The 1980s has witnessed a major breakthrough in the field of macroeconomic modeling. The first examples of this new methodology emerged from the models of real business cycles (RBC), primarily through the groundbreaking work of Kydland and Prescott (1982). Its builders were criticized for focusing the analysis on only one type of shock in a kind of economic structure and for not recognizing any active role for monetary policy. Therefore from the perspective of a central bank, it was difficult to see how these models could bring any positive contribution to the discussion of monetary policy.

Twenty years later, this controversy was completely dissipated. The main reason was that the methodological innovation overlying the RBC models brought the introduction of frictions that allowed the incorporation of Keynesian principles and new shocks to the initial modeling. The success of this new model made it possible for the main economic institutions to develop their own DSGE models as did Central Bank of Brazil (SAMBA), European Central Bank (NAWM), Bank of Canada (Totem), Bank of England (BEQM), Bank of Japan (JEM), Bank of Chile (MAS), European Community (QUESTIII) and the International Monetary Fund (GEM)). Nowadays, DSGE models are used to answer almost any behavior of an economic phenomenon, including issues related to fiscal policy.

This work begins with this introduction and section two presenting the economic model, with section three detailing the calibration process of the model structural parameters. The work continues with the results in section 4 and ends with the conclusions in section 5.

1Dynamic Stochastic General Equilibrium - The DSGE methodology attempts to explain aggregate economic phenomena, such as economic growth, business cycles, and the effects of monetary and fiscal policy. Based on macroeconomic models with microfoundations.

2Overlapping generations models is a modelling type that uses representative agents who live a long enough finite period of time to overlap with at least one period of life to another agent.
2 The Model

The economic model of this work is a small and closed economy with sectors for households (Ricardian and Non-Ricardian), firms, and government (Fiscal Authority, Social Security and Monetary Authority). Besides the inclusion of non-Ricardian agents, this model has two other frictions: monopolistic competition and staggered pricing *a la Calvo*. The latter friction aims to avoid the model to have a very fast adjustment in relation to shocks, a factor noticed in empirical evidence.

2.1 Households

The household sector is divided into two types of representative agents: Ricardian, and non-Ricardian. The Ricardian household represents the active workers who are the contributors to the pension system, forming a fraction of a \((1 - \omega)\) of the total population, while the non-Ricardian household features the inactive workers (retirees) formed by the remaining proportion of the population. The first type of household is able to maximize its intertemporal utility by choosing consumption, savings, investment and leisure. For saving, the household can choose between two different savings instruments - physical capital and government bonds. Briefly, with the disposable income after payment of taxes, the Ricardian household can purchase consumer goods, capital goods, and/or government bonds. On the other hand, the non-Ricardian household just allocates its income (social security benefits) in the acquisition of consumer goods.

2.1.1 Ricardian Households (R) - Workers Active (Taxpayers)

Relying on the behavior described about the households, the Ricardian agent chooses how much to consume, how much to work and how much to acquire financial assets and physical capital to maximize the discounted stream of the expected utility \(^3\)

\[
\max E_t \sum_{t=0}^{\infty} \beta^t S_t^C \left[ \frac{C_{R,t}^{1-\sigma}}{1-\sigma} - \frac{S_t L_t^{1+\psi}}{1+\psi} \right]
\]

subject to their budget constraint,

\(^3\)The most common utility function to represent the choices of Family Representative is the utility function of constant relative risk aversion (CRRA) (Gali, 2008; Lim and McNelis, 2008; Clarida et al, 2008; Gali and Monacelli, 2005; Christoffel and Kuester, 2000; Christoffel et al, 2009; Ravenna and Walsh, 2006). There are other common parameterizations for the utility function in the literature, examples: logarithmic utility function, \(U(C_t, L_t) = \ln C_t + \frac{L_t}{L_0} \ln(1 - L_0)\) (Hansen, 1985); and utility function that would be a combination of the logarithmic and of the CRRA, \(U(C_t, L_t) = \ln(C_t) - \frac{\nu}{1+\chi} L_t^{1+\chi}\) (Gertler and Karadi, 2011).

\(^4\)A utility function must have certain characteristics: \(U_C > 0\) and \(U_L < 0\), this means that consumption and labor have a positive and a negative effects, respectively, over the happiness of the households. On the other hand, \(U_{CC} < 0\) and \(U_{LL} < 0\), indicating that the utility function is concave. This represents that if the consumption increases the utility level also increases, but in a smaller and smaller proportion. Another assumption regarding the utility function says that this function is additionally separable in time. This assumption allows to speak of an instantaneous utility function, wherein the agent receives utility solely from consumption that performs at a given moment in time.
\[ P_t (1 + \tau_c) (C_{R,t} + I_t) + \frac{B_{t+1}}{R_t^t} = W_t L_t \left( 1 - \frac{\tau_l}{\phi_t^l} - \tau_p \right) + R_t K_t \left( 1 - \frac{\tau_k}{\phi_t^k} \right) + B_t \]  

(2)

and in relation to the following law of motion of capital,

\[ K_{t+1} = (1 - \delta) K_t + I_t \]

(3)

where \( E_t \) is the expectations operator, \( \beta \in (0,1) \) is the intertemporal discount factor, \( C_R \) is the consumption of Ricardian household, \( L \) is the labor, \( S^C \) is the intertemporal consumption shock, \( S^L \) is the shock on labor supply, \( \psi \) is the marginal disutility of labor and \( \sigma \) is the coefficient of relative risk aversion.

In the budget constraint, \( P \) is the general price level, \( I \) is the investment, \( B \) is the government bond maturing in one period, \( R^B \) is the rate of return on government bond (basic interest rate), \( W \) is the wage, \( R \) is the return to capital, \( K \) is the stock of capital, \( \phi^l \) and \( \phi^K \) are the stochastic components of the income tax on labor and income tax on capital, respectively. While \( \tau_c, \tau_l, \tau_k, \tau_p \) represent the static components of the tax on consumption, income tax on labor, income tax on capital and on social security contribution, respectively. In this work, is being adopted the convention that \( B_t \) is the nominal bond issued in \((t-1)\) and matured in \( t \). Then, \( B_{t+1} \) and \( K_{t+1} \) are decided in \( t \).

The Ricardian household purchases of consumer goods \( (C_R) \) and investment goods \( (I) \) at the price level \( (P) \), also buys or sells government bonds\( (B) \) maturing in one period. These bonds pay a risk-free rate \( (R^B) \), which is also controlled by the monetary authority.

This kind of household pays three types of taxes (consumption tax, income tax on labor and income tax on capital) and also contributes to social security. Its income comes from three sources: labor income, which depends on the level of nominal wages \( (W) \); return on capital rental to firms, which is a function of the rate of return to capital \( (R) \); and income from government bonds acquired in the previous period.

To solve the problem of the Ricardian household, a Lagrangian function is used:

\[ \mathcal{L} = E_t \sum_{t=0}^{\infty} \beta^t \left\{ S^C_t \left[ \frac{C_{R,t}^{1-\sigma}}{1-\sigma} - S^L_t \frac{K_{t+1}^{1+\psi}}{1+\psi} \right] - \lambda_t \left[ P_t (1 + \tau_c) (C_{R,t} + K_{t+1} - (1 - \delta) K_t) + \frac{B_{t+1}}{R_t^t} \right] - W_t L_t \left( 1 - \frac{\tau_l}{\phi_t^l} - \tau_p \right) - R_t K_t \left( 1 - \frac{\tau_k}{\phi_t^k} \right) - B_t \right\} \]

(4)

The first order conditions associated with the choices of \( C_{R,t}, L_t, K_{t+1} \) and \( B_{t+1} \) are respectively:

\[ \frac{\partial \mathcal{L}}{\partial C_{R,t}} = S^C_t C_{R,t}^{\sigma - 1} - \lambda_t P_t (1 + \tau_c) = 0 \]

(5)
\[ \frac{\partial L}{\partial L_t} = -S^C S_t L_t^\psi + \lambda_t W_t \left( 1 - \frac{\tau_l}{\phi_t} - \tau_p \right) = 0 \]  
\( (6) \)

\[ \frac{\partial L}{\partial K_{t+1}} = -\lambda_t P_t (1 + \tau_c) + \beta E_t \lambda_{t+1} \left[ (1 - \delta) P_{t+1} (1 + \tau_c) + R_{t+1} \left( 1 - \frac{\tau_k}{\phi_{t+1}} \right) \right] = 0 \]  
\( (7) \)

\[ \frac{\partial L}{\partial B_{t+1}} = -\lambda_t R_{B_t} + \beta E_t \lambda_{t+1} = 0 \]  
\( (8) \)

From equation (5),

\[ \lambda_t = \frac{S^C C_{R,t}^{\sigma}}{P_t (1 + \tau_c)} \]  
\( (9) \)

Substituting the equation (9) into (6), it results in the equation of labor supply:

\[ S^L L_t^\psi C_{R,t}^{\sigma} \left[ \frac{(1 + \tau_c)}{1 - \frac{\tau_l}{\phi_t} - \tau_p} \right] = \frac{W_t}{P_t} \]  
\( (10) \)

Substituting equation (9) in equations (7) and (8), we obtain the Euler equations:

\[ S^C C_{R,t}^{\sigma - \sigma} = \beta E_t \frac{S^C C_{R,t+1}^{\sigma - \sigma}}{P_{t+1} (1 + \tau_c)} \left[ (1 - \delta) P_{t+1} (1 + \tau_c) + R_{t+1} \left( 1 - \frac{\tau_k}{\phi_{t+1}} \right) \right] \]  
\( (11) \)

\[ \frac{S^C C_{R,t}^{\sigma - \sigma}}{P_t} = R_B^B \beta E_t \frac{S^C C_{R,t+1}^{\sigma - \sigma}}{P_{t+1}} \]  
\( (12) \)

### 2.1.2 Non-Ricardian Households (NR) - Workers Inactive (Retired)

Non-Ricardian agents[5] have a simpler behavior. Because they do not maximize their intertemporal utility, their consumption is limited to the value of the pension benefit received (PEN). Under this hypothesis:

\[ (1 + \tau_c) P_tC_{NR,t} = PEN \]  
\( (13) \)

### 2.1.3 Aggregate Consumption

The aggregate consumption of this work follows the functional form \( C = (1 - \omega)C_R + \omega C_{NR} \) very common in this type of literature (Boscá et al, 2010; Gali et al, 2007; Itawa, 2009; Coenen and Straub, 2004; Furlanetto, 2007; Dallari, 2012; Mayer et al, 2010; Stahler and Thomas, 2011; Swarbrick, 2012; Motta and Tirelli, 2010).

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5Generally, the DSGE literature treats the non-Ricardian agent as an individual without capacity to maximize the intertemporal utility due to liquidity conditions. In this work, the assumption is that this type of agent does not maximize its utility due to retirement.

6\( C_t = \int_0^1 C_{h,t} dh = (1 - \omega)C_{R,t} + \omega C_{NR,t} \), given that agents belonging to the same group are identical.
Thus, aggregate consumption of the individuals Ricardian and non-Ricardian is performed as follows:

\[ C_t = (1 - \omega)C_{R,t} + \omega C_{NR,t} \]  (14)

### 2.1.4 Shocks to Related Households

There are two shocks related to Ricardian household behavior: the shock in intertemporal preferences \( S^C \) and the shock on labor supply \( S^L \). While the first affects the choice of intertemporal consumption, the second affects labor supply and determination of nominal wages. The shock \( S^C \) was included to capture changes in valuation between the present and the future which the literature on intertemporal behavior suggested as a key to the understanding of aggregate fluctuations (Primiceri et al. 2006). Additionally the shock \( S^L \) was added to model changes in labor supply that Hall (1997) and Chari et al. (2007) identified as responsible for major changes in employment over the business cycle. There are two other shocks in the stochastic components of the taxes on labor income \( \phi^l \) and on capital income \( \phi^k \). These shocks were included to characterize the stochastic component related to these two types of taxes, which are the objects of study in this work.

Thus, the movement rules of such shocks are presented below:

\[
\log S^C_t = (1 - \rho_{sc}) \log S^C_{ss} + \rho_{sc} S^C_{t-1} + \epsilon_{sc,t} \\
\log S^L_t = (1 - \rho_{sl}) \log S^L_{ss} + \rho_{sl} S^L_{t-1} + \epsilon_{sl,t} \\
\log \phi^l_t = (1 - \rho_l) \log \phi^l_{ss} + \rho_l \phi^l_{t-1} + \epsilon_{l,t} \\
\log \phi^k_t = (1 - \rho_k) \log \phi^k_{ss} + \rho_k \phi^k_{t-1} + \epsilon_{k,t} 
\]  (15-18)

where \( \epsilon_{sc,t}, \epsilon_{sl,t}, \epsilon_{l,t}, \epsilon_{k,t} \) are exogenous shocks, and \( \rho_{sc}, \rho_{sl}, \rho_l, \rho_k \) are autoregressive components, of the intertemporal consumption shock, of the shock on labor supply, of the shock of the taxes on labor income and of the shock of the taxes on capital income, respectively.

### 2.2 Firms

The productive sector of the economy in this work is divided into two subsectors: firm producers of finished goods (retail); and firm producers of intermediate goods (wholesale). The wholesale sector is formed by a great number of firms, each producing a different good according to the structure of monopoly competition. In the retail industry, there is a single firm that aggregates intermediate goods in a single good \( Y \) that will be consumed by economic agents. Besides these features, it should be mentioned that the markets for productive factors follow a structure of perfect competition.
2.2.1 Firm Producers of Finished Goods (Retail)

First, it is necessary to define the aggregator behavior of the production function. The finished good is produced by a single firm that operates in perfect competition. For this purpose, the firm combines a continuum of intermediate goods and aggregates them into a single finished good using the following technology:

\[
Y_t = \left( \int_0^1 Y_{j,t}^{\frac{\varphi - 1}{\varphi}} dj \right)^{\frac{\varphi}{\varphi - 1}} \tag{19}
\]

onde \(Y\) is aggregate output, \(Y_j\) is the intermediate product \(j\), \(\varphi\) is the elasticity of substitution between intermediate goods. The form adopted to aggregate the assets is called an Dixit-Stiglitz aggregator (Dixit e Stiglitz, 1977).

As mentioned, the finished goods producer is in perfect competition and maximizes its profit by using the technology of equation (19), whereas the prices of intermediate goods are given. Therefore, the problem of the retail firm is:

\[
\max_{Y_{j,t}} P_t Y_t - \int_0^1 P_{j,t} Y_{j,t} dj \tag{20}
\]

substituting (19) into (20),

\[
\max_{Y_{j,t}} P_t \left( \int_0^1 Y_{j,t}^{\frac{\varphi - 1}{\varphi}} dj \right)^{\frac{\varphi}{\varphi - 1}} - \int_0^1 P_{j,t} Y_{j,t} dj
\]

The first order condition for each intermediate good \(j\) is:

\[
P_t \left( \int_0^1 Y_{j,t}^{\frac{\varphi - 1}{\varphi}} dj \right)^{\frac{\varphi}{\varphi - 1}} - P_{j,t} = 0
\]

\[Y_{j,t} = Y_t \left( \frac{P_t}{P_{j,t}} \right)^{\varphi} \tag{21}\]

Equation (21) demonstrates that the demand for intermediate good \(j\) is a decreasing function of its relative price and increasing in relation to the aggregate output of the economy.

The general price level is obtained by substituting equation (21) in (19):

\[
Y_t = \left\{ \int_0^1 \left[ Y_t \left( \frac{P_t}{P_{j,t}} \right)^{\varphi} \right]^{\frac{\varphi - 1}{\varphi}} \frac{\varphi - 1}{\varphi} dj \right\}^{\frac{\varphi}{\varphi - 1}}
\]

\[
P_t = \left( \int_0^1 P_{j,t} \frac{\varphi - 1}{\varphi} dj \right)^{\frac{\varphi}{\varphi - 1}} \tag{22}\]
2.2.2 Firm Producers of intermediate goods (Wholesalers)

The wholesaler firms solve the problem in two steps. In the first step, firms take as given the prices of production factors: wages \( W \) and return to capital \( R \). They determine the quantities of those inputs that will minimize their costs. In the second stage, firms determine the optimal price of good \( j \) and they determine the quantity that will be produced in accordance with this price.

**First Step**

The objective of the first step is to minimize the cost of production,

\[
\min_{L_{j,t}, K_{j,t}} W_t L_{j,t} + R_t K_{j,t} \tag{23}
\]

subject to the following technology\(^7\)

\[
Y_{j,t} = A_t K_{j,t}^\alpha L_{j,t}^{1-\alpha} \tag{24}
\]

where \( \alpha \) is the share of capital in output, \( e \) \( A \) is the productivity, whose law of motion is:

\[
\log A_t = (1 - \rho_A) \log A_{t-1} + \rho_A \log A_t + \epsilon_{A,t} \tag{25}
\]

where \( \epsilon_{A,t} \) is exogenous shocks and \( \rho_A \) is autoregressive components of the productivity shock.

Using the Lagrangian function to solve the previous problem of wholesaler firm:

\[
\mathcal{L} = W_t L_{j,t} + R_t K_{j,t} - \mu_t (A_t K_{j,t}^\alpha L_{j,t}^{1-\alpha}) \tag{26}
\]

The first order conditions are:

\[
\frac{\partial \mathcal{L}}{\partial L_{j,t}} = W_t - (1 - \alpha) \mu_t A_t K_{j,t}^\alpha L_{j,t}^{1-\alpha} = 0 \tag{27}
\]

\[
\frac{\partial \mathcal{L}}{\partial K_{j,t}} = R_t - \alpha \mu_t A_t K_{j,t}^{\alpha-1} L_{j,t}^{1-\alpha} = 0 \tag{28}
\]

From equations (27) and (28), we arrive at:

\[
W_t = \mu_t (1 - \alpha) \frac{Y_{j,t}}{L_{j,t}} \tag{29}
\]

\[
R_t = \mu_t \alpha \frac{Y_{j,t}}{K_{j,t}} \tag{30}
\]

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\(^7\)As in the case of the utility function of the households, the production function must have some properties: to be strictly increasing \( (F_K > 0 \text{ and } F_L > 0) \); to be strictly concave \( (F_{KK} < 0 \text{ and } F_{LL} < 0) \); and to be twice differentiable. It is also assumed that the production function has constant returns to scale, \( F(zK_t, zL_t) = zY_t \). Still, this function must fulfill the calls Inada conditions: \( \lim_{K \to \infty} = \infty; \lim_{K \to 0} = 0; \lim_{L \to 0} = \infty; \text{ and } \lim_{L \to \infty} = 0.\)
and from equations (29) and (30),

\[
\frac{W_t}{R_t} = \left[ \frac{(1 - \alpha)}{\alpha} \right] \frac{K_{j,t}}{L_{j,t}}
\]  

(31)

**Second Step**

In the second step, the wholesale firm maximizes its profit by choosing the price of its good j,

\[
\max_{P_{j,t}} P_{j,t} Y_{j,t} - W_t L_{j,t} - R_t K_{j,t}
\]

(32)

substituting (21), (29) and (30) in (32):

\[
\max_{P_{j,t}} P_{j,t} Y_{t} \left( P_{t} \frac{P_{j,t}}{P_{j,t}} \right)^{\varphi} - \mu_t Y_t \left( P_{t} \frac{P_{j,t}}{P_{j,t}} \right)^{\varphi} P_{j,t}^{\varphi - 1} = 0
\]

It lies in the following first order condition,

\[
(1 - \varphi) Y_t \left( P_{t} \frac{P_{j,t}}{P_{j,t}} \right)^{\varphi} + \varphi \mu_t Y_t \left( P_{t} \frac{P_{j,t}}{P_{j,t}} \right)^{\varphi} P_{j,t}^{\varphi - 1} = 0
\]

\[
\mu_t = \left( \frac{\varphi - 1}{\varphi} \right) P_{j,t}
\]

(33)

substituting (33) into (29) and (30), and knowing that these firms have the same technology - \( P_{j,t} = P_t \) e \( Y_{j,t} = Y_t \) - the results for prices of the factors of production are:

\[
\frac{W_t}{P_t} = \left( \frac{\varphi - 1}{\varphi} \right) (1 - \alpha) \frac{Y_t}{L_t}
\]

(34)

\[
\frac{R_t}{P_t} = \left( \frac{\varphi - 1}{\varphi} \right) \alpha \frac{Y_t}{K_t}
\]

(35)

**2.2.3 Pricing a la Calvo**

The wholesale firm chooses how much to produce in each period, but following a rule *a la Calvo* (Calvo, 1983) that says they fail to choose the price of their good in all periods. At each period t, a fraction \( 0 < 1 - \theta < 1 \) of firms are randomly selected and allowed to choose the price of their good for period t, \( P_{j,t} \). The remaining firms (the ratio \( \theta \) of firms) keeps the price of the previous period \( P_{j,t} = P_{j,t-1} \) for the product.

Thus, solving equation (31) to \( L_{j,t} \):

\[
L_{j,t} = \left[ \frac{(1 - \alpha)}{\alpha} \right] \frac{R_t K_{j,t}}{W_t}
\]

and substituting this result in the production function (equation (24)):

\[
Y_{j,t} = A_t K_{j,t} \left\{ \left[ \frac{(1 - \alpha)}{\alpha} \right] \frac{R_t}{W_t} \right\}^{1 - \alpha}
\]
getting,

\[ K_{j,t} = \frac{Y_{j,t}}{A_t} \left\{ \frac{\alpha}{(1 - \alpha)} \right\} W_t R_t \}^{1 - \alpha} \]  \tag{36} 

and,

\[ L_{j,t} = \frac{Y_{j,t}}{A_t} \left\{ \frac{\alpha}{(1 - \alpha)} \right\} W_t R_t \}^{-\alpha} \]  \tag{37} 

The wholesale firm has a probability \( \theta \) to keep the price of the previous period for the good and the probability \( (1 - \theta) \) to choose the price optimally. Once fixing the price in period \( t \), there is the probability \( \theta \) that this price will remain fixed in period \( t+1 \), a probability \( \theta^2 \) that this price will remain fixed in period \( t+2 \), and so on. This firm should take into account these probabilities when choosing the price of its own good in its capacity to perform this adjustment.

Thus, the problem of the firm able to adjust the price of the good is:

\[ \max_{P_{j,t}} \mathbb{E}_t \sum_{i=0}^{\infty} (\beta \theta)^i \left\{ P_{j,t}^* Y_{j,t+i} - P_{t+i} R_{t+i} K_{j,t+i} - P_{t+i} W_{t+i} L_{j,t+i} \right\} \]  \tag{38} 

where \( \theta \) is the factor of rigidity in the adjustment of prices and \( P_{j,t}^* \) is the optimal price set by the firm with the ability to adjust the price of your product. Equation (38) is the discounted profit of the firm during the period which the price \( P_{j,t}^* \) is in progress.

Substituting (21), (36) and (37) in (38):

\[ \max_{P_{j,t}} \mathbb{E}_t \sum_{i=0}^{\infty} (\beta \theta)^i Y_{j,t+i} \left\{ \frac{P_{t+i}}{P_{j,t}} A_{t+i} (1 - \alpha) \right\} \left\{ \left( \frac{1 - \alpha}{\alpha} \right) \frac{R_{t+i}}{W_{t+i}} \right\}^{\alpha} \]  \tag{39} 

Arriving at the following first order condition:

\[ 0 = \mathbb{E}_t \sum_{i=0}^{\infty} (\beta \theta)^i Y_{j,t+i} \left\{ 1 - \varphi \frac{P_{t+i} W_{t+i}}{P_{j,t} A_{t+i} (1 - \alpha)} \right\} \left\{ \left( \frac{1 - \alpha}{\alpha} \right) \frac{R_{t+i}}{W_{t+i}} \right\}^{\alpha} \]  \tag{39} 

\[ P_{j,t}^* = \left( \frac{\varphi - 1}{\varphi} \right) \frac{\mathbb{E}_t \sum_{i=0}^{\infty} (\beta \theta)^i Y_{j,t+i} \frac{P_{t+i} W_{t+i}}{A_{t+i} (1 - \alpha)} \left\{ \left( \frac{1 - \alpha}{\alpha} \right) \frac{R_{t+i}}{W_{t+i}} \right\}^{\alpha}}{\mathbb{E}_t \sum_{i=0}^{\infty} (\beta \theta)^i Y_{j,t+i}} \]  \tag{39} 

Combining the pricing rule of equation (22), and the assumption that all firms with the ability to adjust define equal value and that firms without this ability retains the same price, the overall price level is obtained by the equation:

\[ P_t = \left[ \theta P_{t-1}^{1-\varphi} + (1 - \theta) P_{t-1}^{1-\varphi} \right]^{1-\varphi} \]  \tag{40} 

\[ 10 \]
2.3 Government

The government sector in this work is divided into three subsectors: Fiscal Authority, Social Security, and the Monetary Authority.

2.3.1 Fiscal Authority

The government collects taxes and issues bonds to finance its spending on goods and services. The result of the pension system is transferred to the rest of the government. So if social security shows a deficit (or surplus), this is financed (or appropriated) for the remainder of the government. Therefore, the change in public debt is given by the following rule:

\[
\frac{B_{t+1}}{R_t^{B}} - B_t = P_t G_t - BAL_t - TAX_t
\]  

(41)

As could not be otherwise, the expense of the government is sensitive to the size of the public debt (current debt \(B_t\) relative to its steady-state level, \(B_{ss}\)):

\[
G_t - G_{ss} = \chi(B_t - B_{ss})
\]  

(42)

where \(\chi\) is the sensitivity of government spending relative to the size of the public debt.

and tax revenue is obtained by the following equation:

\[
TAX_t = \tau_c P_t(C_t + I_t) + \frac{\tau_l}{\phi_t^l} W_t L_t + \frac{\tau_k}{\phi_t^k} R_t K_t
\]  

(43)

2.3.2 Social Security

Social security is defined as a system of simple allocation, i.e., it is not capitalized (pay-as-you-go). The pension balance is the difference between the total collected with the social security contributions of active workers, \(\tau_p W_t L_t\), and the total payment of benefits to inactive employees (retirees), \(PEN\).

Thus,

\[
BAL_t = \tau_p W_t L_t - PEN
\]  

(44)

2.3.3 Monetary Authority

The Central Bank of Brazil appears in this work following a simple Taylor rule (1993) with the dual goal of output growth and maintenance of price stability:

\[
R_t^B = a(Y_t - Y_{ss}) + b(\pi_t - \pi_{ss}) + R_{ss}^B
\]  

(45)

where \((a)\) and \((b)\) are the sensitivities of the basic interest rate in relation to the product and to the inflation rate, respectively. The inflation rate is defined as:
\[ \pi_t = \frac{P_t}{P_{t-1}} - 1 \]  

(46)

### 2.4 Equilibrium Condition of Goods Market

To complete the model it is necessary to use the equilibrium condition in the goods market. Wherein aggregate production \( Y_t \) is demanded by households (\( C_t \) and \( I_t \)) and Government (\( G_t \)):

\[ Y_t = C_t + I_t + G_t \]  

(47)

### 3 Calibration

Once solved the structural model, next step is to obtain the values of the parameters. For this purpose, there are two possibilities: estimating the model using some econometric technique, or using up the calibration. The latter procedure is to somehow calculate the parameter values arbitrarily through available data or by using values from other works. This technique is the option used by the majority of the works of this type of economic literature. On the other hand, it is possible to estimate the parameters. The two most popular approaches to this modeling are a maximum likelihood and Bayesian. It should be emphasized that this second estimation methodology has gained "space" between macroeconometrists.

This work used the calibration technique. The model equilibrium is a set of twenty one equations representing the behavior of twenty one endogenous variables (\( Y, C, C_R, C_{NR}, I, G, K, L, R, R^B, W, B, TAX, BAL, P, \pi, S^C, S^L, \phi^l \) and \( \phi^k \)). Consequently, it is necessary to assign values somehow for the structural parameters of the model (\( \alpha, \beta, \delta, \theta, \rho_A, \rho_{ac}, \rho_{ad}, \rho_I, \rho_k, \sigma, \varphi, \psi, \omega, \chi, a, b, \) \( PEN, \tau_c, \tau_l, \tau_k \) and \( \tau_p \)).

The main calibration procedure adopted here is to obtain the values of parameters from other relevant DSGE work in the literature. Cavalcanti and Vereda (2010) analyzed the dynamic properties of a DSGE model for Brazil under alternative parameterizations. Therefore, they identified "allowable ranges" of values for some of the key parameters in the literature. Using the results of these authors, it was decided to use the parameters in common between the two studies, which were the discount factor (\( \beta \)); the rate of capital depreciation (\( \delta \)); the coefficient of relative risk aversion (\( \sigma \)); and the marginal disutility of labor (\( \psi \)).

The parameters related to taxation were obtained from Araújo and Ferreira (1999). The procedure adopted by these authors was to find the share of each type of tax to GDP and the ratio of the related variable with the tax to GDP. For the tax rate on consumption (\( \tau_c \)), \( \tau_c C = 0,1282Y \), with \( C/Y = 0,8045 \), resulting in \( \tau_c = 0,1594 \). For the tax rate on capital income (\( \tau_k \)), \( \tau_k R^K = 0,0399Y \), with \( R = 0,1647 \) and \( K/Y = 2,98 \), obtaining \( \tau_k = 0,0813 \). For the tax rate on labor income (\( \tau_l \)), \( \tau_l W^L = 0,0881 \), with \( W^L/Y = 0,5092 \), finding \( \tau_l = 0,1730 \). An aliquot of social security contributions (\( \tau_p \)) was calibrated from Cavalcanti and Silva (2010). For the other parameters related to the government, the sensitivity of the basic interest rate on the product (\( a \)) and on the inflation rate (\( b \)) were obtained.
from Taylor (1993), and the sensitivity of government spending relative to public debt ($\chi$) was calibrated from Lim e Mc Nelis (2008).

The share of consumption of non-Ricardian agents (inactive workers) in aggregate consumption ($\omega$) and the parameter characterizing the benefit payments ($PEN$) were calibrated from Giambiagi and Alêm (2008). In calculating the first parameter, the ratio between the number of workers contributing ($1 - \omega$) and the number of pension beneficiaries ($\omega$) was 1.6 for the year 2010. Thus $\frac{1 - \omega}{\omega} = 1.6$, resulting in $\omega = 0.39$. $PEN$ was obtained from pension expense in % of GDP; the value for 2004 was 9.4%.

Finally, the parameters related to the structure of the firms were calibrated from two studies. The share of capital in output ($\alpha$) was obtained from Kanczuk (2002) while the index of price stickiness ($\theta$) and the elasticity of substitution between intermediate goods ($\phi$) was obtained from Lim and Mc Nelis (2008). Table 1 summarizes the calibration parameters.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.985</td>
<td>Cavalcanti and Vereda (2010)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.025</td>
<td>Cavalcanti and Vereda (2010)</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>2</td>
<td>Cavalcanti and Vereda (2010)</td>
</tr>
<tr>
<td>$\psi$</td>
<td>1.5</td>
<td>Cavalcanti and Vereda (2010)</td>
</tr>
<tr>
<td>$\tau_c$</td>
<td>0.1594</td>
<td>Araújo and Ferreira (1999)</td>
</tr>
<tr>
<td>$\tau_k$</td>
<td>0.0813</td>
<td>Araújo and Ferreira (1999)</td>
</tr>
<tr>
<td>$\tau_l$</td>
<td>0.1730</td>
<td>Araújo and Ferreira (1999)</td>
</tr>
<tr>
<td>$\tau_p$</td>
<td>0.105</td>
<td>Cavalcanti and Silva (2010)</td>
</tr>
<tr>
<td>$\omega$</td>
<td>0.39</td>
<td>Giambiagi and Alêm (2008)</td>
</tr>
<tr>
<td>PEN</td>
<td>0.094</td>
<td>Giambiagi and Alêm (2008)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.39</td>
<td>Kanczuk (2002)</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.85</td>
<td>Lim and Mc Nelis (2008)</td>
</tr>
<tr>
<td>$\phi$</td>
<td>6</td>
<td>Lim and Mc Nelis (2008)</td>
</tr>
</tbody>
</table>

4 Results

This section analyzes the dynamic properties of the model. For this purpose, it will be shown that the variance decomposition and impulse-response functions are a result of shocks to the tax rates on labor income and capital income. This type of analysis is able to tell which variables have a more important behavior for idealized study. The simulations of the model were run on the Dynare platform.

\footnote{Dynare is a software platform for the treatment of a wide class of macroeconomic models, in particular models of Dynamic Stochastic General Equilibrium (DSGE) and Overlapping Generations (OLG). The models solved by Dynare include the rational expectations hypothesis, but Dynare is also able to handle models where expectations are formed differently: on one extreme, models where agents perfectly anticipate the future; at the other extreme, the models where the agents have limited rationality or imperfect knowledge and thus form their expectations through a learning process. In terms of types of agents, it is possible to incorporate in Dynare: consumers, productive enterprises, government, monetary authorities, investors and financial intermediaries. Some level of heterogeneity can be achieved by the}
4.1 Variance Decomposition

Table 2 presents the variance decomposition of the errors of the simulations of the endogenous variables (columns) in relation to exogenous shocks (rows). Note that the shock in the tax rate on income labor ($\epsilon_{l,t}$) was what had the best result, with significant values for product, consumption of Ricardian households, investment, labor, basic interest rate, and inflation rate (Table 2). In other words, affecting the price of hours worked this shock makes the leisure $(1 - L)$ relatively more expensive and the active workers to work more (substitution effect). The result of this increase is a greater aggregate supply. A boom in the production possibilities increases the aggregate demand, so consumption and investment grow. This increased pressure on aggregate demand also pushes the inflation rate, causing the Central Bank to raise the basic interest rate to combat the inflationary process.

<table>
<thead>
<tr>
<th></th>
<th>Y</th>
<th>C</th>
<th>$C_R$</th>
<th>$C_{NR}$</th>
<th>I</th>
<th>G</th>
<th>K</th>
<th>L</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\epsilon_{l,t}$</td>
<td>97.77</td>
<td>90.88</td>
<td>94.41</td>
<td>54.63</td>
<td>89.02</td>
<td>46.14</td>
<td>89.31</td>
<td>98.88</td>
</tr>
<tr>
<td>$\epsilon_{k,t}$</td>
<td>2.23</td>
<td>9.12</td>
<td>5.59</td>
<td>45.37</td>
<td>10.98</td>
<td>53.86</td>
<td>10.69</td>
<td>1.12</td>
</tr>
</tbody>
</table>

Source: Prepared by the author.

On the other hand, the shock in the tax rate on capital income ($\epsilon_{k,t}$) had relatively disappointing results. It demonstrates that this shock barely explained the changes in capital, which affect the poor performance of the product. In other words, the tax reduction related to the capital was not able to increase the disposable income enough to create conditions that stimulated the main macroeconomic variables. Tax reduction on labor income played an important role in increasing the capital stock. Other variables were explained approximately equally between the two tax reductions. This was noted in the variables: wages, return on capital, tax revenue, and the balance of the pension. Briefly, the analysis of variance decomposition showed that tax reduction on labor income was more efficient in almost all results of the macroeconomic variables.

4.2 Impulse-response analysis

Figures 1 and 2 show the impulse response functions for the two shocks that are the objects of study in this work. The shocks are in the stochastic components of the tax rates. Note that these two shocks return to their steady-state level in about 40 periods. The behavior of both functions present similar results for some inclusion of several different classes of agents in each of the categories of the listed agents (Adjemian et al., 2011).

9 With a lower tax on labor income, disposable income increases.

10 The impulse response graph estimates responses to shocks in each of the endogenous variables. These responses are obtained as follows: initially, all variables must be in their steady state levels. At some time $t = 0$, an endogenous variable takes a value equal to its steady-state level over an increase (impulse) in size equal to one standard deviation, and are calculated as all variables evolve after that.

11 $\epsilon_{l,t} = 1$ and $\epsilon_{k,t} = 1$. 

14
variables, while for others the behavior is quite different.

4.2.1 No displacement of some variables regarding the two tax reductions

The consumption of non-Ricardian households shows no changes in relation to any of the shocks (Figure 1a). This inertia is because this consumption is a function of the constant value of benefits \((PEN)\), of the price level \((P)\) (Figure 1b) and of the tax on consumption which do not change with these measures of tax reduction. Prices of factors of production \((W, R)\) do not change with the occurrence of shocks (Figures 1c and 1d). For return of capital, the explanation is simpler, because none of the variables that affect its value \((Y, K, P)\) (Figures 1e, 1f e 1b) suffered displacements due to these events. On other hand, the level of wages should have risen with the occurrence of tax reduction on labor income, since the product has grown with this event (Figure 1e). However there was an increase in labor supply causing a cooling of this effect, maintaining the stable wage level (Figure 1g).

Both tax reductions were not undermined government revenue (Figures 2a). This was due to the growth in revenues on consumption and investment \((\tau_c(C + I))\) (Figures 2b and 2c). In other words, the drop in rates of income taxes stimulated the economy so that the increased demand for goods (consumption and investment) offset the initial tax reductions. This result is in line with the main studies of the literature in relation to Brazil (Salami and Fochezatto, 2009; Araújo and Ferreira, 1999; Cavalcanti and Silva, 2010) that advocated a tax reduction in direct taxes with a compensation increase in indirect taxes keeping tax revenue constant.

4.2.2 Effects related to tax reduction on capital income

In this subsection, the results of tax reduction in the tax rate on capital income will be analyzed. Among the important results, note the low impact of this tax reduction in encouraging the growth of output \((Y)\) (Figure 1e). On the demand side, it is noted that an increase in investment \((I)\) (Figure 2c) is offset by a decrease in government spending \((G)\) (Figure 2d), not changing aggregate demand significantly. Also, there is a perceived neutrality in relation to the labor market, since the level of wages \((W)\) (Figure 1c) nor labor supply \((L)\) (Figure 1g) are amended with this shock.

On the fiscal side, public debt \((B)\) decreases in the same proportion as the decrease in government spending (Figures 2d and 2e). In other words, the main impact on the fiscal front is a reduction of government participation in the economy. Consequently, the stock of capital \((K)\) responds positively to the tax reduction on capital income (Figure 1f). However, this moderate result caused some surprise. On the other hand, the explanation of the poor performance of other macroeconomic variables should be credited to this eventuality.

4.2.3 Effects related to tax reduction on labor income

Considering the returns with greater relevance between the two tax reductions proposed, this work found a better result for the tax reduction on labor income. The
significant growth for the product (Figure 1e) was accompanied by a growth in the consumption of Ricardian households ($CR$) (Figure 2f), increases in investments (Figure 2c), falls in government spending (Figure 2d), falls in the stock of public debt (Figure 2e) and increases in factors of production (capital and labor) (Figures 1f and 1g).

The downside was a slight acceleration in inflation ($pi$) (Figure 2g), corrected by increasing the basic interest rate ($RB$) (Figure 2h). Among these results, there was one surprising event. The result on the stock of capital is more positive from this shock than from the shock of the tax on capital income (also noticed in the variance decomposition). Briefly, using the displacement of any variable as a measure, the result of tax reduction on labor income is much higher than the tax reduction on capital income.

Figure 1: Impulse-response functions for shocks in the taxes on labor income and capital income. The subscript below each variable ($\epsilon_{l,t}, \epsilon_{k,t}$) denominates the shock which is related.

Source: Prepared by the author.
Figure 2: Impulse-response functions for shocks in the taxes on labor income and capital income (continuation).

Source: Prepared by the author.

5 Conclusions

This work aimed to contribute to the discussion on tax reforms in Brazil analyzing tax reductions in the productive sector through a DSGE model. To achieve this purpose, two stochastic shocks were analyzed in the tax rates on labor income and on capital income.

The first relevant result found was the low performance of the tax reduction on capital income. This effect was related to the weak stimulus in increasing the capital stock after the shock. The labor market remained neutral with this tax reduction, showing no impact on the wage level and on labor supply. Anyway, it was still possible to find a satisfactory result for some variables. For example, the government’s participation decreases with lower public spending and falling public debt.
Tax revenues showed an outcome aligned with the major tax reform proposals for Brazil. The results indicated that the increase in revenues on consumption and investment offset the loss of revenue generated by the tax reductions on labor and on capital.

The main result of these simulations concern the tax reduction on labor income. The product grew significantly and it was accompanied by increases in the consumption of Ricardian households and investments, declines in government spending and public debt stock, as well as increases in the quantities of factors of production (capital and labor). The downside was a slight acceleration in inflation; on the other hand, increasing the basic interest rate was fast enough to control this rise in prices.

From this description, note that performance from the elimination of the tax on labor income was significantly higher than from other tax reduction. If the country would adopt such a measure, production and consumption would increase, public finances would not be compromised, and even with an increase in basic interest rate to combat inflation, the economic results would not be cooled.

In future work, it might be relevant to open the economy because the exchange rate could contribute to a better understanding of the tax reduction process in the productive sector.

Appendix A: Steady State

Once obtained the economic equilibrium, the next step is to define the values of the steady state. In fact, the model is stationary, in the sense that there is a value for the variables sustained intertemporally. Thus, an endogenous variable $x_t$ will be in steady state for all $t$, if $E_t x_{t+1} = x_t = x_{t-1} = x_{ss}$.

Some endogenous variables have their values previously determined at steady state. That is, the variables involved in exogenous shocks $S^C, S^L, \phi^l, \phi^k$ and $A$. There was also the need to normalize the steady state value for the public debt, $B$. Another variable with that value previously determined is the inflation rate ($\pi_{ss} = P_{ss} - P_{ss} = 0$). The next step to calculate the steady state is to remove the time indicators of the variables. Therefore, the structural model becomes:

$$I_{ss} = \delta K_{ss} \quad (48)$$

$$L^\psi_{ss} C^\sigma_{R,ss} \left[ \frac{(1 + \tau_c)}{(1 - \tau_l - \tau_p)} \right] = \frac{W_{ss}}{P_{ss}} \quad (49)$$

$$\frac{R_{ss}}{P_{ss}} = \left( \frac{1 + \tau_c}{1 - \tau_k} \right) \left[ \frac{1}{\beta} - (1 - \delta) \right] \quad (50)$$

$$R^B_{ss} = \frac{1}{\beta} \quad (51)$$

$$(1 + \tau_c)P_{ss}C_{NRss} = PEN \quad (52)$$
\[ C_{ss} = (1 - \omega)C_{Rss} + \omega C_{NRss} \]  
\[ Y_{ss} = K_{ss}^\alpha L_{ss}^{1-\alpha} \]  
\[ W_{ss} \frac{P_{ss}}{L_{ss}} = (1 - \alpha) \left( \frac{\varphi - 1}{\varphi} \right) \frac{Y_{ss}}{L_{ss}} \]  
\[ R_{ss} \frac{P_{ss}}{L_{ss}} = \alpha \left( \frac{\varphi - 1}{\varphi} \right) \frac{Y_{ss}}{K_{ss}} \]  
\[ 1 = \left( \frac{\varphi - 1}{\varphi} \right) \frac{W_{ss}}{Y_{ss}} \left[ \frac{(1 - \alpha) R_{ss}}{\alpha W_{ss}} \right]^\alpha \]  
\[ \beta - 1 = P_{ss} G_{ss} - BAL_{ss} - TAX_{ss} \]  
\[ TAX_{ss} = \tau_c P_{ss} (C_{ss} + I_{ss}) + \tau_l W_{ss} L_{ss} + \tau_k R_{ss} K_{ss} \]  
\[ BAL_{ss} = \tau_p W_{ss} L_{ss} - PEN \]  
\[ Y_{ss} = C_{ss} + I_{ss} + G_{ss} \]  
From (56),
\[ K_{ss} = \alpha \left( \frac{\varphi - 1}{\varphi} \right) \frac{Y_{ss}}{R_{ss}} \]  
substituting (62) into (54)
\[ Y_{ss}^{1-\alpha} = \left[ \alpha \left( \frac{\varphi - 1}{\varphi} \right) \frac{P_{ss}}{R_{ss}} \right]^\alpha L_{ss}^{1-\alpha} \]  
\[ \frac{Y_{ss}}{L_{ss}} = \left[ \left( \frac{\varphi - 1}{\varphi} \right) \frac{P_{ss}}{R_{ss}} \right]^\frac{\alpha}{1-\alpha} \]  
\[ L_{ss} = Y_{ss} \left[ \left( \frac{\varphi}{\varphi - 1} \right) \frac{R_{ss}}{\alpha P_{ss}} \right]^\frac{\alpha}{1-\alpha} \]  
substituting (63) into (55),
\[ W_{ss} = P_{ss} (1 - \alpha) \left( \frac{\varphi - 1}{\varphi} \right) \left[ \left( \frac{\varphi - 1}{\varphi} \right) \frac{P_{ss}}{R_{ss}} \right]^\frac{\alpha}{1-\alpha} \]  
\[ W_{ss} = P_{ss}^{\frac{1}{-\alpha}} (1 - \alpha) \left[ \left( \frac{\varphi - 1}{\varphi} \right) \frac{1}{\alpha} \frac{P_{ss}}{R_{ss}} \right]^\frac{\alpha}{1-\alpha} \]  
From (57),
\[ W_{ss} = (1 - \alpha) \left( \frac{\varphi}{\varphi - 1} \right) \left( \frac{\alpha}{R_{ss}} \right)^\frac{\alpha}{1-\alpha} \]  
(65) into (66),
\[ P_{ss}^{1-\alpha} (1-\alpha) \left[ \left( \frac{\varphi}{\varphi - 1} \right)^{1-\alpha} P_{ss} \right]^{\frac{\alpha}{1-\alpha}} = (1-\alpha) \left( \frac{\varphi}{\varphi - 1} \right)^{1-\alpha} \left( \frac{\varphi}{P_{ss}} \right)^{\frac{\alpha}{1-\alpha}} \]

\[ P_{ss} = \left( \frac{\varphi}{\varphi - 1} \right)^{2} \]  \hspace{1cm} (67)

substituting (67) into (50),

\[ R_{ss} = \left( \frac{\varphi}{\varphi - 1} \right)^{2} \left( \frac{1+\tau_{c}}{1-\tau_{k}} \right) \left[ \frac{1}{\beta} - (1-\delta) \right] \]  \hspace{1cm} (68)

substituting (64) into (49),

\[ C_{R,ss}^{\sigma} \left\{ Y_{ss} A_{1}^{\frac{\alpha}{1-\alpha}} \right\}^{\psi} \left[ \frac{(1+\tau_{c})}{(1-\tau_{l}-\tau_{p})} \right] = \frac{W_{ss}}{P_{ss}} \]

with,

\[ A_{1} = \left( \frac{\varphi}{\varphi - 1} \right) \frac{R_{ss}}{\alpha P_{ss}} \]  \hspace{1cm} (69)

\[ C_{R,ss}^{\sigma} \left\{ Y_{ss} A_{1}^{\frac{\alpha}{1-\alpha}} \right\}^{\psi} \left[ \frac{(1+\tau_{c})}{(1-\tau_{l}-\tau_{p})} \right] = \frac{W_{ss}}{P_{ss}} \]

\[ C_{R,ss} = \left( \frac{1}{Y_{ss}} \right) \left[ \frac{1-\tau_{l}-\tau_{p}}{1+\tau_{c}} \right] \left( \frac{1}{\alpha \psi} P_{ss} \right) \frac{W_{ss}}{A_{1}^{\frac{\alpha}{1-\alpha}}} \]  \hspace{1cm} (70)

from (52),

\[ C_{NR,ss} = \frac{PEN}{P_{ss}(1+\tau_{c})} \]  \hspace{1cm} (71)

substituting (70) and (71) into (53),

\[ C_{ss} = (1-\omega) \left( \frac{1}{Y_{ss}} \right) \left[ \frac{1-\tau_{l}-\tau_{p}}{1+\tau_{c}} \right] \left( \frac{1}{\alpha \psi} P_{ss} \right) \frac{W_{ss}}{A_{1}^{\frac{\alpha}{1-\alpha}}} + \omega \frac{PEN}{P_{ss}(1+\tau_{c})} \]  \hspace{1cm} (72)

substituting (59) and (60) into (58),

\[ \beta - 1 = \]

\[ P_{ss} G_{ss} - \tau_{p} W_{ss} Y_{ss} A_{1}^{\alpha} + PEN - \tau_{c} P_{ss} C_{ss} - \tau_{c} P_{ss} Y_{ss} A_{1}^{\alpha} - \tau_{k} R_{ss} Y_{ss} A_{1}^{\alpha} \]

\[ G_{ss} = \frac{1}{P_{ss}} \left\{ \beta - 1 + Y_{ss} \left[ W_{ss} A_{1}^{\alpha} (\tau_{p} + \tau_{l}) + \left( \frac{\tau_{c} P_{ss} Y_{ss} A_{1}^{\alpha}}{A_{1}} \right) \right] - PEN + \tau_{c} P_{ss} C_{ss} \right\} \]  \hspace{1cm} (73)

substituting (48), (72) and (73) into (61),
\[ Y_{ss} = (1 - \omega) \left( \frac{1}{Y_{ss}} \right) \left[ \frac{1}{1 + \tau_c} \left( 1 - \tau_p + \tau_c \right) + \frac{W_{ss}}{P_{ss}} \right]^{\frac{1}{2}} + \omega \frac{PEN}{P_{ss}(1 + \tau_c)} + \frac{1}{P_{ss}} \]

\[ \{ \beta - 1 + Y_{ss} \left[ W_{ss}A_1^{1 - \alpha} (\tau_p + \eta) + \left( \frac{\delta P_{ss}(1 + \tau_c) + \tau_k}{A_1 P_{ss}} \right) \right] - PEN + \tau_c P_{ss} C_{ss} \} + \delta \frac{Y_{ss}}{A_1} \]

assuming that \( \frac{\psi}{\sigma} \approx 1 \), and that,

\[ A_2 = 1 - \left[ \frac{W_{ss}}{P_{ss}} A_1^{1 - \alpha} (\tau_p + \eta) + \left( \frac{\delta P_{ss}(1 + \tau_c) + \tau_k}{A_1 P_{ss}} \right) \right] \] (74)

\[ A_3 = \omega \frac{PEN}{P_{ss}(1 + \tau_c)} + \left( \frac{\beta - 1}{P_{ss}} \right) - PEN \] (75)

\[ A_4 = (1 + \tau_c)(1 - \omega) \left[ \frac{1}{1 + \tau_c} \left( 1 - \frac{\tau_p}{1 + \tau_c} \right) \frac{1}{A_1^{1 - \alpha} P_{ss}} \right]^{\frac{1}{2}} \] (76)

Thus, we have:

\[ A_2 Y_{ss} - A_3 - \frac{A_4}{Y_{ss}} = 0 \]

multiplying this equation by \( Y_{ss} \),

\[ A_2 Y_{ss}^2 - A_3 Y_{ss} - A_4 = 0 \]

\[ Y_{ss} = \frac{A_3 \pm \sqrt{A_3^2 + 4 A_2 A_4}}{2 A_2} \]

whose only admissible response is:

\[ Y_{ss} = \frac{A_3 + \sqrt{A_3^2 + 4 A_2 A_4}}{2 A_2} \] (77)

**Appendix B: Log-linearization - (Uhlig’s Method)**

The conditions for optimizing the model forms a system of nonlinear difference equations. This system has not a closed analytic solution, generally. In most cases, it is easier and more convenient to use approximations to characterize the solution of the dynamic model (Fernandez-Villaverde, 2009). This section summarizes the method of log-linear approximation customarily used in the literature.

Uhlig (1999) recommends a simple method of log-linearization of functions that do not require differentiation. Thus, consider a set of variables \( X_t \) and setting \( \tilde{X}_t = \ln X_t - \ln X_{ss} \). Then we can write the original variables as: \( X_t = X_{ss} e^{\tilde{X}_t} \).

Uhlig also proposes the following block of aid for the log-linearization:

\[ e^{(\tilde{X}_t + a \tilde{Y}_t)} \approx 1 + \tilde{X}_t + a \tilde{Y}_t \]

\[ \tilde{X}_t \tilde{Y}_t \approx 0 \]

\[ E_t \left[ a e^{\tilde{X}_{t+1}} \right] \approx a + a E_t \left[ \tilde{X}_{t+1} \right] \]

21
From (3),
\[ K_{ss}(1 + \tilde{K}_{t+1}) = (1 - \delta)K_{ss}(1 + \tilde{K}_t) + I_{ss}(1 + \tilde{I}_t) \]

\[ K_{ss}\tilde{K}_{t+1} = (1 - \delta)K_{ss}\tilde{K}_t + I_{ss}\tilde{I}_t \]  \hspace{1cm} (78)

From (10),
\[ C_{R,ss}^{\sigma}L_{ss}^{\psi}(1 + \tau)(1 + \tilde{S}_t' + \sigma\tilde{C}_{R,t} + \psi\tilde{L}_t) = \frac{W_{ss}}{P_{ss}}[(1 + \tilde{W}_t - \tilde{P}_t)(1 - \tau_l - \tau_p) + \tau_l\tilde{\phi}_t] \]

\[ C_{R,ss}^{\sigma}L_{ss}^{\psi}(1 + \tau)(\tilde{S}_t' + \sigma\tilde{C}_{R,t} + \psi\tilde{L}_t) = \frac{W_{ss}}{P_{ss}}[(\tilde{W}_t - \tilde{P}_t)(1 - \tau_l - \tau_p) + \tau_l\tilde{\phi}_t] \]  \hspace{1cm} (79)

From (11),
\[ C_{R,ss}^{\sigma}(1 + \tilde{S}_t') - \sigma\tilde{C}_{R,t} = \beta C_{R,ss}^{\sigma}(1 - \delta)(1 + \tilde{S}_t'^{+1} - \sigma\tilde{C}_{R,t+1}) + \frac{\beta C_{R,ss}^{\sigma}}{\tau_{ss}(1 + \tau_c)}[(1 + \tilde{S}_t'^{+1} - \sigma\tilde{C}_{R,t+1} + \tilde{R}_{t+1} - \tilde{P}_{t+1})(1 - \tau_k) + \tau_k\tilde{\phi}_{t+1}^k] \]

\[ (\tilde{S}_t' - \sigma\tilde{C}_{R,t}) = \beta(1 - \delta)(\tilde{S}_t'^{+1} - \sigma\tilde{C}_{R,t+1}) + \frac{\beta R_{ss}}{\tau_{ss}(1 + \tau_c)} \]

\[ [(\tilde{S}_t'^{+1} - \sigma\tilde{C}_{R,t+1} + \tilde{R}_{t+1} - \tilde{P}_{t+1})(1 - \tau_k) + \tau_k\tilde{\phi}_{t+1}^k] \]  \hspace{1cm} (80)

From (12),
\[ \tilde{R}_{B,t} + \tilde{S}_t'^{+1} - \tilde{S}_t' = \sigma(\tilde{C}_{R,t+1} - \tilde{C}_{R,t}) + \tilde{\pi}_{t+1} \]  \hspace{1cm} (81)

From (13),
\[ (1 + \tau_c)P_{ss}C_{NR,ss}(\tilde{P}_t + \tilde{C}_{NR,t}) = 0 \]  \hspace{1cm} (82)

From (14),
\[ C_{ss}\tilde{C}_t = (1 - \omega)C_{R,ss}\tilde{C}_{R,t} + \omega C_{NR,ss}\tilde{C}_{NR,t} \]  \hspace{1cm} (83)

From (15),
\[ \tilde{S}_t' = \rho_{sc}\tilde{S}_{t-1} + \epsilon_{sc,t} \]  \hspace{1cm} (84)

From (16),
\[ \tilde{S}_t' = \rho_{sl}\tilde{S}_{t-1} + \epsilon_{sl,t} \]  \hspace{1cm} (85)
From (17),
\[ \tilde{\phi}_l^t = \rho_l \tilde{\phi}_{l-1}^t + \epsilon_{l,t} \] (86)

From (18),
\[ \tilde{\phi}_k^t = \rho_k \tilde{\phi}_{k-1}^t + \epsilon_{k,t} \] (87)

From (24),
\[ \tilde{Y}_t = \tilde{A}_t + \alpha \tilde{K}_t + (1 - \alpha) \tilde{L}_t \] (88)

From (25),
\[ \tilde{A}_t = \rho_A \tilde{A}_{t-1} + \epsilon_{A,t} \] (89)

From (34),
\[ \tilde{W}_t - \tilde{P}_t = \tilde{Y}_t - \tilde{L}_t \] (90)

From (35),
\[ \tilde{R}_t - \tilde{P}_t = \tilde{Y}_t - \tilde{K}_t \] (91)

From (39) e (40),
\[
P_{*j,t}E_t \sum_{i=0}^{\infty} (\beta \theta)^i Y_{j,t+i} = \left( \frac{\varphi - 1}{\varphi} \right) E_t \sum_{i=0}^{\infty} (\beta \theta)^i Y_{j,t+i} P_{*t+i} \frac{W_{t+i}}{(1-\alpha)A_{t+i}} \left[ \left( \frac{1-\alpha}{\alpha} \right) \frac{R_{s,t+i}}{W_{t+i}} \right]^\alpha
\]

Log-linearizing the left side of the previous equation:
\[
P_{*j,ss}E_t \sum_{i=0}^{\infty} (\beta \theta)^i Y_{j,ss,t+i} = \left( \frac{\varphi - 1}{\varphi} \right) E_t \sum_{i=0}^{\infty} (\beta \theta)^i Y_{j,ss,t+i} \left[ \left( \frac{1-\alpha}{\alpha} \right) \frac{R_{ss,t+i}}{W_{ss}} \right]^\alpha \alpha \sum_{i=0}^{\infty} (\beta \theta)^i \] (92)

Now, log-linearizing the right:
\[
= \left( \frac{\varphi - 1}{\varphi} \right) P_{ss} Y_{j,ss} \frac{W_{ss}}{(1-\alpha)A_{ss}} \left[ \left( \frac{1-\alpha}{\alpha} \right) \frac{R_{ss}}{W_{ss}} \right]^\alpha \alpha \sum_{i=0}^{\infty} (\beta \theta)^i \left[ 1 + \tilde{P}_{t+i} + \tilde{Y}_{j,t+i} + (1 - \alpha)(\tilde{W}_{t+i} + \tilde{R}_{t+i}) - \tilde{A}_{t+i} + \alpha \tilde{R}_{t+i} \right]
\]

at steady state,
\[
\left( \frac{\varphi - 1}{\varphi} \right) = \frac{1}{W_{ss} \left[ \left( \frac{1-\alpha}{\alpha} \right) \frac{R_{ss}}{W_{ss}} \right]^\alpha}
\]

thus,
\[
= P_{ss} Y_{j,ss} E_t \sum_{i=0}^{\infty} (\beta \theta)^i \left[ 1 + \tilde{P}_{t+i} + \tilde{Y}_{j,t+i} + (1 - \alpha)(\tilde{W}_{t+i} - \tilde{A}_{t+i} + \alpha \tilde{R}_{t+i}) \right]
\]

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joining the two sides:

\[
\tilde{P}_{j,t}^* = (1 - \beta \theta) E_t \sum_{i=0}^{\infty} (\beta \theta)^i \left[ \tilde{P}_{t+i} + (1 - \alpha) \tilde{W}_{t+i} - \tilde{A}_{t+i} + \alpha \tilde{R}_{t+i} \right] \tag{92}
\]

and,

\[
\tilde{P}_t = \theta \tilde{P}_{t-1} + (1 - \theta) \tilde{P}_{j,t}^* \tag{93}
\]

substituting (92) into (93),

\[
\tilde{P}_t = \theta \tilde{P}_{t-1} + (1 - \theta)(1 - \beta \theta) E_t \sum_{i=0}^{\infty} (\beta \theta)^i \left[ \tilde{P}_{t+i} + (1 - \alpha) \tilde{W}_{t+i} - \tilde{A}_{t+i} + \alpha \tilde{R}_{t+i} \right] \tag{94}
\]

Multiplying both sides of equation (94) by \((1 - \beta \theta L^{-1})\), knowing that \(LX_t = X_{t-1} \) and \(X_{t+1} = L^{-1}X_t\), and starting from the left side of the equation (94),

\[
(1 - \beta \theta L^{-1}) \tilde{P}_t = \tilde{P}_t - \beta \theta \tilde{P}_{t+1}
\]

Now, working the right side,

\[
= (1 - \beta \theta L^{-1}) \left\{ \theta \tilde{P}_{t-1} + (1 - \theta)(1 - \beta \theta) E_t \sum_{i=0}^{\infty} (\beta \theta)^i \left[ \tilde{P}_{t+i} + (1 - \alpha) \tilde{W}_{t+i} - \tilde{A}_{t+i} + \alpha \tilde{R}_{t+i} \right] \right. \\
= \theta \tilde{P}_{t-1} + (1 - \theta)(1 - \beta \theta) E_t \sum_{i=0}^{\infty} (\beta \theta)^i \left[ \tilde{P}_{t+i} + (1 - \alpha) \tilde{W}_{t+i} - \tilde{A}_{t+i} + \alpha \tilde{R}_{t+i} \right] \\
- \beta \theta \theta \tilde{P}_t - \beta \theta(1 - \theta)(1 - \beta \theta) E_t \sum_{i=0}^{\infty} (\beta \theta)^i \left[ \tilde{P}_{t+i} + (1 - \alpha) \tilde{W}_{t+i} - \tilde{A}_{t+i} + \alpha \tilde{R}_{t+i} \right] \\
= \theta \tilde{P}_{t-1} + (1 - \theta)(1 - \beta \theta) \left[ \tilde{P}_t + (1 - \alpha) \tilde{W}_t - \tilde{A}_t + \alpha \tilde{R}_t \right] - \beta \theta \theta \tilde{P}_t
\]

joining the two sides,

\[
\tilde{P}_t - \beta \theta \tilde{P}_{t+1} = \\
\theta \tilde{P}_{t-1} - \beta \theta \theta \tilde{P}_t + (1 - \theta)(1 - \beta \theta) \left[ (1 - \alpha) \tilde{W}_t - \tilde{A}_t + \alpha \tilde{R}_t \right] + \tilde{P}_t - \beta \theta \tilde{P}_t - \theta \tilde{P}_t + \beta \theta \theta \tilde{P}_t \\
\theta(\tilde{P}_t - \tilde{P}_{t-1}) = \beta \theta(\tilde{P}_{t+1} - \tilde{P}_t) + (1 - \theta)(1 - \beta \theta) \left[ (1 - \alpha) \tilde{W}_t - \tilde{A}_t + \alpha \tilde{R}_t \right] \\
\tilde{H}_t = \beta \tilde{H}_{t+1} + \frac{(1 - \theta)(1 - \beta \theta)}{\theta} \left[ (1 - \alpha) \tilde{W}_t - \tilde{A}_t + \alpha \tilde{R}_t \right] \tag{95}
\]

From (41),

\[
\beta(\tilde{B}_{t+1} - \tilde{B}_t) - \tilde{B}_t = P_{ss}G_{ss}(\tilde{P}_t + \tilde{G}_t) - BAL_{ss} \tilde{B}AL_t - TAX_{ss} \tilde{TAX}_t \tag{96}
\]

From (42),

24
\[ G_{ss} \tilde{G}_t = \chi B_{ss} \tilde{B}_t \]  

From (43),  
\[
\text{TAX}_{ss} \tilde{\text{TAX}}_t = P_{ss} \tau_c \left[ C_{ss}(\tilde{P}_t + \tilde{C}_t) + I_{ss}(\tilde{P}_t + \tilde{I}_t) \right] \\
+ W_{ss} L_{ss} \tau_l (\tilde{W}_t + \tilde{L}_t - \phi^k_l) + R_{ss} K_{ss} \tau_k (\tilde{R}_t + \tilde{K}_t - \phi^k_k) 
\]

From (44),  
\[ \text{BAL}_{ss} \tilde{\text{BAL}}_t = W_{ss} L_{ss} \tau_p (\tilde{W}_t + \tilde{L}_t) \]  

From (45),  
\[ R^B_{ss} \tilde{R}^B_t = a Y_{ss} \tilde{Y}_t + b \pi_{ss} \tilde{\pi}_t \]  

From (46),  
\[ \tilde{\pi}_t = \tilde{P}_t - \tilde{P}_{t-1} \]  

From (47),  
\[ Y_{ss} \tilde{Y}_t = C_{ss} \tilde{C}_t + G_{ss} \tilde{G}_t + I_{ss} \tilde{I}_t \]
Appendix C: Results of Shocks in Dynare.

Figure 3: Stochastic shocks in the tax rates on labor income
Source: Prepared by the authors.
Figure 4: Stochastic shocks in the tax rates on capital income.
Source: Prepared by the authors.
Figure 5: Shock in the productivity.
Source: Prepared by the authors.
Figure 6: Shock in the intertemporal choice.
Source: Prepared by the authors.
Figure 7: Shock in the labor supply.
Source: Prepared by the authors.
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