Dynare Working Papers Series http://www.dynare.org/wp/

Financial frictions in a DSGE model for Latvia

Buss Ginters

Working Paper no. 42

May 2015



centre pour la recherche economique et ses applications 142, rue du Chevaleret — 75013 Paris — France http://www.cepremap.fr

Financial frictions in a DSGE model for Latvia^{*}

Ginters Buss[†] Bank of Latvia

September 17, 2014

Abstract

This paper builds a dynamic stochastic general equilibrium (DSGE) model for Latvia that would be suitable for policy analysis and forecasting purposes at Bank of Latvia. For that purpose, I adapt the DSGE model with financial frictions of Christiano, Trabandt and Walentin (2011) to Latvia's data, estimate it, and study whether adding the financial frictions block to an otherwise identical ('baseline') model is an improvement with respect to several dimensions. The main findings are: i) the addition of financial frictions block provides more appealing interpretation for the drivers of economic activity, and allows to reinterpret their role; ii) financial frictions played an important part in Latvia's 2008-recession; iii) the financial frictions model beats both the baseline model and the random walk model in forecasting both CPI inflation and GDP, and performs roughly the same as a Bayesian structural vector autoregression.

Keywords: DSGE model, financial frictions, small open economy, Bayesian estimation, currency union

JEL code: E0, E3, F0, F4, G0, G1

^{*}I thank Viktors Ajevskis, Rudolfs Bems, Konstantins Benkovskis, Martins Bitans, Dmitry Kulikov and Karl Walentin for feedback. I also thank Andrejs Kurbatskis and several other colleagues at Bank of Latvia for helping with the data. All remaining errors are my own. I have benefited from the program code provided by Lawrence Christiano, Mathias Trabandt and Karl Walentin for their model.

Disclaimer: This report is released to inform interested parties of research and to encourage discussion. The views expressed in this paper are those of the author and do not necessarily reflect the views of the Bank of Latvia.

[†]Address for correspondence: Latvijas Banka, K. Valdemara 2A, Riga, LV-1050, Latvia; e-mail: ginters.buss@gmail.com.

1 Introduction

This work is an attempt to build a dynamic stochastic general equilibrium (DSGE) model for Latvia that would be suitable for policy analysis and forecasting purposes at Bank of Latvia, since the current main macroeconomic model lacks microfoundations. Also, the recent financial crisis has suggested that business cycle modelling should not abstract from financial factors, thus modeling financial frictions is deemed to be requisite.

Therefore, I take the model of Christiano, Trabandt and Walentin (2011) (henceforth, CTW) with financial frictions as a starting point. To assess the effect of having financial frictions mechanism in a DSGE model, I compare the output of the model throughout the paper with an otherwise identical model, called the 'baseline' model, but lacking the mechanism of financial frictions. The baseline model is a standard open economy model, and builds on Christiano, Eichenbaum and Evans (2005) and Adolfson, Laseen, Linde and Villani (2008). The financial frictions model adds the Bernanke, Gertler and Gilchrist (1999, henceforth BGG) financial accelerator mechanism to the baseline model.

I modify the CTW model with respect to monetary policy: since Latvia's currency has been pegged to euro since 2005 and became euro in 2014 when Latvia joined the euro area, I model the monetary policy as a nominal interest rate peg to the foreign interest rate. The foreign economy is modeled as an identified structural vector autoregression (VAR) in foreign output, inflation, nominal interest rate and technology growth.

The main findings are as follows: i) the addition of financial frictions block provides more appealing interpretation for the drivers of economic activity, and allows to reinterpret their role; ii) financial frictions played an important part in Latvia's 2008-recession; iii) the financial frictions model beats both the baseline model and the random walk model in forecasting both CPI inflation and GDP.

The paper is structured as follows. Section 2 overviews the model. Section 3 describes the estimation procedure, and Section 4 - the results. Section 5 concludes. Appendix A contains the figures and tables. Appendix B contains further computational results. Appendices C and D contain a detailed model's description.

2 The model in brief

Since the model is almost a replica of Christiano, Trabandt and Walentin (2011, henceforth CTW), this section is a brief introduction to the model, whereas its formal description is relegated to Appendix C. The only noticeable difference between the CTW model and this one is in the behavior of monetary authority which is modeled as an interest rate peg in this paper.

2.1 Baseline model

The baseline model builds on Christiano, Eichenbaum and Evans (2005) and Adolfson, Laseen, Linde and Villani (2008). The three final goods: consumption, investment and exports, are produced by combining the domestic homogeneous good with specific imported inputs for each type of final good. Specialized domestic importers purchase a homogeneous foreign good, which they turn into a specialized input and sell to domestic import retailers. There are three types of import retailers. One uses the specialized import goods to create a homogeneous good used as an input into the production of specialized exports. Another uses the specialized import goods to create an input used in the production of investment goods. The third type uses specialized imports to produce a homogeneous input used in the production of consumption goods. Exports involve a Dixit-Stiglitz (Dixit and Stiglitz, 1977) continuum of exporters, each of which is a monopolist that produces a specialized export good. Each monopolist produces its export good using a homogeneous domestically produced good and a homogeneous good derived from imports. The homogeneous domestic good is produced by a competitive, representative firm. The domestic good is allocated among the i) government consumption (which consists entirely of the domestic good) and the production of ii) consumption, iii) investment, and iv) export goods. A part of the domestic good is lost due to the real friction in the model economy due to investment adjustment and capital utilization costs.

Households maximize expected utility from a discounted stream of consumption (subject to habit) and hours worked. In the baseline model, the households own the economy's stock of physical capital. They determine the rate at which the capital stock is accumulated and the rate at which it is utilized. The households also own the stock of net foreign assets and determine its rate of accumulation.

The monetary policy is conducted as a hard peg of the domestic nominal interest rate to the foreign nominal interest rate¹. The government expenditures grow exogenously. The taxes in the model economy are: capital tax, payroll tax, consumption tax, labor income tax, and a bond tax. Any difference between government expenditures and tax revenue is offset by lump-sum transfers. The foreign economy is modeled as an identified structural vector autoregression (VAR) in foreign output, inflation, nominal interest rate and technology growth. The model economy has two sources of exogenous growth: neutral technology growth and investment-specific technology growth.

2.2 Financial frictions model

The details are relegated to Appendix C, while a brief summary follows. The financial frictions model adds the Bernanke, Gertler and Gilchrist (1999, henceforth BGG) financial frictions to the above baseline model. Financial frictions reflect that borrowers and lenders are different people, and that they have different information. Thus the model introduces 'entrepreneurs' - agents who have a special skill in the operation and management of capital. Their skill in operating capital is such that it is optimal for them to operate more capital than their own resources can support, by borrowing additional funds. There is financial friction because the management of capital is risky, i.e. entrepreneurs can go bankrupt, and only the entrepreneurs costlessly observe their own idiosyncratic productivity.

In this model, the households deposit money in banks. The interest rate that households receive is nominally non state-contingent.² The banks then lend funds to en-

¹A generalized Taylor rule, including foreign interest rate and nominal exchange rate, was also studied but the results are skipped due to the space constraint. In short, the peg system fits the data better.

²These nominal contracts give rise to wealth effects of unexpected changes in the price level, as emphasized by Fisher (1933). E.g., in the case of a shock driving the price level down, households receive a wealth transfer. This transfer is taken from entrepreneurs whose net worth is thereby reduced. With tightening of their balance sheets, the ability of entrepreneurs to invest is reduced, and this generates an economic slowdown.

trepreneurs using a standard nominal debt contract, which is optimal given the asymmetric information.³ The amount that banks are willing to lend to an entrepreneur under the debt contract is a function of the entrepreneur's net worth. This is how balance sheet constraints enter the model. When a shock occurs that reduces the value of entrepreneurs' assets, this cuts into their ability to borrow. As a result, entrepreneurs acquire less capital and this translates into a reduction in investment and leads to a slowdown in the economy. Although individual entrepreneurs are risky, banks are not.

The financial frictions block brings two new endogenous variables, one related to the interest rate paid by entrepreneurs and the other - to their net worth. There are also two new shocks, one to idiosyncratic uncertainty and the other - to entrepreneurial wealth.

The explicit description of both the baseline and the financial frictions models is relegated to Appendix C.

3 Estimation

I estimate both the baseline and financial frictions models with Bayesian techniques. The equilibrium conditions of the model are reported in Appendix D.

3.1 Calibration

The time unit is a quarter. A subset of model's parameters is calibrated and the rest are estimated using the data for Latvia and the euro area. The calibrated values are displayed in Tables 1 and 2. These are the parameters that are typically calibrated in the literature and are related to "great ratios" and other observable quantities related to steady state values. The values of the parameters are selected such that they would be specific to the data at hand. Sample averages are used when available. The discount factor, β , and the tax rate on bonds, τ_b , are set to match roughly the sample average real interest rate for the euro area. The capital share, α , is set to 0.4.

Table 1 about here

Import shares are set to reasonable values by consulting to the input-output tables and fellow economists - 45%, 65% and 55% for import share in consumption, investment and exports, respectively.⁴ The government expenditure share in the gross domestic product (henceforth GDP) is set to match the sample average, i.e. 20.2%. The steady state growth rates of neutral technology and inflation are set to two percent annually, and correspond to the euro area. The steady state growth rate of investment-specific technology is set to zero. The steady state quarterly bankruptcy rate is calibrated to two percent, up from one percent in the CTW model for the Swedish data. The values of the price markups are set to the typical values found in the literature, i.e., to 1.2 for exports

³Namely, the equilibrium debt contract maximizes the expected entrepreneurial welfare, subject to the zero profit condition on banks and the specified return on household bank liabilities.

⁴The import share in exports might appear to be too high when consulting to the literature of international trade. E.g. the results of Stehrer (2013) suggest, from the value-added perspective, that share closer to 30%. Such a calibration would not change the model's results much but would suggest a slight deterioration of the model's fit to the data, in terms of marginal data density.

and imported exports, and 1.3 for the domestic, imported consumption and imported investment, which is supported by the model's fit in terms of the marginal data density⁵. Wage markup is set to 1.5 as in CTW.

There is full indexation of wages to the steady state real growth, $\vartheta_w = 1$. The other indexation parameters are set to get the full indexation and thereby avoid steady state price and wage dispersion, following CTW. Tax rates are calibrated such that those would represent implicit or effective rates. Three of these are calibrated using Eurostat data⁶: tax rate on capital income is set to 0.1, the value-added tax on consumption, τ^c , and the personal income tax rate that applies to labor, τ^y , are set to $\tau^c = 0.18$ and $\tau^y = 0.3$. Payroll tax rate is set to $\tau^w = 0.33$, down from the official 0.35 (0.24 by employer and 0.11 by employee). The elasticity of country risk to net asset position, $\tilde{\phi}_a$ is set to a small positive number and, in that region, its purpose is to induce a unique steady state for the net foreign asset position. Transfers to entrepreneurs parameter W_e/y is kept the same as in CTW. The country risk adjustment coefficient in the uncovered interest parity condition is set to zero in order to impose the nominal interest rate peg.

Table 2 about here

Three observable ratios are chosen to be exactly matched throughout the estimation, and therefore three corresponding parameters are recalibrated for each parameter draw: the steady state real exchange rate, $\tilde{\varphi}$, to match the export share of GDP in the data, the scaling parameter for disutility of labor, A_L , to fix the fraction of their time that individuals spend working⁷, and the entrepreneurial survival rate, γ , is set to match the net worth to assets ratio⁸. In the earlier steps of calibration, the depreciation rate of capital, δ , was also set to match the ratio of investment over output, but the realized value of depreciation rate turned out to be rather high (unless the capital share in production, α , was substantially increased but that yielded excessively high capital to output ratio) and sensitive to the initial values, therefore it was decided to fix the quarterly depreciation rate to a more reasonable value of three percent.

3.2 Priors

There are 21 structural parameters, eight first-order autoregressive (henceforth, AR(1)) coefficients, 16 VAR parameters for the foreign economy, and 16 shock standard deviations estimated with Bayesian techniques within Matlab/Dynare environment (Adjemian et al, 2011). The priors are displayed in Tables 3 to 6. The priors are similar to CTW. Less agnostic priors are assigned for the foreign structural VAR model since otherwise

⁵In this paper, when I speak of the model's fit, unless otherwise mentioned, I mean the marginal data density and the forecasting performance.

⁶Source: http://epp.eurostat.ec.europa.eu/cache/ITY_PUBLIC/2-29042013-CP/EN/ 2-29042013-CP-EN.PDF, accessed in September 6, 2013

⁷This fraction of time calibrated to 0.27 is somewhat arbitrary but checked against the model fit with respect to its neighboring values.

⁸The net worth to assets ratio for Latvia, if the definition of CTW is taken, yields about 0.15. However, the model fit favors a much larger number, 0.6, which is used in the final calibration. The latter number might be rationalized if the net worth was measured not only by the share price index but if it included also the real estate value.

the foreign monetary policy appears to be weakly identified⁹. The prior means of the estimated standard deviations are set closer to their posteriors, and parameters and shock standard deviations are scaled to be of similar order of magnitude in order to facilitate optimization.

3.3 Data

The model is estimated using data for Latvia ('domestic' part) and the euro area ('foreign' part). The sample period is 1995Q1 - 2012Q4. I use 18 observable time series to estimate the financial frictions model and two less to estimate the baseline model. The variables used in levels are: nominal interest rate, GDP deflator inflation, consumer price index (henceforth CPI) inflation, investment price index inflation, foreign CPI inflation, foreign nominal interest rate and the interest rate spread. The rest of the variables are in terms of the first differences of logs, and these are: GDP, consumption, investment, exports, imports, government expenditures, real wage, real exchange rate, real stock prices, total hours worked, and foreign GDP. All the differenced variables are demeaned except for total hours worked. The domestic inflation rates and the real exchange rate are demeaned as well. All real quantities are in per capita terms. All foreign variables correspond to the euro area data.

3.4 Shocks and measurement errors

In total, there are 18 exogenous stochastic variables in the theoretic financial frictions model: four technology shocks - stationary neutral technology, ϵ , stationary marginal efficiency of investment, Υ , unit-root neutral technology, μ_z , and unit-root investment specific technology, μ_{Ψ} , - a shock on consumption preferences, ζ^c , and for disutility of labor supply, ζ^h , a shock to government expenditure, g, and a country risk premium shock that affects the relative riskiness of foreign assets compared to domestic assets, $\tilde{\phi}$. There are five markup shocks, one for each type of intermediate good, τ^d , τ^x , $\tau^{m,c}$, $\tau^{m,i}$, $\tau^{m,x}$ (d - domestic, x - exports, m, c - imported consumption, m, i - imported investment, m, x - imported exports). The financial frictions model has two more shocks - one to idiosyncratic uncertainty, σ , and one to entrepreneurial wealth, γ . There are also shocks to each of the foreign observed variables - foreign GDP, y^* , foreign inflation, π^* , and foreign nominal interest rate, R^* .

The stochastic structure of the exogenous variables are the following: eight of these evolve according to AR(1) processes:

$$\epsilon_t, \Upsilon_t, \zeta_t^c, \zeta_t^h, g_t, \phi_t, \sigma_t, \gamma_t$$

⁹My unreported results show that this is true regardless of the sample span used in the estimation and whether or not the foreign block is estimated separately from the domestic block. Also, the use of foreign CPI inflation instead of the foreign GDP deflator's inflation (which is used by CTW) improves the identification of the foreign monetary policy only marginally. Therefore the results involving the foreign monetary policy should be interpreted with caution. The replacement of the foreign structural VAR with a full-fledged foreign DSGE block thus might be an improvement but is not considered in this paper.

Five shock processes are i.i.d.:

$$\tau^d_t, \tau^x_t, \tau^{m,c}_t, \tau^{m,i}_t, \tau^{m,x}_t$$

and five shock processes are assumed to follow a first-order VAR:

$$y_t^*, \pi_t^*, R_t^*, \mu_{z,t}, \mu_{\Psi,t}.$$

As in CTW, two shocks are suspended in the estimation: the shock to unit-root investment specific technology, $\mu_{\Psi,t}$, and the idiosyncratic entrepreneur risk shock, σ_t . The first one should correspond to the foreign block but its identification is dubious in the particular SVAR model; the second has been found to have limited importance in CTW.

There are measurement errors except for domestic interest rate and the foreign variables. The variance of the measurement errors is calibrated to correspond to 10% of the variance of each data series.

4 Results

The domestic and foreign blocks are estimated separately since Latvia's economy has minuscule effect on the euro area. The estimation results for the foreign SVAR model are obtained using a single Metropolis-Hastings chain with 100 000 draws after a burn-in of 900 000 draws. For the domestic block, the estimation results are obtained using a single Metropolis-Hastings chain with 100 000 draws after a burn-in of 400 000 draws. Prior-posterior plots are shown in Appendix B.

4.1 Posterior parameter values

The posterior parameter estimates for the foreign block are reported in Tables 3 and 4, and those specific to the domestic block - in Tables 5 and 6. The priors were deliberately fixed to be the same across the two models for a more transparent comparison, and favor the baseline model. The estimated mode of the elasticity of substitution of investment goods parameter, η_i , is close to unity and thus the parameter is calibrated for the financial frictions model to 1.1, similar to the posterior mean in the baseline model, in order to avoid numerical issues. Overall, the estimated posterior means of the parameters are similar between the two models. The most notable difference is in the investment adjustment costs parameter which is about 2.4 times lower for the financial frictions model compared to the baseline specification. They are statistically significantly different at 10% significance level. The lower parameter indicates that the financial frictions model induces the gradual response that the investment adjustment mechanism was introduced to generate. Also, the estimated persistence parameter of the marginal efficiency of investment (henceforth MEI) shock is reduced (from 0.80 to 0.57) with the introduction of the financial frictions block. Regarding the estimated standard deviations of shocks, the financial frictions model assigns a smaller standard deviation to the marginal efficiency of investment shock, which, apparently, is 'crowded out' by the entrepreneurial wealth shock.

Tables 3 - 6 about here

4.2 Model moments and variance decomposition

4.2.1 Model moments

Table 7 presents the data and the model means and standard deviations for the observed time series. The table shows that there is a substantial variation of growth rates in the data, especially between the domestic and foreign variables, which is why real quantities, the domestic inflation rates and the real exchange rate are demeaned before matching the model to the data. The standard deviations are matched rather well but their over-estimation is evident for total hours, GDP, imports, as well as for the interest rate spread¹⁰. The introduction of the financial frictions block appears to slightly lessen this over-estimation issue.

Table 7 about here

4.2.2 Conditional variance decomposition

The conditional variance decomposition at eight quarters forecast horizon is reported in Table 8. (Those at one, four and twenty quarters forecast horizons are reported in Appendix B.).

Table 8 about here

Entrepreneurial wealth shock versus marginal efficiency of investment shock Table 8 shows that the entrepreneurial wealth shock, which is specific to the financial frictions model and absent from the baseline model, 'crowds out' the marginal efficiency of investment (MEI) shock by reducing its share of explaining the variance of investment from 74% (baseline) to 28% (financial frictions model), the variance of net exports to GDP ratio from 60% to 6%, and the variance of GDP from 15% to 4%. As a reminder, MEI shock enters in the capital accumulation equation ((C.38) in Appendix) and affects how (efficiently) investment is transformed into capital. This is the shock whose importance is emphasized in Justiniano, Primiceri and Tambalotti (2011), where one of their interpretations of this shock being a proxy for the effectiveness with which the financial sector channels the flow of the household savings into a new productive capital.

The entrepreneurial wealth shock explains 10% of the variance of GDP, 45% of the variance of investment, 35% of the net exports to GDP ratio, 51% of entrepreneurial net worth and 69% of the spread between the nominal interest rate paid by the entrepreneur and the risk-free one.

CTW do not report the conditional variance decomposition for the baseline model, but with the financial frictions together with the search and matching frictions in labor market (without additional shock added) which are absent in my financial frictions model. Also, their model is estimated for Swedish data with inflation-targeting monetary policy. Nevertheless, it is instructive to compare the results of CTW with ours. The results of CTW suggest that, when financial frictions mechanism is present, MEI shock explains 10% of the variance of investment, 7% of the variance of net exports to GDP ratio, and

¹⁰CTW note that their use of 'endogenous prior' reduces the effect of over-estimated shock standard deviations. I'm not using such a prior.

4% of the variance of GDP. Also, the entrepreneurial wealth shock explains 71% of the variance of investment, 23% of the variance of the net exports to GDP ratio, 25% of the variance of GDP, 64% of entrepreneurial net worth, and 60% of the variance of the spread. CTW briefly mention, but do not report in tables, the effect of shutting down the financial shock in their model. In that case, MEI shock becomes more important in the variance decomposition: it explains 52% of the variance of investment and 6% of GDP. These results are broadly in line with mine except for the variance of investment which appears to be better explained by the entrepreneurial wealth shock than by MEI shock in Sweden compared to Latvia. The difference is likely due to the milder response of entrepreneurial net worth to the wealth shock in Latvia compared to Sweden, reflecting the fact that Swedish financial markets are more developed.

Country risk premium shock Table 8 also reports that the country risk premium shock is the major driving force of the domestic nominal interest rate and a crucial factor in Latvia's business cycles. This is more so in the financial frictions model compared to the baseline. So, for the given sample of 1995Q1-2012Q4, the country risk premium shock explains 92% of the variance of the domestic nominal interest rate (versus 87% in baseline), 11% of the variance of investment (versus 5% in baseline), 3% of the variance of GDP (versus 1% in baseline), 18% of the variance of net exports to GDP ratio (versus 10% in baseline) and 13% of the variance of the entrepreneurial net worth.

Comparing to the results of CTW, there are big differences. For Sweden, this shock explains only 5% of the variance of nominal interest rate, 1% of the variance of investment, and 1% of the variance of net worth, while the variance of GDP is explained by about the same amount as in Latvia, i.e. 3%. The reason for the difference is that, during the specific historic sample, the domestic nominal interest rate in Latvia has been higher than than in the euro area and given that, in the model, Latvia's currency is hard-pegged to the euro, the (huge historic) difference between the actual domestic and foreign interest rates is explained by the country risk premium. It is expected that, since Latvia's joining the euro area in 2014, the weight of the country risk premium shock on the domestic interest rate will diminish, giving more influence to the euro area-wide shocks.

Shocks in the foreign economy block The effect of the foreign interest rate, foreign output and foreign inflation shocks on the domestic economy is estimated to be rather limited, with the greatest influence being to the domestic nominal interest rate. The unitroot technology shock also has been estimated to have little influence on the domestic economy during the particular historic period.

These results are broadly close to the results of CTW who also find negligible role of the shock to foreign interest rate, foreign output and foreign inflation to Swedish economy. Though, their estimated effect of the unit-root technology shock is more influential, explaining 4.1% of the variance of Swedish GDP compared to 0.1% for Latvia's GDP. The latter result might be explained by the fact that, during the particular historic episode, Latvia's economy has been on its more or less idiosyncratic catching-up boom-bust cycle, while the more developed Swedish economy has been more reliant on the world-wide technology growth. Also, CTW estimate this shock based on the trade-weighted foreign variables, while I use the euro area variables, thus the link (the common technology) between the domestic and foreign variables is looser in my case. **Stationary neutral technology shock** While touching upon technology shocks, another difference between CTW results for Sweden and mine for Latvia is in the effect of the stationary neutral technology shock affecting the intermediate goods producers' production function. This shock is estimated to have minor influence on Latvia's economy except for total hours worked (11% of the variance explained by this shock).

CTW estimation shows that this shock explains about the same portion of the variance of hours worked (9%) but also 11% of the variance of consumption (0.1% for Latvia), 9% of the variance of GDP (0.8% for Latvia), 6% of CPI inflation (1% for Latvia) and 8% of the domestic nominal interest rate (0.0% for Latvia). Apparently, other domestic shocks have compensated the lack of influence of the stationary technology shock on Latvia's economy.

Household preference shocks Noticeable, The *consumption* preference shock explains 82% of the variation of consumption in Latvia, whereas 'only' 45% in Sweden. This difference might be explained by the strong consumption-driven boom that Latvia experienced starting around 2004 (see the historic shock decomposition below).

The *labor* preference shock is estimated to have about the same effect on both countries at least with respect to wages; this shock is estimated to explain 39% of the variance of real wages in both Latvia and Sweden. The effects on other labor market variables differ, but this is, most probably, due to the different structure of labor market modeling block in the models.

Domestic markup shock The domestic markup shock, affecting marginal cost of producing the domestic intermediate good, is estimated to explain 27% of Latvia's CPI inflation (45% in Sweden) and 39% of the variance in real wage (31% in Sweden). This completes the similarities of this shock across the countries, since, given Latvia's peg regime, this shock explains 23% of the variance of Latvia's real exchange rate (0.2% in Sweden), while in Sweden, it affects, through Taylor rule, the nominal interest rate, and parts of real economy stronger than in Latvia; e.g. it explains 7% of the variance of Swedish GDP and 3% of the variance of Swedish investment, while these figures are 2% and 0.1% for Latvia.

Export goods markup shock Table 8 shows that the markup shock on export goods is estimated to have little effects on Latvia's economy; the only noticeable ones are the 2.5% (up from 1% in baseline) of the variance of GDP and 2% (up from 1% in baseline) of the variance of hours worked, while in Sweden these figures are 8% and 10%, respectively. Again, given the model differences, it is hard to point exact source of the discrepancy. A small part of the difference¹¹ is due to the higher calibrated imported goods share in exports for Latvia (55%) than for Sweden (35%), resulting in a smaller effect of the exports markup on Latvia's GDP and hence hours worked, since the markup on imported exports is subject to a separate, imported exports markup shock.

Imported markup shocks The *imported exports* markup shock, indeed, has more weight on Latvia's economy than on Swedish: it is estimated to explain 35% of the

¹¹I have checked this claim by recalibrating the model.

variance of Latvia's GDP (16% for Sweden) and 30% of the variance of total hours worked in Latvia (14% in Sweden).

Regarding the rest of the imported goods markup shocks, the *imported consumption* markup shock explains the majority, 51% of the variance of the the domestic CPI inflation (up from 39% in baseline and 34% in Sweden), and hence is the major shock affecting the real exchange rate (it explains 45% (up from 34% in baseline) of the variance of Latvia's real exchange rate, while in Sweden, this shock explains, through Taylor rule, 17% of the variance of the nominal interest rate but less so the real exchange rate. In contrast to the domestic markup shock, the imported consumption markup shock is estimated to have a non-negligible effect on Latvia's GDP - it explains almost 4% (up from 1% in baseline) of the variance of this effect, again, can be explained by the strong consumption-driven boom Latvia's economy experienced during the considered sample span.

Finally, the *imported investment* markup shock explains 7% (down from 10% in baseline) of the variance of investment, 18% (down from 30% in baseline) of the variance of GDP, and 27% (down from 42.5% in baseline) of the variance of total hours worked. Quite differently, this shock is estimated to have negligible effect on Swedish economy. One explanation for the difference might be the higher calibrated imports share in investment goods for Latvia (65%) than in Sweden (43%) but this must be only a part of the answer. Another eye-catching result is the large difference between the results of financial frictions and baseline models. Absent of financial frictions block in the model, the imported investment markup shock would claim to explain almost a third of the variance of Latvia's GDP at two years forecast horizon, whereas it is less than one fifth with the financial frictions block added to the model. The rest of the shock appears to be attracted by consumption-related shocks - the consumption preference shock and the imported consumption markup shock.

Foreign shocks combined Overall, if the foreign shocks are defined as the three foreign (interest rate, output, inflation) stationary shocks, the country risk premium shock, the world-wide unit root neutral technology shock, the markup shocks on imports (imported exports, consumption, investment) and exports - in total, 10 shocks - see the bottom row of Table 8, then they explain 99% of the variance in the domestic nominal interest rate (up from 95% in the baseline and 28% in Sweden), the overwhelming part explained by the country risk premium shock. Also, 53% and 62% of the variations of CPI inflation and GDP, respectively, (versus 43% and 72% in baseline, and 40% and 32% in Sweden) at two year forecast horizon are explained by the foreign shocks, the overwhelming portion coming from markup shocks on imported consumption and domestic goods (for CPI inflation) and on imported exports and imported investment (for GDP).

Since, in the literature, the sources of business cycles are largely related to fluctuations in investment, the major source of the variance of investment in Latvia is estimated to be the entrepreneurial wealth shock. Given the evidence from Sweden, the influence of this shock is to be expected to grow as Latvia's firms become more financially integrated.

4.3 Impulse response functions

Since Table 8 shows that the entrepreneurial wealth shock is the main driver of the variance of investment in the financial frictions model and that it 'crowds out' MEI shock from the baseline model, it is instructive to compare the impulse response functions (henceforth IRF) of these two shocks.

Entrepreneurial wealth shock The IRF to the entrepreneurial wealth shock are plotted in Figure 1, which shows that a positive temporary entrepreneurial wealth shock, γ_t , drives up the net worth, reduces the expected bankruptcy rate and thus the interest rate spread, and increases the investment (by about the same percentage change as in net worth); GDP goes up accordingly, and so do the real wage and total hours worked. Both exports and imports increase but the latter increases more due to the demand for investment goods, thus net exports to GDP ratio decreases slightly. As a consequence, the net foreign assets to GDP ratio worsens, driving up a slight risk premium on the domestic nominal interest rate. The shock causes the cost of investment to decrease, and consumption to pick up only steadily. Therefore, CPI inflation decreases, though by a small amount, and thus the real exchange rate depreciates.

The response of net worth to this and other shocks is quite muted, i.e. its dynamics appear to die out in a few periods. This observation together with the autocorrelated measurement error of net worth suggest that the stock market price index might be a weak proxy for net worth in Latvia, and thus other potential measures, such as the house price index, could be investigated. Such an option is left for future research.

Figure 1 about here

MEI shock Comparing the wealth shock to a temporary MEI shock, Figure 2 shows that the effect of MEI shock in the baseline model is qualitatively similar to the effect of the wealth shock in the financial frictions model (except for the effect on consumption which decreases initially), but the introduction of financial frictions dampens the effect of MEI shock on all plotted variables (and consumption now slightly increases). The effect of these shocks on net worth and the spread is opposite; this is how the two shocks are distinguished in the financial frictions model.

MEI shock increases the amount of capital per investment and thus the price of capital decreases. Consumption barely moves, thus MEI shock has a downward pressure on prices.

Figure 2 about here

Country risk premium shock Figure 3 shows the IRF to a temporary country risk premium shock. As Table 8 shows, this shock is the major cause of the variance of the domestic nominal interest rate. The effects are qualitatively similar across the models but the financial frictions mechanism somewhat amplifies them. The shock increases the domestic nominal interest rate which decays towards its steady state with persistence. This is followed by a decrease in consumption and entrepreneurial net worth, an increase in the spread and the bankruptcy rate (both reverse the sign after a year), and a decrease

in investment (initially, about twice as much with financial frictions mechanism compared to the baseline), GDP, real wage, and total hours worked. Imports decreases more than exports, resulting in a slight increase in net exports to GDP ratio. CPI inflation decreases for about two years, after which the sign is reversed. The real exchange rate thus depreciates for the first two years after the shock.

Figure 3 about here

Foreign nominal interest rate shock Table 8 shows that the foreign nominal interest rate shock has little influence on the domestic economy during the particular historic period; nevertheless, policy-makers are usually interested in what happens after an increase in the policy rate, and it is the European Central Bank's policy rate that matters for Latvia after it joined the euro area in 2014. Figure 4 shows that a positive temporary foreign nominal interest shock increases both the foreign and the domestic nominal interest rate by the same amount, and both decay towards their steady state slowly. Consumption, investment and entrepreneurial net worth decrease, bankruptcy rate increases marginally (for the first year) and, as a result, so does the spread. GDP decreases, so do real wage, and total hours worked. There is a negligible increase in net exports to GDP ratio due to a decrease in imports. Thus, the net foreign assets to GDP ratio increases slightly, fostering a decrease of the domestic country risk premium, and therefore, also of the domestic nominal interest rate. CPI inflation decreases due to the slower domestic activity. The domestic inflation decreases by a larger amount than the foreign inflation, resulting in the initial but small depreciation of the real exchange rate. The effect is similar across the models except for the more persistent dynamics of the nominal interest rate under the financial frictions mechanism.

The impulse response functions are similar between the country risk premium and the foreign nominal interest rate shocks, thus signaling the potential identification issues of these two shocks. The particular procedure of estimating the foreign BVAR separately from the domestic block mitigates the identification problem somewhat. The replacement of the foreign BVAR with a full-blown foreign DSGE block could be a cure since it would identify the foreign monetary policy better but at the cost of model complexity.

The rest of the IRF are plotted in Appendix B.

Figure 4 about here

4.4 Smoothed shock values and historical decomposition

Figures 5 and 6 show the smoothed shock values for the financial frictions model. The table summarizing their means and standard deviations are relegated to Appendix B. These figures show that the means of the shocks are about zero. As to the downside, the measurement errors of the net worth, total hours worked, and real wage appear to be autocorrelated.

Figures 5 and 6 about here

Figures 7 to 13 show the historic shock decomposition of GDP, CPI inflation and the interest rate spread.

Concentrating on the most sizable shocks, Figures 7 - 8 show that the model GDP identifies the shock to household consumption preferences as the most important driving force of the 2004-boom. During the period of 2004-2007, the values of this shock were persistently above the sample average (see Figure 5), signifying that households were especially keen on spending on consumption goods during that period. The shock ceased during the second half of 2007, probably due to the rising costs of living and thus the decreasing consumer confidence (the latter is backed by the ECFIN consumer survey data). At that time several other shocks became adverse, including the stationary and unit-root neutral technology shocks, and the risk premium shock (see Figure 5). Starting from 2008 and up to 2011, a series of negative entrepreneurial wealth shocks is identified to have significantly affected GDP growth (Figure 7). In fact, this shock has become the major source determining the GDP level during the post-recession episode, see Figure 8. In the model, the dynamics of the entrepreneurial wealth is observable and measured by the OMX Riga share price index, which plummeted during the recession. In practice, it is likely that the variable captures also a part of the dynamics in the real estate prices (otherwise, the real estate sector is not present in the model), which also fell sharply during the recession as a result of the burst of the housing bubble.

Figures 7 - 8 about here

For comparison, Figures 9 - 10 show the growth decomposition delivered by the baseline model (smoothed shock figures are skipped due to space constraints). The baseline model identifies MEI shock as one of the most important shocks driving the 2004-boom and the subsequent recession. According to the baseline model, MEI shock has contributed negatively over the whole post-recession period, which is not easy to interpret.

Figures 9 - 10 about here

Therefore, having the financial frictions block in the model both clarifies and changes the story. First, the entrepreneurial wealth shock behaves differently than MEI shock, since the former has played little role during the boom period. Thus, consumption preferences are left as the single most important factor creating the 2004-boom. Second, the entrepreneurial wealth shock is more easily understandable force that has deepened the recession but ceased to be active during the post-recession episode. On the contrary, in the baseline model, the ever-active MEI shock during the post-recession period is harder to explicate.

CPI inflation Figure 11 shows that the model identifies the shock to household labor preferences as the major driving force of Latvia's CPI inflation up in the 2004-boom, coupled with the imported consumption markup shocks in 2007, and these same shocks together with the domestic markup shocks drew CPI inflation down in 2009.

The labor preference shock determines the household willingness to work. The model identifies that, during the period of 2005-2007 households in Latvia were keen to work less (and to consume more), relative to the sample average (see Figure 7). The disutility from work arose probably due to the rapidly growing economy and thus the relatively easy money available for spending. Shirking drove wages up to compensate for the household

disutility from work; and that in turn, pushed the price of consumption goods up. Beginning from late 2008 and continuing until the sample ends in 2012Q4, the labor preference shock is identified to have downward pressure on CPI inflation, which could be explained by the increased necessity to earn a living due to lower wages and fewer vacancies.

The markup shock to imported consumption goods raises the price of imported consumption goods. The model identifies that the level of this shock was persistently above its sample average during the year 2008, the time at which the consumption preference shock had already become flat or even negative, and coinciding with the period of the above average foreign inflation shock (unaffected by the domestic block, since estimated separately) and the peak in both the crude oil and natural gas prices. It is likely that the imported consumption markup shock captures the increase in the cost of energy, since the price of energy is not present in the model but through foreign inflation. Apparently, the foreign inflation variable is not able to fully represent the dynamics of imported costs, thus the rest is absorbed by the markup shock. For example, the price of natural gas affects the heating bills. It was a matter of fact that heating bills rose during the year 2008, constituting up to three percentage points of the annual inflation at that time. Overall, the model suggests that the imported consumption markup shock constituted about a half of the annual CPI inflation during the year 2008.

The domestic markup shock affects the marginal cost of domestic production before it is affected by the foreign markup shocks. The model identifies a series of negative domestic markup shock during 2009 (probably due to the easing in labor market, the reforms in the public sector, postponed investment projects or dividend payments by firms), and partly rebalancing during late 2010-2011, which pushed CPI inflation upwards.

The presence of the financial frictions block in the model reduces slightly the role of MEI shock and stationary technology shock on CPI inflation, see Figure 12.

Figures 11 - 12 about here

Interest rate spread Figure 13 shows that the entrepreneurial wealth shock is the main driving force behind the interest rate spread. The increased spread in the 2008-recession is explained mainly by a negative temporary wealth shock. MEI shock has also contributed to affect the spread but its role has been different from the wealth shock: MEI shock's contribution has been mild during the recession episode. Rather, it has contributed to reduce the spread during the boom period (as the wealth shock but to a greater extent) and during the years 2011-2012 (counteracting the wealth shock). Again, as MEI shock is ad hoc, it is not easy to interpret it.

Figure 13 about here

4.5 Forecasting performance

Figures 14 to 16 show one-step ahead forecasts of the baseline and the financial frictions models for all the observables. These are not true out-of-sample forecasts because the model is calibrated/estimated on the whole sample period 1995Q1-2012Q4. Nevertheless, these figures indicate approximate forecasting performance of the models. Particularly, it is informative to see whether the models tend to yield unbiased forecasts. The results

show that the models forecast relatively well. No crucial biases are evident, except for the CPI inflation which appears to be slightly upward biased. The total hours worked forecasts are rather volatile, inducing this volatility in the GDP series. On the positive side, the pick up of the interest rate spread in 2009 is forecasted in advance thus indicating that the model, potentially, could be applied in forecasting financial stress.

Figures 14 to 16 about here

Table 9 reports the forecasting performance of the baseline and financial frictions models relative to a random walk model (in terms of quarterly growth rates) with respect to predicting CPI inflation and GDP for horizons: one, four, eight and 12 quarters. I also report the forecasting performance of a BSVAR (with the same structure as the foreign BSVAR, and with similar priors), since it is often taken as a benchmark in the literature¹². Table 9 shows that both models forecast both variables at least as precisely as the random walk model at all horizons considered. Both models outperform the random walk by about 30% in forecasting both variables for horizons two to three years, and deliver about the same precision at a one quarter horizon. Moreover, the financial frictions model tends to deliver somewhat more precise forecasts than the baseline model of both CPI inflation and GDP, and a comparable forecasting precision to that of a BSVAR.

Repeating the exercise for only the last ten years of the sample shows the financial frictions model still performs roughly as well as the baseline and a BSVAR models (Table 10). Thus, the model can be used not only for policy studies but also for forecasting purposes. The results from our forecasting exercise are similar to those of CTW who also find that the financial frictions model tends to outperform slightly the baseline model.

Table 9 about here

5 Summary and conclusions

This paper builds a DSGE model for Latvia that would be suitable to replace the traditional macro-econometric model currently employed as the main macroeconomic model at Bank of Latvia. For that purpose, I adapt the Christiano, Trabandt and Walentin (2011, henceforth CTW) financial frictions model to Latvia's data. The monetary policy is altered to become a nominal interest rate peg to the foreign interest rate. I study the model fit, impulse response functions, conditional forecast variance decomposition, shock historic decomposition, and forecasting performance, and compare the outcome to that of the model without the financial accelerator block (the 'baseline' model), as well as to the findings of CTW.

The main findings are as follows. The addition of financial frictions block provides more appealing interpretation for the drivers of economic activity, and allows to reinterpret their role. Financial frictions played an important part in Latvia's 2008-recession. The financial frictions model beats both the baseline model and the random walk model in forecasting both CPI inflation and GDP, and performs roughly the same as a BSVAR.

¹²The particular BSVAR has some economically implausible estimated parameters, since Latvian GDP, CPI inflation and nominal interest rate data do not possess a stable and economically plausible relationship over the particular sample span.

Overall, the results suggest that the financial frictions model is suitable in both policy analysis and forecasting exercises, and is an improvement over the model without the financial frictions block.

References

- Adjemian, Stephane, Bastani, Houtan, Juillard, Michel, Karame, Frederic, Mihoubi, Ferhat, Perendia, George, Pfeifer, Johannes, Ratto, Marco and Villemot, Sebastien, 2011. "Dynare: Reference Manual, Version 4" Dynare Working Papers, 1, CEPREMAP.
- [2] Adolfson, Malin, Laseen, Stefan, Linde, Jesper and Villani, Mattias, 2008. "Evaluating an estimated new Keynesian small open economy model", Journal of Economic Dynamics and Control, vol. 32(8), 2690-2721.
- [3] Bernanke, Ben, Mark Gertler and Gilchrist, Simon, 1999. "The financial accelerator in a quantitative business cycle framework", Handbook of Macroeconomics, edited by John B. Taylor and Michael Woodford, Elsevier Science, 1341-1393.
- [4] Christiano, Lawrence J., Eichenbaum, Martin and Evans, Charles L., 2005. "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy", Journal of Political Economy, vol. 113(1), 1-45.
- [5] Christiano, Lawrence J., Trabandt, Mathias and Walentin, Karl, 2011. "Introducing financial frictions and unemployment into a small open economy model" Journal of Economic Dynamics and Control, vol. 35(12), 1999-2041.
- [6] Dixit, Avinash K. and Stiglitzh, Joseph E., 1977. "Monopolistic Competition and Optimum Product Diversity", American Economic Review, vol. 67(3), 297-308.
- [7] Erceg, Christopher J., Dale W. Henderson and Andrew T. Levin, 2000. "Optimal monetary policy with staggered wage and price contracts", Journal of Monetary Economics, vol. 46(2),281-313.
- [8] Fisher, Irving, 1933. "The debt-deflation theory of great depressions", Econometrica, vol. 1(4), 337-357.
- [9] Justiniano, Alejandro, Giorgio Primiceri and Andrea Tambalotti, 2011. "Investment shocks and the relative price of investment", Review of Economic Dynamics, vol. 14(1), 101-121.
- [10] Stehrer, Robert, 2013. "Accounting Relations in Bilateral Value Added Trade", wiiw Working Papers 101, The Vienna Institute for International Economic Studies, wiiw.

Parameter	Value	Description
α	0.400	Capital share in production
β	0.995	Discount factor
ω_c	0.450	Import share in consumption goods
ω_i	0.650	Import share in investment goods
ω_x	0.550	Import share in export goods
$ ilde{\phi}_a$	0.010	Elasticity of country risk to net asset position
η_g	0.202	Government expenditure share of GDP
$ au_k$	0.100	Capital tax rate
$ au_w$	0.330	Payroll tax rate
$ au_c$	0.180	Consumption tax rate
$ au_y$	0.300	Labor income tax rate
$ au_b$	0.000	Bond tax rate
μ_z	1.005	Steady state growth rate of neutral technology
μ_ψ	1	Steady state growth rate of investment technology
$\bar{\pi}$	1.005	Steady state inflation growth target
λ_w	1.500	Wage markup
$\lambda_{d;m,c;m,i}$	1.300	Price markup for the domestic, imp. consump., imp. investm. goods
$\lambda_{x;m,x}$	1.200	Price markup for exports and imported exports goods
ϑ_w	1.000	Wage indexation to real growth trend
\varkappa^{j}	$1 - \kappa^j$	Indexation to inflation target for $j = d; x; m, c; m, i; m, x; w$
$\breve{\pi}$ $\widetilde{\phi}_S$	1.005	Third indexing base
$ ilde{\phi}_S$	0	Country risk adjustment coefficient
		Financial frictions model
$F(\bar{\omega})$	0.020	Steady state bankruptcy rate
$100W_e/y$	0.100	Transfers to entrepreneurs

Appendix A Tables and Figures

Table 1: Calibrated parameters.

	Parameter description	Posterior baseline		Moment	Moment value
δ	Depreciation rate of capital	0.030	0.030	$p_i i/y$	0.255
$\tilde{\varphi}$	Real exchange rate	2.16	2.02	$SP^{x}X/(PY)$	0.462
A_L	Scaling of disutility of work	16.86	24.46	$L\varsigma$	0.270
γ	Entrepreneurial survival rate		0.96	$n/(p_{k'}k)$	0.600

Table 2: Matched moments and corresponding parameters.

Note: The quarterly depreciation rate of capital is fixed at three percent.

	Demonster de mintien		Prior		Post	erior	HPE) int.
	Parameter description	Distr.	Mean	st.d.	Mean	st.d.	10%	90%
ρ_{μ_z}	Persistence, unit-root tech.	β	0.50	0.075	0.590	0.063	0.487	0.696
a_{11}	Foreign VAR parameter	N	0.90	0.05	0.913	0.034	0.852	0.977
a_{22}	Foreign VAR parameter	N	0.50	0.05	0.521	0.055	0.438	0.605
a_{33}	Foreign VAR parameter	N	0.90	0.05	0.954	0.023	0.919	0.989
a_{12}	Foreign VAR parameter	N	-0.10	0.10	-0.165	0.091	-0.314	-0.016
a_{13}	Foreign VAR parameter	N	-0.10	0.10	-0.045	0.054	-0.124	0.037
a_{21}	Foreign VAR parameter	N	0.10	0.10	0.181	0.043	0.097	0.260
a_{23}	Foreign VAR parameter	N	-0.10	0.10	-0.090	0.055	-0.183	-0.008
a_{24}	Foreign VAR parameter	N	0.05	0.10	0.078	0.041	0.009	0.146
a_{31}	Foreign VAR parameter	N	0.05	0.10	0.080	0.029	0.032	0.131
a_{32}	Foreign VAR parameter	N	-0.10	0.10	-0.095	0.058	-0.198	0.002
a_{34}	Foreign VAR parameter	N	0.10	0.10	0.108	0.026	0.068	0.149
c_{21}	Foreign VAR parameter	N	0.05	0.05	0.021	0.040	-0.048	0.088
c_{31}	Foreign VAR parameter	N	0.10	0.05	0.145	0.031	0.094	0.196
c_{32}	Foreign VAR parameter	N	0.40	0.05	0.374	0.053	0.286	0.459
c_{24}	Foreign VAR parameter	N	0.05	0.05	0.065	0.046	-0.003	0.135
c_{34}	Foreign VAR parameter	N	0.05	0.05	0.048	0.034	-0.002	0.101

Table 3: Estimated foreign SVAR parameters.

Note: Based on a single Metropolis-Hastings chain with 100 000 draws after a burn-in period of 900 000 draws.

	Description		Pri	or	Post	erior	HPD) int.
	Description	Distr.	Mean	st.d.	Mean	st.d.	10%	90%
$100\sigma_{\mu_z}$	Unit root technology	$\mathrm{Inv}\text{-}\Gamma$	0.25	inf	0.328	0.052	0.248	0.406
$100\sigma_{y^*}$	Foreign GDP	$\mathrm{Inv}\text{-}\Gamma$	0.50	\inf	0.317	0.055	0.219	0.415
$1000\sigma_{\pi^*}$	Foreign inflation	$\mathrm{Inv}\text{-}\Gamma$	0.50	\inf	0.593	0.118	0.394	0.805
$100\sigma_{R^*}$	Foreign interest rate	$\operatorname{Inv-}\Gamma$	0.075	\inf	0.067	0.008	0.054	0.079

Table 4: Estimated standard deviations of SVAR shocks.

Note: Based on a single Metropolis-Hastings chain with $100\ 000$ draws after a burn-in period of $900\ 000$ draws.

			Pr	ior		Post	erior		HPD) int.
	Parameter description	Distr.	Mean	st.d.		ean		.d.	10%	90%
					base	finfric	base	finfric	finf	fric
ξ_d	Calvo, domestic	β	0.75	0.075	0.802	0.803	0.024	0.023	0.755	0.856
ξ_x	Calvo, exports	β	0.75	0.075	0.845	0.862	0.036	0.031	0.818	0.906
ξ_{mc}	Calvo, imported consumpt.	β	0.75	0.075	0.778	0.777	0.042	0.049	0.694	0.865
ξ_{mi}	Calvo, imported investment	β	0.65	0.075	0.559	0.418	0.066	0.042	0.324	0.508
ξ_{mx}	Calvo, imported exports	β	0.65	0.10	0.510	0.590	0.069	0.091	0.452	0.727
κ_d	Indexation, domestic	β	0.40	0.15	0.193	0.168	0.064	0.075	0.056	0.279
κ_x	Indexation, exports	β	0.40	0.15	0.330	0.305	0.092	0.107	0.138	0.491
κ_{mc}	Indexation, imported cons.	β	0.40	0.15	0.379	0.398	0.130	0.106	0.168	0.639
κ_{mi}	Indexation, imported inv.	β	0.40	0.15	0.271	0.263	0.123	0.100	0.079	0.444
κ_{mx}	Indexation, imported exp.	β	0.40	0.15	0.328	0.354	0.090	0.115	0.135	0.566
κ_w	Indexation, wages	β	0.40	0.15	0.247	0.247	0.092	0.079	0.073	0.402
$ u^j$	Working capital share	β	0.50	0.25	0.340	0.442	0.217	0.179	0.031	0.829
$0.1\sigma_L$	Inverse Frisch elasticity	Γ	0.30	0.15	0.214	0.254	0.117	0.106	0.085	0.419
b	Habit in consumption	β	0.65	0.15	0.846	0.894	0.033	0.030	0.847	0.945
0.1S''	Investment adjustment costs	Γ	0.50	0.15	0.411	0.171	0.090	0.030	0.105	0.233
σ_a	Variable capital utilization	Γ	0.20	0.075	0.352	0.595	0.084	0.093	0.371	0.827
η_x	Elasticity of subst., exports	Γ_{tr}	1.50	0.25	1.756	1.541	0.186	0.143	1.121	1.971
η_c	Elasticity of subst., cons.	Γ_{tr}	1.50	0.25	1.391	1.337	0.140	0.164	1.021	1.606
η_i	Elasticity of subst., invest.	Γ_{tr}	1.50	0.25	1.111	1.1^{*}	0.074			
η_f	Elasticity of subst., foreign	Γ_{tr}	1.50	0.25	1.548	1.570	0.225	0.159	1.175	1.964
μ	Monitoring cost	β	0.30	0.075		0.271		0.040	0.201	0.340
ρ_{ϵ}	Persistence, stationary tech.	β	0.85	0.075	0.885	0.846	0.034	0.041	0.751	0.939
ρ_{Υ}	Persistence, MEI	β	0.85	0.075	0.804	0.574	0.066	0.106	0.372	0.776
ρ_{ζ^c}	Persist., consumption prefs	β	0.85	0.075	0.860	0.861	0.042	0.038	0.788	0.939
$ ho_{\zeta^h}$	Persistence, labor prefs	β	0.85	0.075	0.807	0.815	0.079	0.048	0.728	0.915
$ ho_{ ilde{\phi}}$	Persist., country risk prem.	β	0.85	0.075	0.904	0.935	0.026	0.025	0.899	0.971
$ ho_g$	Persist., gov. expenditures	β	0.85	0.075	0.753	0.770	0.070	0.083	0.628	0.917
ρ_{γ}	Persistence, entrepren. wealth	β	0.85	0.075		0.767		0.059	0.604	0.921

Table 5: Estimated parameters.

Note: Based on a single Metropolis-Hastings chain with 100 000 draws after a burn-in period of 400 000 draws. * - calibrated in order to avoid numerical issues. Note that truncated priors, denoted by Γ_{tr} , with no mass below 1.01 have been used for the elasticity parameters η_j , $j = \{x, c, i, f\}$.

			Pri	or		Post	erior		HPD) int.
	Description	Distr	Mean	at d	Me	ean	st	.d.	10%	90%
		DISUI.	Mean	st.u.	base	finfric	base	finfric	finf	fric
$10\sigma_{\epsilon}$	Stationary technology	$\mathrm{Inv}\text{-}\Gamma$	0.15	\inf	0.139	0.126	0.016	0.014	0.103	0.149
σ_{Υ}	Marginal efficiency of invest.	$\text{Inv-}\Gamma$	0.15	\inf	0.234	0.162	0.056	0.027	0.093	0.230
σ_{ζ^c}	Consumption prefs	$\mathrm{Inv}\text{-}\Gamma$	0.15	\inf	0.143	0.227	0.029	0.056	0.131	0.320
σ_{ζ^h}	Labor prefs	$\mathrm{Inv}\text{-}\Gamma$	0.50	\inf	0.739	0.804	0.430	0.283	0.300	1.293
$100\sigma_{\tilde{\phi}}$	Country risk premium	$\text{Inv-}\Gamma$	0.50	\inf	0.547	0.554	0.044	0.045	0.475	0.632
$10\sigma_q^{\varphi}$	Government expenditures	$\mathrm{Inv}\text{-}\Gamma$	0.50	\inf	0.468	0.470	0.044	0.041	0.396	0.544
$\sigma_{ au^d}$	Markup, domestic	$\mathrm{Inv}\text{-}\Gamma$	0.50	\inf	0.383	0.374	0.105	0.089	0.179	0.555
$\sigma_{ au^x}$	Markup, exports	$\mathrm{Inv}\text{-}\Gamma$	0.50	\inf	0.813	1.004	0.298	0.391	0.439	1.556
$\sigma_{ au^{m,c}}$	Markup, imports for cons.	$\mathrm{Inv}\text{-}\Gamma$	0.50	\inf	0.887	0.812	0.463	0.329	0.278	1.421
$\sigma_{ au^{m,i}}$	Markup, imports for invest.	$\mathrm{Inv}\text{-}\Gamma$	0.50	\inf	0.895	0.458	0.340	0.078	0.282	0.620
$\sigma_{ au^{m,x}}$	Markup, imports for exports	$\mathrm{Inv}\text{-}\Gamma$	0.50	\inf	1.052	1.447	0.410	0.643	0.523	2.349
$10\sigma_{\gamma}$	Entrepreneurial wealth	$\mathrm{Inv}\text{-}\Gamma$	0.50	\inf		0.307		0.042	0.231	0.384

Table 6: Estimated standard deviations of shocks.

Note: Based on a single Metropolis-Hastings chain with 100 000 draws after a burn-in period of 400 000 draws.

			Mean		Stan	dard devi	ation
Variable	Explanation	Data	Mod	lel	Data	Mod	lel
		Data	baseline	finfric	Data	baseline	finfric
π	Domestic inflation	6.08	2.00	2.00	8.39	8.82	8.61
π^c	CPI inflation	5.62	2.00	2.00	6.29	8.80	8.51
π^i	Investment inflation	6.78	2.00	2.00	51.45	49.57	46.49
R	Nom. interest rate	7.06	6.04	6.04	5.86	5.67	6.40
Δh	Total hours growth	0.02	0.00	0.00	2.20	6.76	5.69
Δy	GDP growth	1.37	0.50	0.50	2.31	5.37	4.56
Δw	Real wage growth	1.06	0.50	0.50	2.35	2.97	2.89
Δc	Consumption growth	1.47	0.50	0.50	2.84	3.16	3.39
Δi	Investment growth	1.73	0.50	0.50	16.32	21.34	21.65
Δq	Real exch. rate growth	-0.88	0.00	0.00	2.51	2.29	2.22
Δg	Gov. expendit. growth	0.44	0.50	0.50	5.46	5.30	5.30
Δx	Export growth	2.19	0.50	0.50	3.41	3.67	3.66
Δm	Import growth	2.22	0.50	0.50	6.30	12.24	9.76
Δn	Stock market growth	1.32		0.50	10.38		14.92
spread	Interest rate spread	4.29		3.01	2.25		5.48
Δy^*	Foreign GDP growth	0.26	0.50	0.50	0.61	0.52	0.52
π^*	Foreign inflation	2.01	2.00	2.00	0.72	0.88	0.88
R^*	Foreign nom. int. rate	3.16	6.04	6.04	1.61	2.58	2.58

Table 7: Data and (first-order approximated) model moments (in percent).

Note: The inflation and interest rates are annualized.

	Description	model	R	π^c	GDP	С	Ι	$\frac{\rm NX}{\rm GDP}$	Η	W	q	Ν	Spread
	Stationary	В	0.0	1.8	0.9	0.4	0.1	0.1	6.1	1.0	1.5		
ϵ_t	technology	\mathbf{F}	0.0	1.2	0.8	0.1	0.0	0.5	10.9	0.7	1.0	0.2	0.1
Υ_t	MEI	В	5.1	1.2	15.1	1.7	73.6	60.2	6.9	1.5	1.0		
1_{t}	MIE/I	\mathbf{F}	0.1	0.1	3.8	0.1	28.5	5.7	5.4	0.5	0.1	19.0	19.2
ζ_t^c	Consumption	В	0.1	0.1	2.0	78.4	0.5	2.1	1.6	0.1	0.1		
S_t	prefs	\mathbf{F}	0.3	0.3	8.7	81.6	0.2	19.1	6.9	0.2	0.2	0.2	0.1
ζ^h_t	Labor prefs	В	0.0	12.0	3.9	3.0	0.8	0.4	4.1	45.3	10.4		
S_t	Labor preis	\mathbf{F}	0.1	8.7	3.1	1.9	0.6	3.5	4.3	39.1	7.5	1.3	0.4
$ au_t^d$	Markup,	В	0.0	32.0	1.2	0.2	0.1	0.1	0.8	37.7	27.5		
't	domestic	\mathbf{F}	0.0	26.6	1.8	0.1	0.1	0.2	1.5	39.2	22.9	0.6	0.1
$ au_t^x$	Markup,	В	0.0	0.0	1.2	0.0	0.0	0.0	1.0	0.0	0.0		
't	exports	\mathbf{F}	0.0	0.0	2.5	0.0	0.0	0.1	2.1	0.0	0.0	0.0	0.0
$ au_t^{mc}$	Markup, imp.	В	0.0	39.0	1.1	0.1	0.0	0.3	0.9	1.4	34.3		
	for cons.	\mathbf{F}	0.0	50.8	3.8	0.0	0.0	0.9	3.1	2.5	44.7	0.1	0.0
τ_t^{mi}	Markup, imp.	В	1.1	3.0	29.6	0.2	9.6	14.6	42.5	0.7	2.5		
't	for inv.	\mathbf{F}	0.1	0.6	17.9	0.0	6.6	5.6	26.6	0.3	0.5	7.1	6.0
$ au_t^{mx}$	Markup, imp.	В	0.3	0.1	38.9	0.1	0.1	6.8	32.2	0.3	0.1		
't	for exp.	\mathbf{F}	0.1	0.1	35.2	0.1	0.1	7.1	29.9	0.2	0.1	0.2	0.1
γ_t	Entrepreneurial												
/t	wealth	\mathbf{F}	0.8	1.0	10.4	0.2	44.8	35.1	1.9	1.1	0.9	51.5	69.2
$ ilde{\phi}_t$	Country risk	В	86.7	0.3	1.2	2.4	5.1	10.5	0.7	1.3	0.2		
φ_t	premium	\mathbf{F}	92.0	0.7	2.7	3.9	11.1	17.8	1.1	3.6	0.6	13.5	2.2
$\mu_{z,t}$	Unit-root	В	1.6	0.1	0.1	0.1	0.2	1.4	0.0	0.4	0.3		
$\mu_{z,l}$	technology	\mathbf{F}	1.6	0.1	0.2	0.0	0.2	1.4	0.0	0.4	0.3	0.1	0.0
$\epsilon_{R^*,t}$	Foreign	В	1.6	0.1	0.1	0.2	0.3	0.9	0.0	0.1	0.0		
$c_{\Lambda^{+},l}$	interest rate	\mathbf{F}	1.5	0.1	0.1	0.2	0.3	0.8	0.0	0.2	0.0	0.3	0.1
$\epsilon_{y^*,t}$	Foreign output	В	3.4	0.2	0.1	0.4	0.7	2.5	0.0	0.2	0.3		
cy^*, ι		\mathbf{F}	3.4	0.1	0.0	0.6	0.3	2.1	0.0	0.3	0.5	0.1	0.1
$\epsilon_{\pi^*,t}$	Foreign	В	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1		
<i>ω</i> η <i>*</i> ,ι	inflation	\mathbf{F}	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.0	0.0
	- c · +	В	93.3	0.7	1.4	3.1	6.4	15.3	0.8	2.0	1.0		
	5 foreign^*	\mathbf{F}	98.6	1.1	3.0	4.7	11.9	22.0	1.1	4.4	1.5	14.0	2.4
	A11 C · **	В	94.8	42.8	72.3	3.5	16.1	37.0	77.3	4.5	38.0		
	All foreign**	\mathbf{F}	98.7	52.6	62.5	4.8	18.7	35.8	62.8	7.5	46.9	21.4	8.5

Table 8: Conditional variance decomposition (percent) given model parameter uncertainty at 8 quarters forecast horizon; posterior mean.

Note: * '5 foreign' is the sum of the foreign stationary shocks, R_t^* , π_t^* , Y_t^* , the country risk premium shock, $\tilde{\phi}_t$, and the world-wide unit root neutral technology shock, $\mu_{z,t}$. ** 'All foreign' includes the above five shocks as well as the markup shocks on imports and exports, i.e. τ_t^{mc} , τ_t^{mi} , τ_t^{mx} and τ_t^x . 'B' - baseline model, 'F' - financial frictions model.

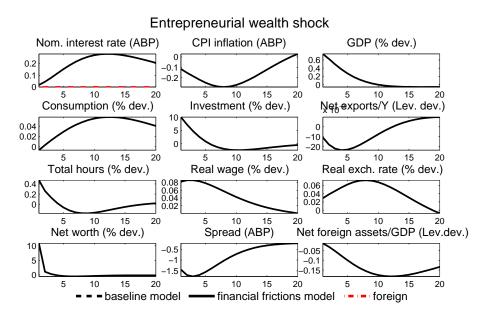


Figure 1: Impulse responses to the entrepreneurial wealth shock, γ_t .

Note: The units on the y-axis are either in terms of percentage deviation (% dev.) from the steady state, annualized basis points (ABP), or level deviation (Lev. dev.).

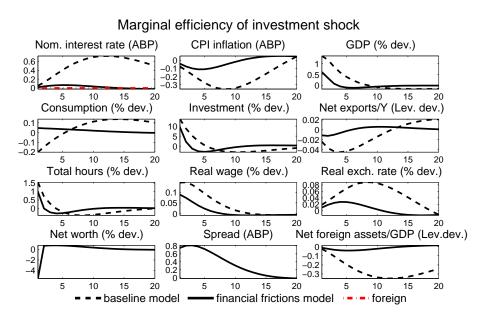


Figure 2: Impulse responses to the marginal efficiency of investment shock, Υ_t .

Note: The units on the y-axis are either in terms of percentage deviation (% dev.) from the steady state, annualized basis points (ABP), or level deviation (Lev. dev.).

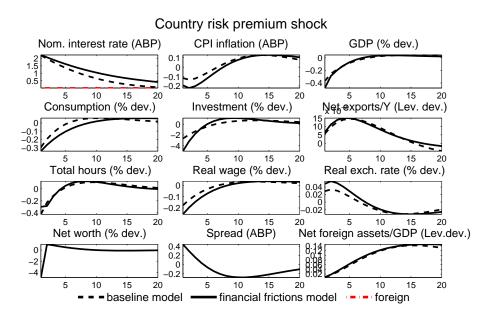


Figure 3: Impulse responses to the country risk premium shock, ϕ_t .

Note: The units on the y-axis are either in terms of percentage deviation (% dev.) from the steady state, annualized basis points (ABP), or level deviation (Lev. dev.).

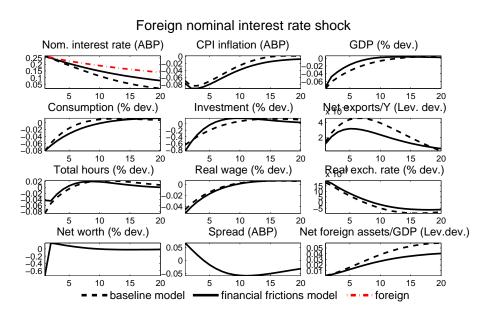


Figure 4: Impulse responses to the foreign nominal interest rate shock, $\epsilon_{R^*,t}$.

Note: The units on the y-axis are either in terms of percentage deviation (% dev.) from the steady state, annualized basis points (ABP), or level deviation (Lev. dev.).

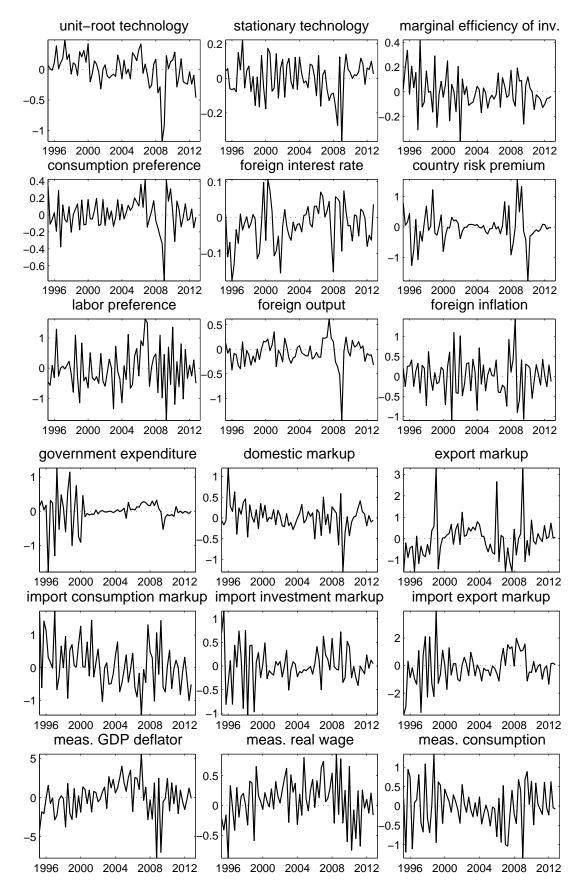


Figure 5: Smoothed shock processes and measurement errors of the financial frictions model.

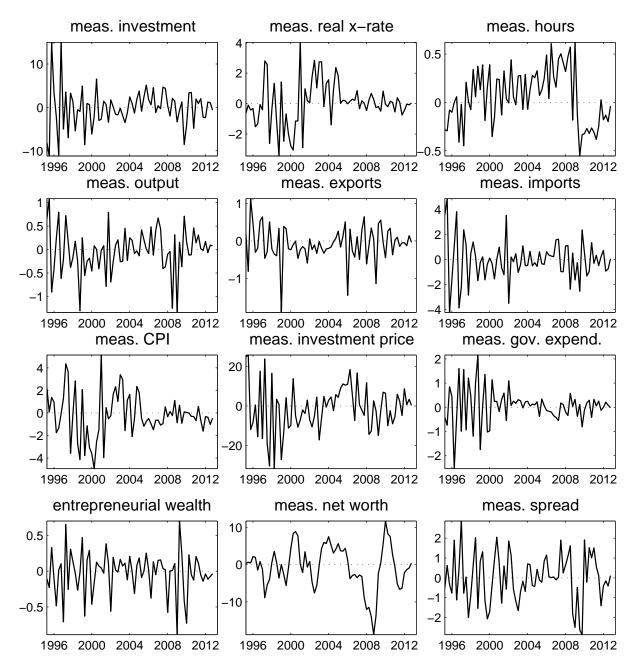


Figure 6: Smoothed shock processes and measurement errors of the financial frictions model (continued).

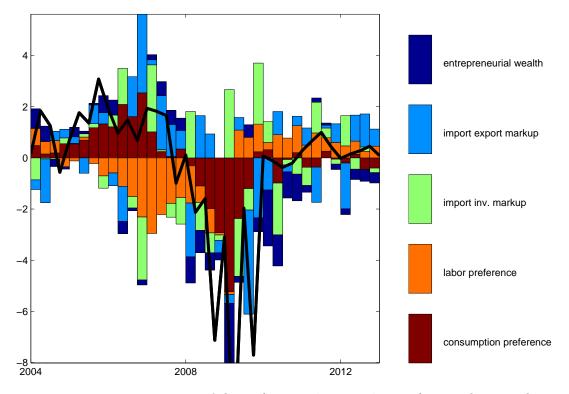


Figure 7: Decomposition of GDP (quarterly growth rates), 2004Q1-2012Q4. Note: Financial frictions model. Only those shocks that are greater than 2pp in at least one period.

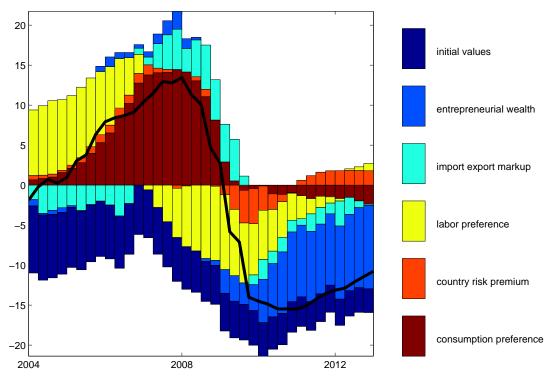


Figure 8: Decomposition of GDP (levels), 2004Q1-2012Q4.

Note: Financial frictions model. Only those shocks that are greater than 4pp in at least one period.

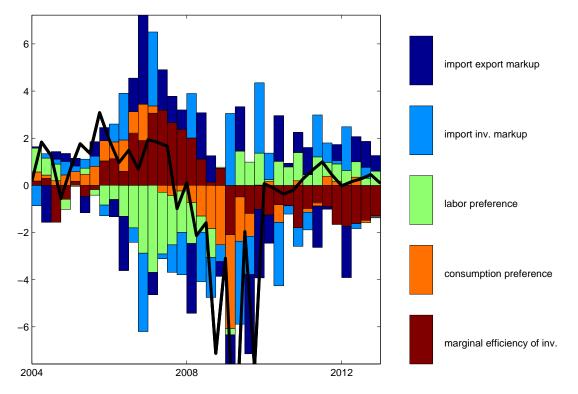


Figure 9: Decomposition of GDP (quarterly growth rates), 2004Q1-2012Q4. Baseline model. Only those shocks that are greater than 2.5pp in at least one period.

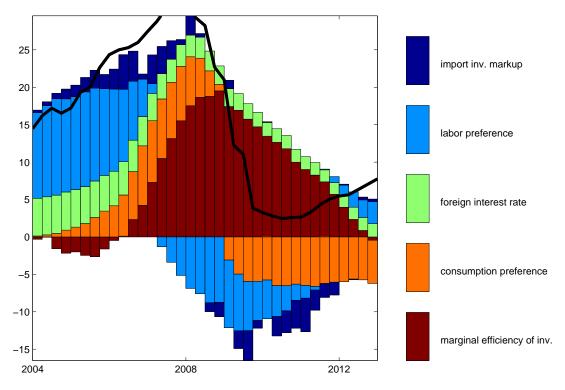


Figure 10: Decomposition of GDP (levels), 2004Q1-2012Q4. Baseline model. Only those shocks that are greater than 4.5pp in at least one period.

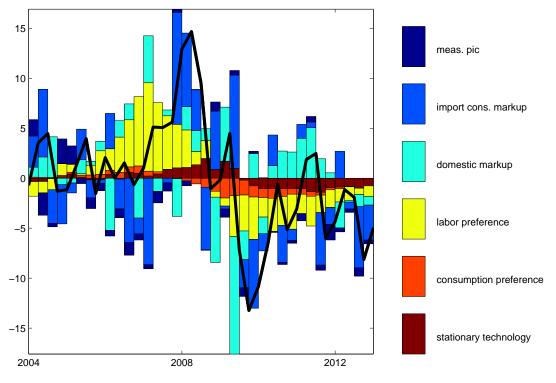


Figure 11: Decomposition of CPI, 2004Q1-2012Q4.

Note: Financial frictions model. Only those shocks that are greater than 1.5pp in at least one period.

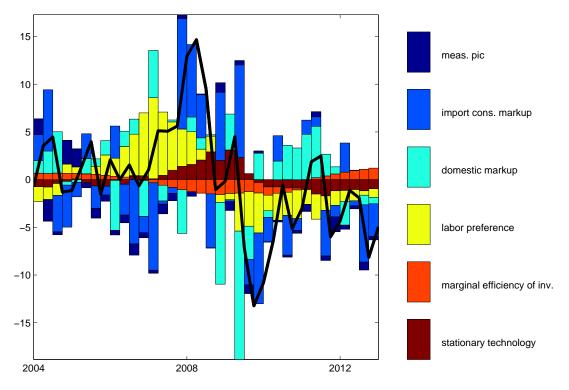


Figure 12: Decomposition of CPI, 2004Q1-2012Q4. Baseline model. Only those shocks that are greater than 1.5pp in at least one period.

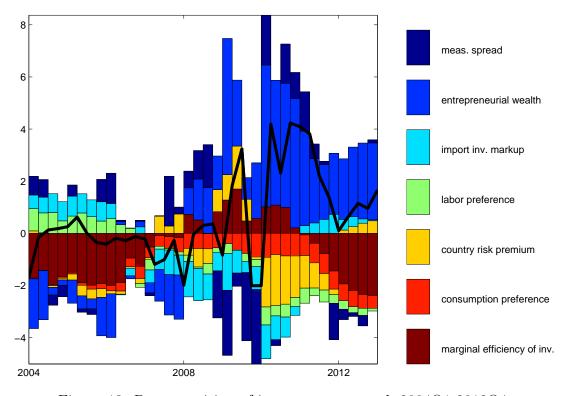


Figure 13: Decomposition of interest rate spread, 2004Q1-2012Q4.

Note: Financial frictions model. Only those shocks that are greater than 0.8pp in at least one period.

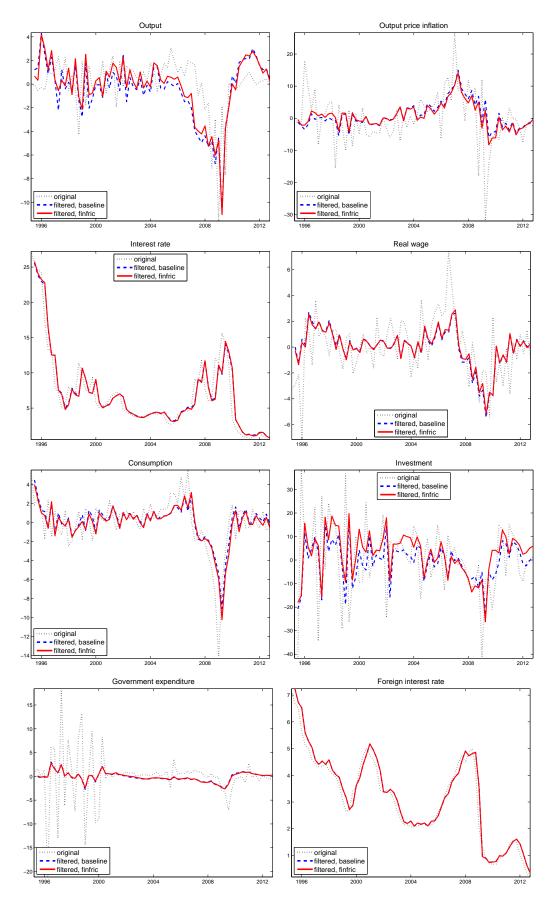


Figure 14: One-step ahead forecasts

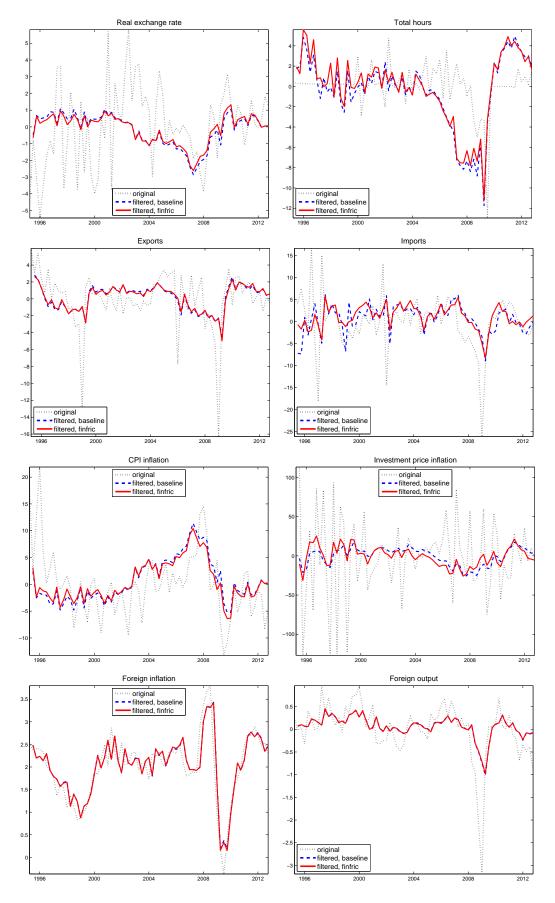


Figure 15: One-step ahead forecasts (continued)

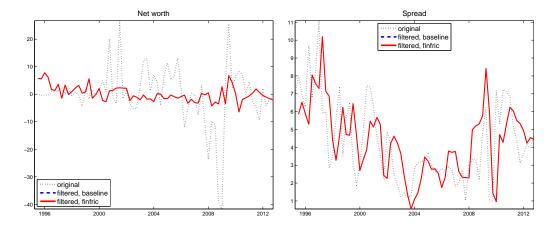


Figure 16: One-step ahead forecasts (continued)

Model	Distance measure	π^c	$\frac{Q}{\Delta y}$	$ \pi^c$	$Q_{\Delta y}$	π^c	$\Delta Q = \Delta y$	12 $ \pi^c$	$2Q \Delta y$
Baseline	RMSE MAE	$1.04 \\ 0.99$	1.03 1.28	0.82	$0.75 \\ 0.79$	0.67 0.74	0.65 0.64	$ 0.71 \\ 0.71 \\ 0.71$	$0.64 \\ 0.67$
Financial fric.	RMSE MAE	$0.99 \\ 0.92$	$0.96 \\ 1.15$	0.79 0.81	$0.70 \\ 0.69$	$0.65 \\ 0.70$	$0.64 \\ 0.58$	0.68	$0.64 \\ 0.60$
BSVAR	RMSE MAE	$\begin{array}{c} 0.86 \\ 0.89 \end{array}$	$0.72 \\ 0.71$	$\begin{array}{c} 0.71 \\ 0.70 \end{array}$	$\begin{array}{c} 0.81 \\ 0.77 \end{array}$	$0.62 \\ 0.62$	$\begin{array}{c} 0.68 \\ 0.62 \end{array}$	$0.63 \\ 0.58$	$\begin{array}{c} 0.66 \\ 0.61 \end{array}$

Table 9: Relative root mean squared error (RMSE) and mean absolute error (MAE) compared to the random walk model.

Note: A number greater than unity indicates that the model makes worse forecasts than the random walk model. Note that this is not a true out-of-sample forecasting performance since the models have been estimated on the whole sample period 1995Q1-2012Q4.

Model	Distance	1	Q	4	_t Q	8	SQ	12Q	
Model	measure	π^c	Δy	π^c	Δy	π^c	Δy	π^c	Δy
Baseline	RMSE	1.04	1.03	0.75	0.74	0.65	0.61	0.75	0.59
Dasenne	MAE	1.11	1.41	0.77	0.80	0.72	0.57	0.70	0.54
Financial fric.	RMSE	0.97	0.95	0.70	0.72	0.64	0.67	0.74	0.64
r manciai mic.	MAE	1.02	1.25	0.72	0.75	0.69	0.78	0.67	0.73
BSVAR	RMSE	0.91	0.74	0.75	0.84	0.68	0.67	0.76	0.64
DOVAN	MAE	0.96	0.75	0.73	0.83	0.69	0.60	0.69	0.53

Table 10: Relative root mean squared error (RMSE) and mean absolute error (MAE) compared to the random walk model, last ten years of the sample.

Note: A number greater than unity indicates that the model makes worse forecasts than the random walk model. Note that this is not a true out-of-sample forecasting performance since the models have been estimated on the whole sample period 1995Q1-2012Q4.

Appendix B Computational appendix - not for publication

$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\begin{array}{c} 7.2 \\ 13.1 \\ 6.4 \\ 5.7 \\ 1.4 \\ 5.1 \\ 2.2 \\ 2.3 \\ 1.0 \\ 1.8 \\ 1.1 \\ 2.5 \\ 1.0 \\ 3.4 \end{array}$	$\begin{array}{c} 0.5\\ 0.4\\ 0.4\\ 0.2\\ 0.0\\ 0.2\\ 40.1\\ 34.5\\ 44.0\\ 46.0\\ 0.0\\ 0.0\\ 0.0\\ 0.0\\ 0.0\\ 0.0\\ 0.0\\ $	$\begin{array}{c} 0.8\\ 0.6\\ 0.0\\ 0.0\\ 0.0\\ 0.1\\ 3.2\\ 2.3\\ 30.5\\ 24.8\\ 0.0\\ 0.0\\ 36.3\\ \end{array}$	0.1 18.2 0.2 0.9 0.6 0.0	0.1 17.4 0.1 0.7 0.0 0.0
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{c} 6.4 \\ 5.7 \\ 1.4 \\ 5.1 \\ 2.2 \\ 2.3 \\ 1.0 \\ 1.8 \\ 1.1 \\ 2.5 \\ 1.0 \\ 3.4 \end{array}$	$\begin{array}{c} 0.4 \\ 0.2 \\ 0.0 \\ 0.2 \\ 40.1 \\ 34.5 \\ 44.0 \\ 46.0 \\ 0.0 \\ 0.0 \\ 0.0 \\ 0.0 \end{array}$	$\begin{array}{c} 0.0 \\ 0.0 \\ 0.0 \\ 0.1 \\ 3.2 \\ 2.3 \\ 30.5 \\ 24.8 \\ 0.0 \\ 0.0 \\ 0.0 \end{array}$	18.2 0.2 0.9 0.6	17.4 0.1 0.7 0.0
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$5.7 \\ 1.4 \\ 5.1 \\ 2.2 \\ 2.3 \\ 1.0 \\ 1.8 \\ 1.1 \\ 2.5 \\ 1.0 \\ 3.4$	$\begin{array}{c} 0.2 \\ 0.0 \\ 0.2 \\ 40.1 \\ 34.5 \\ 44.0 \\ 46.0 \\ 0.0 \\ 0.0 \\ 0.0 \\ 0.0 \end{array}$	$\begin{array}{c} 0.0 \\ 0.0 \\ 0.1 \\ 3.2 \\ 2.3 \\ 30.5 \\ 24.8 \\ 0.0 \\ 0.0 \\ \end{array}$	0.2 0.9 0.6	0.1 0.7 0.0
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$1.4 \\ 5.1 \\ 2.2 \\ 2.3 \\ 1.0 \\ 1.8 \\ 1.1 \\ 2.5 \\ 1.0 \\ 3.4$	$\begin{array}{c} 0.0 \\ 0.2 \\ 40.1 \\ 34.5 \\ 44.0 \\ 46.0 \\ 0.0 \\ 0.0 \\ 0.0 \\ 0.0 \end{array}$	$\begin{array}{c} 0.0 \\ 0.1 \\ 3.2 \\ 2.3 \\ 30.5 \\ 24.8 \\ 0.0 \\ 0.0 \\ \end{array}$	0.2 0.9 0.6	0.1 0.7 0.0
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$5.1 \\ 2.2 \\ 2.3 \\ 1.0 \\ 1.8 \\ 1.1 \\ 2.5 \\ 1.0 \\ 3.4$	$\begin{array}{c} 0.2 \\ 40.1 \\ 34.5 \\ 44.0 \\ 46.0 \\ 0.0 \\ 0.0 \\ 0.0 \\ 0.0 \end{array}$	$\begin{array}{c} 0.1 \\ 3.2 \\ 2.3 \\ 30.5 \\ 24.8 \\ 0.0 \\ 0.0 \end{array}$	0.9 0.6	0.7 0.0
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$2.2 \\ 2.3 \\ 1.0 \\ 1.8 \\ 1.1 \\ 2.5 \\ 1.0 \\ 3.4$	$\begin{array}{c} 40.1 \\ 34.5 \\ 44.0 \\ 46.0 \\ 0.0 \\ 0.0 \\ 0.0 \\ 0.0 \end{array}$	3.2 2.3 30.5 24.8 0.0 0.0	0.9 0.6	0.7 0.0
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$2.3 \\ 1.0 \\ 1.8 \\ 1.1 \\ 2.5 \\ 1.0 \\ 3.4$	$\begin{array}{c} 34.5 \\ 44.0 \\ 46.0 \\ 0.0 \\ 0.0 \\ 0.0 \\ 0.0 \end{array}$	$2.3 \\ 30.5 \\ 24.8 \\ 0.0 \\ 0.0$	0.6	0.0
$ \begin{aligned} & \tau^d_t & \begin{array}{ccccccccccccccccccccccccccccccccccc$	$ 1.0 \\ 1.8 \\ 1.1 \\ 2.5 \\ 1.0 \\ 3.4 $	$\begin{array}{c} 44.0 \\ 46.0 \\ 0.0 \\ 0.0 \\ 0.0 \end{array}$	$30.5 \\ 24.8 \\ 0.0 \\ 0.0$	0.6	0.0
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$ 1.8 \\ 1.1 \\ 2.5 \\ 1.0 \\ 3.4 $	$46.0 \\ 0.0 \\ 0.0 \\ 0.0$	$24.8 \\ 0.0 \\ 0.0$		
$ \tau_t^x \begin{array}{ccccccccccccccccccccccccccccccccccc$	$1.1 \\ 2.5 \\ 1.0 \\ 3.4$	$0.0 \\ 0.0 \\ 0.0$	$\begin{array}{c} 0.0 \\ 0.0 \end{array}$		
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$2.5 \\ 1.0 \\ 3.4$	$\begin{array}{c} 0.0 \\ 0.0 \end{array}$	0.0	0.0	0.0
$\tau_t^{mc} \begin{array}{ccccccccccccccccccccccccccccccccccc$	$\begin{array}{c} 1.0\\ 3.4 \end{array}$	0.0		0.0	0.0
τ_t^{rec} for cons. F 0.0 53.8 4.6 0.0 0.0 2.6	3.4		36.3		
$\begin{array}{cccccccccccccccccccccccccccccccccccc$		0.0			
$_{-mi}$ Markup, imp. B 0.1 2.4 29.0 0.1 10.6 41.7		0.0	46.1	0.1	0.0
	41.8	0.6	2.0		
$101 \text{ mV}. \qquad 1 \qquad 0.0 \qquad 0.0 \qquad 10.5 \qquad 0.0 \qquad 0.1 \qquad 23.0$	24.3	0.3	0.4	6.9	7.1
τ_t^{mx} Markup, imp. B 0.1 0.1 44.8 0.1 0.1 29.8	33.2	0.3	0.1		
$101 \text{ exp.} \qquad \mathbf{F} \qquad 0.0 \qquad 0.1 \qquad 59.8 \qquad 0.0 \qquad 0.1 \qquad 50.9$	31.7	0.2	0.1	0.2	0.2
Entrepreneurial B γ_t much E 0.0 0.0 5.4 0.1 27.7 11.0					
Weatth F 0.0 0.0 5.4 0.1 57.7 11.9	1.5	0.2	0.0	53.2	52.3
$\tilde{\phi}_t$ Country risk B 97.5 0.1 0.9 2.3 4.1 1.9	0.6	0.9	0.1		
premium F 97.0 0.2 2.4 5.0 10.5 5.1	0.6	2.2	0.2	12.9	5.8
Unit-root B 0.1 0.0 0.1 0.0 0.2 0.3 $\mu_{z,t}$ task along F 0.1 0.0 0.2 0.4 0.4	0.0	0.1	0.1		
technology $F = 0.1 = 0.0 = 0.2 = 0.0 = 0.1 = 0.4$	0.0	0.1	0.1	0.1	0.0
Foreign interest B 1.5 0.0 0.0 0.2 0.2 0.1 $\epsilon_{R^*,t}$ meta	0.0	0.1	0.0		
rate F 1.4 0.0 0.1 0.2 0.3 0.2	0.0	0.1	0.0	0.3	0.1
$\epsilon_{y^*,t}$ Foreign output B 0.7 0.0 0.0 0.3 0.3 0.4	0.0	0.1	0.0		
F 0.7 0.0 0.0 0.4 0.2 0.5	0.0	0.2	0.0	0.1	0.0
Foreign B 0.2 0.0 0.0 0.0 0.0 0.0 $e^{\epsilon_{R^*,t}}$ is first B. 0.2 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.	0.0	0.0	0.1		
$e_{R^*,t}$ inflation F 0.2 0.0 0.0 0.0 0.0 0.0	0.0	0.0	0.1	0.0	0.0
B 99.8 0.1 1.0 2.7 4.8 2.8	0.7	1.1	0.3		
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	0.6	2.5	0.4	13.3	6.0
B 100.0 45.4 77.0 2.0 15.5 75.2	77.9	2.1	38.7		
All foreign**B100.043.4 77.9 3.0 13.5 73.2 F100.0 54.6 67.2 3.6 18.1 69.1	62.6	3.1	47.0	20.5	13.3

Table 11: Conditional variance decomposition (percent) given model parameter uncertainty. One quarter forecast horizon. Posterior mean.

* '5 foreign' is the sum of the foreign stationary shocks, R_t^* , π_t^* , Y_t^* , the country risk premium shock, $\tilde{\phi}_t$, and the world-wide unit root neutral technology shock, $\mu_{z,t}$. ** 'All foreign' includes the above five shocks as well as the markup shocks on imports and exports, i.e. τ_t^{mc} , τ_t^{mi} , τ_t^{mx} and τ_t^x . 'B' - baseline model, 'F' - financial frictions model.

	Description	model	R	π^{c}	GDP	С	Ι	$\frac{NX}{GDP}$	Η	W	q	Ν	Spread
6	Stationary	В	0.0	1.9	0.9	0.3	0.1	0.2	6.1	0.9	1.6		
ϵ_t	technology	\mathbf{F}	0.0	1.2	0.8	0.1	0.0	0.7	11.0	0.6	1.0	0.2	0.1

Υ_t	MEI	В	0.9	0.3	14.4	1.6	74.4	53.3	6.2	1.2	0.2		
- 1		\mathbf{F}	0.1	0.1	3.7	0.1	26.2	9.3	4.9	0.4	0.1	18.4	18.8
ζ_t^c	Consumption	В	0.0	0.1	1.9	78.9	0.4	1.7	1.6	0.1	0.1		
	prefs	\mathbf{F}	0.0	0.2	8.8	82.0	0.2	12.9	7.0	0.2	0.2	0.2	0.1
ζ^h_t	Labor prefs	В	0.0	11.2	3.4	2.3	0.4	0.5	3.9	44.6	9.6		
		\mathbf{F}	0.0	8.1	2.7	1.4	0.4	3.0	3.9	38.8	6.9	1.3	0.4
τ^d_t	Markup,	В	0.0	32.7	1.2	0.2	0.1	0.1	0.8	38.5	27.8		
	domestic	\mathbf{F}	0.0	27.3	1.8	0.1	0.1	0.3	1.5	39.9	23.3	0.6	0.0
$ au_t^x$	Markup,	В	0.0	0.0	1.2	0.0	0.0	0.0	0.9	0.0	0.0		
	exports	\mathbf{F}	0.0	0.0	2.5	0.0	0.0	0.2	2.0	0.0	0.0	0.0	0.0
$ au_t^{mc}$	Markup, imp.	В	0.0	39.3	1.1	0.1	0.0	0.5	0.9	1.3	34.2		
	for cons.	\mathbf{F}	0.0	50.9	3.7	0.0	0.0	1.5	3.0	2.3	44.4	0.1	0.0
τ_t^{mi}	Markup, imp.	В	0.5	3.1	30.2	0.2	8.8	23.3	43.1	0.7	2.6		
	for inv.	F	0.0	0.6	18.6	0.0	6.3	9.7	27.4	0.2	0.5	7.0	6.8
τ_t^{mx}	Markup, imp.	В	0.2	0.1	39.7	0.1	0.1	11.0	32.6	0.3	0.1		
	for exp.	F	0.1	0.1	35.8	0.1	0.1	13.0	30.3	0.2	0.1	0.2	0.1
γ_t	Entrepreneurial	В											
	wealth	F	0.1	0.3	9.4	0.1	46.5	31.5	1.7	0.7	0.3	52.0	67.3
~	Country risk	В	93.5	0.2	1.1	2.4	5.2	6.7	0.7	1.3	0.2		
$ ilde{\phi}_t$	premium	F	95.0	0.6	2.7	3.7	11.4	14.7	0.9	3.5	0.5	13.5	2.3
$\mu_{z,t}$	Unit-root	В	1.0	0.1	0.1	0.1	0.2	0.9	0.0	0.3	0.3		
	technology	F	1.0	0.1	0.2	0.0	0.2	1.1	0.0	0.3	0.3	0.1	0.0
	Foreign interest	В	1.6	0.1	0.1	0.2	0.3	0.5	0.0	0.1	0.0	0.1	0.0
$\epsilon_{R^*,t}$	rate	F	1.5	0.1	0.1	0.2	0.4	0.6	0.0	0.1	0.0	0.3	0.1
		В	2.1	0.1	0.1	0.3	0.6	1.2	0.0	0.2	0.2	0.0	
$\epsilon_{y^*,t}$	Foreign output	F	2.0	0.0	0.0	0.5	0.3	1.3	0.0	0.3	0.3	0.1	0.0
$\epsilon_{\pi^*,t}$	Foreign	В	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.1	0.0
	inflation	F	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.0	0.0
												0.0	
	5 for eign*	В	98.3	0.5	1.3	3.0	6.4	9.3	0.7	1.9	0.9		
		F	99.6	0.9	3.1	4.4	12.2	17.7	1.0	4.2	1.3	14.0	2.4
	All foreign ^{**}	В	99.0	43.0	73.4	3.4	15.3	44.1	78.2	4.3	37.8		
	mi loroign	\mathbf{F}	99.7	52.5	63.6	4.5	18.7	42.1	63.7	7.1	46.4	21.4	9.3

Table 12: Conditional variance decomposition (percent) given model parameter uncertainty. Four quarters forecast horizon. Posterior mean.

* '5 foreign' is the sum of the foreign stationary shocks, R_t^* , π_t^* , Y_t^* , the country risk premium shock, $\tilde{\phi}_t$, and the world-wide unit root neutral technology shock, $\mu_{z,t}$. ** 'All foreign' includes the above five shocks as well as the markup shocks on imports and exports, i.e. τ_t^{mc} , τ_t^{mi} , τ_t^{mx} and τ_t^x . 'B' - baseline model, 'F' - financial frictions model.

	Description	model	R	π^c	GDP	С	Ι	$\frac{NX}{GDP}$	Η	w	q	Ν	Spread
ϵ_t	Stationary	В	0.0	1.9	0.9	0.3	0.1	0.1	5.9	1.0	1.6		
	technology	F	0.0	1.3	0.8	0.1	0.0	0.4	10.6	0.7	1.1	0.2	0.1
Υ_t	MEI	В	19.6	2.6	15.5	3.5	75.2	59.6	8.0	1.6	2.2		
		\mathbf{F}	0.1	0.2	3.8	0.2	26.4	6.6	5.4	0.4	0.2	19.2	17.7
ζ_t^c	Consumption	В	0.6	0.2	2.6	78.5	0.5	2.0	1.7	0.2	0.1		
	prefs	\mathbf{F}	2.7	0.7	9.6	82.7	0.3	23.1	7.5	0.4	0.6	0.2	0.6
ζ^h_t	Labor prefs	В	0.1	13.4	4.1	2.8	0.8	0.6	4.7	47.5	11.6		
		\mathbf{F}	0.3	10.0	3.4	1.8	0.6	3.2	4.8	41.3	8.7	1.3	0.8
$ au_t^d$	Markup,	В	0.0	30.6	1.2	0.2	0.1	0.1	0.8	35.9	26.4		
	domestic	\mathbf{F}	0.0	25.4	1.8	0.0	0.1	0.1	1.5	37.4	22.0	0.6	0.1

$ au_t^x$	Markup,	В	0.0	0.0	1.2	0.0	0.0	0.0	1.0	0.0	0.0		
't	exports	\mathbf{F}	0.0	0.0	2.5	0.0	0.0	0.1	2.0	0.0	0.0	0.0	0.0
τ_t^{mc}	Markup, imp.	В	0.0	37.7	1.1	0.1	0.0	0.3	0.9	1.3	33.2		
	for cons.	\mathbf{F}	0.0	49.1	3.7	0.0	0.0	0.7	3.0	2.4	43.5	0.1	0.0
τ_t^{mi}	Markup, imp.	В	1.8	2.9	29.2	0.2	8.7	13.1	41.5	0.7	2.5		
	for inv.	\mathbf{F}	0.1	0.6	17.6	0.0	6.1	4.7	26.0	0.3	0.5	7.1	5.3
τ_t^{mx}	Markup, imp.	В	0.3	0.1	38.2	0.1	0.1	5.8	31.4	0.3	0.1		
	for exp.	\mathbf{F}	0.1	0.1	34.6	0.1	0.1	5.6	29.3	0.2	0.1	0.2	0.1
γ_t	Entrepreneurial	В											
	wealth	\mathbf{F}	3.5	2.1	10.5	0.6	47.4	33.0	2.4	1.4	1.8	51.4	68.0
$ ilde{\phi}_t$	Country risk	В	70.5	0.6	1.2	2.4	5.6	11.4	0.8	1.4	0.6		
	premium	F	84.6	1.0	2.7	3.7	11.6	16.9	1.3	3.6	0.9	13.4	4.5
$\mu_{z,t}$	Unit-root	В	1.5	0.1	0.1	0.2	0.3	1.6	0.0	0.5	0.5		
	technology	F	1.8	0.1	0.2	0.1	0.2	1.5	0.0	0.4	0.4	0.1	0.1
	Foreign interest	В	1.4	0.1	0.1	0.2	0.3	1.2	0.0	0.1	0.1		
$\epsilon_{R^*,t}$	rate	\mathbf{F}	1.6	0.1	0.1	0.2	0.3	0.9	0.0	0.2	0.1	0.3	0.2
$\epsilon_{y^*,t}$	Foreign output	В	4.0	0.4	0.1	0.3	0.7	4.1	0.0	0.2	0.5		
		\mathbf{F}	5.0	0.2	0.0	0.6	0.3	3.2	0.0	0.3	0.5	0.1	0.4
$\epsilon_{\pi^*,t}$	Foreign	В	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1		
	inflation	\mathbf{F}	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.0	0.0
	5 foreign*	В	77.5	1.2	1.5	3.1	6.9	18.3	0.9	2.2	1.7		
		\mathbf{F}	93.1	1.5	3.1	4.5	12.4	22.5	1.4	4.5	2.0	14.0	5.2
	All foreign**	В	79.7	41.9	71.2	3.5	15.7	37.5	75.7	4.6	37.4		
		\mathbf{F}	93.3	51.3	61.4	4.7	18.7	33.5	61.6	7.4	46.0	21.4	10.6

Table 13: Conditional variance decomposition (percent) given model parameter uncertainty. 20 quarters forecast horizon. Posterior mean.

* '5 foreign' is the sum of the foreign stationary shocks, R_t^* , π_t^* , Y_t^* , the country risk premium shock, $\tilde{\phi}_t$, and the world-wide unit root neutral technology shock, $\mu_{z,t}$.

** 'All foreign' includes the above five shocks as well as the markup shocks on imports and exports, i.e. τ_t^{mc} , τ_t^{mi} , τ_t^{mx} and τ_t^x . 'B' - baseline model, 'F' - financial frictions model.

	Description	Me	ean	Std		
	Description	base	finfric	base	finfric	
$100\mu_z$	Unit root technology	-0.00	-0.00	0.27	0.27	
10ϵ	Stationary technology	0.00	0.00	0.12	0.11	
Υ	MEI	0.01	-0.00	0.22	0.15	
ζ^c	Consumption prefs	-0.00	0.01	0.12	0.19	
ζ^h	Labor prefs	-0.04	-0.03	0.66	0.71	
$100 ilde{\phi}$	Country risk premium	-0.02	-0.02	0.50	0.51	
10g	Government expenditures	-0.00	-0.01	0.44	0.44	
$ au^d$	Markup, domestic	0.02	0.02	0.34	0.33	
$ au^x$	Markup, exports	-0.02	-0.03	0.74	0.91	
$ au^{m,c}$	Markup, imports for cons.	0.09	0.08	0.77	0.71	
$ au^{m,i}$	Markup, imports for invest.	0.01	0.01	0.84	0.41	
$ au^{m,x}$	Markup, imports for exports	-0.08	-0.12	0.93	1.28	
100γ	Entrepreneurial wealth		-0.02		0.30	
$100y^{*}$	Foreign GDP	-0.07	-0.07	0.26	0.27	
$1000\pi^*$	Foreign inflation	0.01	0.01	0.49	0.49	
$100R^*$	Foreign interest rate	-0.02	-0.02	0.06	0.06	
$\varepsilon^{me}_{\pi^d}$	measurement π^d	0.03	-0.02	2.08	2.29	

ma		0.00	0.00	2 0 0	1 00
$\varepsilon^{me}_{\pi^c}$	measurement π^c	-0.20	-0.20	2.00	1.98
$\varepsilon^{me}_{\pi^i}$	measurement π^i	-0.42	-0.99	11.01	11.97
ε_w^{me}	measurement w	0.09	0.09	0.35	0.38
ε_c^{me}	measurement c	-0.04	-0.01	0.48	0.57
ε_I^{me}	measurement I	-0.10	-0.14	6.64	4.56
ε_q^{me}	measurement q	0.05	0.05	1.44	1.45
$\varepsilon_{H}^{\hat{m}e}$	measurement H	0.06	0.06	0.26	0.29
ε_y^{me}	measurement y	-0.01	-0.02	0.41	0.47
ε_x^{me}	measurement x	-0.09	-0.08	0.45	0.47
ε_M^{me}	measurement M	-0.14	-0.08	1.55	1.64
ε_g^{me}	measurement g	0.03	0.04	0.69	0.73
ε_n^{me}	measurement n		-0.60		5.83
$\varepsilon^{me}_{spread}$	measurement spread		0.00		1.22

Table 14: The mean and standard deviation of smoothed shocks.

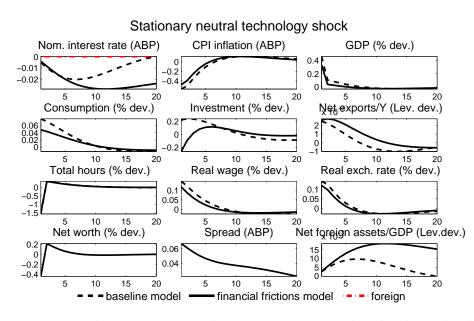


Figure 17: Impulse responses to the stationary neutral technology shock, ϵ_t .

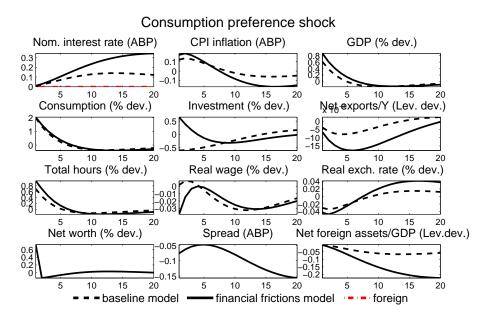


Figure 18: Impulse responses to the consumption preference shock, ζ_t^c .

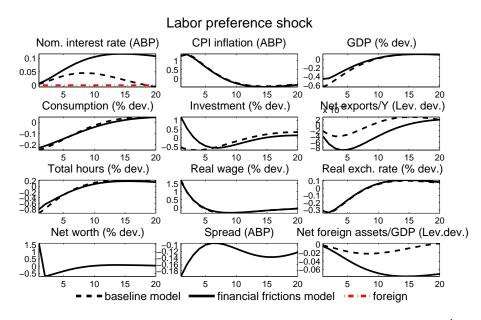


Figure 19: Impulse responses to the labor preference shock, ζ_t^h .

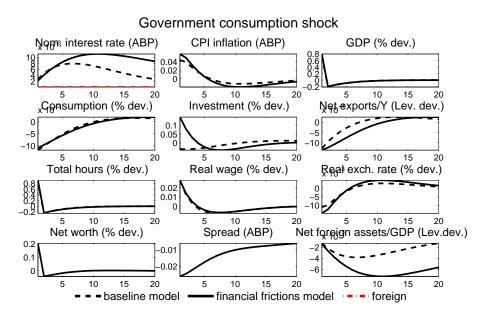


Figure 20: Impulse responses to the government consumption shock, g_t .

Note: The units on the y-axis are either in terms of percentage deviation (% dev.) from the steady state, annualized basis points (ABP), or level deviation (Lev. dev.). In this model, the government consumption crowds out the private consumption. Total consumption falls due to the worsening of the net foreign assets position and a subsequent increase in the risk premium to the nominal interest rate that makes saving activity more appealing.

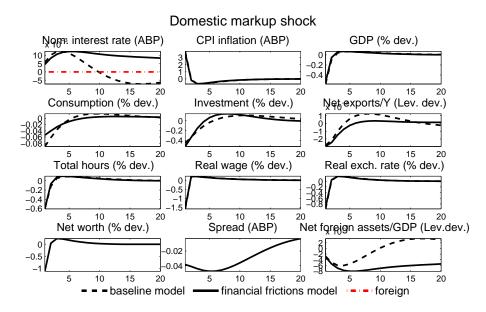


Figure 21: Impulse responses to the domestic markup shock, τ_t^d .

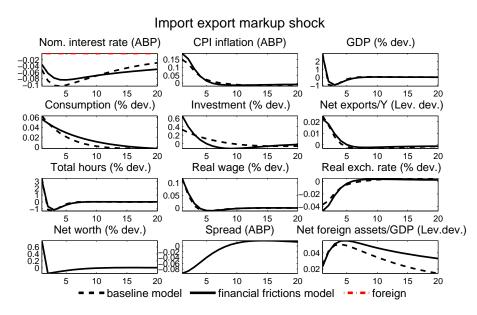


Figure 22: Impulse responses to the imported export markup shock, τ_t^{mx} .

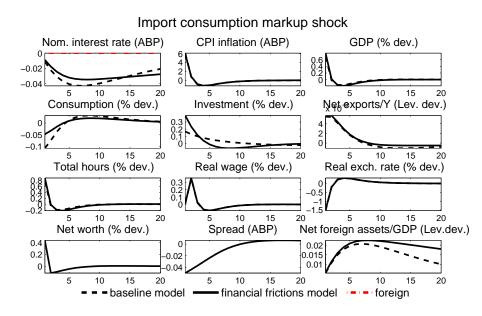


Figure 23: Impulse responses to the import consumption markup shock, τ_t^{mc} .

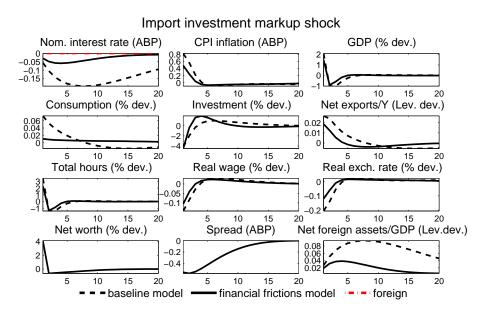


Figure 24: Impulse responses to the import investment markup shock, τ_t^{mi} .

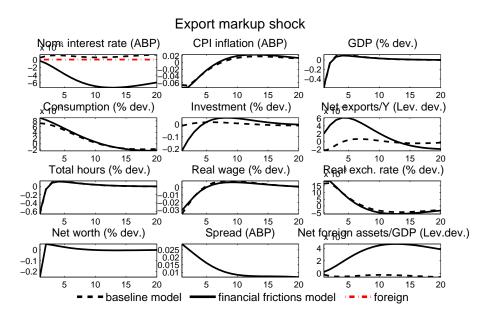


Figure 25: Impulse responses to the export markup shock, τ_t^x .

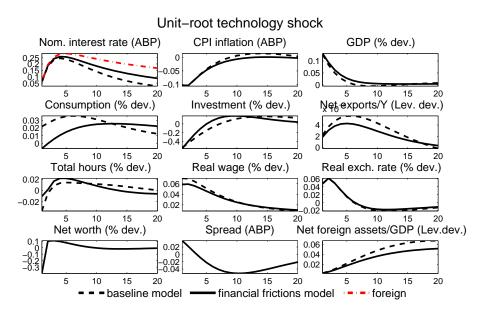


Figure 26: Impulse responses to the unit-root technology shock, $\mu_{z,t}$.

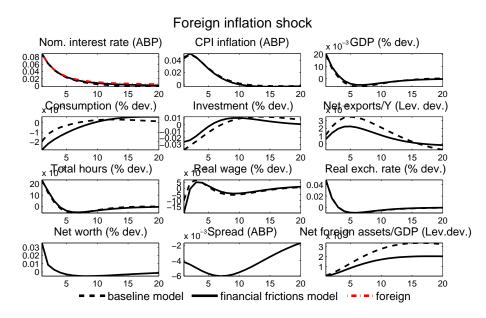


Figure 27: Impulse responses to the foreign inflation shock, $\epsilon_{\pi^*,t}$.

Note: The units on the y-axis are either in terms of percentage deviation (% dev.) from the steady state, annualized basis points (ABP), or level deviation (Lev. dev.). A temporary positive shock to foreign inflation causes the cost of imported consumption and investment to rise. As a result, consumption and investment decrease, imports decrease, and GDP goes up. The effects are small in magnitude, though.

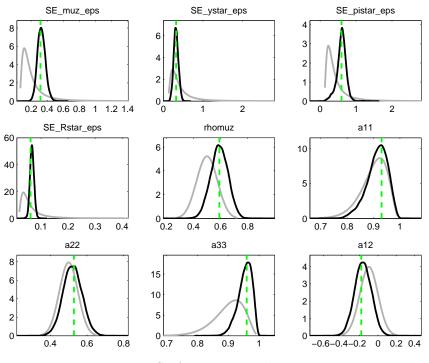


Figure 28: SVAR priors and posteriors.

Note: Prior distribution in gray, simulated distribution in black, and the computed posterior mode in dashed green.

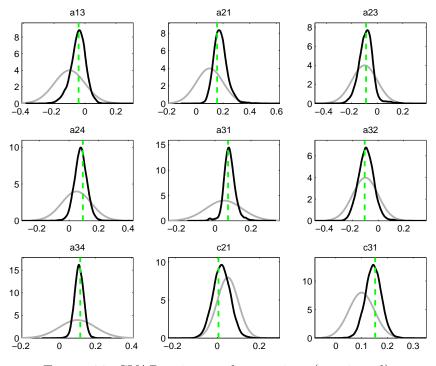


Figure 29: SVAR priors and posteriors (continued).

Note: Prior distribution in gray, simulated distribution in black, and the computed posterior mode in dashed green.

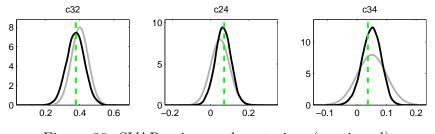


Figure 30: SVAR priors and posteriors (continued).

Note: Prior distribution in gray, simulated distribution in black, and the computed posterior mode in dashed green.

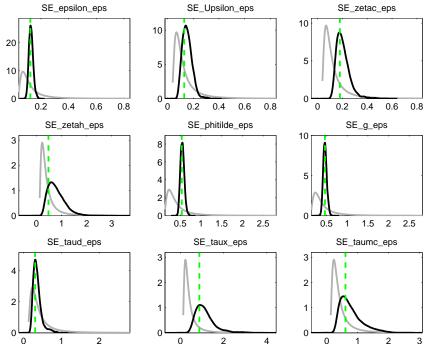


Figure 31: Priors and posteriors.

Note: Financial frictions model. Prior distribution in gray, simulated distribution in black, and the computed posterior mode in dashed green.

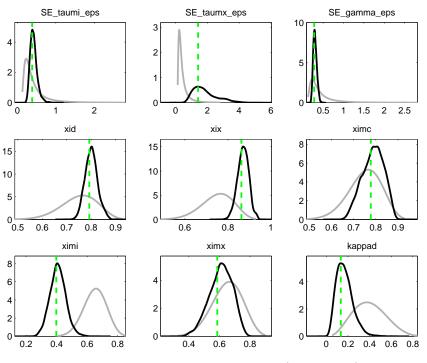


Figure 32: Priors and posteriors (continued).

Note: Financial frictions model. Prior distribution in gray, simulated distribution in black, and the computed posterior mode in dashed green.

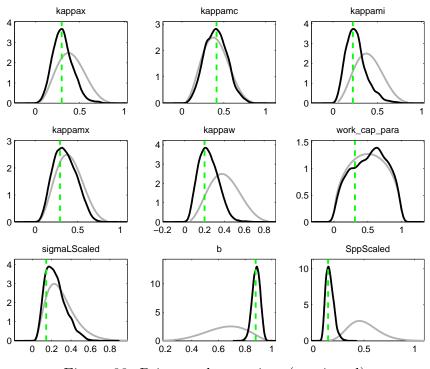


Figure 33: Priors and posteriors (continued).

Note: Financial frictions model. Prior distribution in gray, simulated distribution in black, and the computed posterior mode in dashed green.

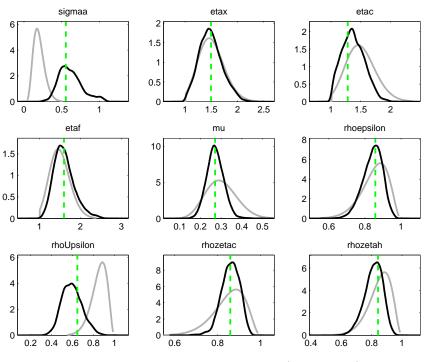


Figure 34: Priors and posteriors (continued).

Note: Financial frictions model. Prior distribution in gray, simulated distribution in black, and the computed posterior mode in dashed green.

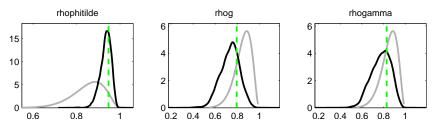


Figure 35: Priors and posteriors (continued).

Note: Financial frictions model. Prior distribution in gray, simulated distribution in black, and the computed posterior mode in dashed green.

Appendix C The model

C.1 The baseline model

As described in Section 2, the three final goods - consumption, investment and exports - are produced by combining the domestic homogeneous good with specific imported inputs for each type of final good. Below we start the model description by going through the production of all these goods.

C.1.1 Production of the domestic homogeneous good

A homogeneous domestic good, Y_t , is produced using

$$Y_t = \left[\int_0^1 Y_{i,t}^{1/\lambda_d} di\right]^{\lambda_d}, \ 1 \le \lambda_d < \infty,$$
(C.1)

where $Y_{i,t}$ denotes intermediate goods and $1/\lambda_d$ their degree of substitutability. The homogeneous domestic good is produced by a competitive, representative firm which takes the price of output, P_t , and the price of inputs, $P_{i,t}$, as given.

The *i*-th intermediate good producer has the following production function:

$$Y_{i,t} = (z_t H_{i,t})^{1-\alpha} \epsilon_t K_{i,t}^{\alpha} - z_t^+ \phi, \qquad (C.2)$$

where $K_{i,t}$ denotes the capital services rented by the *i*-th intermediate good producer. Also, $\log z_t$ is a technology shock whose first difference has a positive mean, $\log \epsilon_t$ is a stationary neutral technology shock and ϕ denotes a fixed production cost. The economy has two sources of growth: the positive drift in $\log z_t$ and a positive drift in $\log \Psi_t$, where Ψ_t is an investment-specific technology shock. The object z_t^+ in (C.2) is defined as¹³

$$z_t^+ = \Psi_t^{\frac{\alpha}{1-\alpha}} z_t.$$

In (C.2), $H_{i,t}$ denotes homogeneous labor services hired by the *i*-th intermediate good producer. Firms must borrow a fraction ν^f of the wage bill, so that one unit of labor costs is denoted by

 $W_t R_t^f$,

with

$$R_t^f = \nu^f R_t + 1 - \nu^f, (C.3)$$

where W_t is the aggregate wage rate, and R_t is the risk-free interest rate that apply on working capital loans.

The firm's marginal cost, divided by the price of the homogeneous good is denoted by mc_t :

$$mc_t = \tau_t^d \left(\frac{1}{1-\alpha}\right)^{1-\alpha} \left(\frac{1}{\alpha}\right)^{\alpha} \left(r_t^k\right)^{\alpha} \left(\bar{w}_t R_t^f\right)^{1-\alpha} \frac{1}{\epsilon_t},\tag{C.4}$$

where r_t^k is the nominal rental rate of capital scaled by P_t and $\bar{w}_t = W_t/(z_t^+P_t)$. Also, τ_t^d is a tax-like shock which affects marginal cost but does not appear in a production function.¹⁴

Productive efficiency dictates that marginal cost is equal to the cost of producing another unit using labor, implying:

$$mc_t = \tau_t^d \frac{\left(\mu_{\Psi,t}\right)^\alpha \bar{w}_t R_t^f}{\epsilon_t (1-\alpha) \left(\frac{k_{i,t}}{\mu_{z^+,t} H_{i,t}}\right)^\alpha} \tag{C.5}$$

The *i*-th firm is a monopolist in the production of the *i*-th good and so it sets its price. Price setting is subject to Calvo frictions. With probability ξ_d the intermediate good firm cannot reoptimize its price, in which case,

$$P_{i,t} = \tilde{\pi}_{d,t} P_{i,t-1}, \ \tilde{\pi}_{d,t} := (\pi_{t-1})^{\kappa_d} (\bar{\pi}_t^c)^{1-\kappa_d - \varkappa_d} (\breve{\pi})^{\varkappa_d},$$

where κ_d , \varkappa_d , $\kappa_d + \varkappa_d \in (0, 1)$ are parameters, π_{t-1} is the lagged inflation rate and $\bar{\pi}_t^c$ is the central bank's (implicit) target inflation rate. Also, $\check{\pi}$ is a scalar which allowing to capture, among other things, the case in which non-optimizing firms either do not change price at all (i.e., $\check{\pi} = \varkappa_d = 1$) or that they index only to the steady state inflation rate (i.e., $\check{\pi} = \bar{\pi}, \varkappa_d = 1$). Note that there is a price dispersion in steady state if $\varkappa_d > 0$ and if $\check{\pi}$ is different from the steady state value of π .

With probability $1 - \xi_d$ the firm can change the price. The problem of the *i*-th domestic intermediate good producer which has the opportunity to change price is to maximize discounted profits:

$$E_t \sum_{j=0}^{\infty} \beta^j v_{t+j} \left\{ P_{i,t+j} Y_{i,t+j} - mc_{t+j} P_{t+j} Y_{i,t+j} \right\},$$
(C.6)

¹³The details regarding the scaling of variables are collected in Appendix D.

¹⁴In the linearized version of the model in which there are no price and wage distortions in the steady state, τ_t^d is isomorphic to a disturbance in λ_d , i.e., a markup shock.

subject to the requirement that production equals demand. In the above expression, v_t is the multiplier on the household's nominal budget constraint. It measures the marginal value to the household of one unit of profits in terms of currency. In states of nature when the firm can reoptimize price, it does so to maximize its discounted profits subject to the price setting frictions and to the requirement that it satisfies demand given by

$$\left(\frac{P_t}{P_{i,t}}\right)^{\frac{\lambda_d}{\lambda_d-1}} Y_t = Y_{i,t}.$$
(C.7)

The equilibrium conditions associated with the price setting problem and their derivation are reported in Appendix D.

The domestic intermediate output good is allocated among alternative uses as follows:

$$Y_t = G_t + C_t^d + I_t^d + \int_0^1 X_{i,t}^d di,$$
 (C.8)

where G_t denotes government consumption (which consists entirely of the domestic good), C_t^d denotes intermediate goods used (together with foreign consumption goods) to produce final household consumption goods, I_t^d is the amount of intermediate domestic goods used in combination with imported foreign investment goods to produce a homogeneous investment good. Finally, the integral in (C.8) denotes domestic resources allocated to exports. The determination of consumption, investment and export demand is discussed below.

C.1.2 Production of final consumption and investment goods

Final consumption goods are purchased by households. These goods are produced by a representative competitive firm using the following linear homogeneous technology:

$$C_{t} = \left[(1 - \omega_{c})^{\frac{1}{\eta_{c}}} \left(C_{t}^{d} \right)^{\frac{\eta_{c}-1}{\eta_{c}}} + \omega_{c}^{\frac{1}{\eta_{c}}} \left(C_{t}^{m} \right)^{\frac{\eta_{c}-1}{\eta_{c}}} \right]^{\frac{\eta_{c}}{\eta_{c}-1}}.$$
 (C.9)

The representative firm takes the price of final consumption goods output, P_t^c , as given. Final consumption goods output is produced using two inputs. The first, C_t^d , is a one-for-one transformation of the homogeneous domestic good and therefore has price, P_t . The second input, C_t^m , is the homogeneous composite of specialized consumption import goods discussed in the next subsection. The price of C_t^m is $P_t^{m,c}$. The representative firm takes the input prices, P_t and $P_t^{m,c}$ as given. Profit maximization leads to the following demand for the intermediate inputs in a scaled form:

$$c_t^d = (1 - \omega_c) (p_t^c)^{\eta_c} c_t$$

$$c_t^m = \omega_c \left(\frac{p_t^c}{p_t^{m,c}}\right)^{\eta_c} c_t,$$
(C.10)

where $p_t^c = P_t^c / P_t$ and $p_t^{m,c} = P_t^{m,c} / P_t$. The price of C_t is related to the price of the inputs by

$$p_t^c = \left[(1 - \omega_c) + \omega_c (p_t^{m,c})^{1 - \eta_c} \right]^{\frac{1}{1 - \eta_c}}.$$
 (C.11)

The rate of inflation of the consumption good is

$$\pi_t^c = \frac{P_t^c}{P_{t-1}^c} = \pi_t \left[\frac{(1 - \omega_c) + \omega_c (p_t^{m,c})^{1 - \eta_c}}{(1 - \omega_c) + \omega_c (p_{t-1}^{m,c})^{1 - \eta_c}} \right]^{\frac{1}{1 - \eta_c}}.$$
(C.12)

Investment goods are produced by a representative competitive firm using the following technology:

$$I_t + a(u_t)\bar{K}_t = \Psi_t \left[(1 - \omega_i)^{\frac{1}{\eta_i}} \left(I_t^d \right)^{\frac{\eta_i - 1}{\eta_i}} + \omega_i^{\frac{1}{\eta_i}} \left(I_t^m \right)^{\frac{\eta_i - 1}{\eta_i}} \right]^{\frac{\eta_i}{\eta_i - 1}},$$

where investment is defined as the sum of investment goods, I_t , used in the accumulation of physical capital plus investment goods used in capital maintenance, $a(u_t)\bar{K}_t$. The maintenance is discussed below.

See Appendix D for the functional form of $a(u_t)$. u_t denotes the utilization rate of capital, with capital services being defined by

$$K_t = u_t \bar{K}_t.$$

In order to accommodate the possibility that the price of investment goods relative to the price of consumption goods declines over time, it is assumed that the investment specific technology shock Ψ_t is a unit root process with a potentially positive drift. As in the consumption good sector, the representative investment goods producers take all relevant prices as given. Profit maximization leads to the following demand for the intermediate inputs in a scaled form:

$$i_t^d = (p_t^i)^{\eta_i} \left(i_t + a(u_t) \frac{k_t}{\mu_{\psi,t} \mu_{z^+,t}} \right) (1 - \omega_i)$$
(C.13)

$$i_t^m = \omega_i \left(\frac{p_t^i}{p_t^{m,i}}\right)^{\eta_i} \left(i_t + a(u_t)\frac{\bar{k}_t}{\mu_{\psi,t}\mu_{z^+,t}}\right) \tag{C.14}$$

where $p_t^i = \Psi_t P_t^i / P_t$ and $p_t^{m,i} = P_t^{m,i} / P_t$. The price of I_t is related to the price of the inputs by

$$p_t^i = \left[(1 - \omega_i) + \omega_i (p_t^{m,i})^{1 - \eta_i} \right]^{\frac{1}{1 - \eta_i}}.$$
 (C.15)

The rate of inflation of the investment good is

$$\pi_t^i = \frac{\pi_t}{\mu_{\Psi,t}} \left[\frac{(1-\omega_i) + \omega_i \left(p_t^{m,i} \right)^{1-\eta_i}}{(1-\omega_i) + \omega_i \left(p_{t-1}^{m,i} \right)^{1-\eta_i}} \right]^{\frac{1}{1-\eta_i}}.$$
 (C.16)

C.1.3 Exports and imports

Both exports and imports activities involve Calvo price setting frictions and therefore require the presence of market power. Dixit-Stiglitz strategy is used to introduce a range of specialized goods. This allows there to be market power without the counterfactual implication that there is a small number of firms in the export and import sector. Thus, exports involve a continuum of exporters, each of which is a monopolist which produces a specialized export good. Each monopolist produces the export good using a homogeneous domestically produced good and a homogeneous good derived from imports. The specialized export goods are sold to foreign competitive retailers which create a homogeneous good that is sold to foreign citizens.

In the case of imports, specialized domestic importers purchase a homogeneous foreign good which they turn into a specialized input and sell to domestic retailers. There are three types of domestic retailers. One uses the specialized import goods to create the homogeneous good used as an input into the production of specialized exports. Another uses the specialized import goods to create an input used in the production of investment goods. The third type uses specialized imports to produce a homogeneous input used in the production of consumption goods. Imported goods are combined with domestic inputs before being passed on to final domestic users. There are pricing frictions in both exports and imports. In all cases it is assumed that prices are set in the currency of the buyer ('pricing to market').¹⁵

Exports. There is a total demand by foreigners for domestic exports, which takes on the following form:

 $^{^{15}}$ Pricing frictions in imports help the model account for the evidence that exchange rate shocks take time to pass into domestic prices. Pricing frictions in exports help the model to produce a hump-shape in the response of output to a domestic monetary shock, though, as seen in Section 4, it is not the case for a currency area-wide monetary policy shock.

$$X_t = \left(\frac{P_t^x}{P_t^*}\right)^{-\eta_f} Y_t^*,\tag{C.17}$$

where Y_t^* is foreign GDP, P_t^* is the foreign currency price of foreign homogeneous goods and P_t^x is an index of export prices defined below. The goods X_t are produced by a representative competitive foreign retailer firm using specialized inputs as follows:

$$X_t = \left[\int_0^1 X_{i,t}^{\frac{1}{\lambda_x}} di\right]^{\lambda_x},\tag{C.18}$$

where $X_{i,t}$, $i \in (0, 1)$ are specialized intermediate goods for export good production. The retailer that produces X_t takes its output price P_t^x and its input prices $P_{i,t}^x$ as given. Optimization leads to the following demand for specialized exports:

$$X_{i,t} = \left(\frac{P_{i,t}^x}{P_t^x}\right)^{\frac{-\lambda_x}{\lambda_x - 1}} X_t.$$
(C.19)

Combining (C.18) and (C.19),

$$P_t^x = \left[\int_0^1 \left(P_{i,t}^x\right)^{\frac{1}{1-\lambda_x}} di\right]^{1-\lambda_x}$$

The *i*-th specialized export is produced by a monopolist using the following technology:

$$X_{i,t} = \left[\omega_x^{\frac{1}{\eta_x}} \left(X_{i,t}^m\right)^{\frac{\eta_x - 1}{\eta_x}} + (1 - \omega_x)^{\frac{1}{\eta_x}} \left(X_{i,t}^d\right)^{\frac{\eta_x - 1}{\eta_x}}\right]^{\frac{\eta_x - 1}{\eta_x - 1}},$$

where $X_{i,t}^m$ and $X_{i,t}^d$ are the *i*-th exporter's use of the imported and domestically produced goods, respectively. The marginal cost associated with the CES production function is derived from the multiplier associated with the Lagrangian representation of the cost minimization problem:

$$C = \min \tau_t^x \left[P_t^{m,x} R_t^x X_{i,t}^m + P_t R_t^x X_{i,t}^d \right] + \lambda \left\{ X_{i,t} - \left[\omega_x^{\frac{1}{\eta_x}} \left(X_{i,t}^m \right)^{\frac{\eta_x - 1}{\eta_x}} + (1 - \omega_x)^{\frac{1}{\eta_x}} \left(X_{i,t}^d \right)^{\frac{\eta_x - 1}{\eta_x}} \right]^{\frac{\eta_x}{\eta_x - 1}} \right\},$$

where $P_t^{m,x}$ is the price of the homogeneous import good and P_t is the price of the homogeneous domestic good. Using the first order conditions of this problem and the production function, the real marginal cost in terms of stationary variables, mc_t^x , is derived as

$$mc_t^x = \frac{\lambda}{S_t P_t^x} = \frac{\tau_t^x R_t^x}{q_t p_t^c p_t^x} \left[\omega_x \left(p_t^{m,x} \right)^{1-\eta_x} + (1-\omega_x) \right]^{\frac{1}{1-\eta_x}},$$
(C.20)

where

$$R_t^x = \nu^x R_t + 1 - \nu^x, (C.21)$$

$$\frac{S_t P_t^x}{P_t} = \frac{S_t P_t^*}{P_t^c} \frac{P_t^c}{P_t} \frac{P_t^x}{P_t^*} = q_t p_t^c p_t^x, \tag{C.22}$$

and q_t denotes the real exchange rate defined as

$$q_t = \frac{S_t P_t^*}{P_t^c}.\tag{C.23}$$

From the solution to the same problem, the demand for domestic inputs for export production is

$$X_{i,t}^d = \left(\frac{\lambda}{\tau_t^x R_t^x P_t}\right)^{\eta_x} X_{i,t}(1-\omega_x).$$
(C.24)

The quantity of the domestic homogeneous good used by specialized exporters is

$$\int_0^1 X_{i,t}^d di$$

which, in terms of aggregates, is [by plugging (C.24) into this integral and derived in Appendix D]

$$X_t^d = \int_0^1 X_{i,t}^d di = \left[\omega_x (p_t^{m,x})^{1-\eta_x} + (1-\omega_x)\right]^{\frac{\eta_x}{1-\eta_x}} (1-\omega_x) (\dot{p}_t^x)^{\frac{-\lambda_x}{\lambda_x-1}} (p_t^x)^{-\eta_f} Y_t^*$$
(C.25)

where \dot{p}_t^x is a measure of the price dispersion and is defined in Appendix D.

Using a similar derivation as for X_t^d ,

$$X_t^m = \omega_x \left(\frac{\left[\omega_x (p_t^{m,x})^{1-\eta_x} + (1-\omega_x) \right]^{\frac{1}{1-\eta_x}}}{p_t^{m,x}} \right)^{\eta_x} (\mathring{p}_t^x)^{\frac{-\lambda_x}{\lambda_x - 1}} (p_t^x)^{-\eta_f} Y_t^*.$$
(C.26)

The *i*-th, $i \in (0, 1)$, export good firm takes (C.19) as its demand curve. This producer sets prices subject to a Calvo sticky-price mechanism. With probability ξ_x , the *i*-th export good firm cannot reoptimize its price, in which case it updates the price as follows:

$$P_{i,t}^{x} = \tilde{\pi}_{t}^{x} P_{i,t-1}^{x}, \ \tilde{\pi}_{t}^{x} = (\pi_{t-1}^{x})^{\kappa_{x}} (\pi^{x})^{1-\kappa_{x}-\varkappa_{x}} (\breve{\pi})^{\varkappa_{x}},$$
(C.27)

where $\kappa_x, \varkappa_x, \kappa_x + \varkappa_x \in (0, 1)$.

The equilibrium conditions associated with price setting by exporters that do get to reoptimize their prices are analogous to the ones derived for the domestic intermediate good producers and are reported in Appendix D.

Imports. Foreign firms sell a homogeneous good to domestic importers. The importers convert the homogeneous good into a specialized input ('brand name it') and supply that input monopolistically to domestic retailers. Importers are subject to Calvo price setting frictions. There are three types of importing firms: (i) one produces goods used to produce an intermediate good for the production of consumption, (ii) one produces goods used to produce an intermediate good for the production of investment, and (iii) one produces goods used to produce an intermediate good for the production of exports.

Consider (i) first. The production function of the domestic retailer of imported consumption goods is

$$C_t^m = \left[\int_0^1 (C_{i,t}^m)^{\frac{1}{\lambda_{m,c}}} di\right]^{\lambda_{m,c}}$$

where $C_{i,t}^m$ is the output of the *i*-th specialized producer and C_t^m is an intermediate good used in the production of the consumption goods. Let $P_t^{m,c}$ denote the price index of C_t^m and let $P_{i,t}^{m,c}$ denote the price of the *i*-th intermediate input. The domestic retailer is competitive and takes $P_t^{m,c}$ and $P_{i,t}^{m,c}$ as given. The demand curve for specialized inputs is given by the domestic retailer's first order necessary condition for profit maximization:

$$C_{i,t}^{m} = C_{t}^{m} \left(\frac{P_{t}^{m,c}}{P_{i,t}^{m,c}}\right)^{\frac{\lambda_{m,c}}{\lambda_{m,c-1}}}$$

We now turn to the producer of $C_{i,t}^m$ who takes the previous equation as a demand curve. This producer buys the homogeneous foreign good and converts it one-for-one into the domestic differentiated good, $C_{i,t}^m$. The intermediate good producer's marginal cost is

$$\tau_t^{m,c} S_t P_t^* R_t^{\nu,*},\tag{C.28}$$

where

$$R_t^{\nu,*} = \nu^* R_t^* + 1 - \nu^*, \tag{C.29}$$

where R_t^* is the foreign nominal rate of interest.¹⁶

As in the homogeneous domestic good sector, $\tau_t^{m,c}$ is a tax-like shock which affects marginal costs but does not appear in a production function.¹⁷

The total value of imports accounted for by the consumption sector is

$$S_t P_t^* R_t^{\nu,*} C_t^m (\mathring{p}_t^{m,c})^{\frac{\lambda_{m,c}}{1-\lambda_{m,c}}},$$

where

$$\mathring{p}_t^{m,c} = \frac{\mathring{P}_t^{m,c}}{P_t^{m,c}}$$

is a measure of the price dispersion in the differentiated good, $C_{i,t}^m$.

Now consider (ii). The production function for the domestic retailer of imported investment goods, I_t^m , is

$$I^m_t = \left[\int_0^1 (I^m_{i,t})^{\frac{1}{\lambda_{m,i}}} di\right]^{\lambda_{m,i}}.$$

The retailer of imported investment goods is competitive and takes output prices, $P_t^{m,i}$, and input prices,

 $P_{i,t}^{m,i}$, as given. The producer of the *i*-th intermediate input into the above production function buys the homogeneous the differentiated good, $I_{i,t}^m$. The marginal cost of $I_{i,t}^m$ is the analogue of (C.28):

$$\tau_t^{m,i} S_t P_t^* R_t^{\nu,*},$$

which implies the importing firm's cost is P_t^* (before borrowing costs, exchange rate conversion and markup shocks), which is the same cost for the specialized inputs used to produce C_t^m .

The total value of imports associated with the production of investment goods is analogous to what was obtained for the consumption good sector:

$$S_t P_t^* R_t^{\nu,*} I_t^m (\mathring{p}_t^{m,i})^{\frac{\lambda_{m,i}}{1-\lambda_{m,i}}}, \, \mathring{p}_t^{m,i} = \frac{P_{i,t}^{m,i}}{P_t^{m,i}}.$$
(C.30)

Now consider (iii). The production function of the domestic retailer of imported goods used in the production of an input, X_t^m , for the production of export goods is

$$X_t^m = \left[\int_0^1 (X_{i,t}^m)^{\frac{1}{\lambda_{m,x}}} di\right]^{\lambda_{m,x}}$$

The imported good retailer is competitive and takes output prices, $P_t^{m,x}$, and input prices, $P_{i,t}^{m,x}$, as given. The producer of the specialized input, $X_{i,t}^m$, has marginal cost

$$\tau_t^{m,x} S_t P_t^* R_t^{\nu,*}.$$

The total value of imports associated with the production of X_t^m is

$$S_t P_t^* R_t^{\nu,*} X_t^m (\mathring{p}_t^{m,x})^{\frac{\lambda_{m,x}}{1-\lambda_{m,x}}}, \, \mathring{p}_t^{m,x} = \frac{P_{i,t}^{m,x}}{P_t^{m,x}}.$$
(C.31)

¹⁶The notion here is that the intermediate good firm must pay the inputs with foreign currency and because they have no resources themselves at the beginning of the period, they must borrow those resources if they are to buy the foreign inputs needed to produce $C_{i,t}^m$. The financing need is in the foreign currency, so the loan is taken in that currency. There is no risk to this working capital loan because all shocks are realized at the beginning of the period and so there is no uncertainty within the duration of the loan about the realization of prices and exchange rates.

¹⁷In the linearization of a version of the model in which there are no price and wage distortions in the steady state, $\tau_t^{m,c}$ is isomorphic to a markup shock.

Each of the above three types of intermediate good firm is subject to Calvo price-setting frictions. With probability $1 - \xi_{m,j}$, the *j*-th type of firm can reoptimize its price and with probability $\xi_{m,j}$ it updates its price according to

$$P_{i,t}^{m,j} = \tilde{\pi}_t^{m,j} P_{i,t-1}^{m,j}, \ \tilde{\pi}_t^{m,j} := (\pi_{t-1}^{m,j})^{\kappa_{m,j}} (\bar{\pi}_t^c)^{1-\kappa_{m,j}-\varkappa_{m,j}} \breve{\pi}^{\varkappa_{m,j}}, \ j = c, i, x.$$
(C.32)

The equilibrium conditions associated with price setting by importers are analogous to the ones derived for domestic intermediate good producers and are reported in Appendix D.

C.1.4 Households

Household preferences are given by

$$E_0^j \sum_{t=0}^{\infty} \beta^t \left[\zeta_t^c \log(C_t - bC_{t-1}) - \zeta_t^h A_L \frac{(h_{j,t})^{1+\sigma_L}}{1+\sigma_L} \right],$$
(C.33)

where ζ_t^c denotes a consumption preference shock, ζ_t^h a disutility of labor shock, b is the consumption habit parameter, h_j denotes the *j*-th household's supply of labor services and σ_L denotes the inverse Frisch elasticity. The household owns the economy's stock of physical capital. It determines the rate at which the capital stock is accumulated and the rate at which it is utilized. The household also owns the stock of net foreign assets and determines its rate of accumulation.

Wage setting. The specialized labor supplied by households is combined by labor contractors into a homogeneous labor services:

$$H_t = \left[\int_0^1 (h_{j,t})^{\frac{1}{\lambda_w}} dj\right]^{\lambda_w}, \ 1 \le \lambda_w < \infty.$$

Households are subject to Calvo wage setting frictions (as in Erceg, Henderson and Levin, 2000). With probability $1 - \xi_w$ the *j*-th household is able to reoptimize its wage and with probability ξ_w it updates its wage according to

$$W_{j,t+1} = \tilde{\pi}_{w,t+1} W_{j,t} \tag{C.34}$$

$$\tilde{\pi}_{w,t+1} = (\pi_t^c)^{\kappa_w} \left(\bar{\pi}_{t+1}^c\right)^{1-\kappa_w - \varkappa_w} (\breve{\pi})^{\varkappa_w} (\mu_{z^+})^{\vartheta_w}, \tag{C.35}$$

where κ_w , \varkappa_w , ϑ_w , $\kappa_w + \varkappa_w \in (0, 1)$.

Consider the j^{th} household that has an opportunity to reoptimize its wage at time t. Denote this wage rate by \tilde{W}_t . This is not indexed by j because the situation of each household that optimizes its wage is the same. In choosing \tilde{W}_t the household considers the discounted utility (neglecting currently irrelevant terms in the household objective) of future histories when it cannot reoptimize (note the i vs j):

$$E_t^j \sum_{i=0}^{\infty} \left(\beta \xi_w\right)^i \left[-\zeta_{t+i}^h A_L \frac{(h_{j,t+i})^{1+\sigma_L}}{1+\sigma_L} + v_{t+i} W_{j,t+i} h_{j,t+i} \frac{1-\tau^y}{1+\tau^w} \right], \tag{C.36}$$

where τ^y is a tax on labor income, τ^w is a payroll tax, v_t is the multiplier on the household's period t budget constraint. The demand for the j^{th} household's labor services, conditional on it having optimized in period t and not again since, is

$$h_{j,t+i} = \left(\frac{\tilde{W}_t \tilde{\pi}_{w,t+i}, \cdots, \tilde{\pi}_{w,t+1}}{W_{t+i}}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}, \tag{C.37}$$

where it is understood that $\tilde{\pi}_{w,t+i}, \dots, \tilde{\pi}_{w,t+1} = 1$ when i = 0. The equilibrium conditions associated with this problem, i.e. wage setting of households that do get to reoptimize, are reported in Appendix D.

Technology for capital accumulation. The law of motion of the stock of physical capital takes into account investment adjustment costs as introduced by Christiano, Eichenbaum and Evans (2005):¹⁸

$$\bar{K}_{t+1} = (1-\delta)\bar{K}_t + \Upsilon_t \left(1 - \tilde{S}\left(\frac{I_t}{I_{t-1}}\right)\right)I_t, \tag{C.38}$$

where Υ_t denotes the marginal efficiency of investment shock that affects how investment is transformed into capital.^19

Household consumption and investment decisions. The first order condition for consumption is

$$\frac{\zeta_t^c}{c_t - bc_{t-1}\frac{1}{\mu_{z^+,t}}} - \beta bE_t \frac{\zeta_{t+1}^c}{c_{t+1}\mu_{z^+,t+1} - bc_t} - \psi_{z^+,t} p_t^c (1+\tau^c) = 0,$$
(C.39)

where

$$\psi_{z^+,t} = \upsilon_t P_t z_t^+$$

is the marginal value of wealth in real terms, in particular in terms of one unit of the homogeneous domestic good at time t.

To define the intertemporal Euler equation associated with the household's capital accumulation decision, define the rate of return on a period t investment in a unit of physical capital, R_{t+1}^k :

$$R_{t+1}^{k} = \frac{(1-\tau^{k}) \left[u_{t+1} r_{t+1}^{k} - \frac{p_{t+1}^{i}}{\Psi_{t+1}} a(u_{t+1}) \right] P_{t+1} + (1-\delta) P_{t+1} P_{k',t+1} + \tau^{k} \delta P_{t} P_{k',t}}{P_{t} P_{k',t}}, \tag{C.40}$$

where

$$\frac{p_t^i}{\Psi_t}P_t = P_t^i$$

is the date t price of the homogeneous investment good, $\bar{r}_t^k = \Psi_t r_t^k$ is the scaled real rental rate of capital, τ^k is the capital tax rate, $P_{k',t}$ denotes the price of a unit of newly installed physical capital which operates in period t+1. This price is expressed in units of the homogeneous good, so that $P_t P_{k',t}$ is the domestic currency price of physical capital. The numerator in the expression for R_{t+1}^k represents the period t+1 payoff from a unit additional physical capital. The expression in square brackets captures the idea that maintenance expenses associated with the operation of capital are deductible from taxes. The last expression in the numerator expresses the idea that physical depreciation is deductible at historical cost. It is convenient to express R_t^k in scaled terms:

$$R_{t+1}^{k} = \frac{\pi_{t+1}}{\mu_{\Psi,t+1}} \frac{(1-\tau^{k}) \left[u_{t+1} \bar{r}_{t+1}^{k} - p_{t+1}^{i} a(u_{t+1}) \right] + (1-\delta) p_{k',t+1} + \tau^{k} \delta \frac{\mu_{\Psi,t+1}}{\pi_{t+1}} p_{k',t}}{p_{k',t}}, \tag{C.41}$$

where $p_{k',t} = \Psi_t P_{k',t}$.²⁰ The first order condition for capital implies

$$\psi_{z^+,t} = \beta E_t \psi_{z^+,t+1} \frac{R_{t+1}^k}{\pi_{t+1}\mu_{z^+,t+1}}.$$
(C.42)

By differentiating the Lagrangian representation of the household's problem with respect to I_t , the investment first order condition in scaled terms is

 $^{^{18}\}text{See}$ Appendix D for the functional form of the investment adjustment costs, $\tilde{S}.$

¹⁹This is the shock whose importance is emphasized by Justiniano, Primiceri and Tambalotti (2011).

 $^{^{20}\}mathrm{A}$ rise in inflation raises the tax rate on capital because of the practice of valuing depreciation at historical cost.

$$-\psi_{z^{+},t}p_{t}^{i}+\psi_{z^{+},t}p_{k',t}\Upsilon_{t}\left[1-\tilde{S}\left(\frac{\mu_{z^{+},t}\mu_{\Psi,t}i_{t}}{i_{t-1}}\right)-\tilde{S}'\left(\frac{\mu_{z^{+},t}\mu_{\Psi,t}i_{t}}{i_{t-1}}\right)\frac{\mu_{z^{+},t}\mu_{\Psi,t}i_{t}}{i_{t-1}}\right]$$
$$+\beta\psi_{z^{+},t+1}p_{k',t+1}\Upsilon_{t+1}\tilde{S}'\left(\frac{\mu_{z^{+},t+1}\mu_{\Psi,t+1}i_{t+1}}{i_{t}}\right)\left(\frac{i_{t+1}}{i_{t}}\right)^{2}\mu_{\Psi,t+1}\mu_{z^{+},t+1}=0.$$
(C.43)

The first order condition associated with capital utilization is, in scaled terms²¹

$$\bar{r}_t^k = p_t^i a'(u_t). \tag{C.44}$$

Financial assets. The household does the domestic economy's saving. Period t saving occurs by the acquisition of net foreign assets, A_{t+1}^* , and a domestic asset. The domestic asset is used to finance the working capital requirements of firms. This asset pays a nominally non-state contingent return from t to t + 1, R_t . The first order condition associated with this domestic asset is

$$\psi_{z^+,t} = \beta E_t \frac{\psi_{z^+,t+1}}{\mu_{z^+,t+1}} \left[\frac{R_t - \tau^b (R_t - \pi_{t+1})}{\pi_{t+1}} \right], \tag{C.45}$$

where τ^b is the tax rate on the real interest rate on bond income.^{22}

The tax treatment of domestic agent's earnings on foreign bonds is the same as the tax treatment of agent's earnings on domestic bonds. The date t first order condition associated with the asset A_{t+1}^* that pays R_t^* in terms of foreign currency is

$$\upsilon_t S_t = \beta E_t \upsilon_{t+1} \left[S_{t+1} R_t^* \Phi_t - \tau^b \left(S_{t+1} R_t^* \Phi_t - \frac{S_t}{P_t} P_{t+1} \right) \right].$$
(C.46)

Recall that S_t is the domestic currency price of a unit foreign currency. The left side of this expression is the cost of acquiring a unit of foreign assets. The currency cost is S_t and this is converted into utility terms by multiplying by the multiplier on the household's budget constraint, v_t . The term in square brackets is the after-tax payoff of the foreign asset in domestic currency units. The period t + 1 pre-tax interest payoff on A_{t+1}^* is $S_{t+1}R_t^*\Phi_t$. Here, R_t^* is the foreign nominal rate of interest, which is risk free in foreign currency units. The term Φ_t represents a relative risk adjustment of the foreign asset return, so that a unit of the foreign asset acquired in t pays off $R_t^*\Phi_t$ units of foreign currency in t + 1. The determination of Φ_t is discussed below. The remaining term in brackets pertains to the impact of taxation on returns on foreign assets.²³

Scaling the first order condition, (C.46), by multiplying both sides by $P_t z_t^+/S_t$ yields

$$\psi_{z^+,t} = \beta E_t \frac{\psi_{z^+,t+1}}{\pi_{t+1}\mu_{z^+,t+1}} \left[s_{t+1}R_t^*\Phi_t - \tau^b (s_{t+1}R_t^*\Phi_t - \pi_{t+1}) \right], \tag{C.47}$$

where

 $s_t = \frac{S_t}{S_{t-1}}.$

The risk adjustment term has the following form:

$$\Phi_t = \Phi\left(a_t, R_t^* - R_t, \tilde{\phi}_t\right) = \exp\left(-\tilde{\phi}_a(a_t - \bar{a}) - \tilde{\phi}_s(R_t^* - R_t - (R^* - R)) + \tilde{\phi}_t\right),$$
(C.48)

²¹The tax rate on capital income does not enter here because of the deductibility of maintenance costs.

 $^{^{22}}$ A consequence of this treatment of the taxation on domestic bonds is that the steady state real after-tax return on bonds is invariant to π .

²³If we ignore the term after the minus sign in parentheses, then the taxation is applied to the whole nominal payoff on the bond, including principal. The term after the minus sign is designed to ensure that the principal is deducted from taxes. The principal is expressed in nominal terms and is set so that the real value at t + 1 coincides with the real value of the currency used to purchase the asset in period t. Recall that S_t is the period t domestic currency cost of a unit (in terms of foreign currency) of foreign assets. So the period t real cost of the asset is S_t/P_t . The domestic currency value in period t+1 of this real quantity is $P_{t+1}S_t/P_t$.

where

$$a_t = \frac{S_t A_{t+1}^*}{P_t z_t^+},$$

 $\tilde{\phi}_t$ is a mean zero country risk premium shock, and $\tilde{\phi}_a$ and $\tilde{\phi}_s$ are positive parameters.²⁴

C.1.5 Fiscal and monetary authorities

The monetary policy is conducted according to a hard peg of the domestic nominal interest rate to the foreign nominal interest rate.

Government expenditures are modeled as

$$G_t = g_t z_t^+,$$

where g_t is an exogenous stochastic process, and z_t^+ ensures a constant government expenditures to GDP ratio. The tax rates in the model are: capital tax rate, τ^k , bond tax rate, τ^b , labor income tax rate, τ^y , consumption tax rate, τ^c , and payroll tax rate, τ^w . Any difference between government expenditures and tax revenues is offset by lump-sum transfers.

C.1.6 Foreign variables

The representation of foreign variables takes into account the assumption that foreign output, Y_t^* , is affected by disturbances to z_t^+ , just as domestic variables are. In particular,

$$\log Y_t^* = \log y_t^* + \log z_t^+$$
$$= \log y_t^* + \log z_t + \frac{\alpha}{1 - \alpha} \log \psi_t,$$

where $\log(y_t^*)$ is assumed to be a stationary process. It is assumed that

$$\begin{pmatrix} \log\left(\frac{y_{t}^{*}}{y^{*}}\right) \\ \pi_{t}^{*} - \pi^{*} \\ R_{t}^{*} - R^{*} \\ \log\left(\frac{\mu_{z,t}}{\mu_{z}}\right) \\ \log\left(\frac{\mu_{\psi,t}}{\mu_{\psi}}\right) \end{pmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & 0 & 0 \\ a_{21} & a_{22} & a_{23} & a_{24} & \frac{a_{24}\alpha}{1-\alpha} \\ a_{31} & a_{32} & a_{33} & a_{34} & \frac{a_{34}\alpha}{1-\alpha} \\ 0 & 0 & 0 & \rho_{\mu_{z}} & 0 \\ 0 & 0 & 0 & 0 & \rho_{\mu_{\psi}} \end{bmatrix} \begin{pmatrix} \log\left(\frac{y_{t-1}}{y^{*}}\right) \\ \pi_{t-1}^{*} - \pi^{*} \\ R_{t-1}^{*} - R^{*} \\ \log\left(\frac{\mu_{\psi,t}}{\mu_{z}}\right) \\ \log\left(\frac{\mu_{\psi,t}}{\mu_{\psi}}\right) \end{pmatrix} \\ + \begin{bmatrix} \sigma_{y^{*}} & 0 & 0 & 0 & 0 \\ c_{21} & \sigma_{\pi^{*}} & 0 & c_{24} & \frac{c_{24}\alpha}{1-\alpha} \\ c_{31} & c_{32} & \sigma_{R^{*}} & c_{34} & \frac{c_{34}\alpha}{1-\alpha} \\ 0 & 0 & 0 & \sigma_{\mu_{z}} & 0 \\ 0 & 0 & 0 & \sigma_{\mu_{z}} & 0 \end{bmatrix} \begin{pmatrix} \varepsilon_{y^{*},t} \\ \varepsilon_{R^{*},t} \\ \varepsilon_{\mu_{z},t} \\ \varepsilon_{\mu_{\psi},t} \end{pmatrix},$$

where ε_t 's are mean zero, unit variance, Gaussian i.i.d. processes uncorrelated with each other. In matrix form,

$$X_t^* = A X_{t-1}^* + C \varepsilon_t$$

in obvious notation. Note that the matrix C has 10 elements, so that the order condition for identification is satisfied, since C'C represents 15 independent equations. The above restrictions assume that the shock

²⁴The dependence of Φ_t on a_t ensures that there is a unique steady state value of a_t that is independent of the initial net foreign assets and the capital stock of the economy. The dependence of Φ_t on the relative level of the interest rate, $R_t^* - R_t$, is designed to allow the model to reproduce two types of observations. The first concerns observations related to uncovered interest parity. The second concerns the humpshaped response of output to a domestic monetary policy shock. The particular calibration sets $\tilde{\phi}_s = 0$ to ensure the nominal interest rate peg regime.

 $\varepsilon_{y^*,t}$ affects the first three variables in X_t^* , while $\varepsilon_{\pi^*,t}$ only affects the second two, and $\varepsilon_{R^*,t}$ only affects the third.²⁵ Also, the zeros in the last two columns of the first row in A and C imply that the technology shocks do not affect y_t^* .²⁶ Third, the A and C matrices capture the notion that innovations to technology affect foreign inflation and the interest rate via their impact on z_t^+ . Fourth, the assumptions on A and C imply that $\log\left(\frac{\mu_{\psi,t}}{\mu_{\psi}}\right)$ and $\log\left(\frac{\mu_{z,t}}{\mu_z}\right)$ are univariate first order autoregressive processes driven by $\epsilon_{\mu_{\psi},t}$ and $\varepsilon_{\mu_z,t}$, respectively.

C.1.7 Resource constraints

The fact that there is potentially steady state price dispersion both in prices and wages complicates the expression for the domestic homogeneous good, Y_t , in terms of aggregate factors of production. The relationship derived in Appendix D can be expressed as

$$y_t = (\mathring{p}_t)^{\frac{\lambda_d}{\lambda_d - 1}} \left[\epsilon_t \left(\frac{1}{\mu_{\Psi, t}} \frac{1}{\mu_{z^*, t}} k_t \right)^{\alpha} \left(\mathring{w}_t^{-\frac{\lambda_w}{1 - \lambda_w}} h_t \right)^{1 - \alpha} - \phi \right], \tag{C.49}$$

where \mathring{p}_t denotes the degree of price dispersion in the intermediate domestic good.

Resource constraint for domestic homogeneous output. Above we defined real, scaled output in terms of aggregate factors of production. It is convenient to also have an expression that exhibits the uses of domestic homogeneous output. Using (C.25),

$$z_t^+ y_t = G_t + C_t^d + I_t^d + \left[\omega_x \left(p_t^{m,x}\right)^{1-\eta_x} + (1-\omega_x)\right]^{\frac{\eta_x}{1-\eta_x}} (1-\omega_x) (\mathring{p}_t^x)^{\frac{-\lambda_x}{\lambda_x-1}} (p_t^x)^{-\eta_f} Y_t^*$$

or, after scaling by z_t^+ and using (C.10)

$$y_{t} = g_{t} + (1 - \omega_{c})(p_{t}^{c})^{\eta_{c}}c_{t} + (p_{t}^{i})^{\eta_{i}} \left(i_{t} + a(u_{t})\frac{\bar{k}_{t}}{\mu_{\psi,t}\mu_{z^{+},t}}\right)(1 - \omega_{i}) \\ + \left[\omega_{x}(p_{t}^{m,x})^{1 - \eta_{x}} + (1 - \omega_{x})\right]^{\frac{\eta_{x}}{1 - \eta_{x}}}(1 - \omega_{x})(\dot{p}_{t}^{x})^{\frac{-\lambda_{x}}{\lambda_{x^{-1}}}}(p_{t}^{x})^{-\eta_{x}}y_{t}^{*}.$$
(C.50)

When GDP is matched to the data, capital utilization costs are subtracted from y_t (see Appendix D):

$$gdp_t = y_t - (p_t^i)^{\eta_i} \left(a(u_t) \frac{\bar{k}_t}{\mu_{\psi,t}\mu_{z^+,t}} \right) (1 - \omega_i)$$

Trade balance. Expenses on imports and new purchases of net foreign assets, A_{t+1}^* , must equal income from exports and from previously purchased net foreign assets:

 $S_t A_{t+1}^* + expenses \ on \ imports_t = receipts \ from \ exports_t + R_{t-1}^* \Phi_{t-1} S_t A_t^*.$

Expenses on imports correspond to the purchases of specialized importers for the consumption, investment and export sectors:²⁷

$$expenses on imports_t = S_t P_t^* R_t^{\nu,*} \left(C_t^m (\mathring{p}_t^{m,c})^{\frac{\lambda_{m,c}}{1-\lambda_{m,c}}} + I_t^m (\mathring{p}_t^{m,i})^{\frac{\lambda_{m,i}}{1-\lambda_{m,i}}} + X_t^m (\mathring{p}_t^{m,x})^{\frac{\lambda_{m,x}}{1-\lambda_{m,x}}} \right)$$

²⁵The assumption about $\varepsilon_{R^*,t}$ corresponds to one strategy for identifying a monetary policy shock, in which it is assumed that inflation and output are predetermined relative to the monetary policy shock.

²⁶This reflects the assumption that the impact of technology shocks on Y_t^* is completely taken into account by z_t^+ , while other shocks to Y_t^* are orthogonal to z_t^+ and they affect Y_t^* via y_t^* .

²⁷Note the presence of the price distortion terms here. To understand these terms, recall that, e.g., C_t^m is produced as a linear homogeneous function of specialized imported goods. Because the specialized importers only buy foreign goods, it is their total expenditures that interests us here. When the imports are distributed evenly across differentiated goods, then the total quantity of those imports is C_t^m , and the value of imports associated with domestic production of consumption goods is $S_t P_t^* R_t^{\nu,*} C_t^m$. When there are price distortion among imported intermediate goods then the sum of the homogeneous import goods is higher for any given value of C_t^m .

The current account can be written as follows in scaled form, using (C.22):

$$a_{t} + q_{t} p_{t}^{c} R_{t}^{\nu,*} \left(c_{t}^{m} (\mathring{p}_{t}^{m,c})^{\frac{\lambda_{m,c}}{1-\lambda_{m,c}}} + i_{t}^{m} (\mathring{p}_{t}^{m,i})^{\frac{\lambda_{m,i}}{1-\lambda_{m,i}}} + x_{t}^{m} (\mathring{p}_{t}^{m,x})^{\frac{\lambda_{m,x}}{1-\lambda_{m,x}}} \right) \\ = q_{t} p_{t}^{c} p_{t}^{x} x_{t} + R_{t-1}^{*} \Phi_{t-1} s_{t} \frac{a_{t-1}}{\pi_{t} \mu_{z^{+},t}}, \tag{C.51}$$

where $a_t = S_t A_{t+1}^* / (P_t z_t^+)$.

This completes the description of the baseline model. Additional equilibrium conditions and the complete list of endogenous variables are in the Appendix D.

C.2 Financial frictions in the model

C.2.1 Overview of the financial frictions model

A number of the activities in the baseline model require financing. Producers of specialized inputs must borrow working capital within the period. The management of capital involves financing because the construction of capital requires a substantial initial outlay of resources, while the return from capital comes in over time as a flow. In the baseline model financing requirements affect the allocations, but not very much. This is because none of the messy realities of actual financial markets are present. There is no asymmetric information between borrower and lender, there is no risk to lenders. In the case of capital accumulation, the borrower and lender are actually the same household who puts up finances and later reaps the rewards. When real-world financial frictions are introduced into the model, then intermediation becomes distorted by the presence of balance sheet constraints and other factors.

This subsection assumes that the accumulation and management of capital involves frictions following Bernanke, Gertler and Gilchrist (1999) (henceforth, BGG). It is assumed that working capital loans are frictionless.

Recall that households deposit money with banks, and that the interest rate that households receive is nominally non state-contingent. This gives rise to potentially interesting wealth effects of the sort emphasized by Fisher (1933). The banks then lend funds to entrepreneurs using a standard nominal debt contract which is optimal given the asymmetric information. The amount that banks are willing to lend to an entrepreneur under the standard debt contract is a function of the entrepreneurial net worth. This is how balance sheet constraints enter the model. When a shock occurs that reduces the value of the entrepreneurial assets, this cuts into their ability to borrow. As a result, they acquire less capital and this translates into a reduction in investment and ultimately into a slowdown in the economy.

Although individual entrepreneurs are risky, banks themselves are not. It is supposed that banks lend to a sufficiently diverse group of entrepreneurs that the uncertainty that exists in individual entrepreneurial loans washes out across all loans. The net worth of entrepreneurs is empirically measured by using a stock market index.

Entrepreneurs all have different histories, as they experience different idiosyncratic shocks. Thus, in general, solving for the aggregate variables would require also solving for the distribution of entrepreneurs according to their characteristics and for the law of motion for that distribution. However, as emphasized in BGG, the right functional form assumption have been made in the model to guarantee the result that the aggregate variables associated with entrepreneurs are not a function of distributions. The loan contract specifies that all entrepreneurs, regardless of their net worth, receive the same interest rate. Also, the loan amount received by an entrepreneur is proportional to his level of net worth. These characteristics are enough to guarantee the aggregation result. The financial frictions bring a net increase of two equations over the equations in the baseline model. In addition, they introduce two new endogenous variables, one related to the interest rate paid by entrepreneurs and the other to their net worth. The financial frictions also allow to introduce two new shocks. A formal discussion of the model follows.

C.2.2 The individual entrepreneur

At the end of period t each entrepreneur has a level of net worth, N_{t+1} . The entrepreneur's net worth, N_{t+1} , constitutes his state at this time, and nothing else about his history is relevant. There are many

entrepreneurs for each level of net worth and for each level of net worth there is a competitive bank with free entry that offers a loan contract. The contract is defined by a loan amount and by an interest rate, both of which are derived as the solution to a particular optimization problem.

Consider a type of entrepreneur with particular level of net worth, N_{t+1} . The entrepreneur combines this net worth with a bank loan, B_{t+1} , to purchase new installed physical capital, \bar{K}_{t+1} , from capital producers. The loan the entrepreneur requires for this is

$$B_{t+1} = P_t P_{k',t} \bar{K}_{t+1} - N_{t+1}. \tag{C.52}$$

The entrepreneur is required to pay a gross interest rate, Z_{t+1} , on the bank loan at the end of period t+1, if it is feasible to do so. After purchasing capital, the entrepreneur experiences an idiosyncratic productivity shock which converts the purchased capital, \bar{K}_{t+1} , into $\bar{K}_{t+1}\omega$, where ω is a unit mean, log-normally and independently distributed random variable across entrepreneurs with $V(\log \omega) = \sigma_t^2$. The t subscript indicates that σ_t is itself the realization of a random variable. This allows to consider the effects of an increase in the riskiness of individual entrepreneurs and we call σ_t the shock to idiosyncratic uncertainty. Denote the cumulative distribution function of ω by $F(\omega; \sigma)$ and its partial derivatives by $F_{\omega}(\omega; \sigma)$.

After observing the period t + 1 shocks, the entrepreneur sets the utilization rate, u_{t+1} , of capital and rents out capital in competitive markets at the nominal rental rate, $P_{t+1}r_{t+1}^k$. In choosing the capital utilization rate, the entrepreneur takes into account that operating one unit of physical capital at rate u_{t+1} requires $a(u_{t+1})$ of domestically produced investment goods for maintenance expenditures, where a is defined in Appendix D. The entrepreneur then sells the undepreciated part of physical capital to capital producers. Per unit of physical capital purchased, the entrepreneur who draws idiosyncratic productivity ω earns a return (after taxes) of $R_{t+1}^k \omega$, where R_{t+1}^k is defined in (C.40). Because the mean of ω across entrepreneurs is unity, the average return across all entrepreneurs is R_{t+1}^k .

After entrepreneurs sell their capital, they settle their bank loans. At this point the resources available to an entrepreneur who has purchased \bar{K}_{t+1} units of physical capital in period t and who experiences an idiosyncratic productivity shock ω are $P_t P_{k',t} R_{t+1}^k \omega \bar{K}_{t+1}$. There is a cutoff value of ω , $\bar{\omega}_{t+1}$, such that the entrepreneur has just enough resources to pay interest:

$$\bar{\omega}_{t+1}R_{t+1}^k P_t P_{k',t}\bar{K}_{t+1} = Z_{t+1}B_{t+1}.$$
(C.53)

Entrepreneurs with $\omega < \bar{\omega}_{t+1}$ are bankrupt and turn over all their resources,

$$R_{t+1}^k \omega P_t P_{k',t} \bar{K}_{t+1},$$

which is less than $Z_{t+1}B_{t+1}$, to the bank. In this case, the bank monitors the entrepreneur at the cost

$$\mu R_{t+1}^k \omega P_t P_{k',t} \bar{K}_{t+1},$$

where $\mu \geq 0$ is a parameter.

Banks obtain the funds loaned in the period t to entrepreneurs by issuing deposits to households at gross nominal rate of interest, R_t . The subscript on R_t indicates that the payoff to households in t + 1 is not contingent on the period t + 1 uncertainty. There is no risk in household bank deposits, and the household Euler equation associated with deposits is exactly the same as in (C.45).

There is competition and free entry among banks and banks participate in no financial arrangements other than the liabilities issued to households and the loans issued to entrepreneurs. It follows that the bank's cash flow in each state of period t + 1 is zero for each loan amount.²⁸ For loans in the amount B_{t+1} , the bank receives gross interest $Z_{t+1}B_{t+1}$ from the fraction $1 - F(\bar{\omega}_{t+1}; \sigma_t)$ of entrepreneurs who are not bankrupt. The bank takes all the resources possessed by bankrupt entrepreneurs, net of monitoring costs. Thus, the state-by-state zero profit condition is

$$[1 - F(\bar{\omega}_{t+1}; \sigma_t)]Z_{t+1}B_{t+1} + (1 - \mu)\int_0^{\bar{\omega}_{t+1}} \omega dF(\omega; \sigma_t)R_{t+1}^k P_t P_{k', t}\bar{K}_{t+1} = R_t B_{t+1}$$

²⁸Absence of state contingent securities markets guarantee that cash flow is non-negative. Free entry guarantees that ex ante profits are zero. Given that each state of nature receives positive probability, the two assumptions imply the state-by-state zero profit condition.

or, after making use of (C.53) and rearranging,

$$[\Gamma(\bar{\omega}_{t+1};\sigma_t) - \mu G(\bar{\omega}_{t+1};\sigma_t)] \frac{R_{t+1}^k}{R_t} \varrho_t = \varrho_t - 1, \qquad (C.54)$$

where

$$\begin{split} G(\bar{\omega}_{t+1};\sigma_t) &= \int_0^{\bar{\omega}_{t+1}} \omega dF(\omega;\sigma_t) \\ \Gamma(\bar{\omega}_{t+1};\sigma_t) &= \bar{\omega}_{t+1}[1 - F(\bar{\omega}_{t+1};\sigma_t)] + G(\bar{\omega}_{t+1};\sigma_t) \\ \varrho_t &= \frac{P_t P_{k',t} \bar{K}_{t+1}}{N_{t+1}}. \end{split}$$

The expression $\Gamma(\bar{\omega}_{t+1}; \sigma_t) - \mu G(\bar{\omega}_{t+1}; \sigma_t)$ is the share of revenues earned by entrepreneurs that borrow B_{t+1} which goes to banks. Note that $\Gamma_{\bar{\omega}}(\bar{\omega}_{t+1}; \sigma_t) = 1 - F(\bar{\omega}_{t+1}; \sigma_t) > 0$ and $G_{\bar{\omega}}(\bar{\omega}_{t+1}; \sigma_t) = \bar{\omega}_{t+1}F_{\bar{\omega}}(\bar{\omega}_{t+1}; \sigma_t) > 0$. Therefore, the share of entrepreneurial revenues accruing to banks is nonmonotone with respect to $\bar{\omega}_{t+1}$.²⁹

The optimal contract is derived in Appendix D. ρ_t and $\bar{\omega}_{t+1}$ are the same for all entrepreneurs regardless of their net worth. This result of the leverage ratio, ρ_t , implies that

$$\frac{B_{t+1}}{N_{t+1}} = \varrho_t - 1,$$

i.e., that an entrepreneur's loan amount is proportional to his net worth. Rewriting (C.52) and (C.53), the rate of interest paid by the entrepreneur is

$$Z_{t+1} = \frac{\bar{\omega}_{t+1} R_{t+1}^k}{1 - \frac{N_{t+1}}{P_t P_{k'} + \bar{K}_{t+1}}} = \frac{\bar{\omega}_{t+1} R_{t+1}^k}{1 - \frac{1}{\varrho_t}},\tag{C.55}$$

which also is the same for all entrepreneurs regardless of their net worth.

C.2.3 Aggregation across entrepreneurs and the external financing premium

The law of motion for the net worth on an individual entrepreneur is

$$V_t = R_t^k P_{t-1} P_{k',t-1} K_t - \Gamma(\bar{\omega}_t; \sigma_{t-1}) R_t^k P_{t-1} P_{k',t-1} K_t.$$

Each entrepreneur faces an identical and independent probability $1 - \gamma_t$ of being selected to exit the economy. With the probability γ_t each entrepreneur remains. Because the selection is random, the net worth of the entrepreneurs who survive is $\gamma_t \bar{V}_t$. A fraction $1 - \gamma_t$ of new entrepreneurs arrive. Entrepreneurs who survive or who are new arrivals receive a transfer W_t^e . This ensures that all entrepreneurs, whether new arrivals or survivors that experienced bankruptcy, have sufficient funds to obtain at least some amount of loans. The average net worth across all entrepreneurs after the W_t^e transfers have been made and exits and entry have occurred, is $\bar{N}_{t+1} = \gamma_t \bar{V}_t + W_t^e$, or

²⁹BGG argue that the expression on the left of (C.54) has an inverted 'U' shape, achieving a maximum value at $\bar{\omega}_{t+1} = \omega^*$. The expression is increasing for $\bar{\omega}_{t+1} < \omega^*$ and decreasing for $\bar{\omega}_{t+1} > \omega^*$. Thus, for any given value of the leverage ratio, ρ_t , and R_{t+1}^k/R_t , there are either no values of $\bar{\omega}_{t+1}$ or two that satisfy (C.54). The value of $\bar{\omega}_{t+1}$ realized in equilibrium must be the one on the left side of inverted 'U' shape. This is because, according to (C.53), the lower value of $\bar{\omega}_{t+1}$ corresponds to a lower interest rate for entrepreneurs which yields them higher welfare. The equilibrium contract is the one that maximizes entrepreneurial welfare subject to the zero profit condition on banks. This reasoning leads to the conclusion that $\bar{\omega}_{t+1}$ falls with a period t + 1 shock that drives R_{t+1}^k up. The fraction of entrepreneurs that experience bankruptcy is $F(\bar{\omega}_{t+1}; \sigma_t)$, so it follows that a shock which drives up R_{t+1}^k has a negative contemporaneous impact on the bankruptcy rate. According to (C.40), shocks that drive R_{t+1}^k up include anything which raises the value of physical capital and/or the rental rate of capital.

$$\bar{N}_{t+1} = \gamma_t \left\{ R_t^k P_{t-1} P_{k',t-1} \bar{K}_t - \left[R_{t-1} + \frac{\mu \int_0^{\bar{\omega}_t} \omega dF(\omega; \sigma_{t-1}) R_t^k P_{t-1} P_{k',t-1} \bar{K}_t}{P_{t-1} P_{k',t-1} \bar{K}_t - \bar{N}_t} \right] \times (P_{t-1} P_{k',t-1} \bar{K}_t - \bar{N}_t) \right\} + W_t^e.$$
(C.56)

where upper bar over a letter denotes its aggregate average value. Because of its direct effect on entrepreneurial net worth, γ_t is referred to as the shock to net worth. For a derivation of the aggregation across entrepreneurs, see Appendix D.

We now turn to the external financing premium for entrepreneurs. The cost to the entrepreneur of internal funds (i.e., his own net worth) is the interest rate R_t which he loses by applying it to capital rather than buying a risk-free domestic asset. The average payment by all entrepreneurs to the bank is the entire object in square brackets in (C.56). So, the term involving μ represents the excess of external funds over the internal cost of funds. As a result, this is one measure of the financing premium in the model. Another is $Z_{t+1} - R_t$, the excess of the interest rate paid by entrepreneurs who are not bankrupt over the risk-free rate. This paper calls this the interest rate spread.

Appendix D Model details - not for publication

D.1 Scaling of variables

We adopt the following scaling of variables. The neutral shock to technology is z_t and its growth rate is $\mu_{z,t}$:

$$\frac{z_t}{z_{t-1}} = \mu_{z,t}.$$

The variable Ψ_t is an investment-specific shock to technology and it is convenient to define the following combination of investment-specific and neutral technology:

$$z_t^+ = \Psi_t^{\frac{\alpha}{1-\alpha}} z_t,$$

$$\mu_{z^+,t} = \mu_{\Psi,t}^{\frac{\alpha}{1-\alpha}} \mu_{z,t}.$$
(D.1)

Capital, \bar{K}_t , and investment, I_t , are scaled by $z_t^+ \Psi_t$. Foreign and domestic inputs into the production of I_t (we denote these by I_t^d and I_t^m , respectively) are scaled by z_t^+ . Consumption goods (C_t^m are imported intermediate consumption goods, C_t^d are domestically produced intermediate consumption goods, and C_t are final consumption goods) are scaled by z_t^+ . Government expenditure, the real wage and real foreign assets are scaled by z_t^+ . Exports (X_t^m are imported intermediate goods for use in producing exports and X_t are final export goods) are scaled by z_t^+ . Also, v_t is the shadow value in utility terms to the household of domestic currency and $v_t P_t$ is the shadow value of one unit of the homogeneous domestic good. The latter must be multiplied by z_t^+ to induce stationarity. \tilde{P}_t is the within-sector relative price of a good. Thus,

$$\begin{aligned} k_{t+1} &= \frac{K_{t+1}}{z_t^+ \Psi_t}, \, \bar{k}_{t+1} = \frac{\bar{k}_{t+1}}{z_t^+ \Psi_t}, \, i_t^d = \frac{I_t^d}{z_t^+}, \, i_t = \frac{I_t}{z_t^+ \Psi_t}, \, i_m^t = \frac{I_t^m}{z_t^+}, \\ c_t^m &= \frac{C_t^m}{z_t^+}, \, c_t^d = \frac{C_t^d}{z_t^+}, \, c_t = \frac{C_t}{z_t^+}, \, g_t = \frac{G_t}{z_t^+}, \, \bar{w}_t = \frac{W_t}{z_t^+ P_t}, \, a_t := \frac{S_t A_{t+1}}{z_t^+ P_t} \\ x_t^m &= \frac{X_t^m}{z_t^+}, \, x_t = \frac{X_t}{z_t^+}, \, \psi_{z^+,t} = v_t P_t z_t^+, \, (y_t =) \tilde{y}_t = \frac{Y_t}{z_t^+}, \, \tilde{p}_t = \frac{\tilde{P}_t}{P_t}, \\ n_{t+1} &= \frac{\bar{N}_{t+1}}{z_t^+ P_t}, \, w^e = \frac{W_t^e}{z_t^+ P_t}. \end{aligned}$$

We define the scaled date t price of new installed physical capital for the start of period t+1 as $p_{k',t}$ and we define the scaled real rental rate of capital as \bar{r}_t^k :

$$p_{k',t} = \Psi_t P_{k',t}, \ \bar{r}_t^k = \Psi_t r_t^k,$$

where $P_{k',t}$ is in units of the domestic homogeneous good. The nominal exchange rate is denoted by S_t and its growth rate is s_t :

$$s_t = \frac{S_t}{S_{t-1}}.$$

We define the following inflation rates:

$$\begin{aligned} \pi_t &= \frac{P_t}{P_{t-1}}, \, \pi_t^c = \frac{P_t^c}{P_{t-1}^c}, \, \pi_t^* = \frac{P_t^*}{P_{t-1}^*}, \\ \pi_t^i &= \frac{P_t^i}{P_{t-1}^i}, \, \pi_t^x = \frac{P_t^x}{P_{t-1}^x}, \, \pi_t^{m,j} = \frac{P_t^{m,j}}{P_{t-1}^{m,j}}. \end{aligned}$$

for j = c, x, i. Here, P_t is the price of a domestic homogeneous output good, P_t^c is the price of the domestic final consumption goods (i.e., the 'consumer price index'), P_t^* is the price of a foreign homogeneous good, P_t^i is the price of the domestic final investment good and P_t^x is the price (in foreign currency units) of a final export good.

With one exception, we define a lower case price as the corresponding uppercase price divided by the price of the homogeneous good. When the price is denominated in domestic currency units, we divide by the price of the domestic homogeneous good, P_t . When the price is denominated in foreign currency units, we divide by P_t^* , the price of the foreign homogeneous good. The exceptional case has to do with handling of the price of investment goods, P_t^i . This grows at a rate potentially slower than P_t , and we therefore scale it by P_t/Ψ_t . Thus,

$$p_t^{m,x} = \frac{P_t^{m,x}}{P_t}, \ p_t^{m,c} = \frac{P_t^{m,c}}{P_t}, \ p_t^{m,i} = \frac{P_t^{m,i}}{P_t},$$

$$p_t^x = \frac{P_t^x}{P_t^x}, \ p_t^c = \frac{P_t^c}{P_t}, \ p_t^i = \frac{\Psi_t P_t^i}{P_t}.$$
(D.2)

Here, m, j means the price of an imported good which is subsequently used in the production of exports in the case j = x, in the production of the final consumption good in the case of j = c and in the production of final investment good in the case of j = i. When there is just a single superscript the underlying good is a final good, with j = x, c, i corresponding to exports, consumption and investment, respectively.

D.2 Functional forms

We adopt the following functional form for capital utilization, a:

$$a(u) = 0.5\sigma_b \sigma_a u^2 + \sigma_b (1 - \sigma_a) u + \sigma_b ((\sigma_a/2) - 1),$$
(D.3)

where σ_a and σ_b are the parameters of this function.

The functional form for investment adjustment costs as well as its derivatives are

$$\tilde{S}(x) = \frac{1}{2} \left\{ \exp\left[\sqrt{\tilde{S}''}(x - \mu_{z^{+}} \mu_{\Psi})\right] + \exp\left[-\sqrt{\tilde{S}''}(x - \mu_{z^{+}} \mu_{\Psi})\right] - 2 \right\}$$

= 0, $x = \mu_{z^{+}} \mu_{\Psi}$, (D.4)

$$\tilde{S}'(x) = \frac{1}{2} \sqrt{\tilde{S}''} \left\{ \exp\left[\sqrt{\tilde{S}''}(x - \mu_{z} + \mu_{\Psi})\right] - \exp\left[-\sqrt{\tilde{S}''}(x - \mu_{z} + \mu_{\Psi})\right] \right\}$$

= 0, $x = \mu_{z} + \mu_{\Psi}$, (D.5)
$$\tilde{S}''(x) = \frac{1}{2} \tilde{S}'' \left\{ \exp\left[\sqrt{\tilde{S}''}(x - \mu_{z} + \mu_{\Psi})\right] + \exp\left[-\sqrt{\tilde{S}''}(x - \mu_{z} + \mu_{\Psi})\right] \right\}$$

= $\tilde{S}'', x = \mu_{z} + \mu_{\Psi}$.

D.3 Baseline model

D.3.1 First order conditions for domestic homogeneous goods price setting Substituting (C.7) into (C.6) and rearranging,

$$E_t \sum_{j=0}^{\infty} \beta^j v_{t+j} P_{t+j} Y_{t+j} \left\{ \left(\frac{P_{i,t+j}}{P_{t+j}} \right)^{1-\frac{\lambda_d}{\lambda_d-1}} - mc_{t+j} \left(\frac{P_{i,t+j}}{P_{t+j}} \right)^{\frac{-\lambda_d}{\lambda_d-1}} \right\},$$

or,

$$E_{t}\sum_{j=0}^{\infty}\beta^{j}v_{t+j}P_{t+j}Y_{t+j}\left\{\left(X_{t,j}\tilde{p}_{t}\right)^{1-\frac{\lambda_{d}}{\lambda_{d}-1}}-mc_{t+j}\left(X_{t,j}\tilde{p}_{t}\right)^{\frac{-\lambda_{d}}{\lambda_{d}-1}}\right\},$$

where

$$\frac{P_{i,t+j}}{P_{t+j}} = X_{t,j}\tilde{p}_t, \ X_{t,j} := \begin{cases} \frac{\tilde{\pi}_{d,t+j}\cdots\tilde{\pi}_{d,t+1}}{\pi_{t+j}\cdots\pi_{t+1}}, & j > 0\\ 1, & j = 0 \end{cases}$$

The *i*-th firm maximizes profits by choice of the within-sector relative price \tilde{p}_t . The fact that this variable does not have an index *i* reflects that all firms that have the opportunity to reoptimize in period *t* solve the same problem, and hence have the same solution. Differentiating its profit function, multiplying the result by $\tilde{p}_t^{\frac{\lambda_d}{\lambda_d-1}+1}$, rearranging, and scaling, yields

$$E_t \sum_{j=0}^{\infty} (\beta \xi_d)^j A_{t+j} [\tilde{p}_t X_{t,j} - \lambda_d m c_{t+j}] = 0,$$

where A_{t+j} is exogenous from the point of view of the firm:

$$A_{t+j} = \psi_{z^+, t+j} \tilde{y}_{t+j} X_{t,j}.$$

After rearranging the optimizing intermediate good firm's first order condition for prices, yields

$$\tilde{p}_{t}^{d} = \frac{E_{t} \sum_{j=0}^{\infty} (\beta \xi_{d})^{j} A_{t+j} \lambda_{d} m c_{t+j}}{E_{t} \sum_{j=0}^{\infty} (\beta \xi_{d})^{j} A_{t+j} X_{t,j}} = \frac{K_{t}^{d}}{F_{t}^{d}},$$

where

$$K_t^d := E_t \sum_{j=0}^{\infty} (\beta \xi_d)^j A_{t+j} \lambda_d m c_{t+j}$$
$$F_t^d := E_t \sum_{j=0}^{\infty} (\beta \xi_d)^j A_{t+j} X_{t,j}.$$

These objects have the following convenient recursive representations

$$E_t \left[\psi_{z^+,t} \tilde{y}_t + \left(\frac{\tilde{\pi}_{d,t+1}}{\pi_{t+1}} \right)^{\frac{1}{1-\lambda_d}} \beta \xi_d F_{t+1}^d - F_t^d \right] = 0$$
$$E_t \left[\lambda_d \psi_{z^+,t} \tilde{y}_t m c_t + \beta \xi_d \left(\frac{\tilde{\pi}_{d,t+1}}{\pi_{t+1}} \right)^{\frac{\lambda_d}{1-\lambda_d}} K_{t+1}^d - K_t^d \right] = 0.$$

Turning to the aggregate price index:

$$P_{t} = \left[\int_{0}^{1} P_{it}^{\frac{1}{1-\lambda_{d}}} di \right]^{1-\lambda_{d}}$$
$$= \left[(1-\xi_{p}) \tilde{P}_{t}^{\frac{1}{1-\lambda_{d}}} + \xi_{p} (\tilde{\pi}_{d,t} P_{t-1})^{\frac{1}{1-\lambda_{d}}} \right]^{1-\lambda_{d}}$$
(D.6)

After dividing by P_t and rearranging

$$\frac{1 - \xi_d \left(\frac{\tilde{\pi}_{d,t}}{\pi_t}\right)^{\frac{1}{1-\lambda_d}}}{1 - \xi_d} = (\tilde{p}_t^d)^{\frac{1}{1-\lambda_d}}.$$
 (D.7)

In sum, the equilibrium conditions associated with price setting for producers of the domestic homogeneous good ${\rm are}^{30}$

$$E_t \left[\psi_{z^+,t} y_t + \left(\frac{\tilde{\pi}_{d,t+1}}{\pi_{t+1}} \right)^{\frac{1}{1-\lambda_d}} \beta \xi_d F_{t+1}^d - F_t^d \right] = 0$$
 (D.8)

$$E_t \left[\lambda_d \psi_{z^+,t} y_t m c_t + \beta \xi_d \left(\frac{\tilde{\pi}_{d,t+1}}{\pi_{t+1}} \right)^{\frac{\lambda_d}{1-\lambda_d}} K_{t+1}^d - K_t^d \right] = 0$$
(D.9)

$$\mathring{p}_{t} = \left[(1 - \xi_{d}) \left(\frac{1 - \xi_{d} \left(\frac{\tilde{\pi}_{d,t}}{\pi_{t}} \right)^{\frac{1}{1 - \lambda_{d}}}}{1 - \xi_{d}} \right)^{\lambda_{d}} + \xi_{d} \left(\frac{\tilde{\pi}_{d,t}}{\pi_{t}} \mathring{p}_{t-1} \right)^{\frac{\lambda_{d}}{1 - \lambda_{d}}} \right]^{\frac{1 - \lambda_{d}}{\lambda_{d}}}$$
(D.10)

$$\left[\frac{1-\xi_d \left(\frac{\tilde{\pi}_{d,t}}{\pi_t}\right)^{\frac{1}{1-\lambda_d}}}{1-\xi_d}\right]^{1-\lambda_d} = \frac{K_t^d}{F_t^d}$$
(D.11)

$$\tilde{\pi}_{d,t} := (\pi_{t-1})^{\kappa_d} (\bar{\pi}_t^c)^{1-\kappa_d - \varkappa_d} (\check{\pi})^{\varkappa_d}$$
(D.12)

³⁰After linearizing about the steady state and setting $\varkappa_d = 0$,

$$\begin{split} \hat{\pi} - \hat{\pi}_t^c = & \frac{\beta}{1 + \kappa_d \beta} E_t(\hat{\pi}_{t+1} - \hat{\pi}_{t+1}^c) + \frac{\kappa_d}{1 + \kappa_d \beta} (\hat{\pi}_{t-1} - \hat{\pi}_t^c) \\ & - \frac{\kappa_d \beta (1 - \rho_\pi)}{1 + \kappa_d \beta} \hat{\pi}_t^c \\ & + \frac{1}{1 + \kappa_d \beta} \frac{(1 - \beta \xi_d) (1 - \xi_d)}{\xi_d} \widehat{mc}_t, \end{split}$$

where a hat indicates log-deviation from steady state.

D.3.2 Export demand

Scaling (C.17), yields

$$x_t = (p_t^x)^{-\eta_f} y_t^*$$
 (D.13)

D.3.3 FOCs for export goods price setting

$$E_t \left[\psi_{z^+,t} q_t p_t^c p_t^x x_t + \left(\frac{\tilde{\pi}_{t+1}^x}{\pi_{t+1}^x} \right)^{\frac{1}{1-\lambda_x}} \beta \xi_x F_{x,t+1} - F_{x,t} \right] = 0$$
 (D.14)

$$E_t \left[\lambda_x \psi_{z^+,t} q_t p_t^c p_t^x x_t m c_t^x + \beta \xi_x \left(\frac{\tilde{\pi}_{t+1}^x}{\pi_{t+1}^x} \right)^{\frac{\lambda_x}{1-\lambda_x}} K_{x,t+1} - K_{x,t} \right] = 0$$
(D.15)

$$\mathring{p}_{t}^{x} = \left[\left(1 - \xi_{x}\right) \left(\frac{1 - \xi_{x}\left(\frac{\tilde{\pi}_{t}^{x}}{\pi_{t}^{x}}\right)\frac{1}{1 - \lambda_{x}}}{1 - \xi_{x}}\right)^{\lambda_{x}} + \xi_{x}\left(\frac{\tilde{\pi}_{t}^{x}}{\pi_{t}^{x}}\mathring{p}_{t-1}^{x}\right)^{\frac{\lambda_{x}}{1 - \lambda_{x}}} \right]^{\frac{1 - \lambda_{x}}{\lambda_{x}}}$$
(D.16)

$$\left[\frac{1-\xi_x \left(\frac{\tilde{\pi}_t^x}{\pi_t^x}\right)^{\frac{1}{1-\lambda_x}}}{1-\xi_x}\right]^{1-\lambda_x} = \frac{K_{x,t}}{F_{x,t}}$$
(D.17)

When linearized around steady state and $\varkappa_{m,j} = 0$, eq. (D.14)-(D.17) reduce to

$$\hat{\pi}_t^x = \frac{\beta}{1 + \kappa_x \beta} E_t \hat{\pi}_{t+1}^x + \frac{\kappa_x}{1 + \kappa_x \beta} \hat{\pi}_{t-1}^x \\ + \frac{1}{1 + \kappa_x \beta} \frac{(1 - \beta \xi_x)(1 - \xi_x)}{\xi_x} \widehat{mc}_t^x$$

where a hat over a variable indicates log-deviation from steady state.

D.3.4 Demand for domestic inputs in export production Integrating (C.24),

$$\int_0^1 X_{i,t}^d di = \left(\frac{\lambda}{\tau_t^x R_t^x P_t}\right)^{\eta_x} (1 - \omega_x) \int_0^1 X_{i,t} di$$
$$= \left(\frac{\lambda}{\tau_t^x R_t^x P_t}\right)^{\eta_x} (1 - \omega_x) X_t \frac{\int_0^1 (P_{i,t}^x)^{\frac{-\lambda_x}{\lambda_x - 1}} di}{(P_t^x)^{\frac{-\lambda_x}{\lambda_x - 1}}}$$
(D.18)

Define $\mathring{P}^x_t,$ a linear homogeneous function of $P^x_{i,t} {:}$

$$\mathring{P}_t^x = \left[\int_0^1 (P_{i,t}^x)^{\frac{-\lambda_x}{\lambda_x - 1}} di\right]^{\frac{\lambda_x - 1}{-\lambda_x}}.$$

Then

$$\left(\mathring{P}_{t}^{x}\right)^{\frac{-\lambda_{x}}{\lambda_{x}-1}} = \int_{0}^{1} (P_{i,t}^{x})^{\frac{-\lambda_{x}}{\lambda_{x}-1}} di$$

and

$$\int_0^1 X_{i,t}^d di = \left(\frac{\lambda}{\tau_t^x R_t^x P_t}\right)^{\eta_x} (1 - \omega_x) X_t (\mathring{p}_t^x)^{\frac{-\lambda_x}{\lambda_x - 1}} \tag{D.19}$$

where

$$\mathring{p}_t^x := \frac{\mathring{P}_t^x}{P_t^x}$$

and the law of motion of
$$\mathring{p}_t^x$$
 is given in (D.16).

We now simplify (D.19). Rewriting the second equality in (C.20), yields

$$\frac{\lambda}{P_t \tau_t^x R_t^x} = \frac{S_t P_t^x}{P_t q_t p_t^x p_t^x} \left[\omega_x (p_t^{m,x})^{1-\eta_x} + (1-\omega_x) \right]^{\frac{1}{1-\eta_x}}$$

or

$$\frac{\lambda}{P_t \tau_t^x R_t^x} = \frac{S_t P_t^x}{P_t \frac{S_t P_t^*}{P_t^c} \frac{P_t^c}{P_t^*} \frac{P_t^x}{P_t^*}} \left[\omega_x (p_t^{m,x})^{1-\eta_x} + (1-\omega_x) \right]^{\frac{1}{1-\eta_x}}$$

or

$$\frac{\lambda}{P_t \tau_t^x R_t^x} = \left[\omega_x (p_t^{m,x})^{1-\eta_x} + (1-\omega_x) \right]^{\frac{1}{1-\eta_x}}.$$

Substituting into (D.19),

$$X_t^d = \int_0^1 X_{i,t}^d di = \left[\omega_x (p_t^{m,x})^{1-\eta_x} + (1-\omega_x)\right]^{\frac{\eta_x}{1-\eta_x}} (1-\omega_x) (\mathring{p}_t^x)^{\frac{-\lambda_x}{\lambda_x-1}} (p_t^x)^{-\eta_x} Y_t^*$$

D.3.5 Demand for imported inputs in export production

Scaling (C.26), yields

$$x_t^m = \omega_x \left(\frac{\left[\omega_x (p_t^{m,x})^{1-\eta_x} + (1-\omega_x) \right]^{\frac{1}{1-\eta_x}}}{p_t^{m,x}} \right)^{\eta_x} (\dot{p}_t^x)^{\frac{-\lambda_x}{\lambda_x - 1}} (p_t^x)^{-\eta_f} y_t^*$$
(D.20)

D.3.6 Value of imports of the intermediate consumption goods producers

It is of interest to have a measure of the value of imports of the intermediate consumption good producers:

$$S_t P_t^* R_t^{\nu,*} \int_0^1 C_{i,t}^m di.$$

In order to relate this to C_t^m , substitute the demand curve into the previous expression:

$$\begin{split} S_t P_t^* R_t^{\nu,*} \int_0^1 C_t^m \left(\frac{P_t^{m,c}}{P_{i,t}^{m,c}}\right)^{\frac{\lambda_{m,c}}{\lambda_{m,c}-1}} di &= S_t P_t^* R_t^{\nu,*} C_t^m (P_t^{m,c})^{\frac{\lambda_{m,c}}{\lambda_{m,c}-1}} \int_0^1 (P_{i,t}^{m,c})^{\frac{-\lambda_{m,c}}{\lambda_{m,c}-1}} di \\ &= S_t P_t^* R_t^{\nu,*} C_t^m \left(\frac{\mathring{P}_t^m}{P_t^m,c}\right)^{\frac{\lambda_{m,c}}{1-\lambda_{m,c}}}, \end{split}$$

where

$$\mathring{P}_{t}^{m,c} = \left[\int_{0}^{1} (P_{i,t}^{m,c})^{\frac{\lambda_{m,c}}{1-\lambda_{m,c}}}\right]^{\frac{1-\lambda_{m,c}}{\lambda_{m,c}}}.$$

Thus the total value of imports accounted for by the consumption sector is

$$S_t P_t^* R_t^{\nu,*} C_t^m (\mathring{p}_t^{m,c})^{\frac{\lambda_{m,c}}{1-\lambda_{m,c}}}$$
(D.21)

where

$$\mathring{p}_t^{m,c} = \frac{\mathring{P}_t^{m,c}}{P_t^{m,c}}.$$

The derivation for the value of imports used by the investment and export production sectors are analogous.

D.3.7 Marginal costs of importers

Real marginal cost is

$$mc_{t}^{m,j} = \tau_{t}^{m,j} \frac{S_{t} P_{t}^{*}}{P_{t}^{m,j}} R_{t}^{\nu,*} = \tau_{t}^{m,j} \frac{S_{t} P_{t}^{*} P_{t}^{c} P_{t}}{P_{t}^{c} P_{t}^{m,j} P_{t}} R_{t}^{\nu,*}$$
$$= \tau_{t}^{m,j} \frac{q_{t} p_{t}^{c}}{p_{t}^{m,j}} R_{t}^{\nu,*}$$
(D.22)

for j = c, i, x.

D.3.8 FOCs for import goods price setting

$$E_t \left[\psi_{z^+,t} p_t^{m,j} \Xi_t^j + \left(\frac{\tilde{\pi}_{t+1}^{m,j}}{\pi_{t+1}^{m,j}} \right)^{\frac{1}{1-\lambda_{m,j}}} \beta \xi_{m,j} F_{m,j,t+1} - F_{m,j,t} \right] = 0$$
(D.23)

$$E_t \left[\lambda_{m,j} \psi_{z^+,t} p_t^{m,j} m c_t^{m,j} \Xi_t^j + \beta \xi_{m,j} \left(\frac{\tilde{\pi}_{t+1}^{m,j}}{\pi_{t+1}^{m,j}} \right)^{\frac{\lambda_{m,j}}{1-\lambda_{m,j}}} K_{m,j,t+1} - K_{m,j,t} \right] = 0$$
(D.24)

$$\mathring{p}_{t}^{m,j} = \left[\left(1 - \xi_{m,j}\right) \left(\frac{1 - \xi_{m,j} \left(\frac{\tilde{\pi}_{t}^{m,j}}{\pi_{t}^{m,j}}\right)^{\frac{1}{1-\lambda_{m,j}}}}{1 - \xi_{m,j}} \right)^{\lambda_{m,j}} + \xi_{m,j} \left(\frac{\tilde{\pi}_{t}^{m,j}}{\pi_{t}^{m,j}} \mathring{p}_{t-1}^{m,j}\right)^{\frac{\lambda_{m,j}}{1-\lambda_{m,j}}} \right]^{\frac{1-\lambda_{m,j}}{\lambda_{m,j}}} \right]$$
(D.25)

$$\left[\frac{1-\xi_{m,j}\left(\frac{\tilde{\pi}_{t}^{m,j}}{\pi_{t}^{m,j}}\right)^{\frac{1}{1-\lambda_{m,j}}}}{1-\xi_{m,j}}\right]^{1-\lambda_{m,j}} = \frac{K_{m,j,t}}{F_{m,j,t}}$$
(D.26)

for $j = c, t, x,^{31}$ and where

$$\Xi_t^j = \begin{cases} c_t^m & j = c \\ x_t^m & j = x \\ i_t^m & j = i \end{cases}.$$

D.3.9 Wage setting conditions in baseline model

Substituting (C.37) into the objective function, (C.36),

$$E_t^j \sum_{0}^{\infty} (\beta \xi_w)^i \left[-\zeta_{t+i}^h A_L \frac{\left(\left(\frac{\tilde{W}_t \tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1}}{W_{t+i}} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \right)^{1+\sigma_L}}{1+\sigma_L} + v_{t+i} \tilde{W}_t \tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1} \left(\frac{\tilde{W}_t \tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1}}{W_{t+i}} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \frac{1-\tau^y}{1+\tau^w} \right]$$

Given the rescaled variables,

$$\frac{\tilde{W}_{t}\tilde{\pi}_{w,t+i}\cdots\tilde{\pi}_{w,t+1}}{W_{t+i}} = \frac{\tilde{W}_{t}\tilde{\pi}_{w,t+1}\cdots\tilde{\pi}_{w,t+1}}{\bar{w}_{t+i}z_{t+i}^{+}P_{t+i}} = \frac{\tilde{W}_{t}}{\bar{w}_{t+i}z_{t}^{+}P_{t}}X_{t,i}$$
$$= \frac{W_{t}\left(\tilde{W}_{t}/W_{t}\right)}{\bar{w}_{t+i}z_{t}^{+}P_{t}}X_{t,i} = \frac{\bar{w}_{t}\left(\tilde{W}_{t}/W_{t}\right)}{\bar{w}_{t+i}}X_{t,i} = \frac{w_{t}\bar{w}_{t}}{\bar{w}_{t+i}}X_{t,i},$$

where

$$X_{t,i} = \begin{cases} \frac{\tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1}}{\pi_{t+i}\pi_{t+i-1} \cdots \pi_{t+1}\mu_{z+,t+i} \cdots \mu_{z+,t+1}}, & i > 0\\ 1, & i = 0 \end{cases}$$

It is interesting to investigate the value of $X_{t,i}$ in steady state, as $i \to \infty$. Thus,

$$X_{t,i} = \frac{(\pi_t^c \cdots \pi_{t+i-1}^c)^{\kappa_w} (\bar{\pi}_{t+1}^c \cdots \bar{\pi}_{t+i}^c)^{1-\kappa_w - \varkappa_w} (\breve{\pi}^i)^{\varkappa_w} (\mu_{z+}^i)^{\vartheta_w}}{\pi_{t+i} \pi_{t+i-1} \cdots \pi_{t+1} \mu_{z+,t+i} \cdots \mu_{z+,t+1}}$$

In steady state,

$$\begin{split} X_{t,i} &= \frac{(\bar{\pi}^i)^{\kappa_w} (\bar{\pi}^i)^{1-\kappa_w -\varkappa_w} (\check{\pi}^i)^{\varkappa_w} (\mu_{z^+}^i)^{\vartheta_w}}{\bar{\pi}^i \mu_{z^+}^i} \\ &= \left(\frac{\check{\pi}^i}{\bar{\pi}^i}\right)^{\varkappa_w} (\mu_{z^+}^i)^{\vartheta_w - 1} \\ &\to 0 \end{split}$$

³¹When linearized around steady state and $\varkappa_{m,j} = 0$,

$$\begin{aligned} \hat{\pi}_{t}^{m,j} - \hat{\pi}_{t}^{c} = & \frac{\beta}{1 + \kappa_{m,j}\beta} E_{t} \left(\hat{\pi}_{t+1}^{m,j} - \hat{\pi}_{t+1}^{c} \right) + \frac{\kappa_{m,j}}{1 + \kappa_{m,j}\beta} \left(\hat{\pi}_{t-1}^{m,j} - \hat{\pi}_{t}^{c} \right) \\ &- \frac{\kappa_{m,j}\beta(1 - \rho_{\pi})}{1 + \kappa_{m,j}\beta} \hat{\pi}_{t}^{c} \\ &+ \frac{1}{1 + \kappa_{m,j}\beta} \frac{(1 - \beta\xi_{m,j})(1 - \xi_{m,j})}{\xi_{m,j}} \widehat{mc}_{t}^{m,j}. \end{aligned}$$

in the no-indexing case, when $\breve{\pi} = 1$, $\varkappa_w = 1$ and $\vartheta_w = 0$. Simplifying using the scaling notation,

$$E_t^j \sum_{i=0}^\infty (\beta \xi_w)^i \left[-\zeta_{t+i}^h A_L \frac{\left(\left(\frac{w_t \bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \right)^{1+\sigma_L}}{1+\sigma_L} + v_{t+i} W_{t+i} \frac{w_t \bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \left(\frac{w_t \bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \frac{1-\tau^y}{1+\tau^w} \right]$$

or

$$E_t^j \sum_{i=0}^{\infty} (\beta \xi_w)^i \left[-\zeta_{t+i}^h A_L \frac{\left(\left(\frac{w_t \bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \right)^{1+\sigma_L} + \sigma_L \right] \\ + \psi_{z^+,t+i} w_t \bar{w}_t X_{t,i} \left(\frac{w_t \bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \frac{1-\tau^y}{1+\tau^w} \right]$$

 or

$$E_t^j \sum_{i=0}^{\infty} (\beta \xi_w)^i \left[-\zeta_{t+i}^h A_L \frac{\left(\left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \right)^{1+\sigma_L}}{1+\sigma_L} w_t^{\frac{\lambda_w}{1-\lambda_w}(1+\sigma_L)} + \psi_{z^+,t+i} w_t^{1+\frac{\lambda_w}{1-\lambda_w}} \bar{w}_t X_{t,i} \left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \frac{1-\tau^y}{1+\tau^w} \right]$$

Differentiating with respect to w_t and solving for the wage rate [skipped some math]

$$w_{t}^{\frac{1-\lambda_{w}(1+\sigma_{L})}{1-\lambda_{w}}} = \frac{E_{t}^{j} \sum_{i=0}^{\infty} (\beta\xi_{w})^{i} \zeta_{t+i}^{h} A_{L} \left(\left(\frac{\bar{w}_{t}}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_{w}}{1-\lambda_{w}}} H_{t+i} \right)^{1+\sigma_{L}}}{E_{t}^{j} \sum_{i=0}^{\infty} (\beta\xi_{w})^{i} \frac{\psi_{z^{+},t+i}}{\lambda_{w}} \bar{w}_{t} X_{t,i} \left(\frac{\bar{w}_{t}}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_{w}}{1-\lambda_{w}}} H_{t+i} \frac{1-\tau^{y}}{1+\tau^{w}}}{e^{\frac{1}{w}t} F_{w,t}}}$$

where

$$K_{w,t} := E_t^j \sum_{i=0}^{\infty} (\beta \xi_w)^i \zeta_{t+i}^h \left(\left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \right)^{1+\sigma_L} F_{w,t} := E_t^j \sum_{i=0}^{\infty} (\beta \xi_w)^i \frac{\psi_{z^+,t+i}}{\lambda_w} X_{t,i} \left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \frac{1-\tau^y}{1+\tau^w}.$$

Thus, the wage set by reoptimizing households is

$$w_t = \left[\frac{A_L K_{w,t}}{\bar{w}_t F_{w,t}}\right]^{\frac{1-\lambda_w}{1-\lambda_w(1+\sigma_L)}}.$$

We now express $K_{w,t}$ and $F_{w,t}$ in recursive form [after some skipped math]:

$$K_{w,t} = \zeta_t^h H_t^{1+\sigma_L} + \beta \xi_w E_t \left(\frac{\tilde{\pi}_{w,t+1}}{\pi_{w,t+1}}\right)^{\frac{\lambda_w}{1-\lambda_w}(1+\sigma_L)} K_{w,t+1}$$

where

$$\pi_{w,t+1} = \frac{W_{t+1}}{W_t} = \frac{\bar{w}_{t+1} z_{t+1}^+ P_{t+1}}{\bar{w}_t z_t^+ P_t} = \frac{\bar{w}_{t+1} \mu_{z^+,t+1} \pi_{t+1}}{\bar{w}_t}$$
(D.27)

.

Also [after some skipped math],

$$F_{w,t} = \frac{\psi_{z^+,t}}{\lambda_w} H_t \frac{1-\tau^y}{1+\tau^w} + \beta \xi_w E_t \left(\frac{\bar{w}_{t+1}}{\bar{w}_t}\right) \left(\frac{\tilde{\pi}_{w,t+1}}{\pi_{w,t+1}}\right)^{1+\frac{\lambda_w}{1-\lambda_w}} F_{w,t+1}.$$

The second restriction on w_t is obtained using the relation between the aggregate wage rate and the wage rates of individual households:

$$W_{t} = \left[(1 - \xi_{w}) \left(\tilde{W}_{t} \right)^{\frac{1}{1 - \lambda_{w}}} + \xi_{w} (\tilde{\pi}_{w,t} W_{t-1})^{\frac{1}{1 - \lambda_{w}}} \right]^{1 - \lambda_{w}}$$

Dividing both sides by W_t and rearranging,

$$w_t = \left[\frac{1 - \xi_w \left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}}\right)^{\frac{1}{1-\lambda_w}}}{1 - \xi_w}\right]^{1-\lambda_w}$$

Substituting out for w_t from the household's FOC for wage optimization,

$$\frac{1}{A_L} \left[\frac{1 - \xi_w \left(\frac{\bar{\pi}_{w,t}}{\pi_{w,t}} \right)^{\frac{1}{1 - \lambda_w}}}{1 - \xi_w} \right]^{1 - \lambda_w (1 + \sigma_L)} \bar{w}_t F_{w,t} = K_{w,t}$$

We now derive the relationship between aggregate homogeneous hours worked, H_t , and aggregate household hours,

$$h_t := \int_0^1 h_{j,t} dj.$$

Substituting the demand for $h_{j,t}$ into the latter expression,

$$h_t = \int_0^1 \left(\frac{W_{j,t}}{W_t}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_t dj$$

$$= \frac{H_t}{(W_t)^{\frac{\lambda_w}{1-\lambda_w}}} \int_0^1 (W_{j,t})^{\frac{\lambda_w}{1-\lambda_w}} dj$$

$$= \mathring{w}_t^{\frac{\lambda_w}{1-\lambda_w}} H_t, \qquad (D.28)$$

where

$$\mathring{w}_t = \frac{\mathring{W}_t}{W_t}, \, \mathring{W}_t = \left[\int_0^1 (W_{j,t})^{\frac{\lambda_w}{1-\lambda_w}} dj\right]^{\frac{1-\lambda_w}{\lambda_w}}$$

and

$$\mathring{W}_{t} = \left[\left(1 - \xi_{w}\right) \left(\tilde{W}_{t}\right)^{\frac{\lambda_{w}}{1 - \lambda_{w}}} + \xi_{w} \left(\tilde{\pi}_{w,t} \mathring{W}_{t-1}\right)^{\frac{\lambda_{w}}{1 - \lambda_{w}}} \right]^{\frac{1 - \lambda_{w}}{\lambda_{w}}},$$

so that

$$\overset{}{w}_{t} = \left[(1 - \xi_{w})(w_{t})^{\frac{\lambda_{w}}{1 - \lambda_{w}}} + \xi_{w} \left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}} \overset{}{w}_{t-1} \right)^{\frac{\lambda_{w}}{1 - \lambda_{w}}} \right]^{\frac{1 - \lambda_{w}}{\lambda_{w}}} \\
= \left[(1 - \xi_{w}) \left(\frac{1 - \xi_{w} \left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}} \right)^{\frac{1}{1 - \lambda_{w}}}}{1 - \xi_{w}} \right)^{\lambda_{w}} + \xi_{w} \left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}} \overset{}{w}_{t-1} \right)^{\frac{\lambda_{w}}{1 - \lambda_{w}}} \right]^{\frac{1 - \lambda_{w}}{\lambda_{w}}}. \quad (D.29)$$

In addition to (D.29), we have the following equilibrium conditions associated with sticky wages:

$$F_{w,t} = \frac{\psi_{z^+,t}}{\lambda_w} \mathring{w}_t^{\frac{-\lambda_w}{1-\lambda_w}} h_t \frac{1-\tau^y}{1+\tau^w} + \beta \xi_w E_t \left(\frac{\bar{w}_{t+1}}{\bar{w}_t}\right) \left(\frac{\tilde{\pi}_{w,t+1}}{\pi_{w,t+1}}\right)^{1+\frac{\lambda_w}{1-\lambda_w}} F_{w,t+1} \tag{D.30}$$

$$K_{w,t} = \zeta_t^h \left(\mathring{w}_t^{\frac{-\lambda_w}{1-\lambda_w}} h_t \right)^{1+\sigma_L} + \beta \xi_w E_t \left(\frac{\tilde{\pi}_{w,t+1}}{\pi_{w,t+1}} \right)^{\frac{\lambda_w}{1-\lambda_w}(1+\sigma_L)} K_{w,t+1}$$
(D.31)

$$\frac{1}{A_L} \left[\frac{1 - \xi_w \left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}} \right)^{\frac{1}{1 - \lambda_w}}}{1 - \xi_w} \right]^{1 - \lambda_w (1 + \sigma_L)} \bar{w}_t F_{w,t} = K_{w,t}.$$
(D.32)

D.3.10 Scaling law of motion of capital

Using (C.38), the law of motion of capital in scaled terms is

$$\bar{k}_{t+1} = \frac{1-\delta}{\mu_{z^+,t}\mu_{\Psi,t}}\bar{k}_t + \Upsilon_t \left(1 - \tilde{S}\left(\frac{\mu_{z^+,t}\mu_{\Psi,t}i_t}{i_{t-1}}\right)\right)i_t.$$
 (D.33)

D.3.11 Output and aggregate factors of production

Below we derive a relationship between total output of the domestic homogeneous good, Y_t , and aggregate factors of production.

Consider the unweighted average of the intermediate goods:

$$Y_t^{sum} = \int_0^1 Y_{i,t} di$$

= $\int_0^1 \left[(z_t H_{i,t})^{1-\alpha} \epsilon_t K_{i,t}^{\alpha} - z_t^+ \phi \right] di$
= $\int_0^1 \left[z_t^{1-\alpha} \epsilon_t \left(\frac{K_{i,t}}{H_{i,t}} \right)^{\alpha} H_{i,t} - z_t^+ \phi \right] di$
= $z_t^{1-\alpha} \epsilon_t \left(\frac{K_t}{H_t} \right)^{\alpha} \int_0^1 H_{i,t} di - z_t^+ \phi$,

where K_t is the economy-wide average stock of capital services and H_t is the economy-wide average of homogeneous labor. The last expression exploits the fact that all intermediate good firms confront the same factor prices, and so they adopt the same capital services to homogeneous labor ratio. This follows from cost minimization, and holds for all firms, regardless whether or not they have an opportunity to reoptimize. Then,

$$Y_t^{sum} = z_t^{1-\alpha} \epsilon_t K_t^{\alpha} H_t^{1-\alpha} - z_t^+ \phi.$$

Recall that the demand for $Y_{j,t}$ is

$$\left(\frac{P_t}{P_{i,t}}\right)^{\frac{\lambda_d}{\lambda_d-1}} = \frac{Y_{i,t}}{Y_t},$$

so that

$$\mathring{Y}_t := \int_0^1 Y_{i,t} di = \int_0^1 Y_t \left(\frac{P_t}{P_{i,t}}\right)^{\frac{\lambda_d}{\lambda_d - 1}} di = Y_t P_t^{\frac{\lambda_d}{\lambda_d - 1}} \left(\mathring{P}_t\right)^{\frac{\lambda_d}{1 - \lambda_d}},$$

where

$$\mathring{P}_{t} = \left[\int_{0}^{1} P_{i,t}^{\frac{\lambda_{d}}{1-\lambda_{d}}} di\right]^{\frac{1-\lambda_{d}}{\lambda_{d}}}.$$
(D.34)

Dividing by P_t ,

$$\mathring{p}_t = \left[\int_0^1 \left(\frac{P_{it}}{P_t} \right)^{\frac{\lambda_d}{1-\lambda_d}} di \right]^{\frac{1-\lambda_d}{\lambda_d}},$$

or,

$$\mathring{p}_{t} = \left[\left(1 - \xi_{p}\right) \left(\frac{1 - \xi_{p}\left(\frac{\tilde{\pi}_{d,t}}{\pi_{t}}\right)^{\frac{1}{1 - \lambda_{d}}}}{1 - \xi_{p}}\right)^{\lambda_{d}} + \xi_{p}\left(\frac{\tilde{\pi}_{d,t}}{\pi_{t}}\mathring{p}_{t-1}\right)^{\frac{\lambda_{d}}{1 - \lambda_{d}}} \right]^{\frac{1 - \lambda_{d}}{\lambda_{d}}}.$$
 (D.35)

The preceding implies

$$Y_t = (\mathring{p}_t)^{\frac{\lambda_d}{\lambda_d - 1}} \mathring{Y}_t = (\mathring{p}_t)^{\frac{\lambda_d}{\lambda_d - 1}} \left[z_t^{1-\alpha} \epsilon_t K_t^{\alpha} H_t^{1-\alpha} - z_t^+ \phi \right],$$

or, after scaling by z_t^+ ,

$$y_t = (\mathring{p}_t)^{\frac{\lambda_d}{\lambda_d - 1}} \left[\epsilon_t \left(\frac{1}{\mu_{\Psi, t} \mu_{z^+, t}} k_t \right)^{\alpha} H_t^{1 - \alpha} - \phi \right],$$

where

$$k_t = \bar{k}_t u_t. \tag{D.36}$$

Plugging H_t from (D.28),

$$y_t = (\mathring{p}_t)^{\frac{\lambda_d}{\lambda_d - 1}} \left[\epsilon_t \left(\frac{1}{\mu_{\Psi, t} \mu_{z^+, t}} k_t \right)^{\alpha} \left(\mathring{w}_t^{\frac{\lambda_w}{1 - \lambda_w}} h_t \right)^{1 - \alpha} - \phi \right].$$

D.3.12 Restrictions across inflation rates

We now consider the restrictions across inflation rates implied by the relative price formulas. In terms of the expressions in (D.2), there are the restrictions implied by $P_t^{m,j}/p_{t-1}^{m,j}$, j = x, c, i, and p_t^x . The restrictions implied by the other two relative prices in (D.2), p_t^i and p_t^c , have already been used in (C.16) and (D.33), respectively. Finally, we also use the restriction across inflation rates implied by q_t/q_{t-1} and (C.23). Thus,

$$\frac{p_t^{m,x}}{p_{t-1}^{m,x}} = \frac{\pi_t^{m,x}}{\pi_t} \tag{D.37}$$

$$\frac{p_t^{m,c}}{p_{t-1}^{m,c}} = \frac{\pi_t^{m,c}}{\pi_t} \tag{D.38}$$

$$\frac{p_t^{m,i}}{p_{t-1}^{m,i}} = \frac{\pi_t^{m,i}}{\pi_t} \tag{D.39}$$

$$\frac{p_t^x}{p_{t-1}^x} = \frac{\pi_t^x}{\pi_t^*} \tag{D.40}$$

$$\frac{q_t}{q_{t-1}} = \frac{s_t \pi_t^*}{\pi_t^c}.$$
 (D.41)

D.3.13 Endogenous variables of the baseline model

In above we derived the following 70 equations:

 $\begin{array}{l} (C.3), (C.4), (C.5), (D.8), (D.9), (D.10), (D.11), (D.12), (D.3), (C.10), (C.11), (C.12), (C.15), (C.16), \\ (C.14), (D.13), (C.21), (C.20), (C.27), (D.14), (D.15), (D.16), (D.17), (D.20), (C.29), (D.23), (D.24), \\ (D.25), (D.26), (C.32), (D.22), (D.4), (D.5), (D.33), (C.39), (C.41), (C.42), (C.43), (C.44), (C.45), (C.47), \\ (D.30), (D.31), (D.32), (D.29), (C.35), (D.27), (D.28), (D.36), (C.49), (C.51), (C.50), (D.37), (D.38), \\ (D.39), (D.40), (D.41), (C.48), \end{array}$

which can be used to solve for the following 70 unknowns:

 $\bar{r}_{t}^{k}, \bar{w}_{t}, R_{t}^{\nu,*}, R_{t}^{f}, R_{t}^{x}, R_{t}, m_{c}, m_{c}^{t}, m_{t}^{m,c}, m_{c}^{m,i}, m_{c}^{m,i}, m_{c}^{m,x}, \pi_{t}, \pi_{t}^{x}, \pi_{t}^{c}, \pi_{t}^{i}, \pi_{t}^{m,c}, \pi_{t}^{m,i}, \pi_{t}^{m,x}, p_{t}^{c}, p_{t}^{x}, p_{t}^{i}, p_{t}^{m,x}, p_{t}^{m,c}, p_{t}^{m,i}, p_{t}^{m,i}, p_{t}^{m,i}, \bar{k}_{t+1}, \bar{k}_{t+1}, u_{t}, h_{t}, H_{t}, q_{t}, i_{t}, c_{t}, x_{t}, a_{t}, \psi_{z^{+},t}, y_{t}, K_{t}^{d}, F_{t}^{d}, \tilde{\pi}_{d,t}, \mathring{p}_{t}, K_{x,t}, F_{x,t}, \tilde{\pi}_{t}^{x}, \mathring{p}_{t}^{x}, \mathring{p}_{t}^{x}, \tilde{s}_{t}^{x}, \tilde{p}_{t}^{x}, \tilde{s}_{t}^{x}, \pi_{t}^{x}, \tilde{s}_{t}^{x}, \tilde{s}_{t}^{x}, \tilde{s}_{t}^{x}, \tilde{s}_{t}^{x}, \tilde{s}_{t}^{x}, \tilde{s}_{t}^{x}, \tilde{s}_{t}^{x}, \pi_{t}^{x}, \pi_{t}^{$

D.4 Equilibrium conditions for financial frictions model

D.4.1 Derivation of optimal contract

As noted in the text, it is supposed that the equilibrium debt contract maximizes entrepreneurial welfare subject to the zero profit condition on banks and the specified required return on household bank liabilities. The date t debt contract specifies a level of debt B_{t+1} and a state t + 1-contingent rate of interest, Z_{t+1} . We suppose that entrepreneurial welfare corresponds to the entrepreneur's expected wealth at the end of the contract. It is convenient to express welfare as a ratio to the amount the entrepreneur could receive by depositing his net worth in a bank:

$$\frac{E_t \int_{\bar{\omega}_{t+1}}^{\infty} \left[R_{t+1}^k \omega P_t P_{k',t} \bar{K}_{t+1} - Z_{t+1} B_{t+1} \right] dF(\omega;\sigma_t)}{R_t N_{t+1}} \\ = \frac{E_t \int_{\omega_{t+1}}^{\infty} [\omega - \bar{\omega}_{t+1}] dF(\omega;\sigma_t) R_{t+1}^k P_t P_{k',t} \bar{K}_{t+1}}{R_t N_{t+1}} \\ = E_t \left\{ [1 - \Gamma(\bar{\omega}_{t+1};\sigma_t)] \frac{R_{t+1}^k}{R_t} \right\} \varrho_t,$$

after making use of (C.52), (C.53) and

$$1 = \int_0^\infty \omega dF(\omega; \sigma_t) = \int_{\bar{\omega}_{t+1}}^\infty \omega dF(\omega; \sigma_t) + G(\bar{\omega}_{t+1}; \sigma_t).$$

We can equivalently characterize the contract by a state-t + 1 contingent set of values for $\bar{\omega}_{t+1}$ and a value of ρ_t . The equilibrium contract is the one involving $\bar{\omega}_{t+1}$ and ρ_t which maximizes entrepreneurial welfare (relative to $R_t N_{t+1}$) subject to the bank zero profits condition. The Lagrangian representation of this problem is

$$\max_{\varrho_{t},\{\bar{\omega}_{t+1}\}} E_{t} \left\{ \left[1 - \Gamma(\bar{\omega}_{t+1};\sigma_{t})\right] \frac{R_{t+1}^{k}}{R_{t}} \varrho_{t} + \lambda_{t+1} \left(\left[\Gamma(\bar{\omega}_{t+1};\sigma_{t}) - \mu G(\bar{\omega}_{t+1};\sigma_{t})\right] \frac{R_{t+1}^{k}}{R_{t}} \varrho_{t} - \varrho_{t} + 1 \right) \right\},$$

where λ_{t+1} is the Lagrange multiplier which is defined for each period t+1 state of nature. The FOCs for this problem are:

$$E_{t}\left\{ \left[1 - \Gamma(\bar{\omega}_{t+1}; \sigma_{t})\right] \frac{R_{t+1}^{k}}{R_{t}} + \lambda_{t+1} \left(\left[\Gamma(\bar{\omega}_{t+1}; \sigma_{t}) - \mu G(\bar{\omega}_{t+1}; \sigma_{t})\right] \frac{R_{t+1}^{k}}{R_{t}} - 1 \right) \right\} = 0$$

$$-\Gamma_{\bar{\omega}}(\bar{\omega}_{t+1}; \sigma_{t}) \frac{R_{t+1}^{k}}{R_{t}} + \lambda_{t+1} [\Gamma_{\bar{\omega}}(\bar{\omega}_{t+1}; \sigma_{t}) - \mu G_{\bar{\omega}}(\bar{\omega}_{t+1}; \sigma_{t})] \frac{R_{t+1}^{k}}{R_{t}} = 0$$

$$[\Gamma(\bar{\omega}_{t+1}; \sigma_{t}) - \mu G(\bar{\omega}_{t+1}; \sigma_{t})] \frac{R_{t+1}^{k}}{R_{t}} \varrho_{t} - \varrho_{t} + 1 = 0,$$

where the absence of λ_{t+1} from the complementary slackness condition reflects that it is assumed that $\lambda_{t+1} > 0$ in each period t+1 state of nature. Substituting out for λ_{t+1} from the second equation into the first, the FOCs reduce to

$$E_t \left\{ \begin{bmatrix} 1 - \Gamma(\bar{\omega}_{t+1}; \sigma_{t+1}) \end{bmatrix} \frac{R_{t+1}^k}{R_t} + \frac{\Gamma_{\bar{\omega}}(\bar{\omega}_{t+1}; \sigma_t)}{\Gamma_{\bar{\omega}}(\bar{\omega}_{t+1}; \sigma_t) - \mu G_{\bar{\omega}}(\bar{\omega}_{t+1}; \sigma_t)} \times \\ \left([\Gamma(\bar{\omega}_{t+1}; \sigma_t) - \mu G(\bar{\omega}_{t+1}; \sigma_t)] \frac{R_{t+1}^k}{R_t} - 1 \right) \right\} = 0$$
(D.42)

$$[\Gamma(\bar{\omega}_{t+1};\sigma_t) - \mu G(\bar{\omega}_{t+1};\sigma_t)] \frac{R_{t+1}^k}{R_t} \varrho_t - \varrho_t + 1 = 0$$
(D.43)

for $t = 0, 1, 2, ..., \infty$ and for t = -1, 0, 1, 2, ... respectively.

Since N_{t+1} does not appear in the last two equations, ϱ_t and $\bar{\omega}_{t+1}$ are the same for all entrepreneurs regardless of their net worth.

D.4.2 Derivation of aggregation of across entrepreneurs

Let $f(N_{t+1})$ denote the density of entrepreneurs with net worth N_{t+1} . Then, aggregate average net worth, \bar{N}_{t+1} , is

$$\bar{N}_{t+1} = \int_{N_{t+1}} N_{t+1} f(N_{t+1}) dN_{t+1}.$$

We now derive the law of motion of N_{t+1} . Consider the set of entrepreneurs who in period t-1 had net worth N. Their net worth after they have settled with the bank in period t is denoted V_t^N , where

$$V_t^N = R_t^k P_{t-1} P_{k',t-1} \bar{K}_t^N - \Gamma(\bar{\omega}_t; \sigma_{t-1}) R_t^k P_{t-1} P_{k',t-1} \bar{K}_t^N,$$
(D.44)

where \bar{K}_t^N is the amount of physical capital that entrepreneurs with net worth N_t acquired in period t-1. Clearing in the market for capital requires:

$$\bar{K}_t = \int_{N_t} \bar{K}_t^N f(N_t) dN_t.$$

Multiplying (D.44) by $f(N_t)$ and integrating over all entrepreneurs,

$$V_t = R_t^k P_{t-1} P_{k',t-1} \bar{K}_t - \Gamma(\bar{\omega}_t; \sigma_{t-1}) R_t^k P_{t-1} P_{k',t-1} \bar{K}_t$$

Writing this out more fully:

$$\begin{split} V_t &= R_t^k P_{t-1} P_{k',t-1} \bar{K}_t - \left\{ [1 - F(\bar{\omega}_t; \sigma_{t-1})] \bar{\omega}_t + \int_0^{\bar{\omega}_t} \omega dF(\omega; \sigma_{t-1}) \right\} R_t^k P_{t-1} P_{k',t-1} \bar{K}_t \\ &= R_t^k P_{t-1} P_{k',t-1} \bar{K}_t \\ &- \left\{ [1 - F(\bar{\omega}_t; \sigma_{t-1})] \bar{\omega}_t + (1 - \mu) \int_0^{\bar{\omega}_t} \omega dF(\omega; \sigma_{t-1}) + \mu \int_0^{\bar{\omega}_t} \omega dF(\omega; \sigma_{t-1}) \right\} R_t^k P_{t-1} P_{k',t-1} \bar{K}_t \end{split}$$

Note that the first two terms in braces correspond to the net revenues of the bank, which must equal $R_{t-1}(P_{t-1}P_{k',t-1}\bar{K}_t - \bar{N}_t)$. Substituting

$$V_{t} = R_{t}^{k} P_{t-1} P_{k',t-1} \bar{K}_{t} - \left\{ R_{t-1} + \frac{\mu \int_{0}^{\bar{\omega}_{t}} \omega dF(\omega;\sigma_{t-1}) R_{t}^{k} P_{t-1} P_{k',t-1} \bar{K}_{t}}{P_{t-1} P_{k',t-1} \bar{K}_{t} - \bar{N}_{t}} \right\} (P_{t-1} P_{k',t-1} \bar{K}_{t} - \bar{N}_{t})$$

which implies (C.56) in the main text.

D.4.3 Adjustment to the baseline model when financial frictions are introduced

Consider the households. Households no longer accumulate physical capital, and the FOC (C.42) must be dropped. No other changes need to be made to the household FOCs. Eq. (C.45) can be interpreted as applying to the household's decision to make bank deposits. The household eq-ns (D.33) and (C.43) pertaining to the law of motion and FOC for investment respectively, can be thought of as reflecting that the household builds and sells physical capital, or it can be interpreted as the FOC of many identical competitive firms that build capital (note that each has a state variable in the form of lagged investment). We must add the three equations pertaining to the entrepreneur's loan contract: the law of motion of net worth, the bank's zero profit condition and the optimality condition. Finally, we must adjust the resource constraints to reflect the resources used in bank monitoring and in consumption by entrepreneurs.

We adopt the following scaling of variables, noting that W_t^e is set so that its scaled counterpart is constant

$$n_{t+1} = \frac{\bar{N}_{t+1}}{P_t z_t^+}, \ w^e = \frac{W_t^e}{P_t z_t^+}$$

Dividing both sides of (C.56) by $P_t z_t^+$, we obtain the scaled law of motion for net worth:

$$n_{t+1} = \frac{\gamma_t}{\pi_t \mu_{z^+,t}} [R_t^k p_{k',t-1} \bar{k}_t - R_{t-1} (p_{k',t-1} \bar{k}_t - n_t) - \mu G(\bar{\omega}_t; \sigma_{t-1}) R_t^k p_{k',t-1} \bar{k}_t] + w^e$$
(D.45)

for $t = 0, 1, 2, \ldots$ Eq. (D.45) has a simple intuitive interpretation. The first object in square brackets is the average gross return across all entrepreneurs in period t. The two negative terms correspond to what the entrepreneurs pay to the bank, including the interest paid by non-bankrupt entrepreneurs and the resources turned over to the bank by the bankrupt entrepreneurs. Since the bank makes zero profits, the payment to the bank by entrepreneurs must equal bank costs. The term involving R_{t-1} represents the cost of funds loaned to entrepreneurs by the bank, and the term involving μ represents the bank's total expenditures on monitoring costs.

The zero profit condition on banks, (D.43), can be expressed in terms of the scaled variables as

$$\Gamma(\bar{\omega}_{t+1};\sigma_t) - \mu G(\bar{\omega}_{t+1};\sigma_t) = \frac{R_t}{R_{t+1}^k} \left(1 - \frac{n_{t+1}}{p_{k',t}\bar{k}_{t+1}}\right)$$
(D.46)

for $t = -1, 0, 1, 2, \dots$ The optimality condition for bank loans is (D.42).

The output equation, (C.49), does not have to be modified. Instead, the resource constraint for domestic homogeneous goods (C.50) needs to be adjusted for the monitoring costs:

$$y_t - d_t = g_t + (1 - \omega_c) (p_t^c)^{\eta_c} c_t + (p_t^i)^{\eta_i} \left(i_t + a(u_t) \frac{\bar{k}_t}{\mu_{\psi,t} \mu_{z^+,t}} \right) (1 - \omega_t) \\ + \left[\omega_x (p_t^{m,x})^{1 - \eta_x} + (1 - \omega_x) \right]^{\frac{\eta_x}{1 - \eta_x}} (1 - \omega_x) (\dot{p}_t^x)^{\frac{-\lambda_x}{\lambda_x - 1}} (p_t^x)^{-\eta_f} y_t^*,$$
(D.47)

where

$$d_t = \frac{\mu G(\bar{\omega}_t; \sigma_{t-1}) R_t^k p_{k', t-1} \bar{k}_t}{\pi_t \mu_{z^+, t}}$$

When the model is brought to the data, measured GDP is y_t adjusted for both monitoring costs and, as in the baseline model, capital utilization costs

$$gdp_t = y_t - d_t - (p_t^i)^{\eta_i} \left(a(u_t) \frac{\bar{k}_t}{\mu_{\psi,t} \mu_{z^+,t}} \right) (1 - \omega_i).$$

Account has to be taken of the consumption by existing entrepreneurs. The net worth of these entrepreneurs is $(1 - \gamma_t)V_t$ and it is assumed that a fraction, $1 - \Theta$, is taxed and transferred in lump-sum form to households, while the complementary fraction, Θ , is consumed by the existing entrepreneurs. This consumption can be taken into account by subtracting

$$\Theta \frac{1-\gamma_t}{\gamma_t} (n_{t+1}-w^e) z_t^+ P_t$$

from the right side of (C.9). In practice we do not make this adjustment because we assume Θ is sufficiently small that the adjustment is negligible.

The financial frictions brings a net increase of two equations (we add (D.42), (D.45) and (D.46), and delete (C.42)) and two variables, n_{t+1} and $\bar{\omega}_{t+1}$. This increases the size of our system to 72 equations in 72 unknowns. The financial frictions also introduce the additional shocks, σ_t and γ_t .

D.5 Measurement equations

Below we report the measurement equations we use to link the model to the data. Our data series for inflation and interest rates are annualized in percentage terms, so we make the same transformation for the model variables, i.e. multiplying by 400:

$$\begin{split} R_t^{data} &= 400(R_t-1) - \vartheta_1 400(R-1) \\ R_t^{*,data} &= 400(R_t^*-1) - \vartheta_1 400(R^*-1) \end{split}$$

$$\begin{aligned} \pi_t^{d,data} &= 400 \log \pi_t - \vartheta_1 400 \log \pi + \varepsilon_{\pi,t}^{me} \\ \pi_t^{c,data} &= 400 \log \pi_t^c - \vartheta_1 400 \log \pi^c + \varepsilon_{\pi^c,t}^{me} \\ \pi_t^{i,data} &= 400 \log \pi_t^i - \vartheta_1 400 \log \pi^i + \varepsilon_{\pi^i,t}^{me} \\ \pi_t^{*,data} &= 400 \log \pi_t^* - \vartheta_1 400 \log \pi^*, \end{aligned}$$

where $\varepsilon_{i,t}^{me}$ denote the measurement errors for the respective variables. In addition, $\vartheta_1 \in \{0, 1\}$ allows us to handle demeaned and non-demeaned data. In particular, the data for interest rates and foreign inflation are not demeaned. The domestic inflation rates are demeaned.

We use undemeaned first differences in total hours worked,

$$\Delta \log H_t^{data} = 100\Delta \log H_t + \varepsilon_{H,t}^{me}$$

We use demeaned first-differenced data for the remaining variables. This implies setting $\vartheta_2=1$ below:

$$\begin{split} \Delta \log Y_t^{data} &= 100 \left(\log \mu_{z^+,t} + \Delta \log \left[y_t - p_t^i a(u_t) \frac{\bar{k}_t}{\mu_{\psi,t} \mu_{z^+,t}} - d_t - \frac{\kappa}{2} \sum_{j=0}^{N-1} (\tilde{v}_t^j)^2 (1 - \mathcal{F}_t^j) l_t^j \right] \\ &\quad - \vartheta_2 100(\log \mu_{z^+}) + \varepsilon_{y,t}^{me} \\ \Delta \log Y_t^{*,data} &= 100(\log \mu_{z^+,t} + \Delta \log y_t^*) - \vartheta_2 100(\log \mu_{z^+}) \\ \Delta \log C_t^{data} &= 100(\log \mu_{z^+,t} + \Delta \log c_t) - \vartheta_2 100(\log \mu_{z^+}) + \varepsilon_{c,t}^{me} \\ \Delta \log X_t^{data} &= 100(\log \mu_{z^+,t} + \Delta \log x_t) - \vartheta_2 100(\log \mu_{z^+}) + \varepsilon_{x,t}^{me} \\ \Delta \log q_t^{data} &= 100\Delta \log q_t + \varepsilon_{q,t}^{me} \\ \Delta \log q_t^{data} &= 100(\log \mu_{z^+,t} + \Delta \log Imports_t) - \vartheta_2 100(\log \mu_{z^+}) + \varepsilon_{M,t}^{me} \\ &= 100 \left[\log \mu_{z^+,t} + \Delta \log \left[mports_t \right) - \vartheta_2 100(\log \mu_{z^+}) + \varepsilon_{M,t}^{me} \\ + i_t^m (\hat{p}_t^{m,c})^{\frac{\lambda_{m,x}}{1 - \lambda_{m,x}}} \\ + i_t^m (\hat{p}_t^{m,x})^{\frac{\lambda_{m,x}}{1 - \lambda_{m,x}}} \right) \right] - \vartheta_2 100(\log \mu_{z^+}) + \varepsilon_{I,t}^{me} \\ \Delta \log I_t^{data} &= 100[\log \mu_{z^+,t} + \log \mu_{\psi,t} + \Delta \log i_t] - \vartheta_2 100(\log \mu_{z^+} + \log \mu_{\psi}) + \varepsilon_{I,t}^{me} \\ \Delta \log G_t^{data} &= 100(\log \mu_{z^+,t} + \Delta \log g_t) - \vartheta_2 100(\log \mu_{z^+}) + \varepsilon_{g,t}^{me} \end{split}$$

Note that neither measured GDP nor measured investment include investment goods used for capital maintenance. To calculate measured GDP we also exclude monitoring costs and recruitment costs.

The measurement equation for demeaned first-differenced wages is

$$\Delta \log(W_t/P_t)^{data} = 100\Delta \log \frac{W_t}{z_t^+ P_t} = 100(\log \mu_{z^+,t} + \Delta \log \bar{w}_t) - \vartheta_2 100(\log \mu_{z^+}) + \varepsilon_{W/P,t}^{me}$$

Finally, we measure demeaned first-differenced net worth and interest rate spread as follows:

$$\Delta \log N_t^{data} = 100(\log \mu_{z^+,t} + \Delta \log n_t) - \vartheta_2 100(\log \mu_{z^+}) + \varepsilon_{N,t}^{me}$$

$$\Delta \log Spread_t^{data} = 100\Delta \log(z_{t+1} - R_t) = 100\Delta \log \left(\frac{\bar{\omega}_{t+1}R_{t+1}^k}{1 - \frac{n_{t+1}}{p_{k',t}k_{t+1}}} - R_t\right) + \varepsilon_{Spread,t}^{me}$$