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# Monetary Policy Rule, Exchange Rate Regime, and Fiscal Policy Cyclicity in a Developing Oil Economy

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## Abstract

This paper constructs a dynamic stochastic general equilibrium model of joint monetary and fiscal policy for a developing oil economy to find an appropriate monetary rule combined with pro-/countercyclical and neutral fiscal stance based on a loss measure. The model captures a set of structural specifics: two monetary instruments—interest rate and foreign exchange interventions, two fiscal instruments—public consumption and public investment, two production sectors—oil and non-oil, and the two types of households—optimizers and rule-of-thumb households. It further includes a Sovereign Wealth Fund, the foreign debt of private sector via collateral constraint, and a world oil price shock. The loss measure is chosen as an equal summation of variances in inflation, output, and real exchange rate to be minimized by Taylor rule's parameters in a small open economy. The foreign exchange interventions distinguish between managed and flexible exchange rate regime. Fiscal policy cyclicity is referred to the oil output response of public consumption and public investment. Impulse response functions to the negative world oil price shock are analyzed at flexible and rigid prices.

*Keywords:* oil economy, monetary policy, fiscal policy, exchange rate, oil price shock, interventions, SWF

*JEL Classification:* E31, E52, E62, E63, F31, F41, H54, H63, Q33, Q38

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# 1 Introduction

Most macroeconomic DSGE models are constructed for the developed world incorporating its advanced market structure and relevant policy environment. Emerging market economies have their own unique features, which can modify the existing core frameworks in several respects. First, public investment should be considered separately from public consumption as a growth inducing instrument of fiscal policy (Berg, Portillo, Yang & Zanna, 2013), since it is usually associated with infrastructure and human capital which developing countries often lack (Rioja, 2003; Sab & Smith, 2002). Second, monetary policy is typically a hybrid of inflation targeting and managed exchange rate regime; thus, interest rate and foreign exchange interventions represent the two separate instruments of monetary policy (Benes et al., 2015). Third, in underdeveloped domestic financial market, the investments of firms are often financed by foreign funds, so that physical capital and foreign debt can be linked through a collateral constraint (Faia & Iliopoulos, 2011). Fourth, households are heterogeneous in their income and access to financial market; a certain portion of the population may be liquidity constrained having only wages, without making savings (Mankiw, 2000; Gali, Lopez-Salido & Valles, 2007). These four structural specifics are incorporated in the model of Algozhina (2012) calibrated for Hungary as a first economy among all emerging markets severely hit by the global financial crisis in mid-October 2008.

This paper extends Algozhina (2012) for a subset of emerging open economies which export oil, but it can be applied to any commodity. Oil exporting developing economies obviously differ from other emerging countries and need to be examined through their own DSGE framework with the following features in addition to those outlined above. The oil and non-oil production sectors should be specified separately. The economy is exposed to a volatile exogenous world oil price shock. A Sovereign Wealth Fund (SWF) is established collecting the oil taxes, saving them abroad, and partly transferring to the government budget<sup>1</sup>. The foreign exchange interventions are related to central bank's reserves that may affect the interest rate according to a mechanism described by Benes et al. (2015). And motivated by Frankel and Catao (2011), hereafter as F&C, monetary policy can follow product price targeting (PPT) as an alternative to consumer price index (CPI); thus, these two anchors need to be compared in a general equilibrium framework jointly with fiscal policy based on some welfare measure.

F&C argue that commodity exporting economies are better off targeting the

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<sup>1</sup>The mechanism of SWF accumulation differs across countries, but since the model is calibrated for Kazakhstan, its experience is specifically captured.

output price index which includes export commodities and excludes import products; such monetary policy is automatically countercyclical against the volatile terms of trade shock. The argument is that, if the world oil price increases and there is PPT, then monetary policy tightens by raising its interest rate, thus causing the exchange rate appreciation which is the objective of offsetting the initial positive terms of trade shock. And vice versa, an adverse terms of trade shock, such as a fall in oil price, can be mitigated by the exchange rate depreciation under PPT. The CPI inflation targeting, in contrast, does not respond to export prices, but to import prices. If there is an adverse terms of trade shock, such as an increase of import prices, CPI targeting brings the exchange rate appreciation exacerbating further the initial negative shock for producers. "Bottom line: a Product Price Targeter would appreciate in response to an increase in world prices of its commodity exports, not in response to an increase in world prices of its imports. CPI targeting gets this backwards." (F&C, p. 4).

The aim of this paper, therefore, is to construct a DSGE model for a developing resource-rich economy capturing its structural specifics, as defined above, to examine the CPI/PPT monetary policy rule at flexible/managed exchange rate regime combined with pro-/countercyclical and neutral fiscal policy. The calibration is based on Kazakhstan as a small open oil exporting economy severely hit by the global financial crisis 2008 due to the high foreign debt of private sector. Since 2006, the IMF has included Kazakhstan in its "fuel exporters" group analyzed in the *World Economic Outlook*. The classification is made on the evidence that over past five years the average share of fuel exports in total exports exceeds 40 percent. In 2000, Kazakhstan established its SWF managed by the National Bank on behalf of the Ministry of Finance. Oil taxes directly accumulate the SWF saved abroad, but regularly on an ad hoc basis there are transfers from SWF to the government budget. Monetary policy is independently conducted by the National Bank pursuing price stability goal and intervening in the foreign exchange market to avoid speculative attacks.

The welfare measure is represented as a loss function of three variables according to De Paoli (2009) for a small open economy: variations in inflation, output, and real exchange rate. Based on this loss measure, two Taylor rule's parameters are searched at managed and flexible exchange rate regime, distinguished by the presence of foreign exchange interventions. Neutral fiscal policy, associated with the zero oil output response of public spending, is taken as a benchmark to calculate loss in deviation from it; thus, pro-/countercyclical fiscal stance corresponds to the positive/negative oil output response of public spending respectively. The impulse

response functions to a fall in world oil price shock, referred to an adverse terms of trade shock throughout as well, are analyzed at flexible and rigid prices.

In section two, the model is outlined with two types of households, standard optimizers and rule-of-thumb households, non-oil firms acting in a monopolistically competitive market, oil sector owned by the foreigners and government, two monetary policy rules for each its instrument, and respective fiscal policy rules. Section three describes the calibrated values for parameters, the list of which is provided in Appendix A. Section four examines the main results followed by conclusion.

## 2 Model

The model has several frictions: an incomplete asset market, investment adjustment costs, collateral constraint, and the Calvo price setting. The crucial underlying assumption is that the foreign world is a saver, while the domestic economy is a borrower; thus, foreign discount factor is higher than domestic discount factor as the domestic households might be relatively impatient compared to the rest of the world. This assumption implies in turn that the interest rate of an emerging economy is always higher than the foreign interest rate, which is consistent with the evidence.

There are two producers in the model: oil firms and non-oil firms. The foreigners and government own the oil firms. Capital-intensive oil production has only capital input affected by FDI that responds to the world oil price which has an exogenous shock as a terms of trade shock. The non-oil firms are monopolistically competitive and set prices on their intermediate goods à la Calvo (1983); their profits are transferred to optimizing households. The government share of oil profits together with taxes on oil sector accumulates the SWF, while the remainder of oil profits goes to the foreigners. The interest income of SWF represents the oil budget revenues interpreted as the SWF transfers discretionary made to the government budget.

Since there are two types of households, only optimizers borrow from abroad and have a collateral constraint on non-oil physical capital. They also hold the domestic government bonds, own the non-oil firms, rent physical capital to these firms, and decide about investment. The liquidity constrained (rule-of-thumb) households consume their wages each period. The labor market is assumed to be competitive, without unions or high households' bargaining power over wages in the emerging market setting.

The CPI Taylor rule includes the lagged interest rate, CPI inflation, and output, yet there is also a rule for the foreign exchange interventions responding to the exchange rate and its change according to Sarno and Taylor (2001). The PPT Taylor

rule, in contrast, involves the oil price inflation and domestic price inflation weighted by the oil and non-oil GDP shares respectively. Public consumption and public investment rules have fiscal debt, oil revenues of the government budget, and oil output to capture pro-/countercyclical and neutral fiscal policy. Public investment is productive in accumulating public capital, which is an additional input in the Cobb-Douglas non-oil production function beyond labor and physical capital.

The foreign world is modeled by its Phillips curve, AR (1) process for output, and the Taylor rule for interest rate. All foreign variables are denoted by an asterisk in this paper.

## 2.1 Households

The economy is populated by a continuum of households on the interval  $[0,1]$ , where the fraction  $\mu$  is rule-of-thumb households. They do not have access to financial markets and consume all of their disposable income each period. In other words, they act myopically without any effect of a future policy on their economic decisions. The other  $(1 - \mu)$  fraction of households are forward-looking optimizers who hold government bonds, borrow from abroad, invest in non-oil physical capital, rent that capital to the non-oil firms, and receive profits from those monopolistic firms. The labor market is competitive, wage is the same across all households, and both types of households work the same number of hours. The superscript  $S$  indicates a variable associated with savers (optimizers), while  $N$  is for non-savers (rule-of-thumb households).

The optimizing household maximizes its utility (Schmitt-Grohe & Uribe, 2003):

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{[C_t^S - \phi^{-1} N_t^\phi]^{1-\sigma} - 1}{1-\sigma}, \quad \phi > 1, \sigma > 1 \quad (1)$$

subject to the following budget constraint:

$$C_t^S + I_t + b_t + R_{t-1}^* \frac{RER_t}{RER_{t-1}} \frac{b_{t-1}^*}{\pi_t^*} + T_t^S = W_t N_t + R_t^{kno} K_{t-1}^{kno} + R_{t-1} \frac{b_{t-1}}{\pi_t} + b_t^* + \Pi_t, \quad (2)$$

where  $b_t = \frac{B_t}{P_t}$  is the real purchase of government bonds,  $RER_t$  is a real exchange rate (the price of foreign goods basket in terms of the domestic goods basket),  $b_t^* = RER_t \frac{B_t^*}{P_t^*}$  is the real foreign borrowings expressed in domestic goods,  $R_{t-1}$  and  $R_{t-1}^*$  are the nominal gross domestic and foreign interest rates respectively,  $T_t^S$  is the real lump-sum taxes,  $W_t$  is a real wage,  $R_t^{kno}$  is the real rental cost of non-oil physical capital,  $\pi_t = \frac{P_t}{P_{t-1}}$  is inflation, and  $\Pi_t$  is the real profits of monopolistic

non-oil firms<sup>2</sup>.

The law of motion for non-oil capital is specified according to Berg, Portillo, Yang, and Zanna (2013) incorporating the investment adjustment costs:

$$K_t^{no} = (1 - \delta)K_{t-1}^{no} + \left[ 1 - \frac{\kappa}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 \right] I_t, \quad \text{where } \kappa > 0 \quad (3)$$

The collateral constraint relates gross foreign liabilities to a future value of capital (Faia & Iliopoulos, 2011) and always binds, assuming that foreign debt is permanently high in this economy<sup>3</sup>:

$$R_t^* b_t^* = E_t \left\{ \Omega \frac{Q_{t+1} \pi_{t+1}^*}{RE R_{t+1} / RE R_t} K_t^{no} \right\}, \quad (4)$$

where  $Q_t$  is a real shadow value of capital (Tobin's Q) and  $\Omega$  is an upper bound of leverage ratio.

The problem of optimizing household is, therefore, to maximize its utility (1) with respect to consumption  $C_t^S$ , investment  $I_t^S$ , capital  $K_t^{no}$ , government bonds holdings  $b_t$ , foreign borrowings  $b_t^*$ , and hours worked  $N_t$  subject to the budget constraint (2), capital accumulation equation (3), and collateral constraint (4). The first-order conditions of this problem are in Appendix B.

The rule-of-thumb household has the same preferences as the optimizer. It chooses only consumption and labor and its budget constraint is simply this:

$$C_t^N + T_t^N = W_t N_t \quad (5)$$

Each  $i \in \{S, N\}$  type of households has the composite CES consumption preference over domestic and foreign goods with  $\eta > 0$  as an elasticity of substitution between goods:

$$C_t(i) = \left[ \gamma^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}}(i) + (1 - \gamma)^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}}(i) \right]^{\frac{\eta}{\eta-1}},$$

where  $\gamma$  is a home-bias parameter, while  $(1 - \gamma)$  is a degree of openness. The standard consumption expenditures minimization by a household delivers the following CPI

<sup>2</sup> $\Pi_t = Y_t^{no}(p_{h,t} - MC_t)$ , where  $Y_t^{no}$  is non-oil output,  $p_{h,t}$  is the relative domestic price of non-oil goods to composite consumption, and  $MC_t$  is the real marginal costs of non-oil firms.

<sup>3</sup>Occasionally binding collateral constraint is ruled out because it requires global solution methods that may be infeasible to apply in this complex model.

index:

$$P_t^{1-\eta} = \gamma P_{h,t}^{1-\eta} + (1-\gamma)P_{f,t}^{1-\eta} \quad \text{or} \quad 1 = \gamma p_{h,t}^{1-\eta} + (1-\gamma)RER_t^{1-\eta}, \quad (6)$$

where  $p_{h,t}$  is a relative price of domestic goods to composite consumption and  $RER_t$  is a relative price of foreign goods to composite consumption.

The aggregate consumption in turn is  $C_t = \mu C_t^N + (1-\mu)C_t^S$ . Similar to private consumption, investment is the CES basket with the same home-bias parameter  $\gamma$  and CPI price index for simplicity.

## 2.2 Firms

Following Gali, Lopez-Salido, and Valles (2007), there are monopolistically competitive non-oil firms producing differentiated intermediate goods, and a perfectly competitive non-oil firm producing a final domestic good. The final domestic non-oil producer has a constant returns technology:

$$Y_t^{no} = \left( \int_0^1 X_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}},$$

where  $X_t(j)$  is the input amount of intermediate good  $j$  and  $\varepsilon > 1$  is the elasticity of substitution between differentiated intermediate goods. It maximizes profit taking as given the domestic final goods price  $P_t^h$  and intermediate goods prices  $P_t^h(j)$  such that the optimal demand allocation is as follows:

$$X_t(j) = \left( \frac{P_t^h(j)}{P_t^h} \right)^{-\varepsilon} Y_t^{no} \quad (7)$$

Each intermediate goods non-oil firm has the identical Cobb-Douglas production function, which includes the private non-oil capital, labor, and public capital:

$$Y_t^{no}(j) = u^{no} K_{t-1}^{no}(j)^\alpha N_t(j)^{1-\alpha} K_{G,t-1}^\psi, \quad (8)$$

where the level of technology  $u^{no}$  is just constant and the usage of public capital are common to all firms.

Intermediate goods producers solve their problem in two stages. First, cost minimization subject to the production function (8) provides the following real marginal costs common to all non-oil firms, taking the real wage and rental cost of capital as



given:

$$MC_t = \frac{W_t^{1-\alpha} (R_t^{kno})^\alpha}{u^{no} K_{G,t-1}^\psi (1-\alpha)^{1-\alpha} \alpha^\alpha} \quad (9)$$

Second, intermediate non-oil producers choose the price  $P_t^{hop}$  to maximize their discounted real profits:

$$\sum_{m=0}^{\infty} \theta^m E_t \left\{ D_{t,t+m} Y_{t+m}^{no}(j) \left( \frac{P_t^{hop}}{P_{t+m}^h} - MC_{t+m} \right) \right\}, \quad (10)$$

where  $D_{t,t+m} = \beta^m E_t \left( \frac{U_{C_{t+m}^S}}{U_{C_t^S}} \right)$  is a stochastic discount factor coming from the optimizing household's problem, subject to the demand constraint according to (7):

$$Y_{t+m}^{no}(j) = \left( \frac{P_t^{hop}}{P_{t+m}^h} \right)^{-\varepsilon} Y_{t+m}^{no}$$

In other words, a fraction  $(1 - \theta)$  of non-oil firms adjusts their prices each period, while the respective fraction  $\theta$  keeps their prices unchanged; thus,  $\theta$  is an index of price stickiness according to Calvo (1983). The domestic price index, therefore, evolves as follows:

$$(P_t^h)^{1-\varepsilon} = \theta (P_{t-1}^h)^{1-\varepsilon} + (1 - \theta) (P_t^{hop})^{1-\varepsilon}$$

The first-order condition of this price setting decision (10) is below:

$$\sum_{m=0}^{\infty} \theta^m E_t \left\{ D_{t,t+m} Y_{t+m}^{no}(j) \left( \frac{P_t^{hop}}{P_{t+m}^h} - \frac{\varepsilon}{\varepsilon - 1} MC_{t+m} \right) \right\} = 0, \quad (11)$$

where  $\frac{\varepsilon}{\varepsilon - 1}$  is a frictionless price markup.

The production function of oil firm has only capital input assuming that oil production is a capital-intensive sector and to avoid any complication originating from possible labor mobility between two sectors:

$$Y_t^o = (K_{t-1}^o)^{\alpha^o} \quad (12)$$

The oil capital is accumulated by FDI which responds to the world oil price:

$$K_t^o = (1 - \delta) K_{t-1}^o + FDI_t^* \quad (13)$$

$$\widehat{FDI}_t^* = \rho_{FDI} \widehat{FDI}_{t-1}^* + (1 - \rho_{FDI}) \widehat{P}_t^{o*} \quad (14)$$

Hats, hereafter, denote the deviation of variables from their steady state.

The world oil price follows AR(1) process and has an exogenous shock referred as the terms of trade shock:

$$\widehat{P}_t^{o*} = \rho_o \widehat{P}_{t-1}^{o*} + \epsilon_t^o \quad (15)$$

The oil firm receives its profits  $\Pi_t^{o*}$  net of royalties levied on production quantity at a rate  $\tau^o$ :

$$\Pi_t^{o*} = (1 - \tau^o) P_t^{o*} Y_t^o \quad (16)$$

The oil sector is owned by the foreigners and government. The dividend share of oil profits that the government receives is denoted by  $\iota^{\text{div}}$ .

### 2.3 Fiscal policy

The government collects lump-sum taxes  $T_t$  and oil revenues  $OR_t$  as the transfers from SWF. It issues one-period bonds to finance the government purchases which include public consumption  $G_t^C$  and public investment  $G_t^I$ . The government budget constraint in real terms can be written as follows:

$$(1 - \mu)b_t + T_t + \underbrace{\left( R_{t-1}^* \frac{1}{\pi_t^*} - \rho_{swf} \right) SWF_{t-1}^* RER_t}_{OR_t} = p_t^g (G_t^C + G_t^I) + (1 - \mu) R_{t-1} \frac{b_{t-1}}{\pi_t}, \quad (17)$$

where  $T_t = (1 - \mu)T_t^S + \mu T_t^N$  and  $p_t^g$  is a relative price of government purchases to composite consumption with its own home-bias parameter  $\gamma_2$ .

$$p_t^g = \left[ \gamma_2 p_{h,t}^{1-\eta} + (1 - \gamma_2) RER_t^{1-\eta} \right]^{\frac{1}{1-\eta}} \quad (18)$$

Public investment is productive so that the law of motion for public capital is given by:

$$K_t^G = (1 - \delta^g) K_{t-1}^G + G_t^I \quad (19)$$

Oil taxes, collected in foreign currency, consist of royalties and government share of oil sector's profits

$$T_t^{o*} = \tau^o P_t^{o*} Y_t^o + \iota^{\text{div}} \Pi_t^{o*} \quad (20)$$

which go directly to SWF accumulated according to the equation below.

$$SWF_t^* = \rho_{swf} SWF_{t-1}^* + T_t^{o*} \quad (21)$$

Two fiscal instruments, public investment and public consumption, have the following rules with their oil output response ( $\vartheta_{GI}$  and  $\vartheta_{GC}$ ) associated with fiscal cyclicity:

$$\widehat{G}_t^I = \rho_{GI}\widehat{G}_{t-1}^I + (1 - \rho_{GI})[\vartheta_{GI}\widehat{Y}_t^o - \gamma_{GI}\widehat{b}_{t-1} + \gamma_{OR}^{GI}\widehat{OR}_t] \quad (22)$$

$$\widehat{G}_t^C = \rho_{GC}\widehat{G}_{t-1}^C + (1 - \rho_{GC})[\vartheta_{GC}\widehat{Y}_t^o - \gamma_{GC}\widehat{b}_{t-1} + \gamma_{OR}^{GC}\widehat{OR}_t] \quad (23)$$

Since fiscal debt clears the government budget constraint, the lump-sum taxes require a separate equation, which includes fiscal debt, public spending similar to Gali, Lopez-Salido, and Valles (2007), and oil revenues specific to this model:

$$\widehat{T}_t = \varphi_b\widehat{b}_{t-1} + \varphi_I\widehat{G}_t^I + \varphi_C\widehat{G}_t^C - \varphi_{OR}\widehat{OR}_t \quad (24)$$

## 2.4 Monetary policy

The nominal interest rate responds to its lagged value, CPI inflation, and aggregate output according to CPI targeting Taylor rule below:

$$\widehat{R}_t = \rho\widehat{R}_{t-1} + (1 - \rho) \left[ \phi_\pi\pi_t + \phi_y\widehat{Y}_t \right] \quad (25)$$

The PPT Taylor rule, in contrast, has the product price inflation which is a weighted average of oil price inflation in real terms  $\pi_t^o = \Delta \widehat{P}_t^{o*} + \Delta \widehat{REER}_t$  and domestic price inflation  $\pi_t^h = \pi_t - \frac{1-\gamma}{\gamma} \Delta \widehat{REER}_t$  according to Appendix D, with weights corresponding to GDP share of oil  $s_o$  and non-oil sector  $(1 - s_o)$  respectively:

$$\widehat{R}_t = \rho\widehat{R}_{t-1} + (1 - \rho) \left[ \phi_\pi [s_o\pi_t^o + (1 - s_o)\pi_t^h] + \phi_y\widehat{Y}_t \right] \quad (26)$$

Foreign exchange interventions represent the purchases/selling of foreign currency by a central bank and accumulate its reserves according to their separate rule (Benes et al., 2015), responding to the exchange rate and its rate of depreciation<sup>4</sup>.

$$\widehat{fxr}_t^* = \rho_{fxr}\widehat{fxr}_{t-1}^* + (1 - \rho_{fxr})(\alpha_1\widehat{REER}_t + \alpha_2 \Delta \widehat{REER}_t), \quad \alpha_1 < 0, \quad \alpha_2 < 0, \quad (27)$$

where  $\widehat{fxr}_t^* = REER_t \frac{FXR_t^*}{P_t^*}$  is the real foreign exchange reserves expressed in domestic goods.

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<sup>4</sup>The higher  $\widehat{REER}_t$ , the more real exchange rate depreciates.

## 2.5 Market clearing conditions

The domestic/non-oil goods market clearing condition is as follows:

$$p_t^h Y_t^{no} = \gamma [C_t + (1 - \mu)I_t] + \gamma_2 p_t^g (G_t^C + G_t^I) \quad (28)$$

The real GDP from supply and demand sides is this:

$$p_t^h Y_t^{no} + Y_t^o P_t^{o*} RER_t = C_t + (1 - \mu)I_t + p_t^g (G_t^C + G_t^I) + NX_t \quad (29)$$

The labor and capital markets clear according to these conditions:

$$N_t = \int_0^1 N_t(j) dj, \quad K_t^{no} = \int_0^1 K_t^{no}(j) dj$$

The balance of payments is below, equalizing its current account with its financial account. The current account includes net exports, interest income of SWF assets, as those assets are saved abroad, minus the foreign share of oil sector's profits, while the financial account represents the foreign borrowings of optimizers and FDI.

$$\begin{aligned} NX_t + (R_{t-1}^* \frac{1}{\pi_t^*} - \rho_{swf}) SWF_{t-1}^* RER_t - (1 - \iota^{\text{div}}) RER_t \Pi_t^{o*} = \\ = (1 - \mu) \left( R_{t-1}^* \frac{RER_t}{RER_{t-1}} \frac{b_{t-1}^*}{\pi_t^*} - b_t^* \right) - RER_t FDI_t^* \end{aligned}$$

## 2.6 The rest of the world

The rest of the world is a relatively large economy governed by three exogenous equations below:

$$\widehat{Y}_t^* = \rho_{Y^*} \widehat{Y}_{t-1}^* + \epsilon_t^{Y^*} \quad (30)$$

$$\widehat{R}_t^* = \phi_\pi^* \pi_t^* + \phi_y^* \widehat{Y}_t^* \quad (31)$$

$$\pi_t^* = \beta^* E_t \pi_{t+1}^* + \lambda^* \left( \sigma + \frac{\phi^* + \alpha^*}{1 - \alpha^*} \right) \widehat{Y}_t^* \quad (32)$$

In total, the model includes 27 variables constituting a system of 27 equations, where the variables are represented in log-deviation from their steady state: inflation  $\pi_t$ , the aggregate consumption of households  $\widehat{C}_t$ , hours worked  $\widehat{N}_t$ , domestic interest rate  $\widehat{R}_t$ , net exports  $\widehat{NX}_t$ , foreign exchange reserves  $\widehat{fxr}_t^*$ , foreign interest rate  $\widehat{R}_t^*$ , foreign inflation  $\pi_t^*$ , foreign output  $\widehat{Y}_t^*$ , foreign debt  $\widehat{b}_t^*$ , oil capital  $\widehat{K}_t^o$ , non-oil capital  $\widehat{K}_t^{no}$ , public capital  $\widehat{K}_{G,t}$ , real exchange rate  $\widehat{RER}_t$ , fiscal debt  $\widehat{b}_t$ , lump-sum taxes  $\widehat{T}_t$ , public consumption  $\widehat{G}_t^C$ , public investment  $\widehat{G}_t^I$ , private investment  $\widehat{I}_t$ , oil output  $\widehat{Y}_t^o$ , non-oil output  $\widehat{Y}_t^{no}$ , aggregate output  $\widehat{Y}_t$ , domestic prices  $\widehat{p}_t^h$ , government purchases'

prices  $\widehat{p}_t^g$ , SWF assets  $\widehat{SWF}_t$ , FDI  $\widehat{FDI}_t^*$ , and the world oil price  $\widehat{P}_t^{o*}$ . The system of log-linear equations consists of the Taylor rule (25), foreign exchange interventions rule (27), public investment rule (22), public consumption rule (23), lump-sum taxes equation (24), FDI process (14), world oil price equation (15), three foreign world expressions (30, 31, and 32), the Phillips curve in Appendix D, and the other 16 equations shown in Appendix E.

### 3 Calibration

The model is calibrated using the averages of Kazakhstani data over 1995Q1-2012Q2 for the steady state values of endogenous variables derived in Appendix C. The data are available from the webpages of the National Bank, Ministry of Finance, Agency of Statistics, and International Financial Statistics. They include real GDP, private consumption, public consumption, fixed capital formation, net exports, oil output, wages, effective T-bill rate, CPI, real US dollar-tenge exchange rate, the external debt of banks and other private sectors, FDI, petroleum UK Brent price, public debt, fiscal capital expenditures, oil and non-oil revenues of the government budget, SWF assets as a stock variable and SWF inflows. The list of calibrated parameters is provided in Appendix A excluding GDP ratios and parameters for the rest of the world which are described in this section.

All parameters can be divided into three sets: standard values borrowed from other studies because of the non-availability of relevant data, estimates from time-series regressions run according to this model's equations, and parameters calibrated based on a steady state of the model. The first set includes the depreciation rates for private and public capital  $\delta = 0.025$ ,  $\delta^g = 0.02$  (Traum & Yang, 2011), the elasticity of substitution between differentiated intermediate goods  $\varepsilon = 9$  (Gali, 2015), price stickiness  $\theta = 0.9$  (Jakab & Vilagi, 2008), the inverse of intertemporal elasticity of substitution for consumption  $\sigma = 2$  (Schmitt-Grohe & Uribe, 2003), investment adjustment costs parameter  $\kappa = 20$  (Berg, Portillo, Yang & Zanna, 2013), and the fiscal debt response of lump-sum taxes  $\varphi_b = 0.4$  (Algozhina, 2012). The foreign parameters are set to their standard values: the elasticity of wages with respect to hours worked  $\phi^* = 1.45$  (Schmitt-Grohe & Uribe, 2003), discount factor  $\beta^* = 0.99$ , inflation  $\phi_\pi^* = 1.5$  and output response in the Taylor rule  $\phi_y^* = 0.125$  (Gali, 2015),  $\theta^* = 0.75$  (Gali, Lopez-Salido & Valles, 2007), output elasticity to capital  $\alpha^* = 0.32$ , and output persistence  $\rho_{Y^*} = 0.8$ .

The second set consists of significant regression estimates according to model's equations, using the seasonally adjusted log of real variables. In particular, the

seasonally adjusted log of real public consumption is regressed on its lagged value, log of oil output, log of real lagged public debt, and real oil revenues of government budget according to the public consumption rule (23). The estimates are as follows with t-statistics in parenthesis, suggesting  $\rho_{GC} = 0.53$  and  $\vartheta_{GC} = \frac{0.14}{1-0.53} = 0.3$ .

$$G_t^C = 0.53G_{t-1}^C + 0.14Y_t^o + 0.03b_{t-1} - 0OR_t \quad \text{Adj. R-sq. 0.915, F-stat 119.7} \\ (3.79) \quad (3.42) \quad (0.63) \quad (-0.07) \quad \text{DW 1.86, N 45 obs.}$$

The regression of public investment rule (22), which is proxied by the data of fiscal capital expenditures, gives its zero persistence  $\rho_{GI} = 0$ , fiscal debt response  $\gamma_{GI}$  of 0.3 that is set for  $\gamma_{GC}$  as well, and oil output response  $\vartheta_{GI}$  of 0.54.

$$G_t^I = 0.2G_{t-1}^I + 0.54Y_t^o - 0.3b_{t-1} + 0OR_t \quad \text{Adj. R-sq. 0.85, F-stat 65.5} \\ (1.4) \quad (4.4) \quad (-2.4) \quad (0.83) \quad \text{DW 2.2, N 45 obs.}$$

According to the lump-sum taxes equation (24), the regression of non-oil fiscal revenues on public consumption, public investment, fiscal debt, and oil revenues of the government budget produces the significant response to public consumption  $\varphi_C = 1$  and public investment  $\varphi_I = 0.2$ .

$$T_t = 0.2G_t^I + G_t^C - 0.17b_{t-1} - 0OR_t \quad \text{Adj. R-sq. 0.83, F-stat 55.4} \\ (2.4) \quad (4.7) \quad (-1.75) \quad (-0.52) \quad \text{DW 1.2, N 45 obs.}$$

The CPI and PPT Taylor rules both give the only significant parameter of interest rate smoothing  $\rho = 0.95$ . The T-bill rate is regressed on its lagged value, CPI inflation, and log of seasonally adjusted real GDP, while the PPT Taylor rule (26) has the world oil price inflation and producer price index inflation.

$$CPI : R_t = 0.95R_{t-1} - 0.02\pi_t - 0.37Y_t \quad \text{Adj. R-sq. 0.98, F-stat 849} \\ (19) \quad (-0.52) \quad (-0.54) \quad \text{DW 1.6, N 57 obs.}$$

$$PPT : R_t = 0.95R_{t-1} - 0.03\pi_t^o + 0.04\pi_t^h - 0.45Y_t \quad \text{Adj. R-sq. 0.98, F-stat 661} \\ (19.2) \quad (-1.7) \quad (1.4) \quad (-0.66) \quad \text{DW 1.72, N 57 obs.}$$

The log of foreign exchange interventions regression gives the exchange rate response  $\alpha_1 = -0.18$  and exchange rate change response  $\alpha_2 = -0.57$ , while the persistence parameter is set to  $\rho_{fxr} = 0.7$  according to Benes et al. (2015). These values for  $\alpha_1$  and  $\alpha_2$  are associated with managed exchange rate regime supported by the foreign exchange interventions as an additional monetary policy instrument, while they are set to zero when pure inflation targeting at flexible exchange rate regime is examined.

$$fxr_t^* = fxr_{t-1}^* - 0.18RER_t - 0.57(RER_t - RER_{t-1}) \quad \text{Adj. R-sq. 0.98, F-stat 1458} \\ (56.8) \quad (-2.3) \quad (-1.88) \quad \text{DW 2.2, N 69 obs.}$$

The autoregressive coefficient in the log of world oil price regression suggests to be  $\rho_o = 0.98$  with standard deviation of its shock 0.15 used to plot the impulse response functions in Appendix F and G.

$$P_t^{o*} = 0.98P_{t-1}^{o*} + \epsilon_t^o \quad \text{Adj. R-sq. 0.95, F-stat 1481}$$

(38.5) (s.d. 0.15) DW 1.4, N 73 obs.

The FDI equation (14) corresponds to the following empirical counterpart with its persistence parameter  $\rho_{FDI} = 1 - 0.8 = 0.2$  given the significant effect of world oil price.

$$FDI_t^* = 0.3FDI_{t-1}^* + 0.8P_t^{o*} \quad \text{Adj. R-sq. 0.53, F-stat 23.7}$$

(1.96) (3.2) DW 2.1, N 41 obs.

The third set includes the parameters calibrated to a steady state of the model which corresponds to data averages<sup>5</sup>. The GDP ratios of consumption, public consumption, net exports, FDI, foreign debt, fiscal debt, oil output, and public investment are as follows respectively:  $c_y = 0.61$ ,  $g_y^C = 0.08$ ,  $nx_y = 0.07$ ,  $fdi_y = 0.09$ ,  $b_y^* = 2.17$ ,  $b_y = 0.5$ ,  $s_o = 0.52$ , and  $g_y^I = 0.07$ . The degree of openness is calculated as a ratio of imports to GDP,  $1 - \gamma = 0.32$ ; thus, home-bias parameter in private consumption and investment  $\gamma$  is equal to 0.68, while it is assumed higher for public spending  $\gamma_2 = 0.9$  as its large share may go to wages of public servants. The domestic discount factor is around 0.978 because the average T-bill rate is used as a proxy for policy interest rate, 2.3 percent per quarter<sup>6</sup>. The upper bound of leverage ratio  $\Omega$  appears to be 0.54. The elasticity of output with respect to private capital  $\alpha$  is equal to 0.3 as a share of capital income to GDP, while with respect to public capital is  $\psi = 0.16$  generated by a steady state wage equation in Appendix C. Using data on wages, the elasticity of wages with respect to hours worked  $\phi$  is 1.45 according to the labor supply condition (38), in which hours are obtained from the non-oil production function (8). The royalties rate levied on oil production quantity  $\tau^o = 0.27$  is calculated as the SWF inflows share in oil output. The dividend share of oil profits that the government receives  $\iota^{\text{div}}$  is set to 0.05, while the elasticity of oil output with respect to oil capital  $\alpha^o$  is technically feasible at 0.7. The persistence in SWF process  $\rho_{swf}$  is equal to 0.755 to match the GDP ratio of SWF assets  $swf_y = 0.65$ .

There are three types of fiscal policy: procyclical, countercyclical, and neutral. The neutral fiscal policy is a benchmark to calculate loss in deviation from it. It is associated with the zero oil output response of public consumption and public investment in their rules ( $\vartheta_{GC} = 0$  and  $\vartheta_{GI} = 0$ ). The procyclical fiscal policy corresponds to the positive oil output response of public spending, as the actual data suggest according to regressions above ( $\vartheta_{GC} = 0.3$  and  $\vartheta_{GI} = 0.54$ ), while the

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<sup>5</sup>The steady state is natural and inefficient, since it is at flexible prices and with monopolistic competition outlined in Appendix C.

<sup>6</sup>The domestic interest rate matters for the government bonds in this model, as investments are financed by the foreign funds rather than domestic financial market.

countercyclical fiscal policy is simulated at their negative values ( $\vartheta_{GC} = -0.3$  and  $\vartheta_{GI} = -0.54$ ). These are the two parameters which differ across fiscal cyclicity, while the rest hold the same. The response of public consumption to oil revenues of government budget is set to 0.2,  $\gamma_{OR}^{GC} = 0.2$  fixing it slightly lower than  $\vartheta_{GC} = 0.3$ , whereas  $\gamma_{OR}^{GI} = 0.1$ <sup>7</sup>. The oil revenues response of lump-sum taxes  $\varphi_{OR}$  appears to be -0.3 according to the taxes rule at steady state<sup>8</sup>.

$$\varphi_{OR} = \frac{\varphi_b \ln \bar{b} + \varphi_I \ln \bar{G}_I + \varphi_C \ln \bar{G}_C - \ln \bar{T}}{\ln \bar{OR}}$$

## 4 Results

According to De Paoli (2009), welfare as a second-order approximation of households' utility is reduced to a loss function consisting of variances in inflation, output, and real exchange rate for a small open economy. In this model, such a loss measure, represented as an equal summation of those three variances, is used to search the two parameters of Taylor rule: inflation  $\phi_\pi$  and output response  $\phi_y$  across pro-/countercyclical and neutral fiscal policy. Monetary policy can be hybrid combining managed exchange rate regime (MER) with CPI/PPT anchor or pure inflation targeting (IT) associated with CPI/PPT without foreign exchange interventions, thus corresponding to flexible exchange rate regime.

Table 1 summarizes the results of this search, according to which fiscal policy cyclicity and exchange rate regime do not matter for the monetary rule's parameters, whereas the inflation response of PPT anchor is higher than under CPI targeting. This is because PPT Taylor rule includes the oil price inflation which is volatile in the presence of its shock and needs to be properly stabilized to achieve the low variations in output and exchange rate as the components of loss function. The output response appears to be close to its standard value of 0.125 commonly accepted by literature (Gali, 2015) given that this search is in a range between 0 and 2 with a step of 0.1 for parameters.

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<sup>7</sup>This value for  $\gamma_{OR}^{GI} = 0.1$  is set to obtain the positive welfare coefficients under procyclical fiscal policy. The utility-based welfare of both types of households to its second-order approximation includes the quadratic terms in aggregate consumption and hours worked, their linear terms, and their joint term. The coefficients in front of quadratic terms especially need to be positive.

<sup>8</sup>This parameter is calculated using the positive value of steady state lump-sum taxes at neutral fiscal stance.



Table 1. **Monetary policy parameters and fiscal stance**

Pro-/countercyclical/neutral fiscal policy		
MER/IT		
	CPI	PPT
$\phi_\pi$	0.9	1.9
$\phi_y$	0.1	0.1

At these policy parameters provided by Table 1, the impulse response functions to a negative world oil price shock at flexible prices are analyzed first to understand the real channels of transmission mechanism<sup>9</sup>. Figure 1 in Appendix F shows that a sudden drop of oil price depreciates the exchange rate because the cash flows in foreign currency from oil output decrease, creating the excessive supply of domestic currency which results in its value loss. The depreciated exchange rate makes the imported goods expensive, foreign debt burden higher, and taxes higher because of the increased oil revenues of government budget in real terms, which all discourage hours worked as labor income declines. The reduced labor, as a main production input, contributes to a fall in non-oil output. Since the final goods output and income is low, private consumption drops decreasing the domestic prices; thus, aggregate output falls as well. The exchange rate depreciation and low consumption, which is associated with a reduction in domestic absorption, improve the net exports. Over time, there is a prolonged recession as all variables decline, while fiscal debt accumulates as automatic stabilizers suggest.

The dynamics of inflation at flexible prices is different across fiscal cyclicity. When fiscal policy is neutral and countercyclical, hours worked have a dominant effect on inflation due to the Phillips curve, so that production costs matter for inflation. Figure 2 in Appendix F shows that inflation significantly drops causing a decrease in interest rate due to high inflation response in the Taylor rule. In the case of procyclical fiscal stance, the exchange rate depreciation leads to inflation emphasizing the pass-through effect in contrast (Figure 1 in Appendix F). This is because public spending positively links the oil output, which is basically external, to domestic non-oil economy by sort of "replicating" this exogenous cycle. In other words, procyclical fiscal policy acts like an additional booster of external shock, strengthening the imported flexible prices channel in inflation determination.

If neutral and procyclical fiscal policy produce the broadly similar impulse response functions, except inflation dynamics as described above, then the effects of

<sup>9</sup>Flexible prices or no nominal rigidities version corresponds to the price stickiness parameter close to zero,  $\theta = 0.0001$ .

countercyclical fiscal stance on exchange rate and foreign debt differ in Figure 3 of Appendix F. The exchange rate depreciates to a lesser extent because fiscal stimulus takes place, counteracting the oil output cycle and thus demanding more the domestic currency, which exerts the appreciating pressure. Foreign debt declines as non-oil output falls, requiring private investment financed by the foreign borrowings. Overall, the impulse response functions at flexible prices suggest that the exchange rate dynamics, its pass-through effect on inflation, and foreign debt behavior depend on fiscal policy cyclicity, whereas monetary policy does not matter for the real effects of terms of trade shock which stay robust across CPI/PPT rule<sup>10</sup>.

Nominal rigidities change the effect of world oil price shock on real exchange rate, private consumption, and taxes. When prices are rigid under neutral and countercyclical fiscal policy, the exchange rate adjustment is delayed in Figure 1 of Appendix G, while procyclical fiscal stance still produces the immediate exchange rate depreciation, but to a lesser extent than at flexible prices which grows over time in Figure 2 of Appendix G. The depreciated exchange rate raises taxes initially, due to the increased oil revenues of government budget in real terms, yet over time taxes decline because public spending procyclically falls in tandem with oil output, suggesting low fiscal revenues to finance it, especially in the environment of rigid prices. Since prices are rigid and inflation with interest rate stay low over time, private consumption rebounds around zero after an initial fall, contributing to the same pattern of aggregate output from its demand side. Neutral fiscal policy, in contrast, causes taxes to drop initially, then rise, following the dynamics of domestic prices and government purchases' prices, which fall due to a reduction in private consumption, then increase over time due to the exchange rate depreciation. Domestic prices determine the non-oil profits of households to be taxed, while government purchases' prices are positively linked to taxes through the government budget constraint. Countercyclical fiscal policy does not generate any changes for taxes, resulting in turn in a lower accumulation of fiscal debt in the long run (Figure 4 of Appendix G).

The dynamics of interest rate do not comply with F&C, who advocated the PPT because its interest rate would rise in response to a positive terms of trade shock, causing the exchange rate appreciation, thus countercyclically offsetting the favorable shock. It appears that the relative magnitude of monetary policy parameters matters for interest rate, but not CPI or PPT anchor per se, while the exchange rate

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<sup>10</sup>Regardless of monetary rules, the impulse response functions stay the same within pro-/countercyclical and neutral fiscal policy. Yet, as a technical note, neutral and procyclical fiscal stance combined with CPI/PPT monetary rule without foreign exchange interventions do not produce the feasible solution at flexible prices.

tends to depreciate in response to an adverse terms of trade shock, but not affected by interest rate itself. Since the loss minimizing inflation response is higher than output response in the Taylor rule, interest rate tightens the economy when there is inflation and stimulates demand when inflation declines. Neutral and countercyclical fiscal policy delay the exchange rate adjustment under rigid prices, therefore, seem to be not welfare preferred according to Table 2.

Table 2 summarizes the numerical results of loss measure  $L$  as an equal summation of variances in inflation, output, and real exchange rate. The results are produced at corresponding Taylor rule's parameters provided by Table 1 for the model with nominal rigidities. All entries are in percent deviation from a benchmark policy combination: neutral fiscal stance and CPI monetary anchor without foreign exchange interventions, i.e.,  $\phi_\pi = 0.9$  and  $\phi_y = 0.1$ . Positive values mean the percentage increase in loss relative to the benchmark, while negative values indicate lower loss contributed by a respective entry than the benchmark delivers.

Table 2. **Loss components**

	Procyclical fiscal policy				Countercyclical fiscal policy			
	MER		IT		MER		IT	
	CPI	PPT	CPI	PPT	CPI	PPT	CPI	PPT
$L$	-26.02	-14.88	-26.5	-15.51	3.6	20.95	3.68	20.77
$Var(\pi_t)$	-1.32	-0.57	-1.41	-0.28	0.18	1.3	0.06	1.17
$Var(\hat{Y}_t)$	-14.97	-8.86	-15.35	-7.82	3.13	13.06	2.84	12.57
$Var(\widehat{RER}_t)$	-9.72	-5.45	-9.74	-7.41	0.29	6.6	0.78	7.03

	Neutral fiscal policy		
	MER		IT
	CPI	PPT	PPT
$L$	0.62	17.57	16.8
$Var(\pi_t)$	0.12	1.23	1.08
$Var(\hat{Y}_t)$	0.53	10.11	9.42
$Var(\widehat{RER}_t)$	-0.03	6.23	6.29

Values are in percent deviation of corresponding entry from the benchmark neutral fiscal policy combined with flexible exchange rate regime and CPI targeting monetary rule.

Table 2 suggests several findings. Procyclical fiscal stance is preferred to countercyclical and neutral fiscal policy because the exchange rate immediately adjusts

in the former case and taxes fall in the long run, causing fiscal debt to accumulate in order to finance public spending. The prospective of low taxes and increased availability of government bonds allow households to properly smooth their consumption and investment, thus aggregate output is relatively well stabilized. This seems better than countercyclical and neutral fiscal stance, where the exchange rate depreciation is delayed and fiscal debt accumulates to a lesser extent over time.

Within each fiscal policy, the CPI targeting monetary rule is robustly preferred to PPT anchor. CPI inflation includes the imported goods' prices in a form of real exchange rate change, while PPT, instead, has the oil price inflation which is basically irrelevant for private consumption. Especially a small open economy with its emerging markets is highly dependent on imported goods, and commodity exporting benefits are not distributed among domestic households. Moreover, the impulse response functions of almost all variables under PPT rule show deeper effects than under CPI rule (Figure 3 in comparison with Figure 2 in Appendix G). This suggests that CPI monetary anchor stabilizes the economy better by cushioning the effects of adverse terms of trade shock. The interest rate of PPT, meanwhile, explicitly responds to that shock captured by the oil price inflation, therefore reacting more and causing higher variations in other variables.

According to Table 2, procyclical and neutral fiscal policy should be combined with CPI inflation targeting at flexible exchange rate regime, while countercyclical fiscal stance is better to couple with managed exchange rate regime and CPI rule as well. This is because foreign exchange interventions serve as an additional buffer to adjust, when exchange rate depreciation is delayed and taxes do not change at countercyclical fiscal stance. The foreign exchange reserves of central bank affect the interest rate, according to its uncovered interest rate parity condition (equation 49 in Appendix E), and private investment, due to collateral constraint (equation 42 in Appendix E), so that in tandem with countercyclical fiscal policy and CPI monetary rule have better stabilizing impact on private consumption and aggregate output from its demand side.

Across all types of fiscal policy, however, PPT with managed exchange rate regime is less preferred to PPT with flexible exchange rate regime. This is driven by the added effects of foreign exchange interventions on interest rate and private investment on top of the PPT rule itself, which, as outlined above, produces the deeper impulse response functions compared to CPI targeting; therefore, volatilities of variables rise in total.

In summary, the best policy combination is procyclical fiscal stance with CPI inflation monetary targeting at flexible exchange rate regime. A small open economy

with its emerging market structure and commodity exporting sector is better off to focus on consumer price stability and let its exchange rate to adjust without central bank's interventions, while fiscal policy can stay procyclical, as the evidence keeps suggesting for developing countries. Countercyclical fiscal policy, observed mostly in the advanced economies, seems not beneficial for those, where the exchange rate adjustment is needed on time, due to high imports share, and the government bonds remain the main domestic assets of private sector.

## 5 Conclusion

This paper develops the DSGE model for an emerging oil economy to study the loss minimizing monetary policy rule jointly with pro-/countercyclical and neutral fiscal policy. The model captures a set of structural specifics: two monetary instruments—interest rate and foreign exchange interventions, two fiscal instruments—public consumption and public investment, non-oil and oil producers with the exogenous world oil price shock, SWF accumulation, and the foreign debt of private sector to finance investment via collateral constraint. The constructed framework combines the New Keynesian model of a small open economy with the two types of households, optimizing individuals and rule-of-thumb households, relaxing the assumption of Ricardian equivalence, integrates the foreign exchange reserves into uncovered interest rate parity (UIP) condition according to Benes et al. (2015), and includes three equations of the rest of the world .

This study reveals the following findings along the joint analysis of monetary rules and fiscal cyclicity in a single oil exporting setting. The best policy combination is procyclical fiscal stance and CPI inflation monetary targeting without foreign exchange interventions. It allows the exchange rate to immediately adjust, the imports to be internalized by CPI monetary anchor, which well cushions the effects of terms of trade shock, and the fiscal taxes to properly adjust in order to bring more government bonds for households over time. The impulse response functions to the negative world oil price shock, as a sudden worsening of the terms of trade, show that monetary policy parameters matter for the interest rate dynamics, but not CPI or PPT anchor as F&C suggest. In fact, PPT rule is less preferred by causing higher variations in output and exchange rate. The volatile terms of trade can be stabilized by an appropriate domestic policy combination or, in other words, fiscal and monetary coordination to smooth their ultimate effects on aggregate output, real exchange rate, and inflation in a small open economy.

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## A Calibration

Parameter	Definition
$\beta = 0.978$	discount factor
$\gamma = 0.68$	home-bias in consumption and investment
$\gamma_2 = 0.9$	home-bias in government purchases
$\Omega = 0.54$	the upper bound of leverage ratio
$\mu = 0.5$	the fraction of rule-of-thumb households
$\alpha = 0.3$	non-oil output elasticity to private capital
$\psi = 0.16$	non-oil output elasticity to public capital
$\alpha^o = 0.7$	oil output elasticity to private capital
$\phi = 1.45$	wage elasticity to hours worked
$\sigma = 2$	the inverse of intertemporal elasticity of substitution for $C_t$
$\delta = 0.025$	the depreciation rate of private capital (oil and non-oil)
$\delta^g = 0.02$	the depreciation rate of public capital
$\theta = 0.9$	the index of price stickiness
$\varepsilon = 9$	the elasticity of substitution b/w differentiated intermediate goods
$\kappa = 20$	investment adjustment costs parameter
$\phi_y = 0.1$	output response in the Taylor rule
$\phi_\pi = 0.9$	inflation response in the Taylor rule
$\alpha_1 = -0.18$	exchange rate response in the intervention rule
$\alpha_2 = -0.57$	exchange rate change response in the intervention rule
$\tau^o = 0.27$	oil royalty rate
$\iota^{\text{div}} = 0.05$	the dividend share of oil profit accrued to the government
$\gamma_{GC} = \gamma_{GI} = 0.3$	the response of public consumption/investment to fiscal debt
$\vartheta_{GI} = 0.54$	the response of public investment to output
$\vartheta_{GC} = 0.3$	the response of public consumption to output
$\gamma_{OR}^{GC} = 0.2$	the response of public consumption to oil revenues
$\gamma_{OR}^{GI} = 0.1$	the response of public investment to oil revenues
$\varphi_b = 0.4$	the response of lump-sum taxes to fiscal debt
$\varphi_{OR} = -0.3$	the response of lump-sum taxes to oil revenues
$\varphi_C = 1$	the response of lump-sum taxes to public consumption
$\varphi_I = 0.2$	the response of lump-sum taxes to public investment
$\rho_{GC} = 0.53$	persistence in public consumption
$\rho_{GI} = 0$	persistence in public investment
$1 - \rho_{FDI} = 0.8$	FDI response to the world oil price
$\rho_{swf} = 0.775$	persistence in SWF process
$\rho = 0.95$	interest rate smoothing in the Taylor rule
$\rho_{fxr} = 0.7$	persistence in the foreign exchange reserves of a central bank
$\rho_o = 0.98$	persistence in the world oil price process
$\sigma_o = 0.15$	standard deviation of the world oil price shock

## B First-order conditions

The first-order conditions of optimizing household's problem are listed in this Appendix, where  $\lambda_t$ ,  $\lambda_t^k$ , and  $\lambda_t fxr_t^*$  are Lagrange multipliers to the budget constraint

(2), capital accumulation (3), and collateral constraint (4) respectively. Note that foreign exchange reserves  $fxr_t^*$  are involved in the Lagrange multiplier to collateral constraint in order to obtain UIP condition in line with Benes et al. (2015), who yet introduced it in an ad hoc fashion. The existence of collateral constraint in this model allows the UIP to base on micro foundations, since the foreign debt of optimizer denominated in foreign currency needs to be backed by the foreign exchange reserves denominated in foreign currency as well on a country level. Effectively, the Lagrange multiplier to collateral constraint  $\lambda_t fxr_t^*$  would suggest this rate of marginal utility produced by a change in foreign debt.

$$U_{C_t^S} = \lambda_t = \frac{1}{\left[ C_t^S - \frac{N_t^\phi}{\phi} \right]^\sigma} \quad (33)$$

$$\frac{1}{Q_t} = 1 - \frac{\kappa}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 - \kappa \left( \frac{I_t}{I_{t-1}} - 1 \right) \frac{I_t}{I_{t-1}} + \beta \kappa E_t \left\{ \frac{Q_{t+1} \lambda_{t+1}}{Q_t \lambda_t} \left( \frac{I_{t+1}}{I_t} - 1 \right) \left( \frac{I_{t+1}}{I_t} \right)^2 \right\}, \quad (34)$$

where  $Q_t = \frac{\lambda_t^k}{\lambda_t}$

$$Q_t = E_t \left\{ \beta \frac{\lambda_{t+1}}{\lambda_t} \left[ R_{t+1}^{kno} + Q_{t+1} (1 - \delta) \right] + fxr_t^* \Omega \frac{Q_{t+1} \pi_{t+1}^*}{RER_{t+1}/RER_t} \right\} \quad (35)$$

$$\frac{1}{R_t} = \beta E_t \left\{ \frac{U_{C_{t+1}^S}}{U_{C_t^S} \pi_{t+1}} \right\} \quad (36)$$

$$\frac{1}{R_t^*} = \beta E_t \left\{ \frac{U_{C_{t+1}^S}}{U_{C_t^S}} \frac{RER_{t+1}}{RER_t \pi_{t+1}^*} \right\} + fxr_t^* \quad (37)$$

$$W_t = N_t^{\phi-1} \quad (38)$$

By dividing (37) into (36), the following UIP condition is obtained:

$$\frac{R_t}{R_t^*} = E_t \left\{ \frac{RER_{t+1} \pi_{t+1}}{RER_t \pi_{t+1}^*} \right\} + \frac{fxr_t^*}{\beta} E_t \left\{ \frac{U_{C_t^S}}{U_{C_{t+1}^S}} \pi_{t+1} \right\} + cov_t, \quad (39)$$

where  $cov_t$  captures covariance terms.

The first-order conditions of rule-of-thumb household with respect to  $N_t$  and  $C_t^N$  are identical to the optimizer's solutions. Thus, non-saver faces the same labor supply condition (38).

## C Steady state

The model's steady state assumes its zero inflation, thus it is at flexible prices. Variables at steady state are denoted by bars and presented in this Appendix.

The first-order condition of optimizing household with respect to the government bonds (36) gives that  $\bar{R} = \frac{1}{\beta}$ , while with respect to the foreign debt (37) suggests

$\overline{fxr^*} = \beta^* - \beta$  at steady state. Similarly,  $\overline{R^*} = \frac{1}{\beta^*}$ .

The first-order condition of oil producer with respect to capital equalizes the marginal factor product to its price:

$$\alpha^o(1 - \tau^o)(\overline{K^o})^{\alpha^o-1} = \overline{R^{ok}} = \frac{1}{\beta} - (1 - \delta),$$

from which the steady state of oil capital can be found.

$$\overline{K^o} = \left[ \frac{1/\beta - (1 - \delta)}{\alpha^o(1 - \tau^o)} \right]^{\frac{1}{\alpha^o-1}}$$

Since oil capital is known, the oil output, FDI, and SWF are obtained from their respective equations (12), (14), and (16, 20, 21):

$$\overline{Y^o} = (\overline{K^o})^{\alpha^o}, \quad \overline{FDI^*} = \delta \overline{K^o}, \quad \overline{SWF} = \frac{[\tau^o + \iota^{\text{div}}(1 - \tau^o)]\overline{Y^o}}{1 - \rho_{swf}}$$

The law of one price holds so that the real exchange rate and relative prices at their steady state equal to 1.

The oil revenues of government budget are as follows:

$$\overline{OR} = (\overline{R^*} - \rho_{swf})\overline{SWF}\overline{RER}$$

The public capital accumulation equation (19) gives public investment at steady state:

$$\overline{G_I} = \delta^g \overline{K_G}$$

Fiscal debt is represented in terms of public capital, using the public investment equation (22) and the expression above:

$$\overline{b} = \left( \frac{\overline{Y^o}^{\vartheta_{GI}} \overline{OR}^{\gamma_{OF}^{GI}}}{\delta^g \overline{K_G}} \right)^{\frac{1}{\gamma_{GI}}}$$

Public consumption is as follows based on its rule (23), in which fiscal debt can be plugged into from the previous equation:

$$\overline{G_C} = \frac{\overline{Y^o}^{\vartheta_{GC}} \overline{OR}^{\gamma_{OF}^{GC}}}{\overline{b}^{\gamma_{GC}}}$$

The lump-sum taxes equation (24) suggests taxes at steady state:

$$\overline{T} = \frac{\overline{b}^{\varphi_b} \overline{G_I}^{\varphi_I} \overline{G_C}^{\varphi_C}}{\overline{OR}^{\varphi_{OR}}}$$

The government budget constraint (17) can be used to obtain public capital, if to substitute the fiscal debt, oil revenues, public consumption, and public investment

with their respective previous expressions:

$$\bar{T} + \overline{OR} = \overline{G_I} + \overline{G_C} + (1 - \mu)(\bar{R} - 1)\bar{b}$$

The first-order condition with respect to non-oil capital (35) yields the following rental cost of capital:

$$\overline{R^{kno}} = \frac{1}{\beta} - (1 - \delta) - \frac{\overline{fxr^*}\Omega}{\beta}$$

The price setting problem of non-oil firm suggests that real marginal costs (9) equate with the inverse of price frictionless mark-up  $\frac{\varepsilon}{\varepsilon - 1}$  at steady state; thus, wages are:

$$\bar{W} = (1 - \alpha) \left[ \frac{\overline{K_G}^\psi \alpha^\alpha (\varepsilon - 1)}{(\overline{R^{kno}})^{\alpha \varepsilon}} \right]^{\frac{1}{1 - \alpha}}$$

The labor supply condition (38) gives  $\bar{N} = \bar{W}^{\frac{1}{\phi - 1}}$ .

As aggregate output is a sum of non-oil and oil output  $\bar{Y} = \overline{Y_{no}} + \overline{REERY_o}$  =  $\overline{N}^{1 - \alpha} \overline{K_G}^\psi \overline{K_{no}}^\alpha + \overline{REERY_o}$ , the non-oil capital is obtained in terms of aggregate output:

$$\overline{K_{no}} = \left( \frac{\bar{Y} - \overline{REERY_o}}{\overline{N}^{1 - \alpha} \overline{K_G}^\psi} \right)^{\frac{1}{\alpha}}$$

The law of motion for capital (3) relates investment with non-oil capital:  $\bar{T} = \delta \overline{K_{no}}$ .

The collateral constraint (4) allows finding the foreign debt:

$$\bar{b}^* = \frac{\Omega \overline{K_{no}}}{\bar{R}^*}$$

The balance of payments equation provides net exports:

$$\overline{NX} = (1 - \mu) \left( \bar{R}^* - 1 \right) \bar{b}^* - \overline{REERFDI^*} - (\bar{R}^* - \rho_{swf}) \overline{SWFRER} + (1 - \iota^{\text{div}}) \overline{REER} (1 - \tau^\circ) \overline{Y^o}$$

The taxes of rule-of-thumb households are equal to:

$$\overline{T^N} = \frac{\bar{T} - (1 - \mu) \overline{T^S}}{\mu}$$

given that  $T_t = \mu T_t^N + (1 - \mu) T_t^S$ , while the taxes of optimizer can be derived from its budget constraint<sup>11</sup> (2), assuming that both types of households have equal consumption at steady state:

$$\overline{T^S} = \mu \left[ (\overline{R^{kno}} - \delta) \overline{K_{no}} + \bar{b}(\bar{R} - 1) + \bar{b}^*(1 - \bar{R}^*) + \left(1 - \frac{\varepsilon - 1}{\varepsilon}\right) \overline{Y^{no}} \right] + \bar{T}$$

<sup>11</sup>According to Benes et al. (2015), the budget constraint of households contains the cash-flow transfers from a central bank, which at steady state in this model would be equal to  $-\frac{(\beta^* - \beta)^2}{\beta^* \beta}$ , therefore represent the small number that wouldn't significantly affect the optimizer's taxes.

The budget constraint of rule-of-thumb household (5) provides its consumption  $\overline{C^N} = \overline{WN} - \overline{T^N}$ , which is assumed to be equal to optimizer's consumption, thus to aggregate consumption as well due to its summation of both types of household consumption:  $\overline{C} = \mu\overline{C^N} + (1 - \mu)\overline{C^S}$ .

The real GDP condition (29) can be utilized to derive the aggregate output by plugging into fiscal variables and the rest components, expressed in terms of output as well according to their steady state equations above:

$$\overline{Y} = \overline{C} + (1 - \mu)\overline{I} + \overline{G_C} + \overline{G_I} + \overline{NX}$$

## D The Phillips curve

The Phillips curve for CPI inflation in a small open economy has been derived according to Galí (2015).

The log-linearized optimal price setting condition (11) delivers a typical equation for domestic inflation  $\pi_t^h$ :

$$\pi_t^h = \beta E_t \pi_{t+1}^h + \lambda \frac{1 - \alpha}{1 - \alpha + \alpha \varepsilon} \widehat{mc}_t,$$

where  $\widehat{mc}_t$  is the log deviation of the economy's average real marginal costs from their steady state and  $\lambda = \frac{(1-\beta\theta)(1-\theta)}{\theta}$ .

The CPI inflation includes the domestic inflation  $\pi_t^h$  and the terms of trade, which can be alternatively represented by the real exchange rate  $REER_t$ :

$$\pi_t = \pi_t^h + \frac{1 - \gamma}{\gamma} \Delta \widehat{REER}_t$$

The Phillips curve then is as follows:

$$\pi_t = \beta E_t \pi_{t+1} + \lambda \frac{1 - \alpha}{1 - \alpha + \alpha \varepsilon} \widehat{mc}_t + \frac{1 - \gamma}{\gamma} \Delta \widehat{REER}_t - \beta \frac{1 - \gamma}{\gamma} E_t \Delta \widehat{REER}_{t+1},$$

where  $\widehat{mc}_t = \widehat{W}_t - (\widehat{Y}_t^{no} - \widehat{N}_t) + \frac{1-\gamma}{\gamma} \widehat{REER}_t$ . Wages can be substituted with the log-linearized labor supply condition (38), so that the Phillips curve used in the model is this:

$$\begin{aligned} \pi_t = & \beta E_t \pi_{t+1} + \frac{1 - \gamma}{\gamma} \left( \lambda \frac{1 - \alpha}{1 - \alpha + \alpha \varepsilon} + \beta + 1 \right) \widehat{REER}_t - \frac{1 - \gamma}{\gamma} \widehat{REER}_{t-1} \quad (40) \\ & - \beta \frac{1 - \gamma}{\gamma} E_t \widehat{REER}_{t+1} + \lambda \frac{1 - \alpha}{1 - \alpha + \alpha \varepsilon} \left( \phi \widehat{N}_t - \widehat{Y}_t^{no} \right) \end{aligned}$$

## E Log-linearized equations

The final 16 log-linearized equations are listed below.

The aggregate consumption equation is derived according to Galí, Lopez-Salido, and Valles (2007) by combining the Euler equation (36), budget constraint of the

rule-of-thumb households (5), and the relationship  $C_t = \mu C_t^N + (1 - \mu)C_t^S$ :

$$\widehat{C}_t = E_t \widehat{C}_{t+1} + \Theta_n (\widehat{N}_t - E_t \widehat{N}_{t+1}) - \Theta_i (\widehat{R}_t - E_t \pi_{t+1}) + \mu \overline{TC}^{-1} (\widehat{T}_{t+1} - \widehat{T}_t), \quad (41)$$

where  $\Theta_n = \left[ \mu \overline{N}^\phi \phi + (1 - \mu) \overline{N}^\phi \right] \overline{C}^{-1}$  and  $\Theta_i = (\sigma \overline{C})^{-1} (1 - \mu) (\overline{C} - \phi^{-1} \overline{N}^\phi)$ .

The combination of first-order condition with respect to non-oil capital (35) and investment (34) given that  $\widehat{R}_t^{kno} = \phi \widehat{N}_t - \widehat{K}_{t-1}^{no}$  delivers the following:

$$\begin{aligned} \kappa(1 + \beta) \widehat{I}_t &= \left( \beta(1 - \delta) + \Omega \overline{fxr}^* \right) \left[ \kappa(1 + \beta) E_t \widehat{I}_{t+1} - \kappa \beta E_t \widehat{I}_{t+2} - \kappa \widehat{I}_t \right] + \kappa \beta E_t \widehat{I}_{t+1} \quad (42) \\ &+ (1 - \beta(1 - \delta) - \Omega \overline{fxr}^*) E_t \left[ \phi \widehat{N}_{t+1} - \widehat{K}_t^{no} \right] - \frac{\overline{C} - \overline{N}^\phi / \phi}{\sigma \overline{C}} \left( \widehat{R}_t - E_t \pi_{t+1} \right) \\ &+ \kappa \widehat{I}_{t-1} + \Omega \overline{fxr}^* (E_t \pi_{t+1}^* + \widehat{RER}_t - E_t \widehat{RER}_{t+1} + \widehat{fxr}_t^*) \end{aligned}$$

The log-linearization of balance of payments equation results in this:

$$\begin{aligned} \widehat{NX}_t &= \left[ \frac{\overline{R}^* b_y^* (1 - \mu)}{nx_y} - \frac{(\overline{R}^* - \rho_{swf}) swf_y}{nx_y} - \frac{fdi_y}{nx_y} + \frac{(1 - \iota^{\text{div}})(1 - \tau^o) s_o}{nx_y} \right] \widehat{RER}_t \quad (43) \\ &- \frac{b_y^* (1 - \mu) \widehat{b}_t^*}{nx_y} + \frac{\overline{R}^* b_y^* (1 - \mu) \widehat{b}_{t-1}^*}{nx_y} + \frac{\overline{R}^* swf_y - (1 - \mu) \overline{R}^* b_y^* \pi_t^*}{nx_y} - \frac{fdi_y \widehat{FDI}_t^*}{nx_y} \\ &- \frac{\overline{R}^* b_y^* (1 - \mu) \widehat{RER}_{t-1}}{nx_y} + \frac{(1 - \iota^{\text{div}})(1 - \tau^o) s_o \widehat{Y}_t^o}{nx_y} - \frac{(\overline{R}^* - \rho_{swf}) swf_y \widehat{SWF}_{t-1}}{nx_y} \\ &+ \frac{(1 - \mu) b_y^* - swf_y}{nx_y} \widehat{R}_{t-1}^* + \frac{(1 - \iota^{\text{div}})(1 - \tau^o) s_o \widehat{P}_t^{o*}}{nx_y} \end{aligned}$$

The collateral constraint (4) combined with the first-order condition with respect to investment (34) yields:

$$\widehat{b}_t^* = E_t \pi_{t+1}^* - \widehat{R}_t^* + \widehat{K}_t^{no} + \widehat{RER}_t - E_t \widehat{RER}_{t+1} + \kappa(1 + \beta) E_t \widehat{I}_{t+1} - \kappa \beta E_t \widehat{I}_{t+2} - \kappa \widehat{I}_t \quad (44)$$

The law of motion for non-oil capital (3) is as follows:

$$\widehat{K}_t^{no} = (1 - \delta) \widehat{K}_{t-1}^{no} + \delta \widehat{I}_t \quad (45)$$

Similarly, the public capital accumulation (19) in its log-linearized form is below:

$$\widehat{K}_t^G = (1 - \delta^g) \widehat{K}_{t-1}^G + \delta^g \widehat{G}_t \quad (46)$$

The oil capital is accumulated by FDI according to its equation (13):

$$\widehat{K}_t^o = (1 - \delta) \widehat{K}_{t-1}^o + \delta \widehat{FDI}_t^* \quad (47)$$

The combination of oil taxes equation (20), SWF accumulation (21), and the

profits of oil producer (16) corresponds to:

$$\widehat{SWF}_t = \rho_{swf} \widehat{SWF}_{t-1} + \frac{[\tau^o + \iota^{\text{div}}(1 - \tau^o)]s_o}{swf_y} (\widehat{Y}_t^o + \widehat{P}_t^{o*}) \quad (48)$$

The UIP condition (39) after some tedious algebra corresponds to:

$$E_t \widehat{RER}_{t+1} = \widehat{R}_t + \widehat{RER}_t - \frac{\beta^*}{\beta} \widehat{R}_t^* + E_t \pi_{t+1}^* - E_t \pi_{t+1} - \left( \frac{\beta^*}{\beta} - 1 \right) \widehat{fxr}_t^* \quad (49)$$

The non-oil and oil production functions (8 and 12) give respectively:

$$\widehat{Y}_t^{no} = \alpha \widehat{K}_{t-1}^{no} + (1 - \alpha) \widehat{N}_t + \psi \widehat{K}_{G,t-1} \quad (50)$$

$$\widehat{Y}_t^o = \alpha^o \widehat{K}_{t-1}^o \quad (51)$$

The aggregate output is as follows:

$$\widehat{Y}_t = (1 - s_o) \widehat{Y}_t^{no} + (1 - s_o) \widehat{p}_t^h + s_o (\widehat{Y}_t^o + \widehat{RER}_t + \widehat{P}_t^{o*}) \quad (52)$$

The government budget constraint (17) in terms of fiscal debt results in:

$$\begin{aligned} \widehat{b}_t = & \bar{R}(\widehat{b}_{t-1} + \widehat{R}_{t-1} - \pi_t) + \frac{g_y^I}{(1 - \mu)b_y} \widehat{G}_t^I + \frac{g_y^C}{(1 - \mu)b_y} \widehat{G}_t^C + \frac{g_y^C + g_y^I}{(1 - \mu)b_y} \widehat{p}_t^g \quad (53) \\ & - \frac{\bar{T}}{\bar{b}(1 - \mu)} \widehat{T}_t - \frac{(\bar{R}^* - \rho_{swf})swf_y}{b_y(1 - \mu)} \widehat{OR}_t, \end{aligned}$$

where oil revenues are as follows  $\widehat{OR}_t = \widehat{RER}_t + \widehat{SWF}_{t-1} + \frac{1}{\bar{R}^* - \rho_{swf}} \widehat{R}_{t-1}^* - \frac{\bar{R}^*}{\bar{R}^* - \rho_{swf}} \pi_t^*$ .

The log-linearized relative price of government purchases to composite consumption (18), assuming  $\eta \rightarrow 1$ , is this:

$$\widehat{p}_t^g = \gamma_2 \widehat{p}_t^h + (1 - \gamma_2) \widehat{RER}_t \quad (54)$$

The domestic goods market clearing condition (28) can be re-written as:

$$\widehat{Y}_t^{no} + \widehat{p}_t^h = \frac{\gamma c_y}{(1 - s_o)} \widehat{C}_t + \frac{(1 - \mu)(1 - i_y)\gamma}{(1 - s_o)} \widehat{I}_t + \frac{\gamma_2 g_y^C}{(1 - s_o)} \widehat{G}_t^C + \frac{\gamma_2 g_y^I}{(1 - s_o)} \widehat{G}_t^I + \frac{\gamma_2 (g_y^C + g_y^I)}{(1 - s_o)} \widehat{p}_t^g, \quad (55)$$

where  $(1 - i_y) = 1 - c_y - g_y^C - g_y^I - nx_y$ .

The real GDP (29) is represented in terms of investment:

$$\widehat{I}_t = \frac{1}{(1 - i_y)(1 - \mu)} \left[ \widehat{Y}_t - c_y \widehat{C}_t - g_y^C \widehat{G}_t^C - g_y^I \widehat{G}_t^I - (g_y^C + g_y^I) \widehat{p}_t^g - nx_y \widehat{NX}_t \right] \quad (56)$$

## F Impulse response functions at flexible prices

Figure 1. Procyclical fiscal policy combined with managed exchange rate and CPI/PPT rule

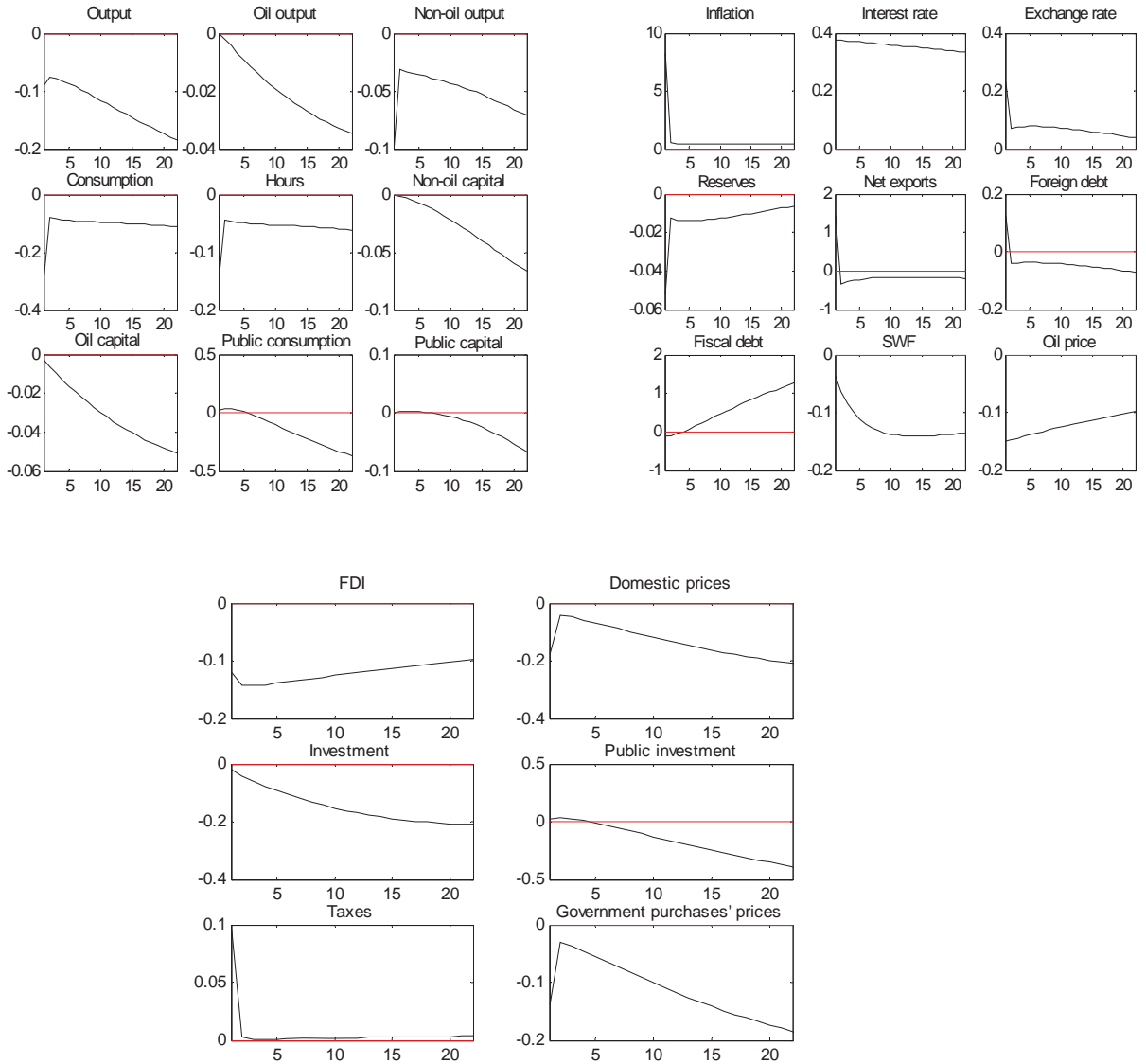


Figure 2. Neutral fiscal policy combined with managed exchange rate and



### CPI/PPT rule

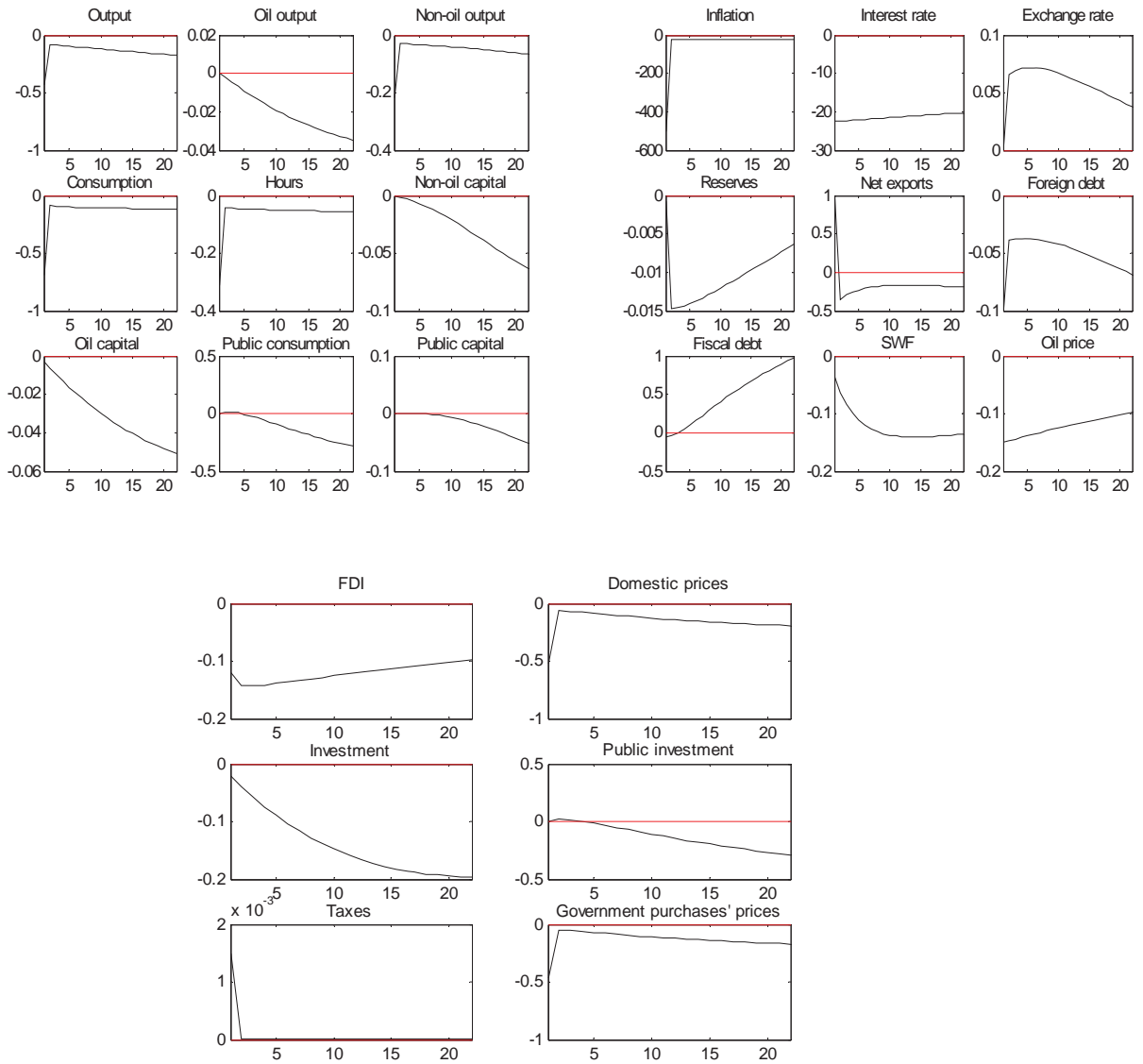
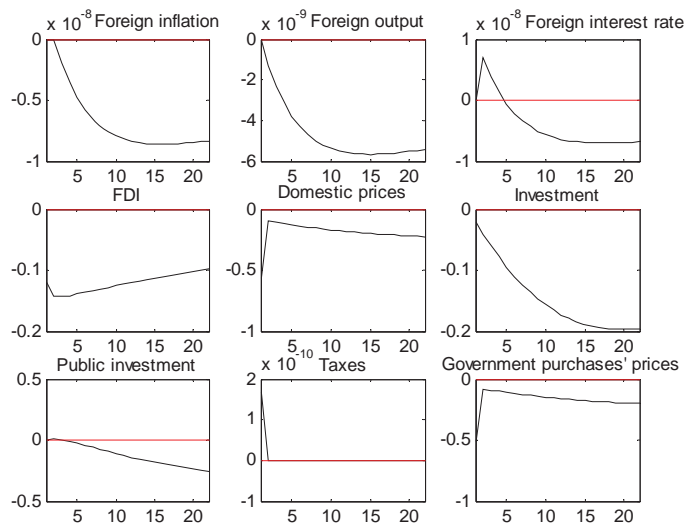
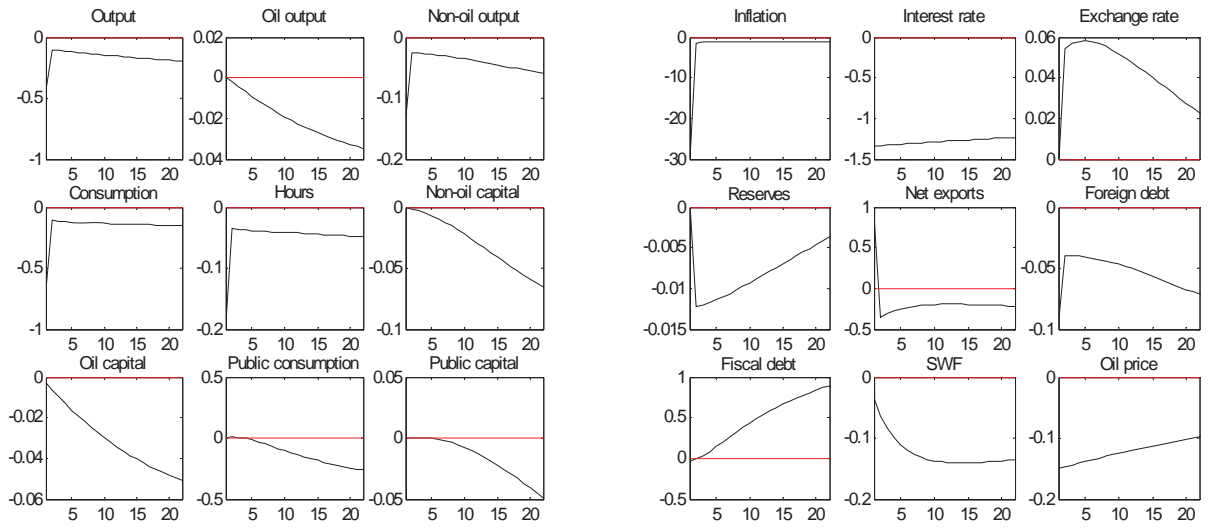


Figure 3. Countercyclical fiscal policy combined with managed/flexible exchange

rate and CPI/PPT rule



## G Impulse response functions with nominal rigidities

Figure 1. Neutral fiscal policy combined with managed/flexible exchange rate and CPI rule

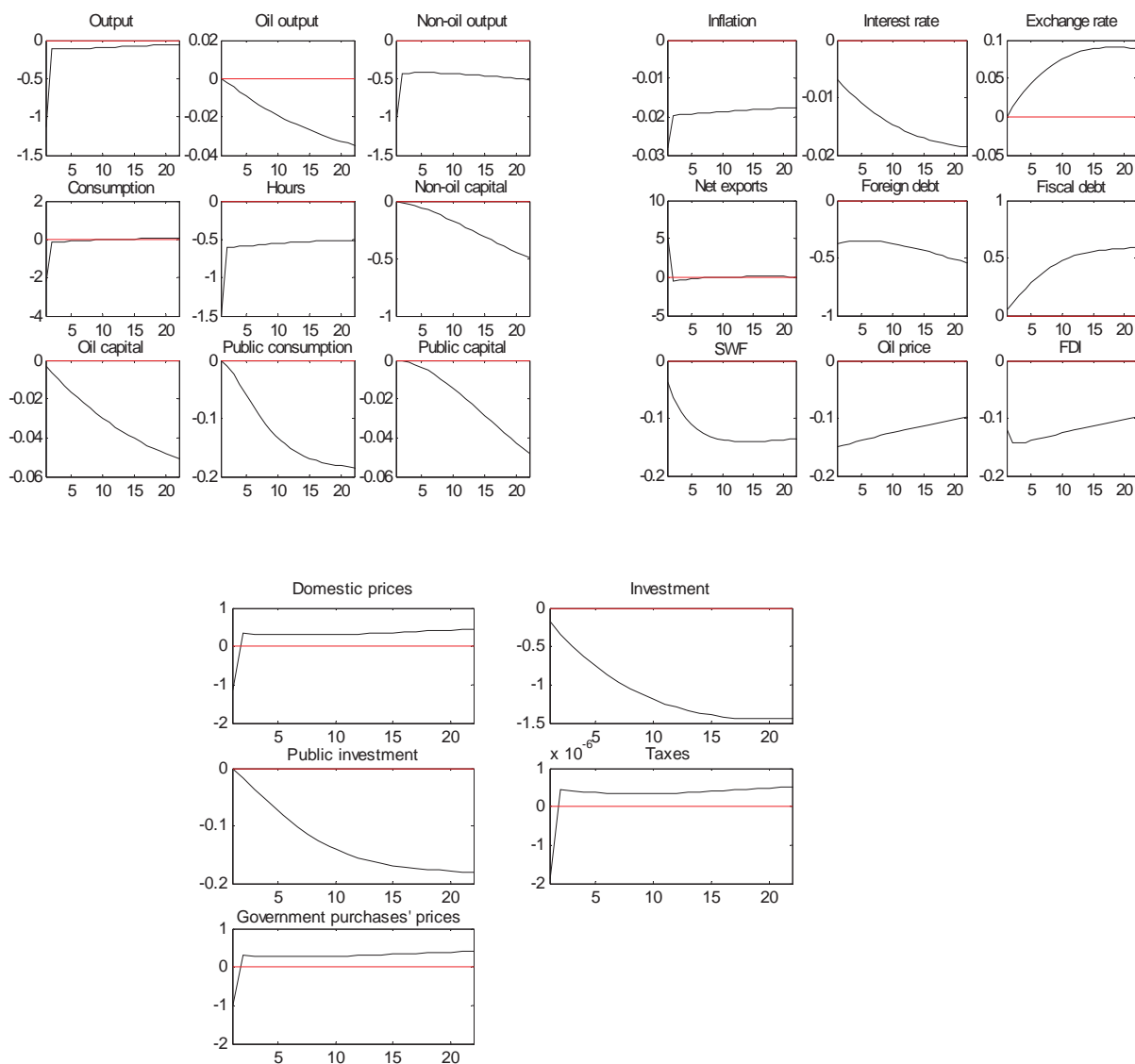


Figure 2. Procyclical fiscal policy combined with managed/flexible exchange rate

and CPI rule

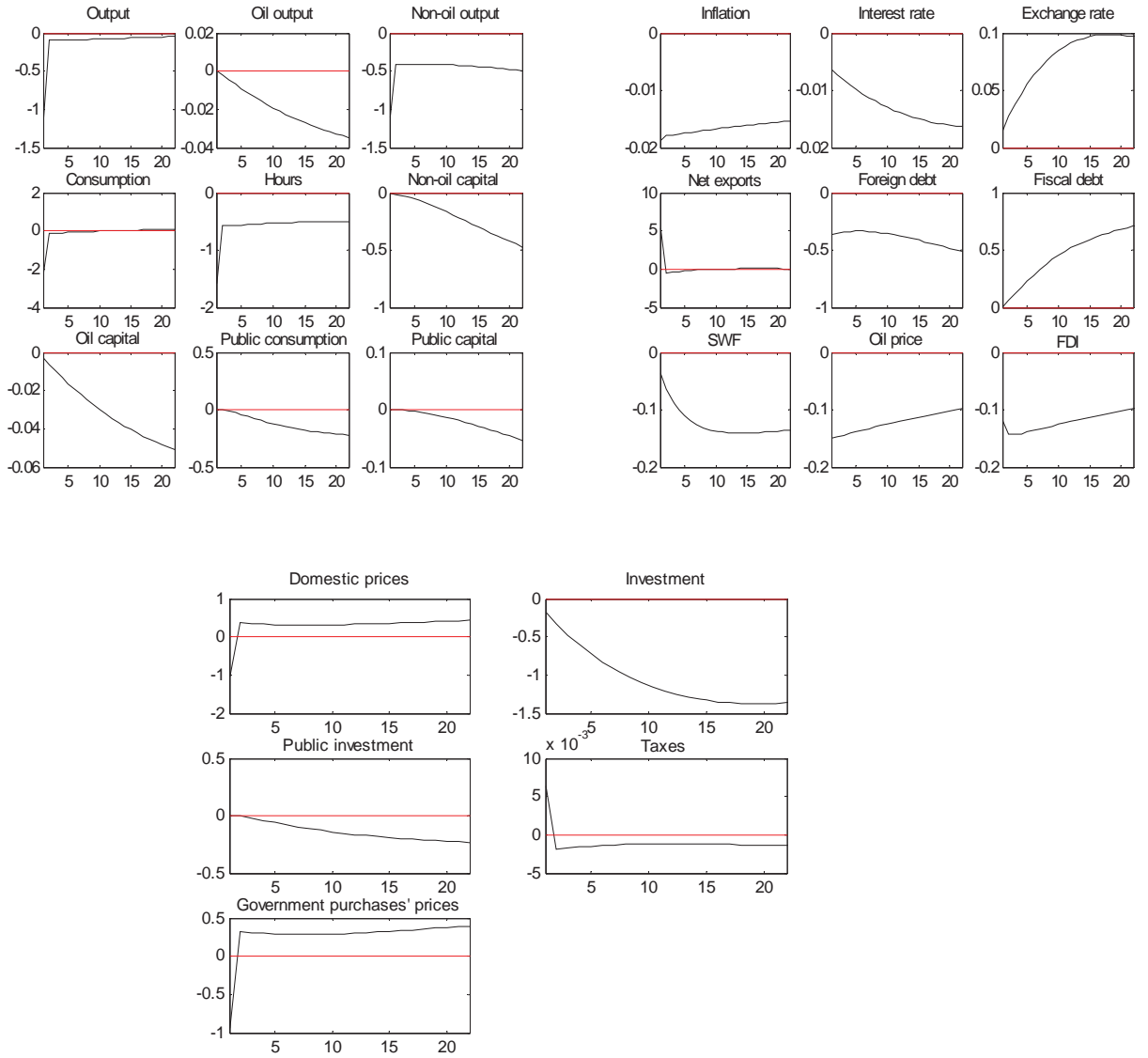


Figure 3. Procyclical fiscal policy combined with managed/flexible exchange rate

and PPT rule

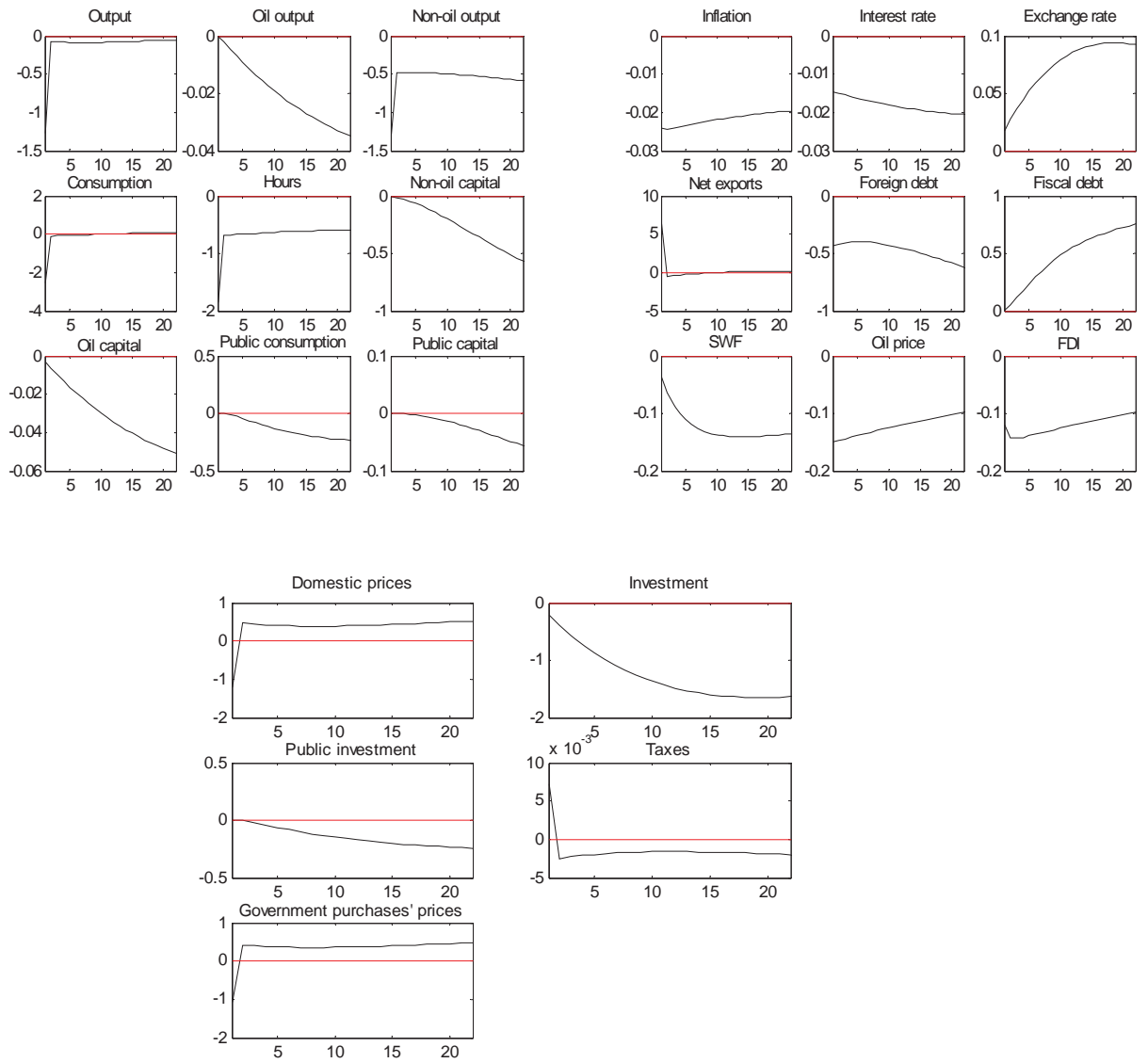


Figure 4. Countercyclical fiscal policy combined with managed/flexible exchange

rate and CPI rule

