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Optimal Monetary Policy and Uncertainty Shocks*

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Abstract

We study optimal monetary policy in response to uncertainty shocks in standard New Keynesian models under Calvo and Rotemberg pricing schemes. We find that optimal monetary policy achieves joint stabilization of inflation and the output gap in both pricing schemes. We show that a simple Taylor rule that puts high weight on inflation stability approximates optimal monetary policy well. This rule mutes firms' precautionary pricing incentive, the key channel that makes responses under Calvo and Rotemberg pricing schemes differ under the empirically calibrated Taylor rule.

Keywords: Optimal monetary policy, Uncertainty shocks

JEL Classification: E12, E52

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1 Introduction

Time-varying uncertainty has recently received considerable attention from policymakers and academics, spurring a burgeoning literature on identifying transmission mechanisms of uncertainty shocks. Different types of nominal rigidities have been shown to affect uncertainty propagation differently. Yet, little is known on whether optimal monetary policy varies depending on the way nominal rigidities are modeled. There are two popular approaches to modeling price rigidities. The first is Calvo (1983) pricing, under which firms face a constant probability of not being allowed to reoptimize their price every period. The second is Rotemberg (1982) pricing, under which firms can always adjust their price upon payment of a quadratic price adjustment cost. It is well-known that the two approaches are observationally equivalent up to a first-order approximation. However, in response to uncertainty shocks, which require at least a third-order perturbation to show up in a policy function, they generate different dynamics under the empirical Taylor rule – see Oh (2020).

This paper explores how optimal monetary policy responds to uncertainty shocks and whether its response varies depending on the way price rigidities are modeled. In particular, we derive the optimal monetary policy in response to uncertainty shocks under Calvo and Rotemberg pricing frictions. We show that, when monetary policy responds optimally, allocations under the two pricing schemes are the same (and efficient). Clarifying what generates different dynamics under the empirical Taylor rule helps us understand why optimal monetary policy is able to achieve the same allocations. When monetary policy follows the empirical Taylor rule, under Rotemberg pricing, uncertainty shocks appear as demand shocks. A rise in uncertainty triggers households' precautionary savings, which causes a fall in both inflation and the output gap. On the contrary, under Calvo pricing, uncertainty shocks appear as cost-push shocks. A rise in uncertainty triggers firms' precautionary pricing motive along with households' precautionary savings. The precautionary pricing incentive stems from firms' exposure to the risk of not being able to set their desired price level in the future. Price-resetting firms raise prices today to hedge against an uncertain future profit stream. This triggers a rise in inflation and a sharper fall in the output gap, as the resulting rise in inflation further compresses aggregate demand. Therefore, the main driver of the different dynamics under the two pricing schemes is the precautionary pricing behavior of firms, which is only present with Calvo-type price rigidities. We show that, when monetary policy is set optimally, not only the households' precautionary motive but also the firms' precautionary pricing motive are eliminated. This results in stabilized inflation and the output gap under the optimal monetary policy, regardless of the type of price friction. We further show that, under both pricing schemes, a simple rule that puts extremely high weight on inflation approximates the optimal monetary policy well.

Our paper is related to two main streams of the literature. The first focuses on the transmission of uncertainty shocks to the macroeconomy and includes works such as Born and Pfeifer (2014), Fernández-Villaverde et al. (2015), Leduc and Liu (2016), Basu and Bundick (2017), and Oh (2020). While these papers show that the form of price rigidity adopted is not innocuous under the empirical Taylor rule, we show that this is not the case under the optimal monetary policy. The second stream of the literature our paper is related to is comparing positive and normative results under the Calvo and Rotemberg pricing assumptions. As for positive studies, Miao and Ngo (2019) and Ngo (2019) show that the two pricing schemes generate different dynamics at the zero lower bound. Ascari and Rossi (2012) argue that the Calvo and Rotemberg models have very different predictions when the models are approximated at a positive steady state inflation rate. As for normative studies, Nisticó (2007) and Lombardo and Vestin (2008) compare the welfare implications of the Calvo and Rotemberg models. Leith and Liu (2016) compares the inflation bias. All these papers have an environment in which monetary policy is suboptimal. In contrast, our work compares the dynamics in response to uncertainty shocks when monetary policy is optimal.

The rest of the paper is structured as follows. Section 2 describes the optimality conditions of a textbook New Keynesian model under Calvo and Rotemberg pricing schemes. Section 3 discusses calibration and responses under the optimal monetary policy and a simple Taylor rule. Section 4 concludes.

2 Textbook New Keynesian Models

We describe the equilibrium conditions of a basic New Keynesian model under Calvo (1983) and Rotemberg (1982) price rigidities. The model features a utility-maximizing household, intermediate good firms that compete monopolistically and face price frictions, and exogenous productivity subject to second moment shocks.

The optimal labor supply and consumption of a representative household are characterized by:

$$\chi {N_t}^{\eta} = {C_t}^{-\gamma} w_t, \tag{1}$$

$${C_t}^{-\gamma} = \beta E_t {C_{t+1}}^{-\gamma} \frac{R_t}{\pi_{t+1}}, \tag{2}$$

where C_t indicates consumption, N_t labor supply, and w_t the real wage. γ is the risk aversion parameter, χ is the labor disutility parameter, and η is the inverse of the Frisch labor supply elasticity. π_t is the gross inflation rate, while R_t is the nominal interest rate.

Differentiated goods are produced by a continuum of intermediate good firms, indexed by $i \in [0, 1]$, according to:

$$Y_t(i) = A_t N_t(i). \tag{3}$$

A_t is the exogenous productivity following:

$$\log A_t = \rho_A \log A_{t-1} + \sigma_t^A \varepsilon_t^A, \quad 0 \le \rho_A < 1, \quad \varepsilon_t^A \sim N(0,1). \tag{4}$$

σ_t^A is the time-varying volatility of productivity and follows:

$$\log \sigma_t^A = (1-\rho_{\sigma^A}) \log \sigma^A + \rho_{\sigma^A} \log \sigma_{t-1}^A + \sigma^{\sigma^A} \varepsilon_t^{\sigma^A}, \quad 0 \le \rho_{\sigma^A} < 1, \quad \varepsilon_t^{\sigma^A} \sim N(0,1), \tag{5}$$

where σ^A indicates the steady state value of σ_t^A.

Intermediate goods are aggregated into final goods using a CES technology with elasticity of substitution $\epsilon > 1$. The average real marginal cost is given by:

$$mc_t = \frac{w_t}{A_t}. \tag{6}$$

The efficient output Y_t^f, the level of output that would prevail under flexible prices and perfect competition, is:

$$\chi \left(\frac{Y_t^f}{A_t} \right)^{\eta} = {Y_t^f}^{-\gamma} \frac{\epsilon - 1}{\epsilon} \frac{A_t}{(1-\tau)}, \tag{7}$$

where $\tau = \frac{1}{\varepsilon}$ indicates the rate at which firms' production is subsidized and ensures the efficient steady state. The output gap is defined by:

$$\widetilde{Y}_t = \log \left(\frac{Y_t}{Y_t^f} \right). \tag{8}$$

2.1 Calvo Pricing

Under Calvo pricing, only a fraction $1-\theta$ of intermediate good firms, are allowed to reset their price in a given period. Denoting the optimal reset price by $P_t^\star$, the optimal relative price, $p_t^\star = \frac{P_t^\star}{P_t}$, solves:

$$p_t^\star = \frac{\epsilon (1-\tau)}{\epsilon - 1} \frac{p_{1,t}}{p_{2,t}}, \tag{9}$$

$$p_{1,t} = {C_t}^{-\gamma} mc_t Y_t + \theta \beta E_t {\pi_{t+1}}^{\epsilon} p_{1,t+1}, \tag{10}$$

$$p_{2,t} = {C_t}^{-\gamma} Y_t + \theta \beta E_t {\pi_{t+1}}^{\epsilon - 1} p_{2,t+1}, \tag{11}$$

where P_t indicates the aggregate price level. Inflation evolves according to:

$$\theta {\pi_t}^{\epsilon-1} = 1 - (1-\theta)\, {p_t^\star}^{1-\epsilon}. \tag{12}$$

The aggregate production function and resource constraint are given by:

$$\Delta_t Y_t = A_t N_t, \tag{13}$$

$$Y_t = C_t, \tag{14}$$

where Δ_t is a measure of price dispersion, which evolves according to:

$$\Delta_t = (1-\theta)\, {p_t^\star}^{-\epsilon} + \theta {\pi_t}^{\epsilon} \Delta_{t-1}. \tag{15}$$

2.2 Rotemberg Pricing

Under Rotemberg pricing, firms can reset their price every period upon payment of a quadratic price adjustment cost, controlled by the parameter $\psi \geq 0$. In equilibrium, all intermediate good firms are symmetric and charge the same price. The inflation rate, π_t, is determined by the firms' optimal pricing condition as follows:

$$\psi\left(\pi_t - 1\right)\pi_t = \psi\beta E_t \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \left(\pi_{t+1} - 1\right)\pi_{t+1}\frac{Y_{t+1}}{Y_t} + 1 - \epsilon + \epsilon\left(1-\tau\right) mc_t. \tag{16}$$

The aggregate production function and resource constraint are given by:

$$Y_t = A_t N_t, \tag{17}$$

$$Y_t = C_t + \frac{\psi}{2}\left(\pi_t - 1\right)^2 Y_t. \tag{18}$$

2.3 Ramsey-Optimal Monetary Policy

Optimal monetary policy is given by the solution to the Ramsey planner's problem. This solution is a sequence of the nominal interest rate that maximizes the discounted sum of the representative agent's utility given the equilibrium conditions of the competitive economy. The Ramsey-optimal equilibrium conditions under the Calvo and Rotemberg pricing assumptions are shown in Appendix A and B.

Table 1: Parameter Values

Parameter	Description	Value
β	Discount factor	0.99
γ	Risk aversion	2.00
η	Inverse labor supply elasticity	1.00
χ	Labor disutility parameter	$N = \frac{1}{3}$
ϵ	Elasticity of substitution between goods	11.00
θ	Calvo price stickiness	0.75
ψ	Rotemberg price stickiness	$\frac{\theta(\epsilon-1)}{(1-\theta)(1-\theta\beta)}$
ρ_A	Technology shock persistence	0.95
σ^A	Steady-state volatility of technology shock	0.01
ρ_{σ^A}	Uncertainty shock persistence	0.76
σ^{σ^A}	Volatility of uncertainty shock	0.392

3 Results

3.1 Calibration and Solution Method

The models are calibrated to a quarterly frequency. Table 1 provides a summary of the key parameters. The discount factor β is set to 0.99. The risk aversion parameter γ is 2. The inverse of labor supply elasticity η is set to 1. The labor disutility parameter χ is calibrated to match a steady state value of hours worked of 1/3. The elasticity of substitution between differentiated intermediate goods ϵ is fixed to 11, implying a steady-state markup of 10%. We parametrize $\theta = 0.75$ to match an average price duration of four quarters. The Rotemberg price adjustment cost ψ is chosen so that the slope of the Phillips curve under the Rotemberg and Calvo assumptions is equivalent upon first-order approximation. We follow Leduc and Liu (2016) to parametrize the shock processes. For the productivity shock, we set $\sigma^Z = 0.01$ and $\rho_Z = 0.95$. For the uncertainty shock, we set $\sigma^{\sigma^Z} = 0.392$ and $\rho_{\sigma^Z} = 0.76$. We solve the models using a third-order approximation (Adjemian et al., 2011 and Fernández-Villaverde et al., 2011) with the pruning scheme (Andreasen et al., 2018).

3.2 Optimal Monetary Policy vs. Simple Taylor Rule

We compare the model predictions under the Ramsey-optimal monetary policy to those under a simple Taylor rule that takes a form of:

$$\log R_t - \log R = \phi_\pi \log \pi_t, \tag{19}$$

where $\phi_\pi = 1.5$ in line with the empirical literature.

Figure 1 shows the impulse responses to a one standard deviation increase in uncertainty, when monetary

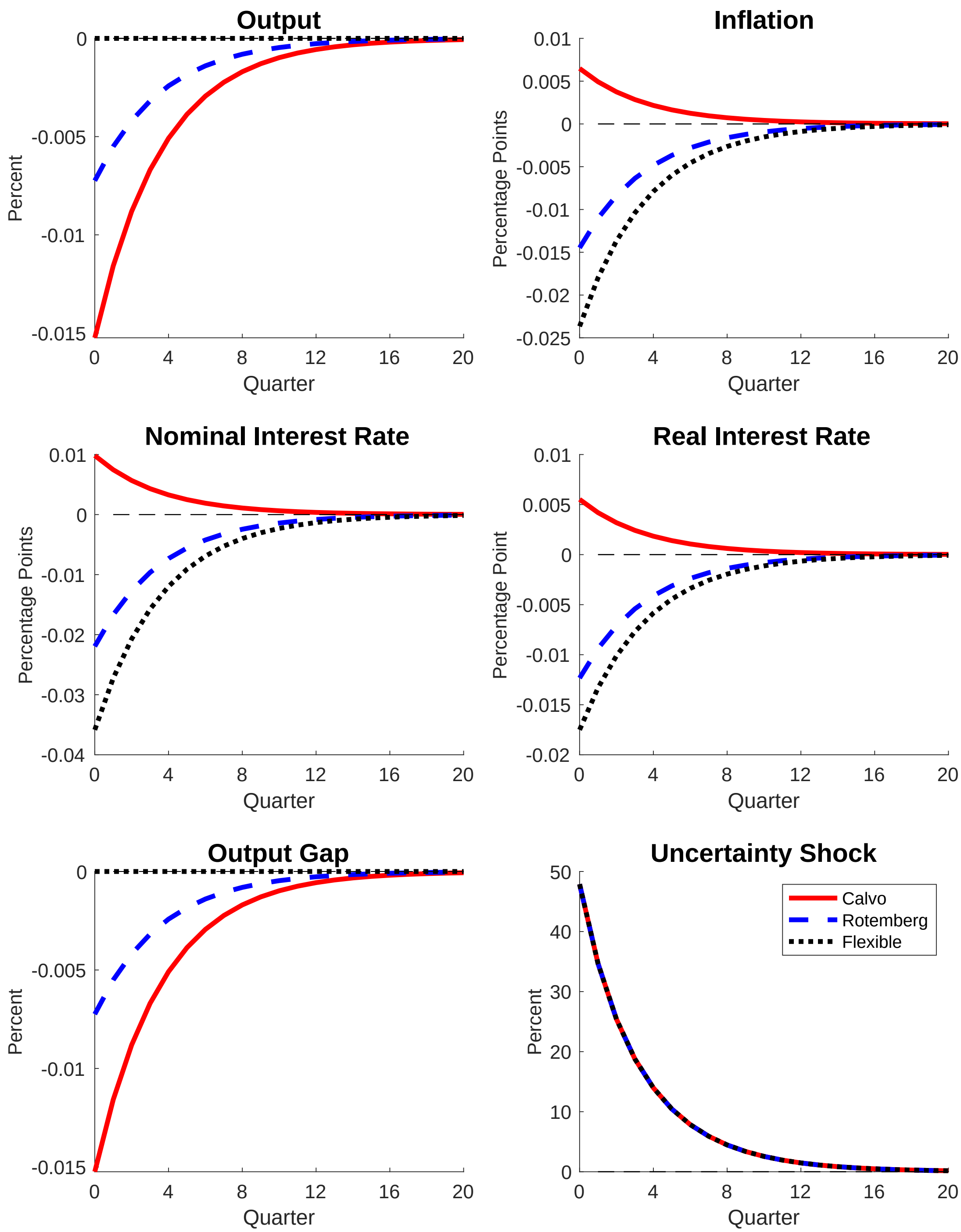

Figure 1: Impulse Responses to Uncertainty Shocks: Empirical Taylor Rule

Note: Impulse responses are in percent deviation from their stochastic steady state.

policy follows Equation (19). An increase in uncertainty induces risk-averse households to cut consumption and engage in precautionary saving. With Rotemberg pricing, the fall in consumption leads to a drop

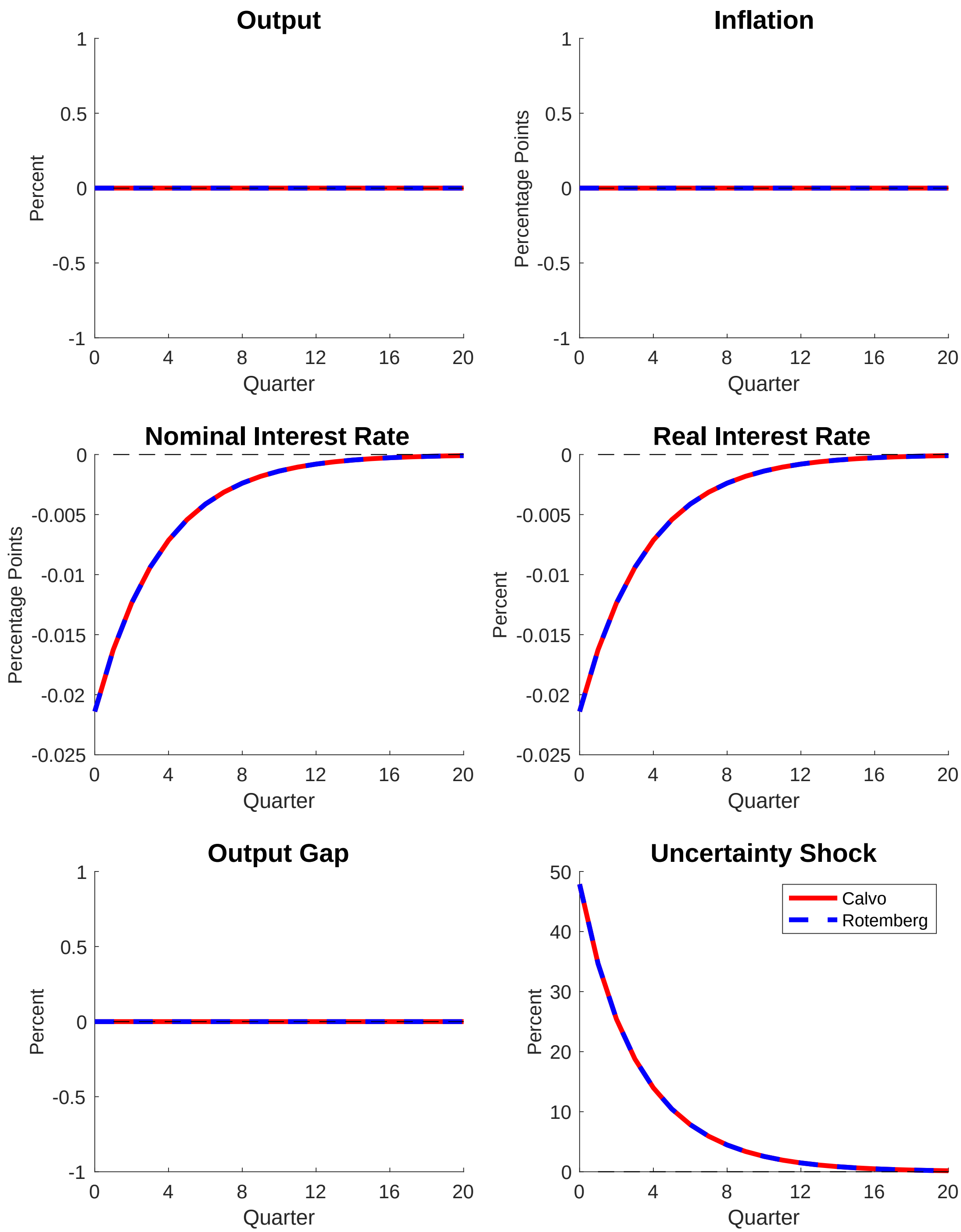

Figure 2: Impulse Responses to Uncertainty Shocks: Ramsey-Optimal Monetary Policy

Note: Impulse responses are in percent deviation from their stochastic steady state.

in output and inflation. The joint decline in prices and quantities implies that uncertainty shocks act as negative demand shocks. As analyzed by Oh (2020) and Oh and Rogantini Picco (2020), with Calvo pricing,

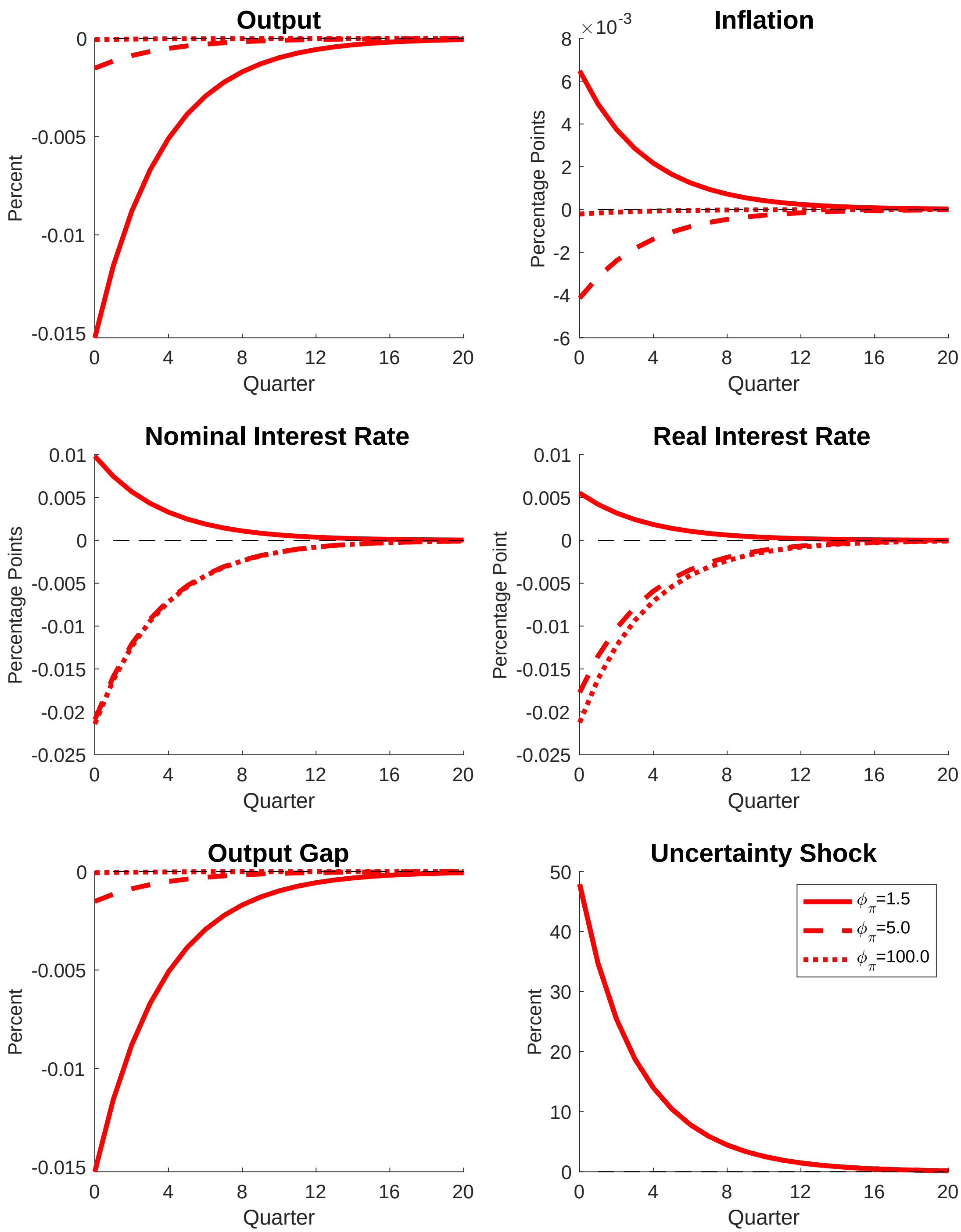

Figure 3: Impulse Responses to Uncertainty Shocks under Different Taylor Rules (Calvo)

Note: Impulse responses are in percent deviation from their stochastic steady state.

there is an additional propagation, which works through the precautionary pricing behavior of firms. When uncertainty increases, firms that are allowed to reset their price raise the price to self-insure against the risk

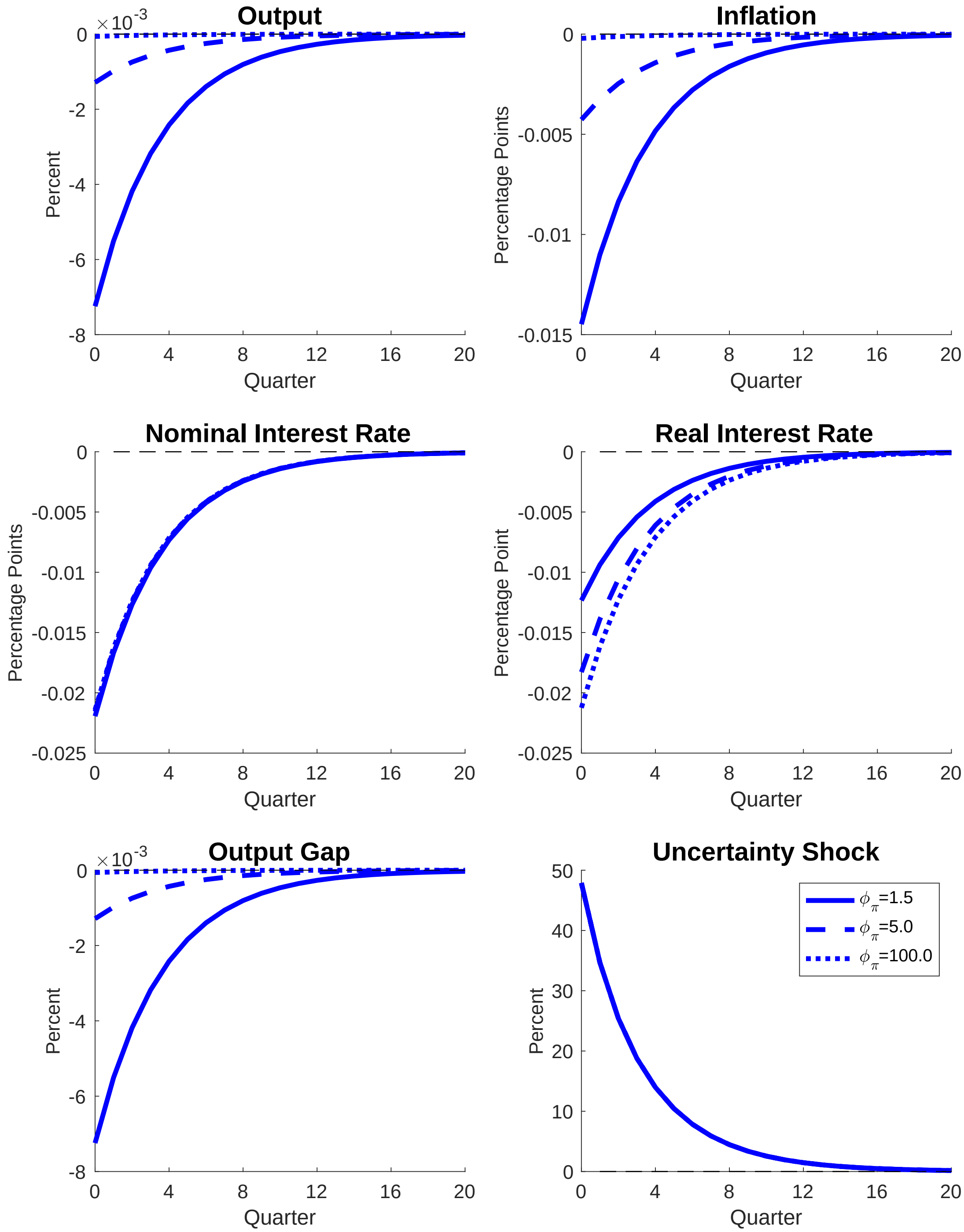

Figure 4: Impulse Responses to Uncertainty Shocks under Different Taylor Rules (Rotemberg)

Note: Impulse responses are in percent deviation from their stochastic steady state.

of being stuck with low prices in the future. Since the increase in prices induced by the precautionary pricing behavior of firms is stronger than the drop in prices induced by the precautionary saving behavior of risk-averse households, inflation increases after a positive uncertainty shock. Hence, with Calvo-type rigidities, uncertainty shocks act as cost-push shocks: inflation rises, and the output gap drops. As monetary policy follows Equation (19), the nominal interest rate falls in the model with Rotemberg-type rigidities, while it rises in the model with Calvo-type rigidities.

Figure 2 displays the impulse responses to an increase in uncertainty under the optimal monetary policy. In this case, inflation and output gaps are fully stabilized in both Calvo and Rotemberg pricing models. This result is surprising, especially for the Calvo model. In fact, as shown in Figure 1, with Calvo-type rigidities, an increase in uncertainty acts as a cost-push shock when monetary policy follows a Taylor rule. Cost-push shocks lead to a standard output-inflation trade-off, making it difficult for the optimal monetary policy to stabilize the output gap and inflation at the same time. Yet, unlike cost-push shocks, uncertainty shocks do not entail an output-inflation trade-off.

To intuitively understand why the optimal monetary policy achieves joint output and inflation stabilization, it is useful to compare predictions under Taylor rules with different inflation coefficients. Figure 3 and Figure 4 compare responses under Taylor rules with different inflation coefficients ($\phi_\pi = 1.5$; $\phi_\pi = 5$; $\phi_\pi = 100$). When the coefficient of inflation is extremely high, the effect of uncertainty is neutralized in both Calvo and Rotemberg models. In fact, in both Calvo and Rotemberg, a Taylor rule that responds very aggressively to inflation ($\phi_\pi = 100$) can generate allocations that are close to the ones under the optimal monetary policy. The higher the value of ϕ_π is, the bigger the drop in the real interest rate is realized upon a fall in inflation. A decrease in the real interest rate on savings weakens the precautionary saving motive and works to stabilize aggregate demand. In Rotemberg, stable aggregate demand implies a stable nominal marginal cost. As a result, firms have no incentive to change prices, and hence inflation is stabilized. In Calvo, stable aggregate demand eliminates uncertainty over future nominal costs. Consequently, firms have no concerns over having their prices fixed at the level that leads to undesired markup. Therefore, the precautionary pricing incentive in Calvo is no longer operative when the central bank has a strong desire to stabilize inflation.

4 Conclusion

We have shown that uncertainty shocks propagate differently under Calvo and Rotemberg pricing assumptions when monetary policy is set according to the empirical Taylor rule: they behave like cost-push shocks under Calvo pricing and negative demand shocks under Rotemberg pricing. However, the optimal monetary policy achieves joint stabilization of inflation and the output gap under both pricing assumptions. This is because the optimal monetary policy eliminates not only the households' precautionary savings motive but also the firms' precautionary pricing incentive, which is the key channel that makes prediction of Calvo price-setting different from those of Rotemberg price-setting under the empirical Taylor rule. We conclude that, while the adopted form of price rigidity does matter under the empirical Taylor rule, it does not matter under the optimal monetary policy.

References

Adjemian, Stéphane, Houtan Bastani, Michel Juillard, Fréderic Karamé, Junior Maih, Ferhat Mihoubi, George Perendia, Johannes Pfeifer, Marco Ratto, and Sébastien Villemot, "Dynare: Reference Manual Version 4," Dynare Working Papers 1, CEPREMAP 2011.

Andreasen, Martin M., Jesús Fernández-Villaverde, and Juan Rubio-Ramírez, "The Pruned State-Space System for Non-Linear DSGE Models: Theory and Empirical Applications," *Review of Economic Studies*, 2018, *85* (1), 1–49.

Ascari, Guido and Lorenza Rossi, "Trend Inflation and Firms Price-Setting: Rotemberg Versus Calvo," *Economic Journal*, 2012, *122*, 1115–1141.

Basu, Susanto and Brent Bundick, "Uncertainty Shocks in a Model of Effective Demand," *Econometrica*, 2017, *85* (3), 937–958.

Born, Benjamin and Johannes Pfeifer, "Policy Risk and the Business Cycle," *Journal of Monetary Economics*, 2014, *68*, 68–85.

Calvo, Guillermo, "Staggered Prices in a Utility Maximizing Framework," *Journal of Monetary Economics*, 1983, *12*, 383–398.

Fernández-Villaverde, Jesús, Pablo Guerrón-Quintana, Juan Rubio-Ramírez, and Martín Uribe, "Risk Matters: The Real Effects of Volatility Shocks," *American Economic Review*, 2011, *101* (6), 2530–2561.

_ , _ , Keith Kuester, and Juan Rubio-Ramírez, "Fiscal Volatility Shocks and Economic Activity," *American Economic Review*, 2015, *105* (11), 3352–3384.

Leduc, Sylvain and Zheng Liu, "Uncertainty Shocks are Aggregate Demand Shocks," *Journal of Monetary Economics*, 2016, *82*, 20–35.

Leith, Campbell and Ding Liu, "The Inflation Bias under Calvo and Rotemberg Pricing," *Journal of Economic Dynamics and Control*, 2016, *73*, 283–297.

Lombardo, Giovanni and David Vestin, "Welfare Implications of Calvo vs. Rotemberg-Pricing Assumptions," *Economics Letters*, 2008, *100*, 275–279.

Miao, Jianjun and Phuong V. Ngo, "Does Calvo Meet Rotemberg at the Zero Lower Bound?," *Macroeconomic Dynamics*, 2019, *Forthcoming.*

Ngo, Phuong V., "Fiscal Multipliers at the Zero Lower Bound: The Role of Government Spending Persistence," *Macroeconomic Dynamics*, 2019, *Forthcoming.*

Nisticó, Salvatore, "The Welfare Loss from Unstable Inflation," *Economics Letters*, 2007, *96*, 51–57.

Oh, Joonseok, "The Propagation of Uncertainty Shocks: Rotemberg vs. Calvo," *International Economic Review*, 2020, *Forthcoming.*

_ **and Anna Rogantini Picco**, "Macro Uncertainty and Unemployment Risk," 2020. Working Paper.

Rotemberg, Julio J., "Sticky Prices in the United States," *Journal of Political Economy*, 1982, *90* (6), 1187–1211.

Appendices

A Calvo Model: Ramsey-Optimal Policy Problem

In the Calvo model, the Ramsey problem under commitment can be described as follows. Let $\{\lambda_{1,t}, \lambda_{2,t}, \lambda_{3,t}, \lambda_{4,t}, \lambda_{5,t}, \lambda_{6,t}, \lambda_{7,t}, \lambda_{8,t}, \lambda_{9,t}, \lambda_{10,t}\}_{t=0}^{\infty}$ represent sequences of Lagrange multipliers on the constraints (1), (2), (6), (9), (10), (11), (12), (13), (14), and (15). Given an initial value for the price dispersion Δ_{t-1} and a set of stochastic processes $\left\{A_t, \sigma_t^A\right\}_{t=0}^{\infty}$, the allocation plans for the control variables $\{C_t, N_t, w_t, R_t, mc_t, \pi_t, Y_t, p_t^\star, p_{1,t}, p_{2,t}, \Delta_t\}_{t=0}^{\infty}$, and for the co-state variables $\{\lambda_{1,t}, \lambda_{2,t}, \lambda_{3,t}, \lambda_{4,t}, \lambda_{5,t}, \lambda_{6,t}, \lambda_{7,t}, \lambda_{8,t}, \lambda_{9,t}, \lambda_{10,t}\}_{t=0}^{\infty}$, represent an optimal allocation if they solve the following maximization problem:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{{C_t}^{1-\gamma}}{1-\gamma} - \chi \frac{{N_t}^{1+\eta}}{1+\eta} \right), \tag{A.1}$$

subject to (1), (2), (6), (9), (10), (11), (12), (13), (14), and (15). The augmented Lagrangian for the optimal policy problem then reads as follows:

$$\begin{aligned}
\max E_0 \sum_{t=0}^{\infty} \beta^t \Bigg[& \left(\frac{{C_t}^{1-\gamma}}{1-\gamma} - \chi \frac{{N_t}^{1+\eta}}{1+\eta} \right) + \lambda_{1,t} \left({C_t}^{-\gamma} w_t - \chi {N_t}^{\eta} \right) + \lambda_{2,t} \left(\beta {C_{t+1}}^{-\gamma} R_t - {C_t}^{-\gamma} \pi_{t+1} \right) \\
& + \lambda_{3,t} \left(mc_t A_t - w_t \right) + \lambda_{4,t} \left(\epsilon \left(1-\tau\right) p_{1,t} - \left(\epsilon - 1\right) p_t^\star p_{2,t} \right) + \lambda_{5,t} \left(p_{1,t} - {C_t}^{-\gamma} mc_t Y_t - \theta \beta {\pi_{t+1}}^{\epsilon} p_{1,t+1} \right) \\
& + \lambda_{6,t} \left({C_t}^{-\gamma} Y_t + \theta \beta {\pi_{t+1}}^{\epsilon-1} p_{2,t+1} - p_{2,t} \right) + \lambda_{7,t} \left(\theta {\pi_t}^{\epsilon-1} - 1 + \left(1-\theta\right) {p_t^\star}^{1-\epsilon} \right) + \lambda_{8,t} \left(A_t N_t - \Delta_t Y_t \right) \\
& + \lambda_{9,t} \left(Y_t - C_t \right) + \lambda_{10,t} \left(\Delta_t - \left(1-\theta\right) {p_t^\star}^{-\epsilon} - \theta {\pi_t}^{\epsilon} \Delta_{t-1} \right) \Bigg].
\end{aligned} \tag{A.2}$$

The first-order conditions are as follows:

$$\begin{aligned}
[C_t]: \quad & {C_t}^{-\gamma} + \gamma E_t \lambda_{2,t} {C_t}^{-\gamma-1} \pi_{t+1} + \gamma \lambda_{5,t} {C_t}^{-\gamma-1} mc_t Y_t \\
& = \gamma \lambda_{1,t} {C_t}^{-\gamma-1} w_t + \gamma \lambda_{6,t} {C_t}^{-\gamma-1} Y_t + \lambda_{9,t} + \gamma \lambda_{2,t-1} {C_t}^{-\gamma-1} R_{t-1},
\end{aligned} \tag{A.3}$$

$$[N_t]: \quad \chi {N_t}^{\eta} + \chi \eta \lambda_{1,t} {N_t}^{\eta-1} = \lambda_{8,t} A_t, \tag{A.4}$$

$$[w_t]: \quad \lambda_{1,t} {C_t}^{-\gamma} = \lambda_{3,t}, \tag{A.5}$$

$$[R_t]: \quad \beta E_t \lambda_{2,t} {C_{t+1}}^{-\gamma} = 0, \tag{A.6}$$

$$[mc_t]: \quad \lambda_{3,t} A_t = \lambda_{5,t} {C_t}^{-\gamma} Y_t, \tag{A.7}$$

$$[\pi_t]: \quad \theta(\epsilon-1)\lambda_{7,t}{\pi_t}^{\epsilon-2} + \theta(\epsilon-1)\lambda_{6,t-1}{\pi_t}^{\epsilon-2}p_{2,t}$$
$$= \theta\epsilon\lambda_{10,t}{\pi_t}^{\epsilon-1}\Delta_{t-1} + \theta\epsilon\lambda_{5,t-1}{\pi_t}^{\epsilon-1}p_{1,t} + \frac{1}{\beta}\lambda_{2,t-1}{C_{t-1}}^{-\gamma}, \tag{A.8}$$

$$[Y_t]: \quad \lambda_{5,t}{C_t}^{-\gamma}mc_t + \lambda_{8,t}\Delta_t = \lambda_{6,t}{C_t}^{-\gamma} + \lambda_{9,t}, \tag{A.9}$$

$$[p_t^\star]: \quad (\epsilon-1)\lambda_{4,t}p_{2,t} = (1-\theta)(1-\epsilon)\lambda_{7,t}{p_t^\star}^{-\epsilon} + (1-\theta)\epsilon\lambda_{10,t}{p_t^\star}^{-\epsilon-1}, \tag{A.10}$$

$$[p_{1,t}]: \quad \epsilon(1-\tau)\lambda_{4,t} + \lambda_{5,t} = \theta\lambda_{5,t-1}{\pi_t}^{\epsilon}, \tag{A.11}$$

$$[p_{2,t}]: \quad (\epsilon-1)\lambda_{4,t}p_t^\star + \lambda_{6,t} = \theta\lambda_{6,t-1}{\pi_t}^{\epsilon-1}, \tag{A.12}$$

$$[\Delta_t]: \quad \lambda_{8,t}Y_t = \lambda_{10,t} - \theta\beta E_t\lambda_{10,t+1}{\pi_{t+1}}^{\epsilon}, \tag{A.13}$$

$$[\lambda_{1,t}]: \quad \chi {N_t}^{\eta} = {C_t}^{-\gamma}w_t, \tag{A.14}$$

$$[\lambda_{2,t}]: \quad {C_t}^{-\gamma} = \beta E_t {C_{t+1}}^{-\gamma}\frac{R_t}{\pi_{t+1}}, \tag{A.15}$$

$$[\lambda_{3,t}]: \quad mc_t = \frac{w_t}{A_t}, \tag{A.16}$$

$$[\lambda_{4,t}]: \quad p_t^\star = \frac{\epsilon}{\epsilon-1}\frac{(1-\tau)p_{1,t}}{p_{2,t}}, \tag{A.17}$$

$$[\lambda_{5,t}]: \quad p_{1,t} = {C_t}^{-\gamma}mc_tY_t + \theta\beta E_t{\pi_{t+1}}^{\epsilon}p_{1,t+1}, \tag{A.18}$$

$$[\lambda_{6,t}]: \quad p_{2,t} = {C_t}^{-\gamma}Y_t + \theta\beta E_t{\pi_{t+1}}^{\epsilon-1}p_{2,t+1}, \tag{A.19}$$

$$[\lambda_{7,t}]: \quad \theta{\pi_t}^{\epsilon-1} = 1 - (1-\theta){p_t^\star}^{1-\epsilon}, \tag{A.20}$$

$$[\lambda_{8,t}]: \quad \Delta_tY_t = A_tN_t, \tag{A.21}$$

$$[\lambda_{9,t}]: \quad Y_t = C_t, \tag{A.22}$$

$$[\lambda_{10,t}]: \quad \Delta_t = (1-\theta){p_t^\star}^{-\epsilon} + \theta{\pi_t}^{\epsilon}\Delta_{t-1}. \tag{A.23}$$

B Rotemberg Model: Ramsey-Optimal Policy Problem

In the Rotemberg model, the Ramsey problem under commitment can be described as follows. Let $\{\lambda_{1,t}, \lambda_{2,t}, \lambda_{3,t}, \lambda_{4,t}, \lambda_{5,t}, \lambda_{6,t}\}_{t=0}^{\infty}$ represent sequences of Lagrange multipliers on the constraints (1), (2), (6), (16), (17), and (18). Given a set of stochastic processes $\{A_t, \sigma_t^A\}_{t=0}^{\infty}$, the allocation plans for the control variables $\{C_t, N_t, w_t, R_t, mc_t, \pi_t, Y_t\}_{t=0}^{\infty}$, and for the co-state variables $\{\lambda_{1,t}, \lambda_{2,t}, \lambda_{3,t}, \lambda_{4,t}, \lambda_{5,t}, \lambda_{6,t}\}_{t=0}^{\infty}$, represent an optimal allocation if they solve the following maximization problem:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{{C_t}^{1-\gamma}}{1-\gamma} - \chi \frac{{N_t}^{1+\eta}}{1+\eta} \right), \tag{B.1}$$

subject to (1), (2), (6), (16), (17), and (18). The augmented Lagrangian for the optimal policy problem then reads as follows:

$$\begin{aligned}
&\max E_0 \sum_{t=0}^{\infty} \beta^t \Bigg[\left(\frac{{C_t}^{1-\gamma}}{1-\gamma} - \chi \frac{{N_t}^{1+\eta}}{1+\eta} \right) + \lambda_{1,t} \left({C_t}^{-\gamma} w_t - \chi {N_t}^{\eta} \right) + \lambda_{2,t} \left(\beta {C_{t+1}}^{-\gamma} R_t - {C_t}^{-\gamma} \pi_{t+1} \right) \\
&\qquad + \lambda_{3,t} \left(mc_t A_t - w_t \right) \\
&+ \lambda_{4,t} \left(\psi {C_t}^{-\gamma} (\pi_t - 1) \pi_t Y_t - \psi \beta {C_{t+1}}^{-\gamma} (\pi_{t+1} - 1) \pi_{t+1} Y_{t+1} - (1-\epsilon) {C_t}^{-\gamma} Y_t - \epsilon (1-\tau) ({C_t}^{-\gamma} mc_t Y_t) \right) \\
&\qquad + \lambda_{5,t} \left(A_t N_t - Y_t \right) + \lambda_{6,t} \left(Y_t - C_t - \frac{\psi}{2} (\pi_t - 1)^2 Y_t \right) \Bigg].
\end{aligned} \tag{B.2}$$

The first-order conditions are as follows:

$$\begin{aligned}
[C_t]: \quad & {C_t}^{-\gamma} + \gamma E_t \lambda_{2,t} {C_t}^{-\gamma-1} \pi_{t+1} + \psi \gamma \lambda_{4,t-1} {C_t}^{-\gamma-1} (\pi_t - 1) \pi_t Y_t \\
&= \gamma \lambda_{1,t} {C_t}^{-\gamma-1} w_t + \gamma \lambda_{4,t} {C_t}^{-\gamma-1} Y_t \left(\psi (\pi_t - 1) \pi_t - 1 + \epsilon - \epsilon (1-\tau) mc_t \right) + \lambda_{6,t} + \gamma \lambda_{2,t-1} {C_t}^{-\gamma-1} R_{t-1},
\end{aligned} \tag{B.3}$$

$$[N_t]: \quad \chi {N_t}^{\eta} + \chi \eta \lambda_{1,t} {N_t}^{\eta-1} = \lambda_{5,t} A_t, \tag{B.4}$$

$$[w_t]: \quad \lambda_{1,t} {C_t}^{-\gamma} = \lambda_{3,t}, \tag{B.5}$$

$$[R_t]: \quad \beta E_t \lambda_{2,t} {C_{t+1}}^{-\gamma} = 0, \tag{B.6}$$

$$[mc_t]: \quad \lambda_{3,t} A_t = \epsilon (1-\tau) \lambda_{4,t} {C_t}^{-\gamma} Y_t, \tag{B.7}$$

$$[\pi_t]: \quad \psi \lambda_{4,t} {C_t}^{-\gamma} (2\pi_t - 1) Y_t = \psi \lambda_{6,t} (\pi_t - 1) Y_t + \frac{1}{\beta} \lambda_{2,t-1} {C_{t-1}}^{-\gamma} + \psi \lambda_{4,t-1} {C_t}^{-\gamma} (2\pi_t - 1) Y_t, \tag{B.8}$$

$$\begin{aligned}
[Y_t]: \quad & \lambda_{4,t} {C_t}^{-\gamma} \left(\psi (\pi_t - 1) \pi_t - 1 + \epsilon - \epsilon (1-\tau) mc_t \right) + \lambda_{6,t} \left(1 - \frac{\psi}{2} (\pi_t - 1)^2 \right) \\
&= \lambda_{5,t} + \psi \lambda_{4,t-1} {C_t}^{-\gamma} (\pi_t - 1) \pi_t,
\end{aligned} \tag{B.9}$$

$$[\lambda_{1,t}]: \quad \chi {N_t}^{\eta} = {C_t}^{-\gamma} w_t, \tag{B.10}$$

$$[\lambda_{2,t}]: \quad {C_t}^{-\gamma} = \beta E_t {C_{t+1}}^{-\gamma} \frac{R_t}{\pi_{t+1}}, \tag{B.11}$$

$$[\lambda_{3,t}]: \quad mc_t = \frac{w_t}{A_t}, \tag{B.12}$$

$$[\lambda_{4,t}]: \quad \psi\left(\pi_t - 1\right)\pi_t = \psi\beta E_t\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\left(\pi_{t+1} - 1\right)\pi_{t+1}\frac{Y_{t+1}}{Y_t} + 1 - \epsilon + \epsilon\left(1-\tau\right)mc_t, \tag{B.13}$$

$$[\lambda_{5,t}]: \quad Y_t = A_t N_t, \tag{B.14}$$

$$[\lambda_{6,t}]: \quad Y_t = C_t + \frac{\psi}{2}\left(\pi_t - 1\right)^2 Y_t. \tag{B.15}$$