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# Monetary policy rules for an open economy with financial frictions: A Bayesian approach

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# Monetary policy rules for an open economy with financial frictions: A Bayesian approach

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**Abstract**

This paper evaluates optimal monetary policy in a new Keynesian model for an open economy with financial frictions. In the model, aggregate demand is made up of the weighted average of the short and long-term interest rates. A comprehensive set of monetary policy rules is established, all suitable for small open economies, such as Peru. A domestic inflation forecast based rule and an exchange rate based rule are found to work well. Furthermore, international shocks can affect competitiveness and involve co-movements in domestic interest rates. Finally, the estimates suggest that adding the nominal exchange rate to the monetary rule significantly improves the model fit. Consequently, the estimated parameters indicate that international shocks introduced in this model can replicate key empirical facts observed in the domestic business cycle.

Keywords: Bayesian estimation, Financial frictions, Open economy, Optimal monetary policy, Rules comparison.

JEL classification: C11, E31, E32, E44, E52, E58.

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I dedicate this research to the memory of my grandparents and my parents who lived through a time of high inflation.

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# 1 Introduction

This document evaluates the performance of different monetary policy rules in a new Keynesian model for an open economy with imperfect substitution of short and long term government bonds. The most popular rule, the Taylor rule was designed for closed economies, in which it is assumed that the exchange rate channel is unimportant and that financial assets are perfectly substituted. So here it is possible to introduce these two concepts in an economy like Peru, which being a country that trades intensively with the world, the usual Taylor rule, may not be the most efficient. On the other hand, being a shallow financial market, short and long-term government bonds are not necessarily perfect substitutes.

After the 2008 financial crisis, the economic environment has evolved in ways that were previously considered uncommon. The relevance of financial frictions in the business cycle dynamics is the subject of a broad research activity. With the development of new estimation techniques for stochastic dynamic general equilibrium models, it is possible to highlight the role of the financial sector as a source of fluctuation of the business cycle and as a propagator of adverse shocks. There is a wide debate on how to ensure the stability of the economic system with financial friction prone to external shocks, this has led to the development of a set of monetary and macroprudential policy tools to ensure economic stability.

On the theoretical side, it is possible to focus on an extension of the canonical new Keynesian model, in which aggregate demand is driven by a weighted average of the interest rates on short-term and long-term bonds. Aggregate supply, characterized by the Phillips curve, assumes that prices are rigid and do not immediately adjust to changes in costs or demand. The nominal exchange rate and the uncovered parity of interest rates fulfill the role of propagating international shocks in the domestic economy. Following Batini, Harrison, and Millard (2003), it is possible to establish a comprehensive set of ten monetary policy rules and the calculation of the welfare-based loss function.

On the estimation side, I combine my prior information with the maximum likelihood function to estimate the posterior distribution of the parameters that are the source of fluctuation in the model. The Kalman filter is used to evaluate the likelihood function and the Random Walk Metropolis-Hastings Monte Carlo Markov Chains (RWMH-MCMC) algorithm to draw the posterior distribution of each model rule. In addition, the marginal probability, the Bayes factor, and the posterior marginal probability are used to compare each rule in the model. In line with the established, it is possible to determine which rule best fits the data.

My findings suggest that, for an open economy like Peru, a domestic inflation forecast based rule and an exchange rate based rule work well in minimizing the loss function. In contrast to the Taylor rule that has a greater loss function. Although the Peruvian economy has an explicit inflation targeting scheme, its central bank uses a set of monetary policy rules for each particular shock.

On the other hand, international shocks can affect competitiveness and involve co-movements in domestic interest rates. Banerjee, Devereux, and Lombardo (2015), mention that an international contractionary monetary shock leads to a reduction in capital flows and an exchange rate depreciation in emerging countries. Furthermore, optimal policies do not need to be coordinated between countries. Kolasa and Wesołowski (2018), mention that monetary stimuli with low interest rates by advanced economies, cause collateral effects related to capital flows, leading to an increase in international co-movements of the term premium and an appreciation of the exchange rate in emerging economies.

Next, it is possible to use Bayesian methods developed by Geweke (1998), Lubik and Schorfheide (2005), Rabanal and Rubio-Ramírez (2005), An and Schorfheide (2006), and Smets and Wouters (2007). According to my model simulations, the rules based on the nominal exchange rate provide a better fit. Consequently, the estimated parameters indicate that there is less volatility in the shock of international interest rates and greater persistence in the shock of the world price level. This empirically contrasts the initial results and what is found in Batini, Levine, and Pearlman (2009), showing that a simple rule of nominal exchange rate improves welfare gains, better fits the data, and is optimal.

My contribution consists in adding the adjustment costs of the short and long term government bond portfolio to the analysis of an open economy. Other contributions use portfolio adjustment costs in a context of large-scale asset purchase when the economy crosses the zero lower bound in its interest rates. Andrés, López-Salido, and Nelson (2004), evaluate optimal monetary policy in a closed economy stochastic dynamic general equilibrium model, which allows imperfect substitution between financial assets. This modification provides an additional channel through which the relative prices of financial assets can affect long-term returns and aggregate demand. Harrison (2012), uses a new Keynesian closed economy model with imperfect short and long-term bond substitution, thereby evaluating a policy in which the central bank uses asset purchases as an unconventional policy instrument to improving the stabilization of the output gap and inflation, which due to a negative demand shock, pushes the short-term policy rate to its lower limit.

In an economy like Peru, interest rates have not yet reached their lower limit, but there is a narrow margin to cover. The interest rate pass-through is incomplete because the impact on long-term rates is slower and less than the adjustment in the monetary policy rate, this reduces the power of conventional monetary policy. It should be noted that the Peruvian central bank has maintained an expansive monetary policy since April 2017, so that the main interest rates maintain a downward trajectory.

Currently, the domestic capital market is the destination of a large flow of funds as a result of a greater demand for sovereign bonds by non-resident investors. Some distinguishing features with respect to Castillo, Montoro, and Tuesta (2009) and Vega (2015) is that the optimal monetary policy rule is evaluated with greater emphasis on capital flows, the process of de-dollarization of the domestic economy, and the long-term bonds market. Assuming these suppositions is possibly more realistic considering monetary stimulus around the world.

The rest of the document has a sequential structure. Section 2, describes the new Keynesian model with financial friction for an open economy, establish equilibrium, the dynamics of each monetary policy rule, the loss function, and calibration. Section 3 describes the data, the estimate, the comparison for each rule in the model, and the priors. Section 4, presents and discusses the results. Section 5, concludes.

# 2 Model construction

This model provides an overview of an open economy with financial friction which is based on Galí (2015, chap. 8) and Harrison (2017). In this economy the uncertainty comes from the demand and supply side, shocks of monetary policy, productivity, preferences, international interest rate, and international price level are assumed. In each case, the monetary policy implications are addressed with a special emphasis on rules that central banks currently use for their monetary policy design. Considering that the domestic economy trades with the world, through Galí (2015, chap. 8), it is possible to use a simple extension of the new Keynesian model in an open economy to highlight the determination of optimal monetary policy and its contribution to the stability of inflation, economic fluctuations and welfare. Modifying the Harrison (2017) model, it is possible to focus on evaluating the optimal monetary policy when families face adjustment costs of the government bond portfolio, in the budget constraint. The complete derivation of the model is presented in Appendix A.

## 2.1 Budget constraint

A typical small, open economy inhabited by families have resource constraints; with the income they collect, they allocate their spending to a set of goods and assets. There are three assets in the economy: nominal short-term government bonds, nominal long-term government bonds, and state-contingent assets. Long-term nominal bonds are taken as instruments of infinite maturity and state-contingent assets are traded in full international markets. Considering what is proposed by Woodford (2003, chap. 2), the budget constraint is made up of nominal assets, in an economy without monetary balances.

$$\mathrm{B}_t^s+\mathrm{B}_t^l+\mathbb{E}_t\mathrm{Q}_{t,t+1}\mathrm{A}_{t+1}+\mathrm{P}_t\mathrm{C}_t=\mathrm{R}_{t-1}\mathrm{B}_{t-1}^s+\mathrm{R}_t^l\mathrm{B}_{t-1}^l+\mathrm{A}_t+\mathrm{W}_t\mathrm{N}_t+\mathrm{T}_t+\mathrm{D}_t-\Psi_t \tag{1}$$

The left side captures the spending, $\mathrm{B}_t^s$ are the nominal short-term government bonds, $\mathrm{B}_t^l$ are the nominal long-term government bonds, $\mathrm{A}_{t+1}$ are the holdings of state-contingent assets, $\mathrm{Q}_{t,t+1}$ is the price of the asset (also considered the stochastic discount factor) and $\mathbb{E}_t$ their conditional expectation, $\mathrm{C}_t$ are consumer goods, and $\mathrm{P}_t$ is the consumer price index. The right side captures income, $\mathrm{B}_{t-1}^s$ are nominal short-term government bonds purchased in the prior period that mature in the current period with a nominal payment of $\mathrm{R}_{t-1}$ per bond, $\mathrm{B}_{t-1}^l$ are nominal long-term government bonds purchased in the previous period that mature in the current period with a nominal payment of $\mathrm{R}_t^l$ per bond, $\mathrm{N}_t$ are the hours worked, $\mathrm{W}_t$ is the nominal salary, $\mathrm{T}_t$ are net transfers or taxes, $\mathrm{D}_t$ are company dividends, and $\Psi_t$ are the portfolio adjustment costs. $\mathrm{R}_t$ is the short-term interest rate and $\mathrm{R}_t^l$ is the long-term interest rate.

These nominal government bonds, being imperfect substitutes, maintain quadratic adjustment costs.

$$\Psi_t=\frac{\mathrm{P}_t\left(\mathrm{b}^s+\mathrm{b}^l\right)}{2}\left[\delta\frac{\mathrm{B}_t^s}{\mathrm{B}_t^l}-1\right]^2+\frac{\mathrm{P}_t\left(\mathrm{b}^s+\mathrm{b}^l\right)}{2}\left[\frac{\mathrm{B}_t^s/\mathrm{B}_t^l}{\mathrm{B}_{t-1}^s/\mathrm{B}_{t-1}^l}-1\right]^2 \tag{2}$$

Portfolio adjustment costs are a convex function that reflects brokerage costs. $\delta$ is set equal to the ratio of long-term government bonds to short-term government bonds. These portfolio costs are zero in the steady state, $\mathrm{b}^s$ are the nominal short-term government bonds in the steady state, and $\mathrm{b}^l$ are the nominal long-term government bonds in the steady state.

These adjustment costs have two components. The first component is a function of the deviation of the portfolio mix, $B^s_t/B^l_t$ of their desired level. These adjustment costs attempt to capture changes in the supply of these bonds that may have direct effects on their rates. The second component is a function of the deviation of the change in the portfolio mix. These adjustment costs show that changes in bonds supplies associated with their acquisition have an effect on the rate of the purchased bonds and their close substitutes. Later this relationship can be shown in aggregate demand, in imperfect markets central banks have an additional channel to boost aggregate demand.

## 2.2 Households

The utility function plays an important role in building a general equilibrium model, it is assumed that the economy is populated by a set of identical households that have infinite life, this assumption allows analyzing their behavior through the study of a representative agent seeking to maximize its discounted utility function.

$$\max_{\left\{B^s_t,B^l_t,A_{t+1},C_t,N_t\right\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t g_t \left\{ \frac{C_t^{1-\sigma}-1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right\} \tag{3}$$

Where $\beta$ is the discount factor, $\sigma$ is the risk aversion coefficient, $\varphi$ corresponds to the inverse labor supply elasticity or commonly known as the Frisch elasticity, and $g_t$ represents the shock of preferences that influences the behavior of the natural real interest rate (discussed later).

On the other hand, maximization is subject to budget constraint, which includes the formulation of portfolio adjustment costs, $\Psi$.

$$\begin{aligned} B^s_t+B^l_t+\mathbb{E}_t Q_{t,t+1}A_{t+1}+P_tC_t = {} & R_{t-1}B^s_{t-1}+R^l_tB^l_{t-1}+A_t+W_tN_t+D_t \\ & -\frac{P_t\left(B^s+b^l\right)}{2}\left[\delta\frac{B^s_t}{B^l_t}-1\right]^2-\frac{P_t\left(B^s+b^l\right)}{2}\left[\frac{B^s_t/B^l_t}{B^s_{t-1}/B^l_{t-1}}-1\right]^2 \end{aligned} \tag{4}$$

The condition of not excessive accumulation of debt.

$$\lim_{T\to\infty} \mathbb{E}_t Q_{t,T+1}\left\{R_T B^s_T+R^l_{T+1}B^l_T+A_{T+1}\right\} \tag{5}$$

This restriction establishes that in the long term the household's net nominal liabilities must grow at a lower rate than the nominal interest rate. This condition rules out schemes in which households renew their net debts forever.

### 2.2.1 Intratemporal

Consumer spending is made up of consumer spending on domestic goods and spending on imported goods.

$$P_tC_t = P_{d,t}C_{d,t} + P_{m,t}C_{m,t} \tag{6}$$

Where $P_{d,t}$ is the domestic price index, $P_{m,t}$ is the imported price index expressed in domestic currency, $C_{d,t}$ and $C_{m,t}$ are consumption indices of domestic and imported goods respectively.

Consumption and the consumer price index are defined by the Constant Elasticity of Substitution (CES) function.

$$\mathrm{C_t} \equiv \left[ (1-\alpha)^{\frac{1}{\eta}} \mathrm{C_{d,t}}^{\frac{\eta-1}{\eta}} - \alpha^{\frac{1}{\eta}} \mathrm{C_{m,t}}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \tag{7}$$

$$\mathrm{P_t} \equiv \left[ (1-\alpha)^{\frac{1}{\eta}} \mathrm{P_{d,t}}^{1-\eta} - \alpha^{\frac{1}{\eta}} \mathrm{P_{m,t}}^{1-\eta} \right]^{\frac{1}{1-\eta}} \tag{8}$$

The parameter $\eta$ measures the degree of substitution between domestic and imported goods and the parameter $\alpha$ can be interpreted as a measure of openness.

### Consumption of domestic and imported goods

$$\mathrm{C_{d,t}} \equiv \left( \int_0^1 \mathrm{C_{d,t}}\,(\mathrm{i})^{\frac{\varepsilon-1}{\varepsilon}} \mathrm{di} \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{9}$$

$$\mathrm{C_{m,t}} \equiv \left( \int_0^1 \mathrm{C_{m,t}}\,(\mathrm{j})^{\frac{\varepsilon-1}{\varepsilon}} \mathrm{dj} \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{10}$$

Where $\mathrm{i} \in [0,1]$ represents the variety of household goods and $\mathrm{j} \in [0,1]$ represents the variety of imported goods. Each consumption represents the consumption of differentiated goods. $\mathrm{C_{d,t}}\,(\mathrm{i})$ is individual household consumption and $\mathrm{C_{m,t}}\,(\mathrm{j})$ is the imported individual consumption. The parameter $\varepsilon$ denotes the elasticity of substitution between varieties produced domestically.

### Price index of domestic and imported goods

$$\mathrm{P_{d,t}} \equiv \left( \int_0^1 \mathrm{P_{d,t}}\,(\mathrm{i})^{1-\varepsilon} \mathrm{di} \right)^{\frac{1}{1-\varepsilon}} \tag{11}$$

$$\mathrm{P_{m,t}} \equiv \left( \int_0^1 \mathrm{P_{m,t}}\,(\mathrm{j})^{1-\varepsilon} \mathrm{dj} \right)^{\frac{1}{1-\varepsilon}} \tag{12}$$

Where $\mathrm{P_{d,t}}\,(\mathrm{i})$ is the domestic individual price and $\mathrm{P_{m,t}}\,(\mathrm{j})$ is the imported individual price expressed in domestic currency.

### Demand for consumption of domestic and imported goods

$$\mathrm{C_{d,t}}\,(\mathrm{i}) = (1-\alpha) \left( \frac{\mathrm{P_{d,t}}\,(\mathrm{i})}{\mathrm{P_{d,t}}} \right)^{-\varepsilon} \left( \frac{\mathrm{C_{d,t}}}{\mathrm{P_t}} \right)^{-\eta} \mathrm{C_t} \tag{13}$$

$$\mathrm{C_{m,t}}\,(\mathrm{j}) = \alpha \left( \frac{\mathrm{P_{m,t}}\,(\mathrm{i})}{\mathrm{P_{m,t}}} \right)^{-\varepsilon} \left( \frac{\mathrm{C_{m,t}}}{\mathrm{P_t}} \right)^{-\eta} \mathrm{C_t} \tag{14}$$

These identities are the optimal allocations through the variety in domestic and imported goods that results in the consumption demand function of domestic and imported goods.

### 2.2.2 Intertemporal

Each variable with a circumflex is expressed in its log-linear form. Following the method of Uhlig (1995), $\widehat{\mathfrak{a}}_t = \log(\mathfrak{a}_t) - \log(\mathfrak{a}_{ss})$ is the deviation of the variable $\mathfrak{a}_t$ from its steady state value $\mathfrak{a}_{ss}$.

Using the first-order conditions of the optimization process, the main consumption equation is reached. This equation is represented as the log-deviation of its steady state.

**Euler equation**

$$\begin{aligned}\widehat{c}_t = \mathbb{E}_t\widehat{c}_{t+1} - \frac{1}{\sigma}\left[\widehat{R}_t - \mathbb{E}_t\widehat{\pi}_{t+1}\right] + \frac{\delta(1+\delta)}{\sigma}\left[\widehat{b}^s_t - \widehat{b}^l_t\right] + \frac{(1+\delta)}{\sigma}\Delta\left[\widehat{b}^s_t - \widehat{b}^l_t\right] \\ - \frac{(1+\delta)}{\sigma}\beta\mathbb{E}_t\Delta\left[\widehat{b}^s_{t+1} - \widehat{b}^l_{t+1}\right] + \frac{1}{\sigma}(1-\rho_g)\widehat{g}_t\end{aligned} \tag{15}$$

Aggregate consumption is made up of expected consumption $\mathbb{E}_t\widehat{c}_{t+1}$, the nominal interest rate $\widehat{R}_t$, expected inflation $\mathbb{E}_t\widehat{\pi}_{t+1}$, short-term real bond holding $\widehat{b}^s_t$, long-term real bond holding $\widehat{b}^l_t$, and the preferences shock $\widehat{g}_t$. This equation can also be represented as a weighted average of short and long-term bond interest rates.

$$\widehat{c}_t = \mathbb{E}_t\widehat{c}_{t+1} - \frac{1}{\sigma}\left[\frac{1}{1+\delta}\widehat{R}_t + \frac{\delta}{1+\delta}\mathbb{E}_t\widehat{R}^l_{t+1} - \mathbb{E}_t\widehat{\pi}_{t+1}\right] + \frac{1}{\sigma}(1-\rho_g)\widehat{g}_t \tag{16}$$

**Expected long-term nominal interest rate**

$$\mathbb{E}_t\widehat{R}^l_{t+1} = \widehat{R}_t - \delta\gamma\left[\widehat{b}^s_t - \widehat{b}^l_t\right] - \gamma\Delta\left[\widehat{b}^s_t - \widehat{b}^l_t\right] + \beta\gamma\mathbb{E}_t\Delta\left[\widehat{b}^s_{t+1} - \widehat{b}^l_{t+1}\right] \tag{17}$$

Where $\gamma \equiv (1+\delta)^2/\delta$. This equation indicates that the expected long-term interest rate depends on the household's relative holdings of short-term and long-term bonds. Consequently, an increase in the household's relative holdings of short-term bonds acts like a reduction in the nominal long-term interest rate, increasing demand for consumption.

## 2.3 Important identities

**Terms of trade**

$$\widehat{tt}_t = \widehat{p}_{m,t} - \widehat{p}_{d,t} \tag{18}$$

This identity is defined as the difference between the imported price index and the domestic price index.

**Consumer's price index (CPI)**

$$\widehat{p}_t = (1-\alpha)\widehat{p}_{d,t} + \alpha\widehat{p}_{m,t} \tag{19}$$

$$\widehat{p}_t = \widehat{p}_{d,t} + \alpha\widehat{tt}_t \tag{20}$$

The CPI (19) is made up of the domestic price index and the imported price index. This aggregate price index can be expressed using the terms of trade in the identity (20).

### CPI inflation and domestic inflation

$$\widehat{\pi}_t = \widehat{p}_t - \widehat{p}_{t-1} \tag{21}$$

$$\widehat{\pi}_{d,t} = \widehat{p}_{d,t} - \widehat{p}_{d,t-1} \tag{22}$$

$$\widehat{\pi}_t = \widehat{\pi}_{d,t} + \alpha \left( \widehat{tt}_t - \widehat{tt}_{t-1} \right) \tag{23}$$

The CPI inflation (21) is defined as the rate of chance in the CPI, domestic inflation (22) is defined as the rate of change in the domestic price index. The previous two identities can be related using the terms of trade (23).

### Nominal exchange rate

$$\widehat{tt}_t = \widehat{e}_t + \widehat{p}^*_t - \widehat{p}_{d,t} \tag{24}$$

It is assumed that the law of one price is maintained in all periods, $\widehat{e}_t$ is the nominal exchange rate, $\widehat{p}^*_t$ is the world price level expressed in foreign currency, this variable is represented by a shock. The nominal exchange rate is expressed as the price of a foreign currency in terms of domestic currency.

### Real exchange rate

$$\widehat{q}_t = \widehat{p}_{m,t} - \widehat{p}_t \tag{25}$$

$$\widehat{q}_t = (1-\alpha)\, \widehat{tt}_t \tag{26}$$

Real exchange rate (25) is defined as the difference between the imported price index and the CPI, both terms are expressed in domestic currency, just as the other variables can also be related to the terms of trade (26).

### International risk sharing

$$\widehat{c}_t = \widehat{y}^*_t + \frac{1}{\sigma} g_t + \left( \frac{1-\alpha}{\sigma} \right) \widehat{tt}_t \tag{27}$$

This identity assumes a complete set of internationally traded state-contingent assets, where $\widehat{y}^*_t$ is the world product.

### Uncovered interest rates parity

$$\widehat{R}_t = \widehat{R}^*_t + \mathbb{E}_t \widehat{e}_{t+1} - \widehat{e}_t \tag{28}$$

A flexible exchange rate regime is assumed with perfect capital mobility. Where $\widehat{R}^*_t$ is the international interest rate, this rate is represented by a shock. This equation starts from the differential between the domestic and international interest rates, the exchange rate adjusts slowly. If the return in domestic currency is greater than the return in foreign currency, investors only have to wait for the domestic currency to depreciate.

### Exports

$$X_t = \alpha TT^{\eta}_{ss} Y^*_t \tag{29}$$

Where $TT_{ss}$ are the steady state terms of trade and $Y^*_t$ is the world product level. This identity assumes that the preferences of domestic households are identical to the preferences of households in the rest of the world, and also implies international consumption equivalent to the world output, $C^*_t = Y^*_t$.

## 2.4 Firms

It features monopolistic competition in goods markets. Each company is indexed by $i \in [0,1]$, which produces differentiated goods that are purchased by households.

$$Y_t \equiv \left( \int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{30}$$

Where $Y_t$ is the aggregate domestic product, analogous to domestic consumption.

### 2.4.1 Technology

Firms produce using constant returns production function.

$$Y_t(i) = A_t N_t(i) \tag{31}$$

Where $Y_t(i)$ is firm i's output, $A_t$ represents the level of technology that evolves exogenously through time, and $N_t(i)$ is the level of labor contracted by each firm.

### 2.4.2 Optimal price setting

Following Calvo (1983), the domestic firm sets prices in staggered fashion, the objective function to be re-optimized is,

$$\max_{\widetilde{P}_{d,t}} \mathbb{E}_t \sum_{k=0}^{\infty} Q_{t,t+k}\theta^k \left\{ \widetilde{P}_{d,t} - (1-\mathbb{S}) \frac{W_{t+k}}{A_{t+k}} \right\} Y_{t+k|t} \tag{32}$$

Firms choose the price $\widetilde{P}_{d,t}$ that maximizes the present value of the market benefits generated while that price remains effective, $Q_{t,t+k}$ represents the stochastic discount factor for the period $t+k$, $\theta$ is the natural index of prices rigidities or also known as the probability that the firm will not re-optimize its price, $\mathbb{S}$ is a subsidy that neutralizes the distortion associated with firms' market power.

#### Phillips curve with price rigidities

$$\widehat{\pi}_{d,t} = \beta \mathbb{E}_t \widehat{\pi}_{d,t+1} - \lambda \widehat{\mu}_t \tag{33}$$

This equation shows that domestic inflation is made up of expected domestic inflation, $\mathbb{E}_t \widehat{\pi}_{d,t+1}$, and the markup gap, $\widehat{\mu}_t$. Initially, it follows that for this economy domestic inflation does not depend on any parameter that characterizes an open economy. Where $\lambda \equiv (1-\theta)(1-\beta\theta)/\theta$.

## 2.5 Equilibrium

#### Aggregate demand

Domestic goods market equilibrium requires the following identity.

$$Y(i) = \left( \frac{P_{d,t}(i)}{P_{d,t}} \right)^{-\varepsilon} \left\{ (1-\alpha) \left( \frac{P_{d,t}}{P_t} \right)^{-\eta} C_t + \alpha TT_{ss}^{\eta} Y_t^* \right\} + \Psi_t \tag{34}$$

Putting the aggregate domestic product (30) in the definition of aggregate domestic demand, the following condition is reached.

$$\mathrm{Y_t}=(1-\alpha)\left(\frac{\mathrm{P_{d,t}}}{\mathrm{P_t}}\right)^{-\eta}\mathrm{C_t}+\alpha \mathrm{TT_t^{\eta}Y_t^{*}}+\Psi_{\mathrm{t}} \tag{35}$$

Log-linear portfolio adjustment costs do not affect the equilibrium in goods market, the aggregate product in equilibrium can be approximated through its symmetric steady state.

$$\widehat{\mathrm{y}}_{\mathrm{t}}=(1-\alpha)\,\widehat{\mathrm{c}}_{\mathrm{t}}+\alpha\,(2-\alpha)\,\eta\widehat{\mathrm{tt}}_{\mathrm{t}}+\alpha \mathrm{y_t^{*}} \tag{36}$$

Terms of trade can be derived using (27) and (36).

$$\widehat{\mathrm{tt}}_{\mathrm{t}}=\sigma_{\alpha}\left(\widehat{\mathrm{y}}_{\mathrm{t}}-\widehat{\mathrm{y}}_{\mathrm{t}}^{*}\right)-(1-\alpha)\,\Phi\widehat{\mathrm{g}}_{\mathrm{t}} \tag{37}$$

Where $\sigma_{\alpha}\equiv\sigma\Phi$, $\Phi\equiv1/(1+\alpha\,(\varpi-1))$, and $\varpi\equiv\sigma\eta+(1-\alpha)\,(\sigma\eta-1)$.

Consumption in an open economy is the result of combining Euler's equation (16) and domestic inflation (23)

$$\widehat{\mathrm{c}}_{\mathrm{t}}=\mathbb{E}_{\mathrm{t}}\widehat{\mathrm{c}}_{\mathrm{t+1}}-\frac{1}{\sigma}\left\{\frac{1}{1+\delta}\widehat{\mathrm{R}}_{\mathrm{t}}-\mathbb{E}_{\mathrm{t}}\widehat{\pi}_{\mathrm{d,t+1}}+\frac{\delta}{1+\delta}\mathbb{E}_{\mathrm{t}}\widehat{\mathrm{R}}^{\mathrm{l}}_{\mathrm{t+1}}\right\}+\frac{\alpha}{\sigma}\mathbb{E}_{\mathrm{t}}\Delta\widehat{\mathrm{tt}}_{\mathrm{t+1}}+\frac{1}{\sigma}\left(1-\rho_{\mathrm{g}}\right) \tag{38}$$

Combining (36), (37), and (38) results in the first version of the dynamic IS equation for a small and open economy.

$$\widehat{\mathrm{y}}_{\mathrm{t}}=\mathbb{E}_{\mathrm{t}}\widehat{\mathrm{y}}_{\mathrm{t+1}}-\frac{1}{\sigma_{\alpha}}\left\{\frac{1}{1+\delta}\widehat{\mathrm{R}}_{\mathrm{t}}-\mathbb{E}_{\mathrm{t}}\widehat{\pi}_{\mathrm{d,t+1}}+\frac{\delta}{1+\delta}\mathbb{E}_{\mathrm{t}}\widehat{\mathrm{R}}^{\mathrm{l}}_{\mathrm{t+1}}\right\}+\Gamma_{\varpi}\mathbb{E}_{\mathrm{t}}\Delta\widehat{\mathrm{y}}^{*}_{\mathrm{t+1}}+\frac{1-\alpha}{\sigma}\left(1-\rho_{\mathrm{g}}\right)\widehat{\mathrm{g}}_{\mathrm{t}} \tag{39}$$

Where $\Gamma_{\varpi}\equiv\alpha\varpi-\alpha$.

The so-called business cycle is characterized by the output gap.

$$\widehat{\mathrm{x}}_{\mathrm{t}}=\widehat{\mathrm{y}}_{\mathrm{t}}-\widehat{\mathrm{y}}^{\mathrm{n}}_{\mathrm{t}} \tag{40}$$

This identity corresponds to the deviation between the product and the natural product. The output gap is positive if the current product exceeds the natural product, and negative if the current product does not reach the natural product.

In general, equation (39) can be rewritten in terms of the output gap,

$$\widehat{\mathrm{x}}_{\mathrm{t}}=\mathbb{E}_{\mathrm{t}}\widehat{\mathrm{x}}_{\mathrm{t+1}}-\frac{1}{\sigma_{\alpha}}\left\{\frac{1}{1+\delta}\widehat{\mathrm{R}}_{\mathrm{t}}-\mathbb{E}_{\mathrm{t}}\widehat{\pi}_{\mathrm{d,t+1}}+\frac{\delta}{1+\delta}\mathbb{E}_{\mathrm{t}}\widehat{\mathrm{R}}^{\mathrm{l}}_{\mathrm{t+1}}-\widehat{\iota}^{\mathrm{n}}_{\mathrm{t}}\right\} \tag{41}$$

The dynamic IS curve is made up of the weighted average of the short and long term interest rates. This equation represents the aggregate demand of the economy. The natural real interest rate, $\widehat{\iota}^{\mathrm{n}}_{\mathrm{t}}$, has an implicit equilibrium concept that allows determining whether monetary policy is contractive or expansive. Through changes in the short-term nominal interest rate, the central bank can correct the deviations of the product or maintain the domestic inflation in a determined range, with imperfect bond substitution it is possible to have an additional channel of monetary policy that influences the aggregate demand.

**Consumption determined by budget constraint**

$$\widehat{\mathrm{nx}}=\widehat{\mathrm{y}}_{\mathrm{t}}-\widehat{\mathrm{c}}_{\mathrm{t}}-\alpha\widehat{\mathrm{tt}}_{\mathrm{t}} \tag{42}$$

**Net exports**

$$\widehat{\mathrm{nx}}_{\mathrm{t}} = \alpha\left(\frac{\varpi}{\sigma} - 1\right)\widehat{\mathrm{tt}}_{\mathrm{t}} - \frac{\alpha}{\sigma}\widehat{\mathrm{g}}_{\mathrm{t}} \tag{43}$$

This identity indicates that a negative preferences shock affects net exports, causing a trade deficit.

**Production function**

$$\widehat{\mathrm{y}}_{\mathrm{t}} = \widehat{\mathrm{a}}_{\mathrm{t}} + \widehat{\mathrm{n}}_{\mathrm{t}} \tag{44}$$

**Natural product**

$$\widehat{\mathrm{y}}_{\mathrm{t}}^{\mathrm{n}} = \Gamma_{\mathrm{a}}\widehat{\mathrm{a}}_{\mathrm{t}} + \Gamma_{\mathrm{g}}\widehat{\mathrm{g}}_{\mathrm{t}} + \Gamma_{*}\widehat{\mathrm{y}}_{\mathrm{t}}^{*} \tag{45}$$

This identity indicates that a positive productivity shock, a positive preference shock, and an increase in world product temporarily increases natural product, $\widehat{\mathrm{y}}_{\mathrm{t}}^{\mathrm{n}}$. In the long-run an economy that is in a recession or overheated must return to its natural level of production. Where $\Gamma_{\mathrm{a}} \equiv (1+\varphi)/(\sigma_{\alpha}+\varphi)$, $\Gamma_{\mathrm{g}} \equiv -\alpha\varpi\Phi/(\sigma_{\alpha}+\varphi)$, and $\Gamma_{*} \equiv -\alpha(\varpi-1)\sigma_{\alpha}/(\sigma_{\alpha}+\varphi)$.

**Natural terms of trade**

$$\widehat{\mathrm{tt}}_{\mathrm{t}}^{\mathrm{n}} = \sigma_{\alpha}\left(\widehat{\mathrm{y}}_{\mathrm{t}}^{\mathrm{n}} - \widehat{\mathrm{y}}_{\mathrm{t}}^{*}\right) - (1-\alpha)\Phi\widehat{\mathrm{g}}_{\mathrm{t}} \tag{46}$$

This identity indicates that an increase in world product and a negative productivity shock temporarily increases natural terms of trade, $\widehat{\mathrm{tt}}_{\mathrm{t}}^{\mathrm{n}}$.

**Terms of trade gap**

Next, terms of trade gap $\widetilde{\mathrm{tt}}$, can be represented using (37) and (46).

$$\widetilde{\mathrm{tt}}_{\mathrm{t}} = \sigma_{\alpha}\widehat{\mathrm{x}}_{\mathrm{t}} \tag{47}$$

**Markup gap**

$$\widehat{\mu}_{\mathrm{t}} = -(\sigma+\varphi)\widehat{\mathrm{y}}_{\mathrm{t}} + \alpha(\varpi-1)\widehat{\mathrm{tt}}_{\mathrm{t}} + (1+\varphi)\widehat{\mathrm{a}}_{\mathrm{t}} - \alpha\widehat{\mathrm{g}}_{\mathrm{t}} \tag{48}$$

$$\widehat{\mu}_{\mathrm{t}} = -(\sigma+\varphi)\widehat{\mathrm{x}}_{\mathrm{t}} + \alpha(\varpi-1)\widetilde{\mathrm{tt}}_{\mathrm{t}} \tag{49}$$

$$\widehat{\mu}_{\mathrm{t}} = -(\sigma_{\alpha}+\varphi)\widehat{\mathrm{x}}_{\mathrm{t}} \tag{50}$$

**Aggregate supply**

Combining (33) with (48) yields the Phillips curve for an open economy.

$$\widehat{\pi}_{\mathrm{d,t}} = \beta\mathbb{E}_{\mathrm{t}}\widehat{\pi}_{\mathrm{d,t+1}} + \kappa\widehat{\mathrm{y}}_{\mathrm{t}} - \lambda\alpha(\varpi-1)\widehat{\mathrm{tt}}_{\mathrm{t}} - \lambda(1+\varphi)\widehat{\mathrm{a}}_{\mathrm{t}} + \lambda\alpha\widehat{\mathrm{g}}_{\mathrm{t}} \tag{51}$$

This equation reflects that domestic inflation adjusts slowly from one period to another, there is a positive relationship with the product, it has a negative relationship with the terms of trade, it is exposed to productivity shocks (supply) and preference shocks (demand). Where $\kappa \equiv \lambda(\sigma+\varphi)$.

Combining (33) with (50) yields the new Keynesian Phillips curve.

$$\widehat{\pi}_{\mathrm{d,t}} = \beta\mathbb{E}_{\mathrm{t}}\widehat{\pi}_{\mathrm{d,t+1}} + \kappa_{\alpha}\widehat{\mathrm{x}}_{\mathrm{t}} \tag{52}$$

This equation indicates that domestic inflation is made up of expected domestic inflation and the output gap. In which $\kappa_{\alpha}\equiv\lambda\left(\sigma_{\alpha}+\varphi\right)$, shows that the dynamics of inflation is influenced by the degree of openness of the country, $\alpha$, and the substitution between domestic and imported goods, $\eta$. Thus, greater trade openness reduces the sensitivity of domestic inflation to the output gap.

**Real interest rate**

$$\widehat{\iota}_{t}=\widehat{R}_{t}-\mathbb{E}_{t}\widehat{\pi}_{d,t+1} \tag{53}$$

**Natural real interest rate**

$$\widehat{\iota}_{t}^{n}=-\sigma_{\alpha}\Gamma_{a}\left(1-\rho_{a}\right)\widehat{a}_{t}+\Theta_{*}\mathbb{E}_{t}\Delta\widehat{y}_{t+1}^{*}+\Theta_{g}\left(1-\rho_{g}\right)\widehat{g}_{t} \tag{54}$$

This identity indicates that a negative productivity shock, an increase in the expected world product growth, and a positive preferences shock temporarily increases the natural real interest rate. With $\Theta_{*}\equiv\sigma_{\alpha}\left(\alpha\left(\varpi-1\right)+\Gamma_{*}\right)$ and $\Theta_{g}\equiv\left(1-\alpha\right)\Phi-\sigma_{\alpha}\Gamma_{g}$.

**Government bonds**

Equilibrium for short-term bonds.

$$\frac{B_{t}^{s}}{P_{t}}=\widehat{b}_{t}^{s}=b \tag{55}$$

Equilibrium for long-term bonds.

$$\frac{B_{t}^{l}}{P_{t}}=\widehat{b}_{t}^{l}=\delta b \tag{56}$$

In equilibrium, the government bond market.

$$\widehat{b}_{t}^{s}-\widehat{b}_{t}^{l}=-\widehat{b}_{t}^{l} \tag{57}$$

Consequently, the relationship between long-term government bonds and the expected long-term interest rate.

$$\mathbb{E}_{t}\widehat{R}_{t+1}^{l}=\widehat{R}_{t}+\left(1+\delta+\beta\right)\gamma\widehat{b}_{t}^{l}-\gamma\widehat{b}_{t-1}^{l}-\beta\gamma\mathbb{E}_{t}\widehat{b}_{t+1}^{l} \tag{58}$$

**Wage**

$$\widehat{w}_{t}-\widehat{p}_{t}=\sigma\widehat{c}_{t}+\varphi\widehat{n}_{t} \tag{59}$$

**Terms of trade**

$$\widehat{tt}_{t}=\widetilde{tt}_{t}+\widehat{tt}_{t}^{n} \tag{60}$$

**World product**

$$\widehat{y}_{t}^{*}=0 \tag{61}$$

## 2.6 Monetary policy

Monetary policy rules are mathematical formulas that describe how central banks adjust their monetary policy rate to changes in main macroeconomic variables. I assume that changes in monetary policy rate are transmitted to short-term nominal interest rate and real interest rate in the economy.

**Rule 1 (R1)**

$$\widehat{\mathrm{R}}_t = \varphi_\pi \widehat{\pi}_t + \widehat{\xi}_t \tag{62}$$

This rule imposes a CPI inflation targeting scheme. Where $\varphi_\pi$ is the monetary rule inflation coefficient.

**Rule 2 (R2)**

$$\widehat{\mathrm{R}}_t = \varphi_\pi \widehat{\pi}_{d,t} + \widehat{\xi}_t \tag{63}$$

This rule imposes a domestic inflation targeting scheme.

**Rule 3 (R3)**

$$\widehat{\mathrm{e}}_t = 0 \tag{64}$$

Fixed nominal exchange rate regime. Banerjee et al. (2015), mention that in an emerging economy with financial frictions, an inflation targeting scheme has little advantage over a fixed exchange rate regime.

**Rule 4 (R4)**

$$\widehat{\mathrm{R}}_t = \varphi_\pi \widehat{\pi}_t + \varphi_x \widehat{\mathrm{x}}_t + \widehat{\xi}_t \tag{65}$$

Following Taylor (1993), this rule was designed for a closed economy, where the monetary policy exchange rate channel does not have a significant role in propagating monetary impulses. Formally, Taylor rule reacts to deviations from inflation and output gap. Where $\varphi_x$ is the monetary rule output gap coefficient.

**Rule 5 (R5)**

$$\widehat{\mathrm{R}}_t = \varphi_\pi \widehat{\pi}_{d,t} + \varphi_x \widehat{\mathrm{x}}_t + \widehat{\xi}_t \tag{66}$$

It consists of domestic inflation Taylor rule.

**Rule 6 (R6)**

$$\widehat{\mathrm{R}}_t = \varphi_R \widehat{\mathrm{R}}_{t-1} + \varphi_\pi \widehat{\pi}_{d,t} + \varphi_x \widehat{\mathrm{x}}_t + \widehat{\xi}_t \tag{67}$$

Following Orphanides (2003), this rule incorporates an inertial component in the interest rate. Where $\varphi_R$ is the monetary rule interest rate smoothing coefficient.

**Rule 7 (R7)**

$$\widehat{R}_t = \varphi_R \widehat{R}_{t-1} + \varphi_{\pi F} \widehat{\pi}_{d,t+1} + \widehat{\xi}_t \tag{68}$$

Following Batini and Haldane (1998), a domestic inflation forecast based (DIFB) rule, is a rule that reacts to deviations of expected domestic inflation from target. Where $\varphi_{\pi F}$ is the monetary rule DIFB coefficient. According to Batini et al. (2009), the stabilization of this rule deteriorates if the future horizon $\widehat{\pi}_{d,t+j}$, increases for periods greater than 2 quarters.

**Rule 8 (R8)**

$$\widehat{R}_t = \varphi_R \widehat{R}_{t-1} + \varphi_\pi \widehat{\pi}_{d,t} + \varphi_e \widehat{e}_t + \widehat{\xi}_t \tag{69}$$

This rule incorporates the nominal exchange rate channel in the transmission of monetary policy. In this economy, it is taken into account that changes in the nominal interest rate affect not only the IS equation through the real interest rate, but also net exports through their relation to the nominal exchange rate. Where $\varphi_e$ is the monetary rule exchange rate coefficient.

**Rule 9 (R9)**

$$\widehat{R}_t = \varphi_{R1} \widehat{R}_{t-1} + \varphi_{\pi 0} \widehat{\pi}_{d,t} + \varphi_{\pi 1} \widehat{\pi}_{d,t-1} + \varphi_{x0} \widehat{x}_t - \varphi_{x1} \widehat{x}_{t-1} + \varphi_{\Delta e} \Delta \widehat{e}_t + \varphi_{tt} \widehat{tt}_t + \widehat{\xi}_t \tag{70}$$

Modifying the short-term optimal rule of Cabrera, Bejarano, and Savino (2011), this rule implies that the interest rate responds to the movements of domestic inflation, the output gap, fluctuations in the nominal exchange rate, and the terms of trade. Where $\varphi_{R1}$, $\varphi_{\pi 0}$, $\varphi_{\pi 1}$,$\varphi_{x0}$, $\varphi_{x1}$, $\varphi_{\Delta e}$, and $\varphi_{tt}$ are its associated coefficients. In emerging economies, it is useful to incorporate the variation of the nominal exchange rate, commonly called the exchange rate float.

**Rule 10 (R10)**

$$\widehat{R}_t = \varphi_R \widehat{R}_{t-1} + \varphi_\pi \widehat{\pi}_{d,t} + \varphi_x \widehat{x}_t + \varphi_e \widehat{e}_t + \widehat{\xi}_t \tag{71}$$

This compound rule incorporates the inertial component of the interest rate, domestic inflation, the output gap, and the nominal exchange rate in monetary policy decisions.

## 2.7 Shocks representation

**Monetary policy shock**

$$\widehat{\xi}_t = \rho_\xi \widehat{\xi}_{t-1} + \varepsilon_t^\xi \tag{72}$$

Where $\rho_\xi$ and $\varepsilon_t^\xi$ are the autocorrelation coefficient and the innovation of the monetary policy shock, respectively. This innovation is normal, independent and identically distributed with a standard deviation, $\sigma_\xi$.

**Productivity shock**

$$\widehat{\mathrm{a}}_{\mathrm{t}}=\rho_{\mathrm{a}}\widehat{\mathrm{a}}_{\mathrm{t}-1}+\varepsilon_{\mathrm{t}}^{\mathrm{a}} \tag{73}$$

Where $\rho_{\mathrm{a}}$ and $\varepsilon_{\mathrm{t}}^{\mathrm{a}}$ are the coefficient of autocorrelation and productivity shock innovation respectively. This innovation is normal, independent and identically distributed with a standard deviation, $\sigma_{\mathrm{a}}$.

**Preferences shock**

$$\widehat{\mathrm{g}}_{\mathrm{t}}=\rho_{\mathrm{g}}\widehat{\mathrm{g}}_{\mathrm{t}-1}+\varepsilon_{\mathrm{t}}^{\mathrm{g}} \tag{74}$$

Where $\rho_{\mathrm{g}}$ and $\varepsilon_{\mathrm{t}}^{\mathrm{g}}$ are the coefficient of autocorrelation and innovation of the preference shock respectively. This innovation is normal, independent and identically distributed with a standard deviation, $\sigma_{\mathrm{g}}$.

**International interest rate shock**

$$\widehat{\mathrm{R}}_{\mathrm{t}}^{*}=\rho_{\mathrm{R}^{*}}\widehat{\mathrm{R}}_{\mathrm{t}-1}^{*}+\varepsilon_{\mathrm{t}}^{\mathrm{R}^{*}} \tag{75}$$

Where $\rho_{\mathrm{R}^{*}}$ and $\varepsilon_{\mathrm{t}}^{\mathrm{R}^{*}}$ are the autocorrelation coefficient and the innovation of the international interest rate shock, respectively. This innovation is normal, independent and identically distributed with a standard deviation, $\sigma_{\mathrm{R}^{*}}$.

**World price level shock**

$$\widehat{\mathrm{p}}_{\mathrm{t}}^{*}=\rho_{\mathrm{p}^{*}}\widehat{\mathrm{p}}_{\mathrm{t}-1}^{*}+\varepsilon_{\mathrm{t}}^{\mathrm{p}^{*}} \tag{76}$$

Where $\rho_{\mathrm{P}^{*}}$ and $\varepsilon_{\mathrm{t}}^{\mathrm{P}^{*}}$ are the autocorrelation coefficient and the innovation of the shock of the world price level respectively. This innovation is normal, independent and identically distributed with a standard deviation, $\sigma_{\mathrm{p}^{*}}$.

## 2.8 Welfare

To assess the welfare of the different monetary policy rules it is possible to use the profit-based loss function. To obtain an appropriate welfare criterion, it is possible to derive a second order approximation of the loss of discounted utility of the domestic representative consumer associated with the deviations from optimal policy.

$$\mathcal{L}=\sum_{\mathrm{t}=0}^{\infty}\beta^{\mathrm{t}}\left\{\frac{(1-\alpha)\,\epsilon}{2\lambda}\widehat{\pi}_{\mathrm{d,t}}^{2}+\frac{(1-\alpha)\,(1+\varphi)}{2}\widehat{\mathrm{x}}_{\mathrm{t}}^{2}\right\} \tag{77}$$

In an open economy with financial friction, for the special case of $\sigma$=$\eta$= 1. The loss function $\mathcal{L}$, is expressed in terms of domestic inflation and the output gap.

$$\mathbb{V}=\left\{\frac{(1-\alpha)\,\epsilon}{2\lambda}\mathrm{var}\left(\widehat{\pi}_{\mathrm{d,t}}\right)+\frac{(1-\alpha)\,(1+\varphi)}{2}\mathrm{var}\left(\widehat{\mathrm{x}}_{\mathrm{t}}\right)\right\} \tag{78}$$

Taking unconditional expectation at (77) and leaving $\beta\rightarrow 1$, the expected welfare loss is represented in terms of the variance of inflation and output gap. In contrast, the loss function $\mathcal{L}$, is a considerable source of welfare loss. These losses cannot be avoided, but they are considerably reduced when the central bank can commit to policy plans that consist of keeping interest rates low for an extended period.

## 2.9 Calibration

Table 1: Calibration

| Parameter | Description | Value |
|---|---|---|
| $\boldsymbol{\beta}$ | Discount factor | 0.9852 |
| $\boldsymbol{\sigma}$ | Inverse of the intertemporal substitution elasticity | 1 |
| $\boldsymbol{\varphi}$ | Inverse of the elasticity of labor supply | 0.45 |
| $\boldsymbol{\varepsilon}$ | Elasticity of substitution between domestic goods | 6 |
| $\boldsymbol{\theta}$ | Calvo probability | 0.75 |
| $\boldsymbol{\alpha}$ | Degree of openness | 0.6 |
| $\boldsymbol{\eta}$ | Substitution between domestic and imported goods | 2 |
| $\boldsymbol{\delta}$ | Bond holding rate | 3 |
| $\boldsymbol{\varphi_{R}}$ | Interest rate smoothing coefficient | 0.5 |
| $\boldsymbol{\varphi_{\pi}}$ | Inflation coefficient | 1.5 |
| $\boldsymbol{\varphi_{\pi F}}$ | DIFB coefficient | 5 |
| $\boldsymbol{\varphi_{x}}$ | Output gap coefficient | 0.5 |
| $\boldsymbol{\varphi_{e}}$ | Exchange rate coefficient | 0.5 |
| $\boldsymbol{\varphi_{R1}}$ | Interest rate smoothing coefficient of R9 | 0.763 |
| $\boldsymbol{\varphi_{\pi 0}}$ | Inflation coefficient of R9 | 0.107 |
| $\boldsymbol{\varphi_{\pi 1}}$ | Inflation smoothing coefficient of R9 | 0.028 |
| $\boldsymbol{\varphi_{x0}}$ | Output gap coefficient of R9 | 0.346 |
| $\boldsymbol{\varphi_{x1}}$ | Output gap smoothing coefficient of R9 | 0.062 |
| $\boldsymbol{\varphi_{\Delta e}}$ | Exchange rate variation coefficient of R9 | 0.053 |
| $\boldsymbol{\varphi_{tt}}$ | Terms of trade coefficient of R9 | 0.082 |
| $\boldsymbol{\rho_{\xi}}$ | Autocorrelation of monetary policy shock | 0.5 |
| $\boldsymbol{\rho_{a}}$ | Autocorrelation of productivity shock | 0.9 |
| $\boldsymbol{\rho_{g}}$ | Autocorrelation of preferences shock | 0.9 |
| $\boldsymbol{\rho_{R^*}}$ | Autocorrelation of international interest rate shock | 0.9 |
| $\boldsymbol{\rho_{p^*}}$ | Autocorrelation of world price level shock | 0.95 |

Calibration can be considered as an estimation strategy, Gregory and Smith (1987). This procedure allows me to assign values to the parameters of the general equilibrium model based on various sources. Considering the standard literature, the Peruvian economy is characterized by parameters estimated by various authors. Some parameters reflect their historical values and others my own estimates.

The subjective discount factor $\beta$, is set to 0.9852. In Vega (2015), this parameter implies a steady state domestic real interest rate equivalent to 6% per year.

The inverse of the elasticity of intertemporal substitution $\sigma$, is set to 1. This parameter is canonical and shows that the intertemporal substitution elasticity of consumption is invariant on a consumption scale.

The inverse of the elasticity of labor supply $\varphi$, is set to 0.45. measures the percentage change in the labor supply with the real wage. In Vega (2015), this parameter indicates a very inelastic labor offer. For the Peruvian economy, it reflects that labor demand could be more sensitive to wages.

The elasticity of substitution between domestic goods $\varepsilon$, is established at 6. In Castillo et al. (2009), this parameter is consistent with a margin of 15% on marginal costs in all sectors.

The probability that an individual firm will not change its price $\theta$, is set to 0.75. The average duration of this price quote is $1/(1-\theta)$ quarters. In Castillo et al. (2009), this choice implies that companies keep their prices fixed for 4 quarters.

The degree of openness $\alpha$, is set to 0.6. In Castillo et al. (2009), this parameter implies a 60% share of household goods in the CPI.

The elasticity of intratemporal substitution between imported and domestic goods $\eta$, is 2. In Vega (2015), this parameter suggests an environment where families have difficulty substituting imported goods for domestic goods.

The long-term government bond holding rate relative to the short-term government bond holdings $\delta$, is set to 3 for model purposes. Harrison (2012), use this parameter based on the study by Kuttner (2006).

The following parameters have a non-negativity restriction. The coefficients of the monetary rules are consistent with the findings in Taylor (1993), Henderson and McKibbin (1993) and Ball (1999). Shocks are introduced as first order autoregressive processes.

Under Batini and Haldane (1998) and Batini et al. (2003), the DIFB coefficient $\varphi_{\pi\mathrm{F}}$, is set to 5. According to Cabrera et al. (2011), the parameters $\varphi_{\mathrm{R1}}$, $\varphi_{\pi 0}$, $\varphi_{\pi 1}$, $\varphi_{\mathrm{x0}}$, $\varphi_{\mathrm{x1}}$, $\varphi_{\Delta\mathrm{e}}$, and $\varphi_{\mathrm{ti}}$, are set to 0.763, 0.107, 0.028, 0.346, 0.062, 0.053, and 0.082 respectively.

The interest rate smoothing coefficient $\varphi_{\mathrm{R}}$, is set to 0.5. The inflation coefficient $\varphi_{\pi}$, is established in 1.5. The output gap coefficient $\varphi_{\mathrm{x}}$, is set to 0.5. The exchange rate coefficient $\varphi_{\mathrm{e}}$, is set to 0.5.

The autocorrelation of the monetary policy shock $\rho_{\xi}$, is set to 0.5. The autocorrelation of the technological shock $\rho_{\mathrm{a}}$, is set to 0.9. The autocorrelation of the preference shock $\rho_{\mathrm{g}}$, is set to 0.9. The autocorrelation of the international interest rate shock $\rho_{\mathrm{R}^*}$, is set to 0.9. The world price level shock autocorrelation $\rho_{\mathrm{p}^*}$, is set to 0.95.

# 3 Model estimation

The following estimate is based on Lubik and Schorfheide (2005), Rabanal and Rubio-Ramírez (2005), and An and Schorfheide (2006). Here the prior distributions play an important role in estimating the general equilibrium model. In principle, the prior information can be extracted from personal introspection to reflect strongly sustained beliefs about the validity of the model. In practice, prior information that I chose is based on my own previous experience estimating autoregressive models for the Peruvian economy.

Consequently, priors and the likelihood function are combined to obtain the posterior distribution of the parameter vector. The Kalman filter is used to evaluate the likelihood function of the model's linear logarithmic approximation and the RWMH-MCMC algorithm to draw the posterior distribution[2]. Then, logarithmic marginal probability, the Bayes factor, and the posterior probability of the model are used to compare each monetary policy rule. By doing so, it is possible to determine which rule helps explain the data better, as well as compare each rule in the model.

[2]These simulation and estimation methods are implemented within the platform Dynare software, Adjemian et al. (2011).

## 3.1 Data

In the empirical analysis, the observables have a quarterly frequency that goes from the first quarter of 2004 to the fourth quarter of 2019[3]. This choice corresponds to the validity of the explicit inflation targeting scheme.

Let $\mathcal{Y}_{\mathrm{T}}=\{y_{\mathrm{t}}\}_{\mathrm{t}=1}^{\mathrm{T}}$ the set of observables.

$$y_{\mathrm{t}}=[\log \mathrm{Y}_{\mathrm{t}}, \log \mathrm{C}_{\mathrm{t}}, \log \pi_{\mathrm{t}}, \log \mathrm{R}_{\mathrm{t}}]'$$

It is assumed that the period $t$ in the model corresponds to one quarter, $y_{\mathrm{t}}$ is the vector of observables, $\mathrm{Y}_{\mathrm{t}}$ is the Gross Domestic Product (GDP), $\mathrm{C}_{\mathrm{t}}$ is the private consumption, $\pi_{\mathrm{t}}$ is the CPI inflation, and $\mathrm{R}_{\mathrm{t}}$ is the monetary policy rate.

The parameters to estimate are contained in $\Theta$.

$$\Theta=[\rho_{\xi}, \rho_{\mathrm{a}}, \rho_{\mathrm{g}}, \rho_{\mathrm{R}^*}, \rho_{\mathrm{p}^*}, \sigma_{\xi}, \sigma_{\mathrm{a}}, \sigma_{\mathrm{g}}, \sigma_{\mathrm{R}^*}, \sigma_{\mathrm{p}^*}]'$$

The parameter vector $\Theta$, is made up of the autocorrelation slopes and the standard deviations of the exogenous shocks that are a source of fluctuation in the general equilibrium model.

## 3.2 Bayesian estimation

The prior density $\mathrm{p}(\Theta \mid \mathrm{M}_{\mathrm{R}})$, assumes that prior information about the parameter vector can be summarized by a joint probability density function. These have a Beta and Inverse Gamma distribution respectively.

The likelihood function describes the density of the observed data given the model and the parameter vector. It is estimated using the Kalman filter, which evaluates the likelihood function associated with the solution of the space-state system of the model. This function can be represented recursively.

$$\mathscr{L}(\Theta \mid \mathcal{Y}_{\mathrm{T}}, \mathrm{M}_{\mathrm{R}}) \equiv \mathrm{p}(y_0 \mid \Theta, \mathrm{M}_{\mathrm{R}}) \prod_{\mathrm{t}=1}^{\mathrm{T}} \mathrm{p}(y_{\mathrm{t}} \mid \mathcal{Y}_{\mathrm{t}-1}, \Theta, \mathrm{M}_{\mathrm{R}}) \tag{79}$$

Where $\mathscr{L}(\Theta \mid \mathcal{Y}_{\mathrm{T}}, \mathrm{M}_{\mathrm{R}})$ is the likelihood function and $\mathrm{p}(y_{\mathrm{t}} \mid \mathcal{Y}_{\mathrm{t}-1}, \Theta, \mathrm{M}_{\mathrm{R}})$ is the density conditional on the information available up to $\mathrm{t}-1$.

The posterior distribution is given by Bayes' theorem.

$$\mathrm{p}(\Theta \mid \mathcal{Y}_{\mathrm{T}}, \mathrm{M}_{\mathrm{R}}) = \frac{\mathscr{L}(\Theta \mid \mathcal{Y}_{\mathrm{T}}, \mathrm{M}_{\mathrm{R}})\, \mathrm{p}(\Theta \mid \mathrm{M}_{\mathrm{R}})}{\mathrm{p}(\mathcal{Y}_{\mathrm{T}} \mid \mathrm{M}_{\mathrm{R}})} \tag{80}$$

The term $\mathrm{p}(\mathcal{Y}_{\mathrm{T}} \mid \mathrm{M}_{\mathrm{R}})$ is the marginal density of the data and appears as a normalization constant in the denominator of (80). The logarithm of the marginal density of the data can be interpreted as a function of maximized log-likelihood penalized by the dimension of the model.

The term $\mathrm{p}(\Theta \mid \mathcal{Y}_{\mathrm{T}}, \mathrm{M}_{\mathrm{R}})$, is the posterior density proportional to the product of the likelihood function and the prior.

$$\mathrm{p}(\Theta \mid \mathcal{Y}_{\mathrm{T}}, \mathrm{M}_{\mathrm{R}}) \propto \mathscr{L}(\Theta \mid \mathcal{Y}_{\mathrm{T}}, \mathrm{M}_{\mathrm{R}})\, \mathrm{p}(\Theta \mid \mathrm{M}_{\mathrm{R}}) \equiv \mathbb{K}(\Theta \mid \mathcal{Y}_{\mathrm{T}}, \mathrm{M}_{\mathrm{R}}) \tag{81}$$

[3]The data are obtained from the historical series of the Central Bank of Perú.

This equation is of fundamental interest, because it summarizes everything that is known about $\Theta$, after using the data. The posterior kernel $\mathbb{K}(\Theta \mid \mathcal{Y}_{\mathrm{T}}, \mathrm{M}_{\mathrm{R}})$, corresponds to the numerator of the posterior density.

## 3.3 Metrópolis-Hastings algorithm

In order to obtain the posterior moments, the RWMH-MCMC algorithm, part of the posterior mode $\widetilde{\Theta}$.

**First step**

Choose a starting point $\widehat{\Theta}^0$, where $\widetilde{\Theta}$ is typically found.

**Second step**

Draw a proposed $\widehat{\Theta}^*$ of a jump distribution.

$$\mathrm{J}\left(\widehat{\Theta}^* \,\middle|\, \widehat{\Theta}^{t-1}\right) = \mathrm{N}\left(\widehat{\Theta}^{t-1}, w^2\boldsymbol{\Omega}_{\mathrm{d}}\right) \tag{82}$$

Where N is the normal distribution, $w$ is a scalar that controls the jump size of the proposed algorithm update, $\boldsymbol{\Omega}$ is the inverse of the Hessian calculated at $\widetilde{\Theta}$, and d is the dimension of $\Theta$.

**Third step**

Compute an acceptance rate

$$r = \frac{\mathrm{p}\left(\widehat{\Theta}^* \,\middle|\, \mathcal{Y}_{\mathrm{T}}, \mathrm{M}_{\mathrm{R}}\right)}{\mathrm{p}\left(\widehat{\Theta}^{t-1} \,\middle|\, \mathcal{Y}_{\mathrm{T}}, \mathrm{M}_{\mathrm{R}}\right)} = \frac{\mathbb{K}\left(\widehat{\Theta}^* \,\middle|\, \mathcal{Y}_{\mathrm{T}}, \mathrm{M}_{\mathrm{R}}\right)}{\mathbb{K}\left(\widehat{\Theta}^{t-1} \,\middle|\, \mathcal{Y}_{\mathrm{T}}, \mathrm{M}_{\mathrm{R}}\right)} \tag{83}$$

Compute its associated probability.

$$\mathfrak{P} = \min\left\{1, \frac{\mathbb{K}\left(\widehat{\Theta}^* \,\middle|\, \mathcal{Y}_{\mathrm{T}}, \mathrm{M}_{\mathrm{R}}\right)}{\mathbb{K}\left(\widehat{\Theta}^{t-1} \,\middle|\, \mathcal{Y}_{\mathrm{T}}, \mathrm{M}_{\mathrm{R}}\right)}\right\} \tag{84}$$

**Fourth step**

Accept or discard the proposal $\widehat{\Theta}^*$. Draw an uniform random variable $\mathfrak{q} \sim \mathrm{U}(0,1)$. The proposal is accepted $\widehat{\Theta}^t = \widehat{\Theta}^*$, if $\mathfrak{q} \leq \mathfrak{P}$, otherwise redraw a new candidate.

This process is repeated $\mathcal{H}$ times to generate the posterior density $\mathrm{p}\left(\widehat{\Theta} \,\middle|\, \mathcal{Y}_{\mathrm{T}}, \mathrm{M}_{\mathrm{R}}\right)$. The convergence rate is sensitive to $w$ as well as $\mathcal{H}$. To allow convergence I set $\mathcal{H} = 250000$ draws, as in Smets and Wouters (2007). I set up a $w$ that leads to an acceptance rate close to a third, starting with $w=2.4/\sqrt{\mathrm{d}}$.

## 3.4 Bayes Factor

For each rule in the model, the posterior marginal density is calculated using the modified harmonic mean estimator of Geweke (1998). Therefore, the Bayes factor is considered as a tool to determine which model rule best explains the behavior of the set of variables. To do this, I define the Bayes factor of rule k for rule $\ell$, $\mathrm{FB}_{\mathrm{k},\ell}$.

$$\mathrm{FB}_{\mathrm{k},\ell}=\frac{\mathrm{p}\left(\mathcal{Y}_{\mathrm{T}} \mid \mathrm{M}_{\mathrm{R,k}}\right)}{\mathrm{p}\left(\mathcal{Y}_{\mathrm{T}} \mid \mathrm{M}_{\mathrm{R},\ell}\right)} \tag{85}$$

Where $\mathrm{p}\left(\mathcal{Y}_{\mathrm{T}} \mid \mathrm{M}_{\mathrm{R,k}}\right)$ is the posterior marginal density of rule k and $\mathrm{p}\left(\mathcal{Y}_{\mathrm{T}} \mid \mathrm{M}_{\mathrm{R},\ell}\right)$ is the posterior marginal density of rule $\ell$. Using the Bayes factors, it is possible to calculate the posterior probability of each rule, $\mathrm{p}_{\mathrm{k}}$.

Considering that $\sum_{\mathrm{k}=1}^{10} \mathrm{p}_{\mathrm{k}}= 1$,

$$\frac{1}{\mathrm{p}_1}=\sum_{\mathrm{k}=2}^{10} \mathrm{FB}_{\mathrm{k},1} \tag{86}$$

Where $\mathrm{p}_1$ is the posterior probability of rule 1. Then $\mathrm{p}_{\mathrm{k}}=\mathrm{p}_1\mathrm{FB}\left(\mathrm{k},1\right)$ gives the remaining probabilities of the model. If the posterior probability of rule k is greater than the posterior probability of rule $\ell$, then rule k is better than rule $\ell$.

## 3.5 Choice of priors

Table 2: Priors

| Parameter | Distribution [Limits] | Mean | SD |
|---|---|---|---|
| $\boldsymbol{\rho_{\xi}}$ | Beta [0.1] | 0.50 | 0.01 |
| $\boldsymbol{\rho_{\mathrm{a}}}$ | Beta [0.1] | 0.90 | 0.01 |
| $\boldsymbol{\rho_{\mathrm{g}}}$ | Beta [0.1] | 0.90 | 0.01 |
| $\boldsymbol{\rho_{\mathrm{R}^*}}$ | Beta [0.1] | 0.90 | 0.01 |
| $\boldsymbol{\rho_{\mathrm{p}^*}}$ | Beta [0.1] | 0.95 | 0.01 |
| $\boldsymbol{\sigma_{\xi}}$ | Inv-Gamma $[0.\infty]$ | 0.10 | 0.02 |
| $\boldsymbol{\sigma_{\mathrm{a}}}$ | Inv-Gamma $[0.\infty]$ | 0.40 | 0.02 |
| $\boldsymbol{\sigma_{\mathrm{g}}}$ | Inv-Gamma $[0.\infty]$ | 0.20 | 0.02 |
| $\boldsymbol{\sigma_{\mathrm{R}^*}}$ | Inv-Gamma $[0.\infty]$ | 0.20 | 0.02 |
| $\boldsymbol{\sigma_{\mathrm{p}^*}}$ | Inv-Gamma $[0.\infty]$ | 1.00 | 0.02 |

Autocorrelation coefficients are assumed to have a Beta distribution with limits of [0.1]. It is also assumed that the standard deviations of the shocks impose an Inverse Gamma distribution with limits of $[0.\infty]$. Considering the standard literature, dogmatic priors are imposed over Standard Deviation (SD) parameters.

The standard deviation of the monetary policy shock $\sigma_{\xi}$, is set to 0.1. The standard deviation of the productivity shock $\sigma_{\mathrm{a}}$, is set to 0.4. The standard deviation of the preferences shock $\sigma_{\mathrm{g}}$, is set to 0.2. The standard deviation of the international interest rate shock $\sigma_{\mathrm{R}^*}$, is set to 0.2. The standard deviation of the world price level shock $\sigma_{\mathrm{p}^*}$, is set to 1.

# 4 Results

This section provides the theoretical and empirical results.

## 4.1 Optimal monetary policy

**Robustness analysis to all shocks**

Table 3: Comparison between monetary policy rules

| | $\sigma(\widehat{\mathrm{y}})$ | $\sigma(\widehat{\mathrm{x}})$ | $\sigma(\widehat{\pi}_{\mathrm{d}})$ | $\sigma(\widehat{\pi})$ | $\sigma(\widehat{\mathrm{tt}})$ | $\sigma(\Delta\widehat{\mathrm{e}})$ | $\mathcal{L}$ |
|---|---|---|---|---|---|---|---|
| **Rule 1** | 5.645 | 4.934 | 3.718 | 3.907 | 3.213 | 4.197 | 1.977 |
| **Rule 2** | 6.180 | 5.209 | 3.476 | 3.852 | 3.515 | 4.379 | 1.744 |
| **Rule 3** | 3.270 | 2.500 | 0.519 | 0.742 | 1.982 | 0.000 | 0.055 |
| **Rule 4** | 4.231 | 2.560 | 1.826 | 1.967 | 2.524 | 2.346 | 0.479 |
| **Rule 5** | 4.411 | 2.590 | 1.759 | 2.003 | 2.626 | 2.499 | 0.446 |
| **Rule 6** | 4.104 | 1.786 | 0.658 | 1.109 | 2.483 | 1.853 | 0.069 |
| **Rule 7** | 4.385 | 2.351 | 0.505 | 1.200 | 2.621 | 2.043 | 0.051 |
| **Rule 8** | 3.771 | 2.578 | 0.528 | 1.039 | 2.261 | 1.489 | 0.058 |
| **Rule 9** | 4.606 | 3.607 | 1.153 | 1.744 | 2.685 | 2.498 | 0.221 |
| **Rule 10** | 3.521 | 1.588 | 0.385 | 0.741 | 2.163 | 1.253 | 0.028 |

Table 3 shows the standard deviations of the variables analyzed for each monetary policy rule. Where the loss function, $\mathcal{L}$, is used to find the optimal monetary policy.

R1 and R2, which consist of the inflation targeting scheme for the CPI and domestic inflation, respectively. Variable deviations are high compared to the other rules. The greatest source of volatility comes from reacting to CPI inflation. Consequently, reacting to domestic inflation is more efficient in minimizing the loss function. In general, central banks usually impose inflation tolerance ranges to prevent them from fluctuating too much.

R3, which consists of a fixed nominal exchange rate regime. Variable deviations decrease compared to the first two rules. The volatility of the nominal exchange rate is zero. However, this regime reduces the power of monetary policy, so this rule will not be considered later.

R4 and R5, consisting of the Taylor rule and the Taylor rule for domestic inflation respectively. Variable deviations decrease compared to the first two rules. Consequently, reacting to the output gap is more efficient in minimizing the loss function. In general, the central banks of advanced economies use the Taylor rule for adverse shocks.

R6, which reacts to the interest rate smoothing, the domestic inflation, and the output gap. Variable deviations decrease compared to the previous two rules. Consequently, reacting to the smoothing component of the nominal interest rate, to domestic inflation and the output gap is more efficient in minimizing the loss function.

R7, which reacts to the interest rate smoothing and deviations of expected domestic inflation from target. The deviations of domestic inflation decrease compared to R6. Consequently, in a context with more complex dynamics, in which the central bank is committed to containing future inflation, it would probably generate a smaller loss function. Batini et al. (2009), mention that in a dollarized economy there are substantial gains by including the exchange rate in the monetary rule. Here I demonstrate that for a model that captures the relevant facts of the Peruvian economy, dollarization is not necessary to reach the same conclusions.

R8, which reacts to the interest rate smoothing, domestic inflation, and the nominal exchange rate. The deviations of the product, CPI inflation, terms of trade and variation of the nominal exchange rate decrease compared to R7. Consequently, reacting to the exchange rate is more efficient in minimizing the loss function for an economy exposed to external shocks. In general, this open economy rule is parsimonious.

R9, which reacts to the movements of domestic inflation, the output gap, fluctuations in the nominal exchange rate, and the terms of trade. Variable deviations increase compared to R8. Consequently, reacting to the variation of the nominal exchange rate is less efficient in minimizing the loss function.

R10, which reacts to the interest rate smoothing, domestic inflation, the output gap, and the nominal exchange rate. The deviations of the variables decrease compared to the other rules. In addition to R8, reacting to the output gap is more efficient in minimizing the loss function.

**Robustness analysis to individual shocks**

To provide more insight into why certain rules work better than others, it is possible to re-evaluate the rules that had a lower loss function. Assuming that the economy was hit by one type of shock at a time, it is possible to find out which rules produce sensitive responses to each of the shocks and it is possible to analyze if the rules that work well do so because they are robust for various shocks. This is a test of robustness of the specification of each rule.

Table 4: Monetary policy shock

| $\widehat{\xi}_t$ | **Rule 2** | **Rule 5** | **Rule 7** | **Rule 8** | **Rule 10** |
|---|---|---|---|---|---|
| $\sigma(\widehat{y})$ | 2.612 | 1.226 | 1.872 | 1.563 | 0.923 |
| $\sigma(\widehat{x})$ | 2.612 | 1.226 | 1.872 | 1.563 | 0.923 |
| $\sigma(\widehat{\pi}_d)$ | 0.445 | 0.209 | 0.356 | 0.207 | 0.126 |
| $\sigma(\widehat{\pi})$ | 1.141 | 0.536 | 0.795 | 0.661 | 0.392 |
| $\sigma(\widehat{tt})$ | 1.419 | 0.666 | 1.017 | 0.850 | 0.502 |
| $\sigma(\Delta\widehat{e})$ | 1.686 | 0.791 | 1.147 | 0.992 | 0.588 |
| $\mathcal{L}$ | 0.047 | 0.010 | 0.028 | 0.013 | 0.005 |

Assuming an individual monetary policy shock. In table 4, R10, it is more efficient to minimize the loss function. The deviations of the variables are smaller compared to the other rules.

Table 5: Productivity shock

| $\widehat{a}_t$ | **Rule 2** | **Rule 5** | **Rule 7** | **Rule 8** | **Rule 10** |
|---|---|---|---|---|---|
| $\sigma(\widehat{y})$ | 2.993 | 3.169 | 3.269 | 2.677 | 2.884 |
| $\sigma(\widehat{x})$ | 0.355 | 0.180 | 0.112 | 0.865 | 0.548 |
| $\sigma(\widehat{\pi}_d)$ | 0.271 | 0.137 | 0.028 | 0.216 | 0.159 |
| $\sigma(\widehat{\pi})$ | 0.459 | 0.452 | 0.439 | 0.185 | 0.294 |
| $\sigma(\widehat{tt})$ | 1.627 | 1.722 | 1.776 | 1.455 | 1.567 |
| $\sigma(\Delta\widehat{e})$ | 0.717 | 0.751 | 0.741 | 0.339 | 0.524 |
| $\mathcal{L}$ | 0.011 | 0.003 | 0.000 | 0.009 | 0.004 |

Assuming an individual productivity shock. In table 5, R7, it is more efficient to minimize the loss function. The deviations of the output gap and domestic inflation are smaller compared to the other rules.

Table 6: Preferences shock

| $\widehat{g}_t$ | **Rule 2** | **Rule 5** | **Rule 7** | **Rule 8** | **Rule 10** |
|---|---|---|---|---|---|
| $\sigma(\widehat{y})$ | 1.518 | 1.661 | 1.743 | 1.287 | 1.436 |
| $\sigma(\widehat{x})$ | 0.289 | 0.146 | 0.091 | 0.704 | 0.446 |
| $\sigma(\widehat{\pi}_d)$ | 0.221 | 0.112 | 0.023 | 0.175 | 0.129 |
| $\sigma(\widehat{\pi})$ | 0.374 | 0.368 | 0.357 | 0.151 | 0.239 |
| $\sigma(\widehat{tt})$ | 1.324 | 1.401 | 1.446 | 1.184 | 1.275 |
| $\sigma(\Delta\widehat{e})$ | 0.584 | 0.623 | 0.603 | 0.276 | 0.426 |
| $\mathcal{L}$ | 0.007 | 0.002 | 0.000 | 0.006 | 0.003 |

Assuming an individual preferences shock. In table 6, R7, it is more efficient to minimize the loss function. The deviations of the output gap and domestic inflation are smaller compared to the other rules.

Table 7: International interest rate shock

| $\widehat{R}^*_t$ | **Rule 2** | **Rule 5** | **Rule 7** | **Rule 8** | **Rule 10** |
|---|---|---|---|---|---|
| $\sigma(\widehat{y})$ | 4.479 | 2.267 | 1.414 | 1.394 | 0.860 |
| $\sigma(\widehat{x})$ | 4.479 | 2.267 | 1.414 | 1.394 | 0.860 |
| $\sigma(\widehat{\pi}_d)$ | 3.418 | 1.729 | 0.356 | 0.259 | 0.184 |
| $\sigma(\widehat{\pi})$ | 3.620 | 1.832 | 0.698 | 0.619 | 0.391 |
| $\sigma(\widehat{tt})$ | 2.434 | 1.232 | 0.768 | 0.757 | 0.468 |
| $\sigma(\Delta\widehat{e})$ | 3.812 | 1.929 | 1.003 | 0.909 | 0.566 |
| $\mathcal{L}$ | 1.669 | 0.427 | 0.023 | 0.015 | 0.007 |

Assuming an individual international interest rate shock. In table 7, R10, it is more efficient to minimize the loss function. The deviations of the variables are smaller compared to the other rules.

Table 8: World price level shock

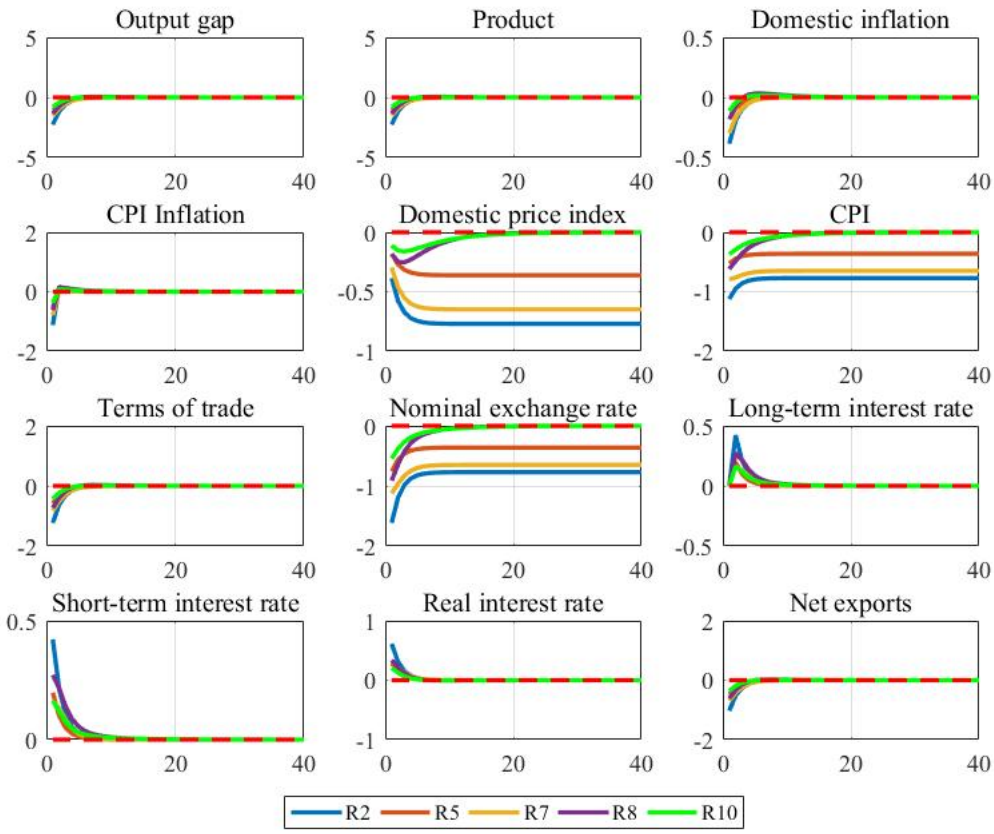
| $\widehat{p}^*_t$ | **Rule 2** | **Rule 5** | **Rule 7** | **Rule 8** | **Rule 10** |
|---|---|---|---|---|---|
| $\sigma(\widehat{y})$ | 0.208 | 0.126 | 0.064 | 1.006 | 0.656 |
| $\sigma(\widehat{x})$ | 0.208 | 0.126 | 0.064 | 1.006 | 0.656 |
| $\sigma(\widehat{\pi}_d)$ | 0.281 | 0.170 | 0.022 | 0.301 | 0.237 |
| $\sigma(\widehat{\pi})$ | 0.285 | 0.173 | 0.035 | 0.450 | 0.315 |
| $\sigma(\widehat{tt})$ | 0.113 | 0.069 | 0.035 | 0.547 | 0.357 |
| $\sigma(\Delta\widehat{e})$ | 0.973 | 0.979 | 0.971 | 0.464 | 0.670 |
| $\mathcal{L}$ | 0.011 | 0.004 | 0.000 | 0.015 | 0.009 |

Assuming an individual world price level shock. In table 8, R7, it is more efficient to minimize the loss function. Without considering the variation of the nominal exchange rate, the deviations of the variables are smaller compared to the other rules.

## 4.2 Impulse response function

### Monetary policy shock

Figure 1: Monetary policy shock




Note: Impulse response to a positive monetary policy shock with a standard deviation of 100 basis points (bp). The shocks was normalized to present a 1 percentage point (pp) deviation, and responses of the variables are expressed in percentage deviations in relation to their steady state (dashed red lines).

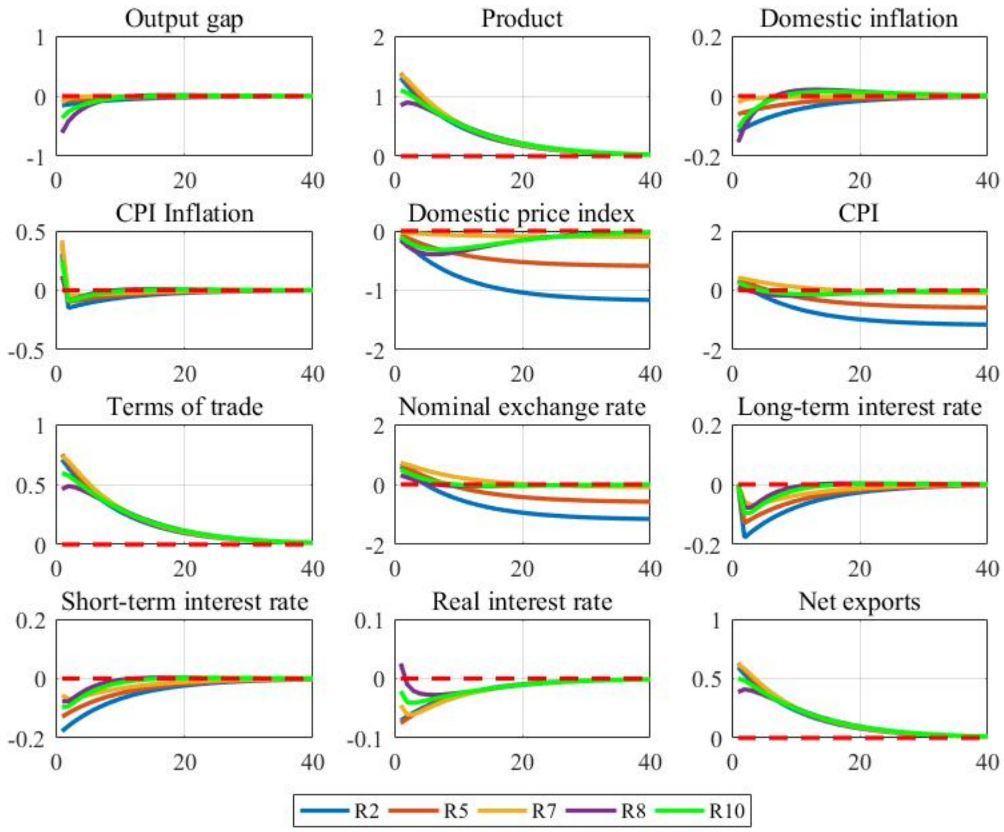
An increase in the monetary policy rate, increases the nominal interest rate, increases the real interest rate, decreases consumption, decreases output. Consequently, the output gap narrows, decreasing prices in the economy. In the external sector, the terms of trade fall and the nominal exchange rate appreciates. On the other hand, the demand for short-term bonds decreases, the demand for long-term bonds decreases, causing an excess supply of bonds that leads to an increase in the long-term interest rate.

The domestic price index gradually falls. For R2, it maintains a permanent drop below 100 bp. For R5, it maintains a permanent drop below 50 bp. For R7, it maintains a permanent drop equivalent to 50 bp. For R8 and R10, the initial response follows a hump-shaped pattern, when it reaches a certain point close to 25 bp, it falls until it converges to a steady state value of zero.

It can be seen that for R7, the CPI and the nominal exchange rate, fall permanently, but in greater magnitude than R5. This behavior reflects that the central bank must commit to contain future inflation. Consequently the magnitude of $\varphi_{\pi F}$ is important. Also in R10, the variables fluctuate less than R8.

**Productivity shock**

Figure 2: Productivity shock



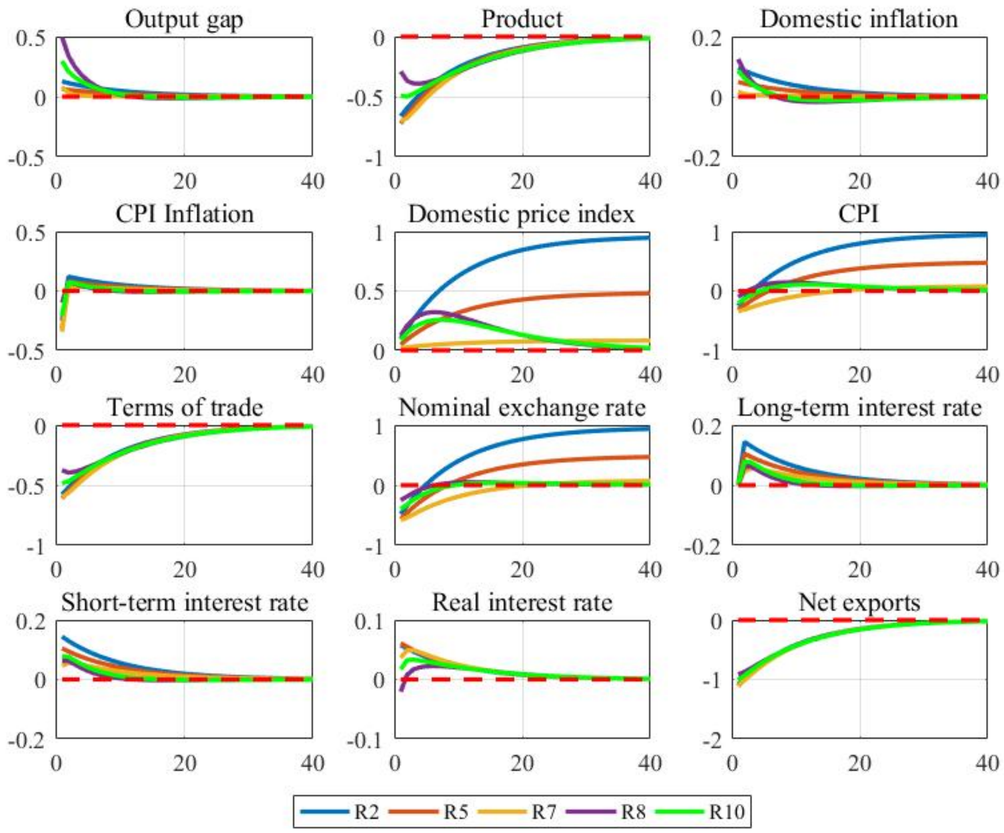

Note: Impulse response from a positive productivity shock with a standard deviation of 100 bp.

Increased productivity decreases the natural real interest rate, increases the natural product and increases the product through the production function. Consequently, the output gap decreases, decreasing domestic inflation and the domestic price index in the economy. In the external sector, the terms of trade increase and the nominal exchange rate initially depreciates. On the other hand, the central bank decreases the nominal interest rate, the demand for short-term bonds increases, the demand for long-term bonds increases, causing an excess demand for bonds that leads to a decrease in the long-term interest rate.

The domestic price index gradually falls. For R2, it maintains a permanent drop of more than 100 bp. For R5, it maintains a permanent drop above 50 bp. For R7, it maintains a permanent drop of more than 5 bp. For R8 and R10, the initial response follows a hump-shaped pattern, when it reaches a specific point close to 25 bp, it drops until it converges to a steady state value of zero.

### Preferences shock

Figure 3: Preferences shock



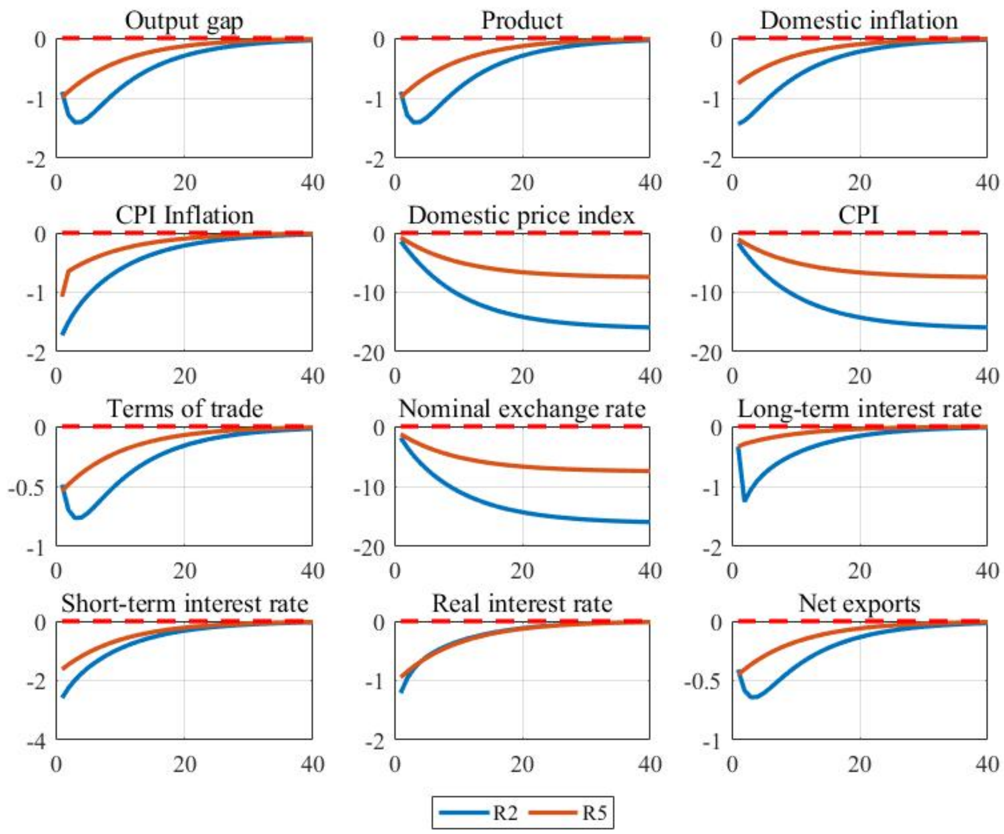

Note: Impulse response from a positive preference shock with a standard deviation of 100 bp.

A positive preferences shock increases the natural real interest rate, increases the natural product and decreases the natural terms of trade. Consequently, the output gap increases, increasing domestic inflation and the domestic price index in the economy. In the external sector, the terms of trade fall and the nominal exchange rate initially appreciates. On the other hand, the central bank increases the nominal interest rate, the demand for short-term bonds decreases, the demand for long-term bonds decreases, causing an excess supply of bonds that leads to an increase in the long-term interest rate.

The domestic price index gradually increases. For R2, it maintains a permanent increase close to 100 bp. For R5, it maintains a permanent increase equivalent to 50 bp. For R7, it maintains a permanent increase of more than 5 bp. For R8 and R10, the initial response follows a hump-shaped pattern, when it reaches a certain point close to 25 bp, it falls until it converges to a steady state value of zero.

#### International interest rate shock

Figure 4: International interest rate shock R2 and R5



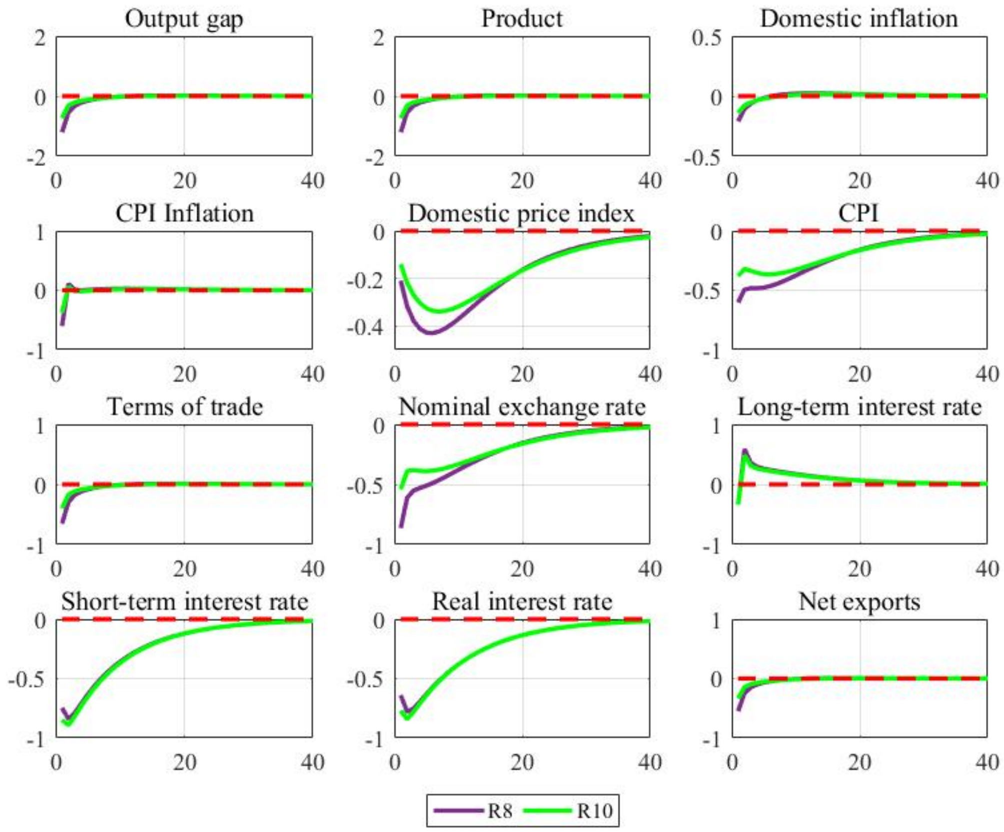

Note: Impulse response of a negative international interest rate shock with a standard deviation of 100 bp.

A decrease in the international interest rate causes a massive flow of accumulated capital to the domestic economy, which leads to a persistent appreciation of the nominal exchange rate, which deteriorates competitiveness and leads to a drop in net exports. In the country the terms of trade decrease, the product decreases, consequently, the output gap decreases, decreasing prices in the economy. On the other hand, the central bank lowers the nominal interest rate, which leads to a decrease in the real interest rate.

Regarding a possible international monetary stimulus, the results are consistent with that documented in Banerjee et al. (2015) and Kolasa and Wesołowski (2018). The effects on long-term interest rates have a different reaction for each monetary policy rule. In the financial sector, it generates co-movements in short and long-term interest rates. In particular, it leads to the long-term interest rate decreasing, but to a lesser extent, considering the conventional monetary policy of the domestic economy.

Thus, a lower interest rate in the external economy induces its investors to seek returns abroad. Consequently, the participation of non-residents in the long-term bond market of the domestic economy increases. Capital flows to the domestic bond market are accompanied by a drop in the long-term interest rate, this drop is driven by the supply of bonds and by lower current and future rates expected in the short term.

Figure 5: International interest rate shock R8 and R10



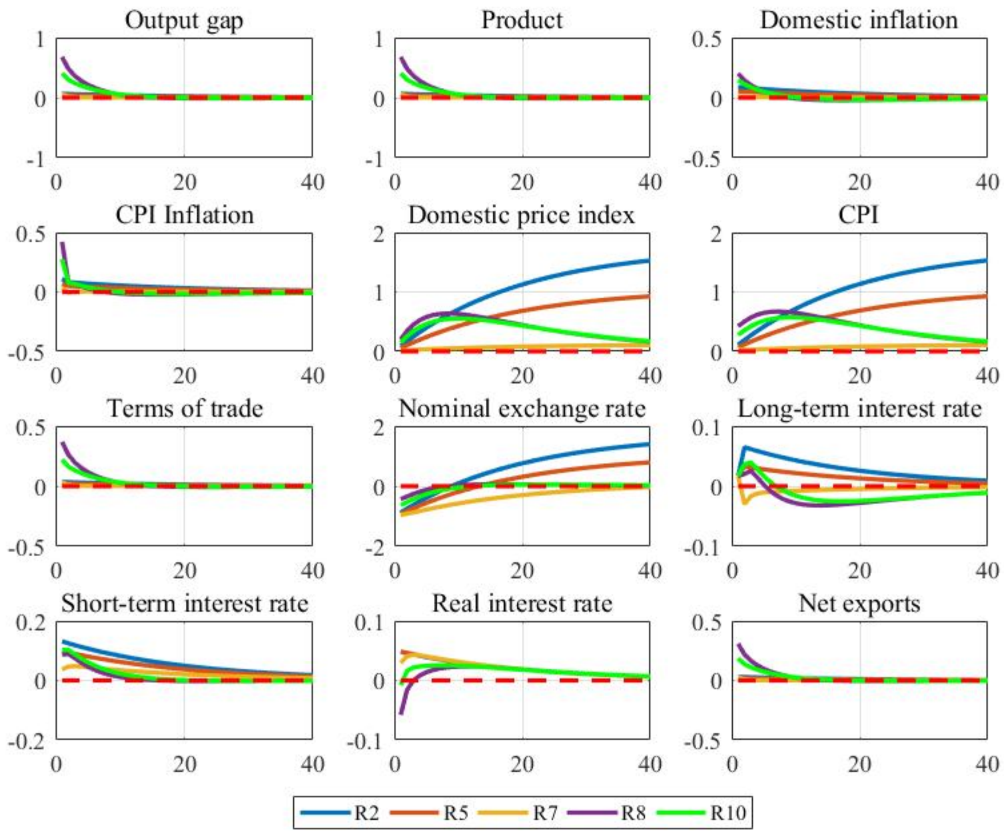

Note: Impulse response of a negative international interest rate shock with a standard deviation of 100 bp.

At the same time, through the supply of bonds and by higher current and future rates expected in the short term, a higher short-term interest rate decreases the long-term interest rate by about 4 bp. For R2, the long-term interest rate gradually increases. For R5, the long-term interest rate maintains a downward trajectory, and then gradually increases. For R8 and R10, since the central bank reacts to the nominal exchange rate, the long-term interest rate of the domestic economy increases, then gradually decreases. This shows that an exchange intervention rule allows reducing the excessive volatility of the exchange rate, avoiding large variations in the prices of the economy.

Consequently, due to financial frictions, R2 or R5, have little advantage over exchange rate rules. In addition, Adler and Tovar (2014), recognizes the importance of the use of foreign exchange intervention by central banks in emerging economies to deal with excessive fluctuations in capital flows and associated exchange rate volatility.

### World price level shock

Figure 6: World price level shock




Note: Impulse response of a positive shock to the world price level with a standard deviation of 100 bp.

An increase in the world price level, increases the terms of trade and initially appreciates the nominal exchange rate. In addition, the product increases, consequently, the output gap increases, increasing prices in the economy. The central bank increases the nominal interest rate which leads to an increase in the real interest rate (due to the greater effect in $\widehat{\pi}_{\mathrm{d,t+1}}$, the real interest rate falls for R8 and R10).

At the same time, through the supply of bonds and by higher current and future rates expected in the short term, a higher short-term interest rate increases the long-term interest rate by about 2 bp. For R2, the initial response follows a hump-shaped pattern, then gradually decreases. For R5, the initial response follows a hump-shaped pattern, then gradually decreases. For R7, since the central bank reacts to projected inflation, the long-term interest rate of the domestic economy decreases, then gradually increases. For R8 and R10, follow a hump-shaped pattern, fluctuating until they converge to a steady state value of zero.

The domestic price index and the CPI increase gradually. For R2, it maintains a permanent increase close to 150 bp. For R5, it maintains a permanent increase equivalent to 100 bp. For R7, it maintains a permanent increase of less than 10 bp. For R8 and R10, the initial response follows a hump-shaped pattern. Upon reaching a certain point close to 50 bp, they fall until they converge to a steady state value of zero.

Consequently, due to financial frictions, R2 or R5, have little advantage over the other rules. However, a strong monetary policy towards managing projected domestic inflation notably improves the performance of the domestic economy.

### 4.3 Posterior estimation

Table 9: Posterior probability of the model

| | $\log \widehat{p}(\mathcal{Y}_T \mid M_R)$ | FB | p |
|---|---|---|---|
| **Rule 1** | 11.767 | 1.00E+00 | 6.36E−14 |
| **Rule 2** | -25.838 | 4.66E−17 | 2.96E−30 |
| **Rule 3** | -13.508 | 1.06E−11 | 6.71E−25 |
| **Rule 4** | -13.700 | 8.71E−12 | 5.53E−25 |
| **Rule 5** | -34.717 | 6.49E−21 | 4.12E−34 |
| **Rule 6** | -41.744 | 5.76E−24 | 3.66E−37 |
| **Rule 7** | -33.428 | 2.36E−20 | 1.50E−33 |
| **Rule 8** | 42.146 | 1.56E+13 | 9.92E−01 |
| **Rule 9** | 33.794 | 3.68E+09 | 2.34E−04 |
| **Rule 10** | 37.274 | 1.20E+10 | 7.60E−03 |

The rule that best explains the behavior of the set of variables is R8, which react to the interest rate smoothing, domestic inflation, and the nominal exchange rate.

Table 9 shows that the logarithmic marginal probability, the Bayes factor and the posterior probability clearly favor R8 over the others. R10, which also reacts to the output gap, has a better fit than R9 but worse than R8.

There is a difference between R8, R9, and R10 but this difference cannot be accepted as decisive evidence in favor of one rule over the other. Rabanal and Rubio-Ramírez (2005), mention that an attractive feature of the Bayes factor is that it embodies a strong preference for parsimony. It can also be considered that these three factors are too large, so the inclusion of the exchange rate considerably improves the fit of the rule in the model. Batini et al. (2009), mention that there is an intrinsic interest in knowing to what extent an economy can be stabilized with the simplest rule, so R8 can be a point of reference to analyze the Peruvian economy.

Table 10: R8 Estimate

| | **Prior** | **Posterior** | | **Intervals** | |
|---|---|---|---|---|---|
| **Parameter** | **Mean** | **Mode** | **Mean** | **Lower** | **Higher** |
| $\rho_\xi$ | 0.50 | 0.5007 | 0.5007 | 0.4843 | 0.5173 |
| $\rho_a$ | 0.90 | 0.9058 | 0.9049 | 0.8902 | 0.9200 |
| $\rho_g$ | 0.90 | 0.9339 | 0.9331 | 0.9229 | 0.9438 |
| $\rho_{R^*}$ | 0.90 | 0.8981 | 0.8974 | 0.8809 | 0.9137 |
| $\rho_{p^*}$ | 0.95 | 0.9769 | 0.9760 | 0.9685 | 0.9836 |
| $\sigma_\xi$ | 0.10 | 0.0791 | 0.0826 | 0.0639 | 0.1015 |
| $\sigma_a$ | 0.40 | 0.3927 | 0.3953 | 0.3675 | 0.4243 |
| $\sigma_g$ | 0.20 | 0.1566 | 0.1591 | 0.1397 | 0.1775 |
| $\sigma_{R^*}$ | 0.20 | 0.1566 | 0.1590 | 0.1412 | 0.1766 |
| $\sigma_{p^*}$ | 1.00 | 0.9945 | 0.9956 | 0.9645 | 1.0284 |

Table 10 shows the posterior moments of the autocorrelation coefficients and the standard deviations of the shocks. The last five columns present the prior mean, the posterior mode, the posterior mean, and the 90% highest posterior density intervals (inferior and superior respectively).

In $\rho_{\xi}$, $\rho_{\mathrm{a}}$, and $\rho_{\mathrm{R}^*}$, the prior mean is very similar to its posterior estimates.

In $\rho_{\mathrm{g}}$ and $\rho_{\mathrm{p}^*}$, the prior mean is slightly lower than its posterior estimates. These coefficients fall outside the 90% probability intervals, therefore it implies a greater persistence in the preferences shock and the world price level shock.

In $\sigma_{\xi}$, the prior mean is slightly higher than its posterior estimates. This coefficient falls within the 90% probability interval, very close to the upper bound.

In $\sigma_{\mathrm{a}}$, the prior mean is very similar to its posterior estimates.

In $\sigma_{\mathrm{g}}$ and $\sigma_{\mathrm{R}^*}$, the prior mean is greater than its posterior estimates. Therefore it implies a low volatility in the preferences shock and the international interest rate shock. Thus it reveals that exogenous variation matter less for R8 to explain variation in $\mathcal{Y}_{\mathrm{T}}$.

In $\sigma_{\mathrm{p}^*}$, the prior mean is very similar to the posterior mode and mean. This coefficient falls within the 90% probability interval, making it stable. The data clearly provides support to believe that the world price level has greater volatility compared to other shocks. Overall for the Peruvian economy, which is exposed to commodities price shock, volatility is high.

# 5 Conclusion

My main findings suggest that, for a small and open economy with financial frictions like Peru, a DIFB rule, R7, and an exchange rate based rule, R8 and R10, work well in minimizing the loss function. These rules are associated with less variability in the main variables analyzed. In contrast to the Taylor rule that has a greater loss function.

Individually, a marked monetary policy towards a DIFB rule seems quite robust facing productivity shocks, preferences shocks and world price level shocks. On the other hand, a monetary policy marked to manage the nominal exchange rate seems quite robust facing monetary policy shocks and international interest rate shocks. Although the Peruvian economy has an explicit inflation targeting scheme, its central bank can use a set of monetary policy rules for each particular shock.

On the other hand, international shocks can affect competitiveness and involve comovements in domestic short and long-term interest rates. It is possible to find that, as in the data until December 2019, under financial frictions, a negative international interest rate shock generates a contraction in the domestic economy, along with an increase in the flow of capital, as well as a fall in the exchange rate nominal. conversely a positive world price level shock generates a stimulus in the domestic economy, as well as an increase in the nominal exchange rate. A conventional monetary policy response has a smaller magnitude in the domestic economy.

According to the model simulations, the rules based on the exchange rate provide a better fit. Consequently, the estimated parameters indicate that there is less volatility in international interest rates and greater volatility in the world price level shock, making it more likely that fluctuations in the commodities prices will have a greater magnitude in the domestic economy. I conclude that this model can replicate key empirical facts observed in the domestic business cycle.

# A Derivation of the model

## Households

The optimization problem:

$$\max_{\left\{B_t^s,B_t^l,A_{t+1},C_t,N_t\right\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t g_t \left\{ \frac{C_t^{1-\sigma}-1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right\} \tag{A.1}$$

Subject to:

$$\begin{aligned} B_t^s+B_t^l+\mathbb{E}_t Q_{t,t+1} A_{t+1}+P_t C_t =& R_{t-1} B_{t-1}^s + R_t^l B_{t-1}^l + A_t + W_t N_t + D_t \\ & -\frac{P_t\left(b^s+b^l\right)}{2}\left[\delta\frac{B_t^s}{B_t^l}-1\right]^2 - \frac{P_t\left(b^s+b^l\right)}{2}\left[\frac{B_t^s}{B_{t-1}^s}\frac{B_{t-1}^l}{B_t^l}-1\right]^2 \end{aligned} \tag{A.2}$$

$$\lim_{T\to\infty} \mathbb{E}_t Q_{t,T+1}\left\{R_T B_T^s + R_{T+1}^l B_T^l + A_{T+1}\right\} \tag{A.3}$$

First order conditions:

$$\begin{aligned} \omega_t+\omega_t\frac{P_t\left(b^s+b^l\right)}{B_t^l}\left[\delta\frac{B_t^s}{B_t^l}-1\right] + \omega_t\frac{P_t\left(b^s+b^l\right)}{B_{t-1}^s}\frac{B_{t-1}^l}{B_t^l}\left[\frac{B_t^s}{B_{t-1}^s}\frac{B_{t-1}^l}{B_t^l}-1\right] =& \beta\mathbb{E}_t R_t \omega_{t+1} \\ & + \beta\mathbb{E}_t\omega_{t+1}\frac{P_{t+1}\left(b^s+b^l\right)B_{t+1}^s}{\left(B_t^s\right)^2}\frac{B_t^l}{B_{t+1}^l}\left[\frac{B_{t+1}^s}{B_t^s}\frac{B_t^l}{B_{t+1}^l}-1\right] \end{aligned} \tag{A.4}$$

$$\begin{aligned} \omega_t+\omega_t\frac{P_t\left(b^s+b^l\right)B_t^s}{\left(B_t^l\right)^2}\left[\delta\frac{B_t^s}{B_t^l}-1\right] + \omega_t\frac{P_t\left(b^s+b^l\right)B_t^s}{B_{t-1}^s}\frac{B_{t-1}^l}{\left(B_t^l\right)^2}\left[\frac{B_t^s}{B_{t-1}^s}\frac{B_{t-1}^l}{B_t^l}-1\right] =& \beta\mathbb{E}_t R_{t+1}^l \omega_{t+1} \\ & + \beta\mathbb{E}_t\omega_{t+1}\frac{P_{t+1}\left(b^s+b^l\right)}{B_t^s}\frac{B_{t+1}^s}{B_{t+1}^l}\left[\frac{B_{t+1}^s}{B_t^s}\frac{B_t^l}{B_{t+1}^l}-1\right] \end{aligned} \tag{A.5}$$

$$\mathbb{E}_t\omega_t Q_{t,t+1} = \beta\omega_{t+1} \tag{A.6}$$

$$g_t C_t^{-\sigma} = \omega_t P_t \tag{A.7}$$

$$g_t N_t^{\varphi} = \omega_t W_t \tag{A.8}$$

Given $\omega_t$ is the Lagrange multiplier of the nominal budget constraint. Let $\Lambda_t$ be the Lagrange multiplier of the actual budget constraint:

$$\Lambda_t = \omega_t P_t \tag{A.9}$$

The first order conditions can be written in real terms:

$$\begin{aligned} \Lambda_t+\Lambda_t\frac{\left(b^s+b^l\right)}{b_t^l}\left[\delta\frac{b_t^s}{b_t^l}-1\right] + \Lambda_t\frac{\left(b^s+b^l\right)}{b_{t-1}^s}\frac{b_{t-1}^l}{b_t^l}\left[\frac{b_t^s}{b_{t-1}^s}\frac{b_{t-1}^l}{b_t^l}-1\right] =& \beta\mathbb{E}_t R_t \Lambda_{t+1}\pi_{t+1}^{-1} \\ & + \beta\mathbb{E}_t\Lambda_{t+1}\frac{\left(b^s+b^l\right)b_{t+1}^s}{\left(b_t^s\right)^2}\frac{b_t^l}{b_{t+1}^l}\left[\frac{b_{t+1}^s}{b_t^s}\frac{b_t^l}{b_{t+1}^l}-1\right] \end{aligned} \tag{A.10}$$

$$\Lambda_t+\Lambda_t\frac{\left(b^s+b^l\right)b^s_t}{\left(b^l_t\right)^2}\left[\delta\frac{b^s_t}{b^l_t}-1\right]+\Lambda_t\frac{\left(b^s+b^l\right)b^s_t}{b^s_{t-1}}\frac{b^l_{t-1}}{\left(b^l_t\right)^2}\left[\frac{b^s_t}{b^s_{t-1}}\frac{b^l_{t-1}}{b^l_t}-1\right]=\beta\mathbb{E}_t R^l_{t+1}\Lambda_{t+1}\pi^{-1}_{t+1}$$
$$+\beta\mathbb{E}_t\Lambda_{t+1}\frac{\left(b^s+b^l\right)}{b^s_t}\frac{b^s_{t+1}}{b^l_{t+1}}\left[\frac{b^s_{t+1}}{b^s_t}\frac{b^l_t}{b^l_{t+1}}-1\right] \tag{A.11}$$

$$\mathbb{E}_t\Lambda_t Q_{t,t+1}\pi_{t+1}=\beta\Lambda_{t+1} \tag{A.12}$$

$$g_t C_t^{-\sigma}=\Lambda_t \tag{A.13}$$

$$g_t N_t^{\varphi}=\Lambda_t\frac{W_t}{P_t} \tag{A.14}$$

Euler's equation for consumption:

$$g_t C_t^{-\sigma}=\beta\mathbb{E}_t R_t g_{t+1}C^{-\sigma}_{t+1}\pi^{-1}_{t+1}+\beta\mathbb{E}_t g_{t+1}C^{-\sigma}_{t+1}\frac{\left(b^s+b^l\right)b^s_{t+1}}{\left(b^s_t\right)^2}\frac{b^l_t}{b^l_{t+1}}\left[\frac{b^s_{t+1}}{b^s_t}\frac{b^l_t}{b^l_{t+1}}-1\right]$$
$$-g_t C_t^{-\sigma}\frac{\left(b^s+b^l\right)}{b^l_t}\left[\delta\frac{b^s_t}{b^l_t}-1\right]-g_t C_t^{-\sigma}\frac{\left(b^s+b^l\right)}{b^s_{t-1}}\frac{b^l_{t-1}}{b^l_t}\left[\frac{b^s_t}{b^s_{t-1}}\frac{b^l_{t-1}}{b^l_t}-1\right] \tag{A.15}$$

Labor supply:

$$\frac{W_t}{P_t}=C_t^{\sigma}N_t^{\varphi} \tag{A.16}$$

Next, each variable with a circumflex is expressed in its log-linear form. Where: $b^l(b^s)^{-1}=\delta$, $b^s(b^s+b^l)^{-1}=(1+\delta)^{-1}$, and $b^l(b^s+b^l)^{-1}=\delta(1+\delta)^{-1}$.

Euler's equation for log-linear consumption:

$$\widehat{c}_t=\mathbb{E}_t\widehat{c}_{t+1}-\frac{1}{\sigma}\left[\widehat{R}_t-\mathbb{E}_t\widehat{\pi}_{t+1}\right]\frac{\delta\left(1+\delta\right)}{\sigma}\left[\widehat{b}^s_t-\widehat{b}^l_t\right]+\frac{(1+\delta)}{\sigma}\Delta\left[\widehat{b}^s_t-\widehat{b}^l_t\right]$$
$$-\frac{(1+\delta)}{\sigma}\beta\mathbb{E}_t\Delta\left[\widehat{b}^s_{t+1}-\widehat{b}^l_{t+1}\right]+\frac{1}{\sigma}\left(1-\rho_g\right)\widehat{g}_t \tag{A.17}$$

Log-linear Labor offer:

$$\widehat{w}_t-\widehat{p}_t=\sigma\widehat{c}_t+\varphi\widehat{n}_t \tag{A.18}$$

First order condition for holding log-linear short-term bonds:

$$\widehat{\Lambda}_t=\mathbb{E}_t\left[R_t+\widehat{\Lambda}_{t+1}-\widehat{\pi}_{t+1}\right]-\delta\left(1+\delta\right)\left[\widehat{b}^s_t-\widehat{b}^l_t\right]-(1+\delta)\Delta\left[\widehat{b}^s_t-\widehat{b}^l_t\right]$$
$$+\beta\left(1+\delta\right)\mathbb{E}_t\Delta\left[\widehat{b}^s_{t+1}-\widehat{b}^l_{t+1}\right] \tag{A.19}$$

First order condition for long-term log-linear bond holding:

$$\widehat{\Lambda}_t=\mathbb{E}_t\left[\widehat{R}^l_{t+1}+\widehat{\Lambda}_{t+1}-\widehat{\pi}_{t+1}\right]+(1+\delta)\left[\widehat{b}^s_t-\widehat{b}^l_t\right]+\frac{(1+\delta)}{\delta}\Delta\left[\widehat{b}^s_t-\widehat{b}^l_t\right]$$
$$-\beta\frac{(1+\delta)}{\delta}\mathbb{E}_t\Delta\left[\widehat{b}^s_{t+1}-\widehat{b}^l_{t+1}\right] \tag{A.20}$$

Combining the two previous equations, I arrive at the equation of the expected long-term interest rate:

$$\mathbb{E}_{\mathrm{t}}\widehat{\mathrm{R}}^{\mathrm{l}}_{\mathrm{t+1}}=\widehat{\mathrm{R}}_{\mathrm{t}}-(1+\delta)\left[\widehat{\mathrm{b}}^{\mathrm{s}}_{\mathrm{t}}-\widehat{\mathrm{b}}^{\mathrm{l}}_{\mathrm{t}}\right]-\frac{(1+\delta)}{\delta}\left\{\Delta\left[\widehat{\mathrm{b}}^{\mathrm{s}}_{\mathrm{t}}-\widehat{\mathrm{b}}^{\mathrm{l}}_{\mathrm{t}}\right]+\beta\mathbb{E}_{\mathrm{t}}\Delta\left[\widehat{\mathrm{b}}^{\mathrm{s}}_{\mathrm{t+1}}-\widehat{\mathrm{b}}^{\mathrm{l}}_{\mathrm{t+1}}\right]\right\} \tag{A.21}$$

Euler's equation can be rewritten:

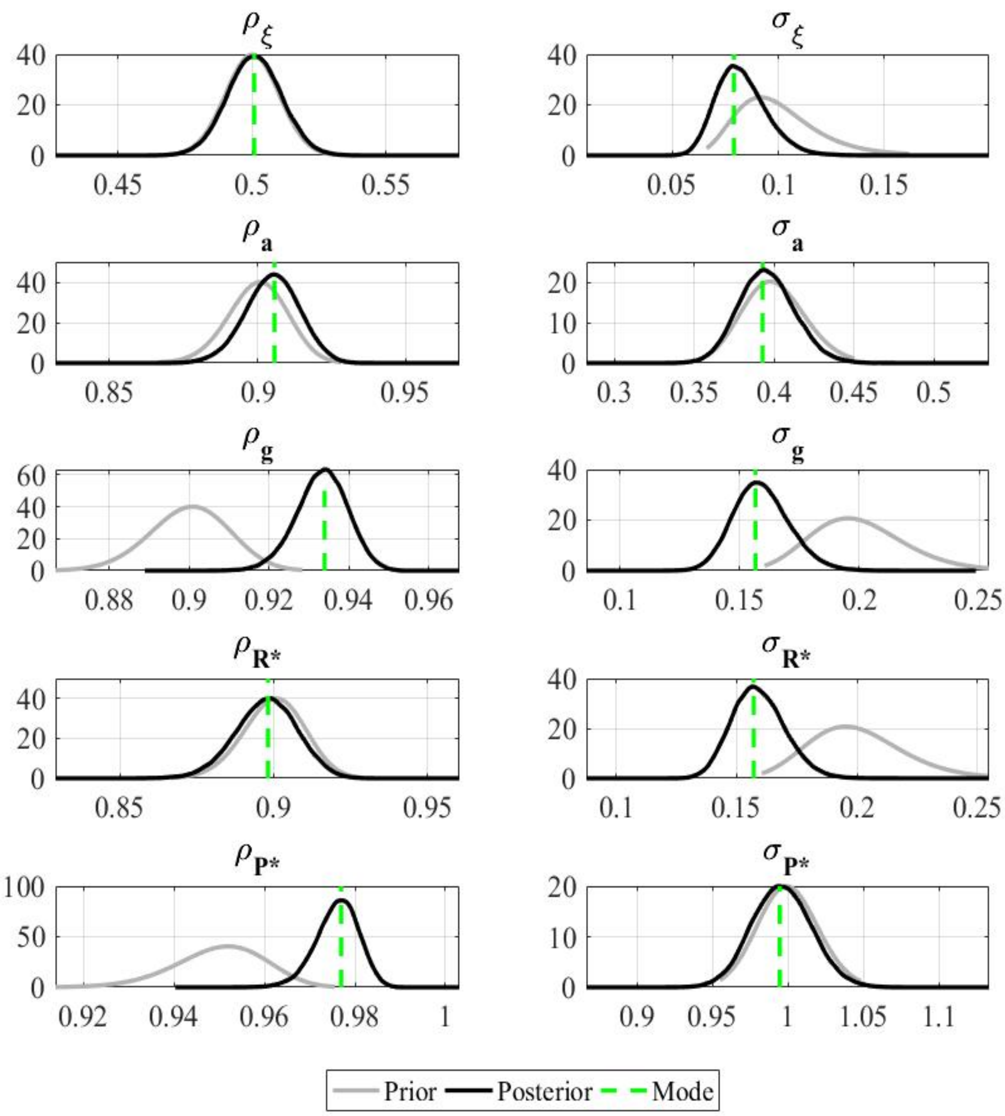
$$\widehat{\mathrm{c}}_{\mathrm{t}}=\mathbb{E}_{\mathrm{t}}\widehat{\mathrm{c}}_{\mathrm{t+1}}-\frac{1}{\sigma}\left[\widehat{\mathrm{R}}_{\mathrm{t}}-\mathbb{E}_{\mathrm{t}}\widehat{\pi}_{\mathrm{t+1}}\right]+\frac{\delta}{\sigma(1+\delta)}\left[\widehat{\mathrm{R}}_{\mathrm{t}}-\mathbb{E}_{\mathrm{t}}\widehat{\mathrm{R}}^{\mathrm{l}}_{\mathrm{t+1}}\right]+\frac{1}{\sigma}(1-\rho_{\mathrm{g}})\,\widehat{\mathrm{g}}_{\mathrm{t}} \tag{A.22}$$

$$\widehat{\mathrm{c}}_{\mathrm{t}}=\mathbb{E}_{\mathrm{t}}\widehat{\mathrm{c}}_{\mathrm{t+1}}-\frac{1}{\sigma}\left[\frac{1}{1+\delta}\widehat{\mathrm{R}}_{\mathrm{t}}+\frac{\delta}{1+\delta}\mathbb{E}_{\mathrm{t}}\widehat{\mathrm{R}}^{\mathrm{l}}_{\mathrm{t+1}}-\mathbb{E}_{\mathrm{t}}\widehat{\pi}_{\mathrm{t+1}}\right]+\frac{1}{\sigma}(1-\rho_{\mathrm{g}})\,\widehat{\mathrm{g}}_{\mathrm{t}} \tag{A.23}$$

# B Posterior distributions

## Rule 8

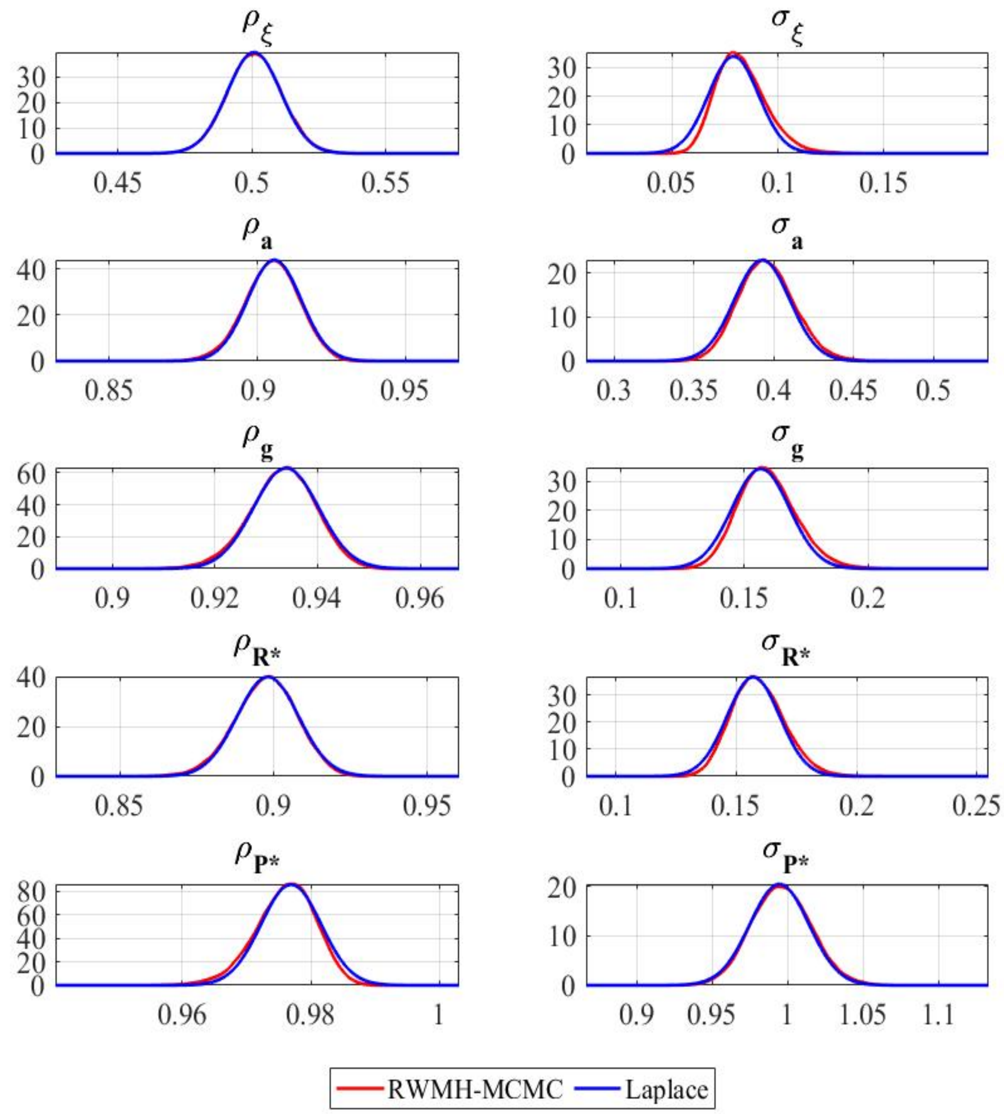
Figure 7: R8 Posterior distributions



## Comparison between RWMH-MCMC and Laplace

The posterior distributions can be calculated by the RWMH-MCMC algorithm and the Laplace approximation. Christiano, Trabandt, and Walentin (2010), mention that the calculation of the posterior density of the RWMH-MCMC algorithm can be very intensive and recommend using this approach during the early and intermediate phases of the research project. For the estimated parameters, the results of both methods are very similar.

Figure 8: Comparison between RWMH-MCMC and Laplace



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