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The Effects of Money-financed Fiscal Stimulus in a Small Open Economy*

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Abstract

In this paper, we analyze the effects of money-financed (*MF*) fiscal stimulus and compare them with those resulting from a conventional debt-financed (*DF*) fiscal stimulus in a small open economy. We find that in normal times which is a period when a zero lower bound (ZLB) on the nominal interest rate is not applicable, *MF* fiscal stimulus is effective in increasing output. In a liquidity trap where the ZLB is applicable, even though the decrease in both consumer price index (CPI) inflation and output is more severe than in a closed economy when there is no fiscal response, *MF* fiscal stimulus is effective in stabilizing both. Accordingly, we show that even in an imperfect pass-through environment including a liquidity trap, an increase in government expenditure under *MF* fiscal stimulus is effective. In contrast, our policy implications concerning an increase in government expenditure under *DF* fiscal stimulus lie opposite to Gali, Jordi (2020), "The Effects of a Money-financed Fiscal Stimulus," *Journal of Monetary Economics*, 115, 1-19, assuming a closed economy. In normal times, an increase in government expenditure under the *DF* scheme in a small open economy is more effective than in a closed economy, although Gali (2020) argues that it is much less effective. In a liquidity trap, an increase in government expenditure under the *DF* scheme is less effective, also in contrast to Gali (2020). We find that even in an imperfect pass-through environment, an increase in government expenditure under *DF* fiscal stimulus is not effective. Thus, in a small open economy, *MF* fiscal stimulus is not always essential in normal times, and in a liquidity trap, *MF* fiscal stimulus is more important than what Gali (2020) suggests because *DF* fiscal stimulus is not effective, irrespective of nominal exchange rate pass-through.

Keywords: Fiscal Stimulus, Money Financing, Debt Financing, Zero Lower Bound, Imperfect Pass-through

JEL Classification: E31, E32, E52; E62; F41

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1 Introduction

Gali[13] instituted a theoretical framework concerning money-financed (*MF*) fiscal stimulus and demonstrates that it is effective in stabilizing both gross domestic product (GDP) and inflation, even in an economy becoming stuck in a liquidity trap. Currently, while US Federal Reserve Board officials expect the Federal Funds Rate to rise to 0.6% by the end of 2023, the overnight rates in member countries of the Organization for Economic Co-operation and Development (OECD) (except for countries in the Eurozone comprising the 19 member countries of the European Monetary Union) and the Eurozone in Quarter 1, 2021 are all less than 1%, except for Colombia, Turkey, and Mexico, as shown in Table 1, which details the share of GDP to gross world product (GWP) and the overnight interest rate in the OECD countries and the Eurozone (Column 7, Table 1).[1] On this basis, many OECD countries could become stuck in a liquidity trap, and this policy implication is then potentially very timely.

However, one problem is that Gali[13] derives his *MF* fiscal stimulus policy implication under the assumption of a closed economy. This provides our motivation to extend his analysis by applying a more plausible assumption concerning the openness and size of economies, especially as the share of GDP in GWP for most countries is less than 1%, or 2% at the most, except for the United States, the Eurozone, Japan, and the United Kingdom (Column 2, Table 1). Unfortunately, we cannot always decide which countries are a small open economy using just their share of GDP in GWP because the definition of one is a country that is sufficiently small compared with its trading partners such that its policies do not alter world prices, interest rates, or incomes. However, a country whose share of GDP in GWP is negligible could be regarded as a small open economy because it is very difficult for it to alter world prices, interest rates, or incomes. Instead, a country like this is affected by the world economy in a more one-sided fashion. For this reason, nearly all countries in the OECD are small open economies and therefore assuming a small open economy in analyzing *MF* fiscal stimulus is a logical step forward.

We extend the model in Gali[13] to a small open economy model and calculate the responses under *MF* fiscal stimulus, conventional debt-financed (*DF*) fiscal stimulus, and the case of *no response* in which there is no fiscal stimulus such as a tax cut or an increase in government expenditure to an adverse demand shock that causes the nominal interest rate to become stuck at the zero lower bound (ZLB) or in a liquidity trap. Note that the cost of the fiscal stimulus is financed by the issuance of money and the balance of government debt is unchanged under the *MF* fiscal stimulus scenario. In contrast, the cost of the *DF* fiscal stimulus expense is financed by changing the real balance of government debt. The scenario we use for the liquidity trap is identical to that in Gali[13] where an adverse shock strikes a small open economy six periods in succession.

Our findings are as follows. To start, we find that in the case of *no response*, the cumulative decrease in output in a small open economy is 1.77 times larger than in the closed economy assumed by Gali[13] under our benchmark parameterization where the openness of the economy takes a value of 0.4. Likewise, the fall in consumer price index (CPI) inflation when an adverse shock strikes a small open economy is some 3.24 times larger than in a closed economy. The reason for this more serious decline in GDP and CPI inflation stems from the ‘negative’ repercussions of a decrease in domestic goods inflation and appreciation in the nominal exchange rate. When an

[1] See Board of the Governor of the Federal Reserve System[10] and The New York Times[22].

adverse demand shock strikes, output diminishes through a decrease in consumption from shifts in the Euler equation. This decrease in output then decreases domestic goods inflation through an increase in the average markup (i.e., a decrease in marginal cost).

This decrease in domestic goods inflation then applies downward pressure on CPI inflation in a small open economy, unlike a closed economy where there is no further influence because there are no import goods and domestic goods inflation is then the same as CPI inflation. In a small open economy, the nominal exchange rate is an important factor for deciding CPI inflation because the nominal exchange rate is related to the CPI through purchasing power parity (PPP). Although PPP is not applicable in every period in a small open economy, PPP is applicable in the long run in our model similar to the small open economy model in Gali and Monacelli[15]. Thus, successive decreases in CPI inflation decrease (i.e., appreciate) the nominal exchange rate, and this causes further decreases in CPI inflation. This results in a severe fall (i.e., improvement) in the terms of trade (TOT), being the price of import goods in terms of the price of domestic goods, because import goods inflation (or the price) has no nominal rigidity and only depends on the nominal exchange rate (provided there is perfect nominal exchange rate pass-through). This decrease (i.e., improvement) in the TOT again decreases output through the expenditure-switching effect and decreases domestic goods inflation further through an increase in the average markup (i.e., a decrease in marginal cost). Further changes ensue, resulting in severe decreases in output and CPI inflation. Notice that this severe decrease in the CPI inflation amplifies the burden of redeeming government debt and makes government debt larger than in a closed economy.

Similar to Gali[13], in our model, during the period when the adverse demand shock strikes, money growth falls in the case of *no response*. However, unlike Gali[13], which shows that the nominal interest rate is stuck in the ZLB as long as the adverse demand shock strikes a closed economy, we find that the nominal interest rate is hiked and exceeds its steady-state value, that is, it overshoots, before the adverse demand shock disappears in the case of *no response*. This response of nominal interest rates is then reminiscent of the lifting of the zero-interest-rate policy in August 2000 in Japan. This was based on the view of many members of the Bank of Japan policy board that deflationary concerns were dispelled, even though it was considered a premature decision by at least one government official.[2]

Although we find that the fall in CPI inflation is more severe in a small open economy in the case of *no response*, its pace of recovery is fast because the recovery in import goods inflation, which has no nominal rigidity and only depends on the nominal exchange rate, is very fast. This rapid recovery in CPI inflation makes fiscal reconstruction faster than a closed economy because the burden of redeeming government debt is mitigated by rapid recovery in CPI inflation. Similarly, higher tax revenue, resulting from larger government debt due to severe decrease in CPI inflation and a simple tax rule intending to adjust path of taxes to attain the long run debt target, contributes to fiscal reconstruction. Then, seignorage is no longer necessary. The nominal interest rate is hiked and money growth decreases. This increase in the nominal interest rate occurs before the adverse demand shock disappears, as mentioned, and results in an increase in the real consumption interest rate and so the recovery in consumption is weaker than in a closed economy. As a result, cumulative decrease in output in a small open economy is more severe than in a closed economy. A small open economy thus faces the peril of a severe depression made more terrible by the absence of fiscal stimulus, and this is one of our most important findings.

[2]See Bank of Japan[3] and Nihon Keizai Shimbun[23] for details.

However, *MF* fiscal stimulus is effective regardless of whether the means is a tax cut or an increase in government expenditure because the 'negative' repercussion is mitigated by a 'positive' repercussion in which the direction of the repercussion is opposite to the 'negative' repercussion. The *MF* fiscal stimulus does this by applying pressure to increase (i.e., depreciate) the nominal exchange rate through an increase in both CPI and domestic inflation. As a result, the TOT returns to stability and output and CPI inflation recover without delay, similar to that in the closed economy in Gali[13]. The increase in the nominal interest rate in the middle of the adverse demand shock striking a small open economy has then gone owing to issuing money to finance a tax cut or an increase in government expenditure. Instead, recovery or an increase in the nominal interest rate begins after the adverse demand shock disappears, and the pace of recovery is slow.

Conducting *DF* fiscal stimulus is then not effective in preventing the 'negative' repercussions brought about by the adverse demand shock. A tax cut is definitely not effective because of Ricardian equivalence and this result is identical to Gali[13]. However, although Gali[13] appreciates the increase in government expenditure under *DF* scheme in a liquidity trap, the responses to an increase in government expenditure we identify are closer to those under *no response* in a small open economy. Different from a closed economy, there is 'negative' repercussion that causes a severe decrease (i.e., appreciation) in the nominal exchange rate. Given PPP in the long run, a severe decrease (i.e., appreciation) in the nominal exchange rate causes a severe decline in CPI inflation. This amplifies the burden of redeeming government debt, and an increase in the burden of debt redemption makes the balance of real government debt higher than in a closed economy. Given the simple tax rule which has mentioned, tax revenue increases so that the government has incentive to reduce seignorage, similar to the case of *no response*. Then, the real consumption interest rate increases, and consumption stagnates. In addition, the severe decrease (i.e., appreciation) in the nominal exchange rate causes a decrease (i.e., improvement) in the TOT through a decrease in the import goods price and output also stagnates because of the expenditure-switching effect. Thus, an increase in government expenditure under *DF* scheme is no longer effective in stabilizing output and CPI inflation. Our result on the effectiveness of an increase in government expenditure under *DF* fiscal stimulus contrasts with that in Gali[13] and is another of our most important findings.

Additionally, we focus on how the nominal exchange rate pass-through affects our results. As argued by Monacelli[20], there is a known limitation in assuming perfect pass-through because of two well-established empirical facts: the overwhelming failure of the law of one price for tradables, and more rapid exchange rate pass-through with wholesale import prices. To consider this limitation, we introduce into our model foreign retailers that choose their optimal prices to maximize their profits following Calvo pricing and sell their goods in a small open economy. This enables us to analyze the effectiveness of *MF* fiscal stimulus in a small open economy with an imperfect nominal exchange rate pass-through environment where the changes in the nominal exchange rate are less effective in changing the import goods price as well as import goods inflation. In an imperfect pass-through environment, import goods inflation has nominal rigidity and the decrease in CPI inflation is mitigated, even in the case of *no response*, when the adverse demand shock hits a small open economy.

An increase in government expenditure under *MF* fiscal stimulus is then effective in stabilizing output and CPI inflation, similar to existing analysis assuming perfect pass-through. However, the increase in government expenditure under *DF* scheme is not necessarily effective in stabilizing output and CPI inflation. This is because the increase in government expenditure decreases (i.e., improves) the TOT through an increase in domestic goods inflation in the imperfect pass-through environment and this decrease (i.e., improvement) offsets the pressure to increase output through the expenditure-switching effect. Our result that the increase in government expenditure under *DF* fiscal stimulus in a perfect pass-through environment is even applicable in an imperfect pass-through environment is yet another of our most important findings. Overall, the presence of the TOT peculiar to an open economy interferes with the materialization of the policy implication derived by Gali[13] in a closed economy.

Following Gali[13], before deriving the responses in a liquidity trap, we obtain those for normal times when the ZLB does not apply. Unlike Gali[13], we find that the increase in government expenditure under *DF* fiscal stimulus is more effective in boosting output because of less crowding-out in consumption in a small open economy than in a closed economy. This result is opposite to Gali[13] in showing that the effects of this policy are much less than those for an increase in government expenditure under *MF* fiscal stimulus. This is also an important finding. However, like Gali[13], we reveal that *MF* fiscal stimulus is effective in boosting output but that a tax cut under *DF* fiscal stimulus is definitely not effective because of Ricardian equivalence. We then calculate fiscal multipliers under various degrees of openness in normal times. We find that except for a tax cut under the *DF* scheme in which Ricardian equivalence applies and multipliers are definitely zero regardless of the degree of openness, the fiscal multipliers increase as openness increases.

Overall, we conclude that in normal times in a small open economy, *MF* fiscal stimulus is not necessarily essential for increasing output, and an increase in government expenditure under *DF* scheme remains effective. However, the *MF* fiscal stimulus is more important than what Gali[13] suggests in a small open economy in a liquidity trap. This is because the absence of a fiscal response is more dangerous, especially in a perfect pass-through environment, and because an increase in government expenditure under *DF* scheme is no longer effective regardless of nominal exchange rate pass-through.

The remainder of the paper is organized as follows. Section 2 discusses the related literature and Section 3 formulates the model. Section 4 provides the steady state and the set of equilibrium dynamics and defines the fiscal and monetary policies examined. Section 5 details the effects of fiscal stimulus in normal times when the ZLB is not available and calculates the fiscal multipliers. Section 6 considers the effects of fiscal stimulus in a liquidity trap where there is a ZLB. Section 7 concludes the paper.

2 Related Literature

We extend Gali[13], which previously extends Auerbach and Obstfeld[1] and Buiter[5]. Auerbach and Obstfeld[1] study the effects of open market operations in raising inflation and output when the economy is at the ZLB as a result of some temporary adverse shock. These effects are dependent on whether the increase in liquidity is permanent and expected to be so by agents. Buiter[5] analyzes the impact of the *MF* transfer to households in a relatively general setting and emphasizes the importance of the irredeemability of money as the ultimate source of the expansionary effect on consumption of such a policy. Turner[26] conceded the effectiveness of the *MF* fiscal stimulus.[3] In addition, Turner[26] shows that while political issues such as the excessive use of money financing may result, these can be managed by rules, norms, or belief systems. All these studies derive

[3] A not necessarily formal analysis.

positive results concerning *MF* fiscal stimulus, as we do here.

Alternatively, several studies focus on the case in which *MF* fiscal stimulus fails to stabilize the economy. Drawing on Gali[13], Tsuruga and Wake[25] analyze how an implementation lag modifies the effectiveness of *MF* fiscal stimulus and find that it can result in a recession in normal times. In addition, with a liquidity trap, the recession will deepen. English et al.[9] develop a conventional macroeconomic model and address the credibility of the central bank. They show that *MF* fiscal stimulus is effective if communicated successfully and seen as credible by the public. However, if the public doubt the central bank's commitment to the policy, *MF* fiscal stimulus will be ineffective.

Similar to Gali[13], the present paper relates to the literature on monetary policy in a New Keynesian framework in the presence of a ZLB, as do Jung et al.[18], Eggertsson and Woodford[8], Werning[27], Buiter[4], Svensson[24], Nakajima[21], and Fujiwara et al.[12] who examine monetary policy in an environment similar to that in Section 6 demonstrating the perils associated with the absence of fiscal stimulus and the effectiveness of *MF* fiscal stimulus in a liquidity trap. Among these studies, Buiter[4], Svensson[24], Nakajima[21], and Fujiwara et al.[12] assume an open economy as here. Elsewhere, Buiter[4] analyzes monetary policy in a liquidity trap in a small open economy by developing a general equilibrium model without a micro foundation while Svensson[24], Nakajima[21], and Fujiwara et al.[12] derive well micro-founded models, similar to our analysis. However, unlike this analysis, their focus is only on monetary policy.

To extend the closed economy model in Gali[13] to a small open economy, we refer to Gali and Monacelli[15]. The models in Gali and Monacelli[15] are quite precise and to simplify these without loss of generality, we refer to Gali and Monacelli[16] as it succeeds in aggregating individual behavior in a more simplified manner.[4] An alternative way of deriving a small open economy model is by following De Paoli[7] who derives a small open economy model by reducing a two-country model which includes home bias. However, to simplify the derivation, we straightforwardly follow Gali and Monacelli[15].

As noted, we include additional analysis of an imperfect pass-through environment. To generate this, we refer to the analysis of monetary policy in a small open economy with imperfect pass-through in Monacelli[20]. Corsetti et al.[6] also derive a local currency pricing model that generates incomplete pass-through. However, we choose to follow Monacelli[20] as Corsetti et al.[6] assume a two-country economy.

3 The Model

We extend the closed economy model in Gali[13] to a small open economy model using the seminal New Keynesian small open economy model in Gali and Monacelli[15].[5] However, unlike Gali and Monacelli[15], our model consists not only of households and domestic producers but also a government issuing government debt and money to finance expenditure. As discussed in Section 2, because the model in Gali and Monacelli[15] is very precise, to simplify it without loss of generality, we also refer to Gali and Monacelli[16], which succeeds in aggregating individual behavior in a more simplified way. The presentation of the model and the notation closely parallels the model in Gali[13].

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[4] Their purpose is to analyze the effectiveness of wage rigidity in a currency union.

[5] See the corresponding author's website https://www.econ.nagoya-cu.ac.jp/~eiji_okano/papers_e.html for details of the derivation.

3.1 Households

There is a representative household in a small open economy and the household has a continuum of members indexed by $j \in [0,1]$.

The household's utility function is given by:

$$\sum_{t=0}^{\infty} \beta^t \mathrm{E}_0 \left[\mathcal{U}\left(C_t, L_t, N_t; Z_t\right)\right], \tag{1}$$

where $C_t \equiv \frac{1}{(1-\nu)^{1-\nu}\nu^{\nu}} C_{H,t}^{1-\nu} C_{F,t}^{\nu}$ denotes a consumption index, $C_{H,t} \equiv \left[\int_0^1 C_{H,t}(j)^{\frac{\epsilon-1}{\epsilon}} dj\right]^{\frac{\epsilon}{\epsilon-1}}$ is an index of domestic goods consumption, $C_{F,t} \equiv \left[\int_0^1 C_{F,t}(j)^{\frac{\epsilon-1}{\epsilon}} dj\right]^{\frac{\epsilon}{\epsilon-1}}$ is the quantity consumed of a composite foreign good, $L_t \equiv \frac{M_t}{P_t}$ is the households' holding of real money balances, M_t is (non-interest-bearing) money, $P_t \equiv P_{H,t}^{1-\nu} P_{F,t}^{\nu}$ is consumer price index (CPI) in units of domestic currency, $P_{H,t} \equiv \left[\int_0^1 P_{H,t}(j)^{1-\epsilon} di\right]^{\frac{1}{1-\epsilon}}$ is the domestic price index and $P_{F,t} \equiv \left[\int_0^1 P_{F,t}(j)^{1-\epsilon} di\right]^{\frac{1}{1-\epsilon}}$ is the price of import goods in units of domestic currency, $\epsilon > 0$ is the elasticity of substitution between goods, $\nu \in [0,1]$ is a measure of openness, $N_t \equiv \int_0^1 N_t(j)\, dj$ is hours of labor and Z_t is an exogenous preference shifter, $\beta \equiv \frac{1}{1+\rho} \in (0,1)$ denotes the subjective discount factor, and ρ is the rate of time preference.

Period utility is given by:

$$\mathcal{U}\left(C_t, L_t, N_t; Z_t\right) \equiv \left[U\left(C_t, L_t\right) - V\left(N_t\right)\right] Z_t,$$

with $V(\cdot)$ increasing and convex and $U(\cdot)$ increasing and concave.

The optimal allocation of any given expenditure within each category of goods yields the demand functions:

$$C_{H,t}(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\epsilon} C_{H,t} \quad ; \quad C_{F,t}(j) = \left(\frac{P_{F,t}(j)}{P_{F,t}}\right)^{-\epsilon} C_{F,t}, \tag{2}$$

for all j. The optimal allocation of expenditures between domestic and foreign goods implies:

$$C_{H,t} = (1-\nu)\, \mathcal{S}_t^{\nu} C_t \quad ; \quad C_{F,t} = \nu \mathcal{S}_t^{-(1-\nu)} C_t, \tag{3}$$

where $\mathcal{S}_t \equiv \frac{P_{F,t}}{P_{H,t}}$ denotes the terms of trade (TOT) which is the price of foreign goods in terms of the price of domestic goods.

The sequence of budget constraints has the following form:

$$P_t C_t + B_t + \mathcal{E}_t B_t^* + M_t = B_{t-1}\left(1 + i_{t-1}\right) + \mathcal{E}_t B_{t-1}^* \left(1 + i_{t-1}^*\right) + M_{t-1} + W_t N_t - P_t TR_t + D_t,$$

where B_t denotes a nominal riskless one-period domestic government bond in units of domestic currency, B_t^* is the nominal riskless one-period foreign government bond in units of foreign currency, i_t is the domestic nominal interest rate, i_t^* is the foreign nominal interest rate, $\mathcal{E}_t$ is the nominal exchange rate, which is the price of foreign currency in units of domestic currency, W_t is the nominal wage, TR_t are lump-sum taxes, and D_t are the nominal dividends paid by firms.

Dividing both sides of the previous expression by the CPI P_t yields:

$$\begin{aligned} C_t + \mathcal{B}_t + \mathcal{Q}_t \mathcal{B}_t^* + L_t \quad &= \quad \Pi_t^{-1} \mathcal{B}_{t-1}\left(1 + i_{t-1}\right) + \left(\Pi_t^*\right)^{-1} \mathcal{Q}_t \mathcal{B}_{t-1}^* \left(1 + i_{t-1}^*\right) + \Pi_t^{-1} L_{t-1} + \frac{W_t}{P_t} N_t \\ &\quad - TR_t + \frac{D_t}{P_t}, \end{aligned} \tag{4}$$

where $\mathcal{B}_t \equiv \frac{B_t}{P_t}$ denotes real domestic government debt, $\mathcal{B}_t^* \equiv \frac{B_t^*}{P_t^*}$ is real foreign government debt, $\mathcal{Q}_t \equiv \frac{\mathcal{E}_t P_t^*}{P_t}$ is the real exchange rate, that is, the ratio of the CPIs expressed in domestic currency, $\Pi_t^* \equiv \frac{P_t^*}{P_{t-1}^*}$ denotes the (gross) foreign inflation, P_t^* is the foreign price index, and $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ is (gross) CPI inflation.

Under the assumption of complete international financial markets, the equilibrium price (in units of domestic currency) of a riskless bond denominated in foreign currency is given by $\mathcal{E}_t (1 + i_t^*)^{-1} = \mathrm{E}_t (Q_{t,t+1}\mathcal{E}_{t+1})$ where $\mathrm{E}_t (Q_{t,t+1})$ denotes the price of a one-period discount bond paying off one unit of domestic currency. The previous pricing equation can be combined with the domestic bond pricing equation, $(1 + i_t)^{-1} = \mathrm{E}_t (Q_{t,t+1})$ to obtain a version of the uncovered interest parity (UIP) condition:

$$\mathrm{E}_t \left\{ Q_{t,t+1} \left[(1+i_t) - (1+i_t^*) \left(\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \right) \right] \right\} = 0. \tag{5}$$

Let us define $\mathcal{A}_t \equiv \left[(1+i_{t-1}) \mathcal{B}_{t-1} + \mathcal{Q}_{t-1}\mathcal{B}_{t-1}^* \frac{\mathcal{E}_t}{\mathcal{E}_{t-1}} \left(1 + i_{t-1}^*\right) + L_{t-1} \right] \Pi_t^{-1}$ as the representative household's real financial wealth at the beginning of period t. Then, Eq. (4) can be rewritten as:

$$C_t + \frac{1}{1+i_t}\mathcal{A}_{t+1}\Pi_{t+1} + L_t \left(1 - \frac{1}{1+i_t}\right) = \mathcal{A}_t + \frac{W_t}{P_t}N_t - TR_t + \frac{D_t}{P_t}, \tag{6}$$

where we assume a standard solvency constraint $\lim_{k\to\infty}\Lambda_{t,t+k}\mathcal{A}_{t+k} \geq 0$ with $\Lambda_{t,t+k} \equiv \prod_{j=0}^{k-1} \mathcal{R}_{t+j}^{-1}$ being the domestic discount factor and $\mathcal{R}_t \equiv (1+i_t)\,\Pi_{t+1}^{-1}$, thus ruling out a Ponzi scheme.

Households maximize Eq. (1) subject to Eq. (6) and we have optimality conditions as follows:

$$U_{c,t} = \beta (1+i_t)\, \Pi_{t+1}^{-1} U_{c,t+1} \frac{Z_{t+1}}{Z_t}, \tag{7}$$

$$\frac{W_t}{P_t} = \frac{V_{n,t}}{U_{c,t}}, \tag{8}$$

$$h\left(\frac{L_t}{C_t}\right) = \frac{i_t}{1+i_t}, \tag{9}$$

with $h\left(\frac{L_t}{C_t}\right) \equiv \frac{U_{l,t}}{U_{c,t}}$. $h\left(\frac{L}{C}\right) \equiv \frac{U_l}{U_c}$ is a continuous decreasing function and satisfies $h(\bar{\chi}) = 0$ for some $0 < \bar{\chi} < \infty$, which guarantees that the demand for the real money balances is bounded as the interest rate approaches zero with a satiation point attained at $L = \bar{\chi}C$. Eqs. (7), (8), and (9) are the consumption Euler equation and the intertemporal optimality condition that determines the labor supply under the assumption of a competitive labor market and money demand schedule, respectively. These optimality conditions must be complemented with the transversality condition $\lim_{k\to\infty}\Lambda_{t,t+k}\mathcal{A}_{t+k} = 0$.

3.2 International Risk-sharing Condition

Given the assumption of complete financial markets, a condition analogous to Eq. (4) must also hold for the representative household in the foreign country. Combining that condition and Eq. (7) together with the UIP and the definition of the real exchange rate, we have an international risk-sharing condition as follows:

$$U_{c,t}^{-1} = \vartheta \left(U_{c,t}^*\right)^{-1} \mathcal{Q}_t \frac{Z_t}{Z_t^*}, \tag{10}$$

where $U_{c,t}^*$ denotes the counterpart of $U_{c,t}$ in the foreign country and Z_t^* denotes the foreign exogenous preference shifter and ϑ is a constant that depends on initial conditions.

We assume the law of one price (LOOP), that is, $P_{F,t}(j) = \mathcal{E}_t P_{F,t}^*(j)$ for all j where $P_{F,t}^*(j)$ denotes the price of foreign good j in units of foreign currency. Integrating over all goods, we obtain:

$$P_{F,t} = \mathcal{E}_t P_{F,t}^*, \tag{11}$$

where $P_{F,t}^*$ denotes the foreign currency price of foreign goods. Our treatment of the rest of the world as an (approximately) closed economy (with goods produced in the small open economy representing a negligible fraction of the world's consumption basket) implies that the foreign price index coincides with the foreign currency price of foreign goods, namely, $P_t^* = P_{F,t}^*$.

By plugging the definition of the CPI into that of the real exchange rate, we have:

$$\mathcal{Q}_t = \mathcal{S}_t^{1-\nu}, \tag{12}$$

which implies that the assumption of complete markets at the international level leads to a simple relationship linking consumption at home and abroad and the TOT. In fact, by plugging Eq. (12) into Eq. (10), we have:

$$U_{c,t}^{-1} = \vartheta \left(U_{c,t}^*\right)^{-1} \mathcal{S}_t^{1-\nu} \frac{Z_t}{Z_t^*}. \tag{13}$$

3.3 Domestic Producers

A typical domestic firm produces a differential good using the technology:

$$Y_t(j) = N_t(j)^{1-\alpha},$$

where $Y_t(j)$ is the output of generic good j and α denotes the index of decreasing returns to labor. Note that an index for aggregate domestic output is given by $Y_t \equiv \left[\int_0^1 Y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj\right]^{\frac{\epsilon}{\epsilon-1}}$. By integrating the previous expression, we have:

$$N_t^{1-\alpha} = Y_t \left[\int_0^1 \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\frac{\epsilon}{1-\alpha}} dj\right]^{1-\alpha}, \tag{14}$$

where $\int_0^1 \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\frac{\epsilon}{1-\alpha}} dj$ is price dispersion.

In each period, a subset of firms of measure $1-\theta$, with $\theta \in [0,1]$ being an index of price rigidities drawn randomly from the population, reoptimizes the price of their good, subject to a sequence of isoelastic demand schedules for the latter. The remaining θ firms keep their price unchanged. That is, firms are subject to Calvo pricing. Prices are set in domestic currency and are the same for both the domestic and export markets and the LOOP also applies to exports.

The first-order necessary condition (FONC) for domestic producers is given by:

$$\sum_{k=0}^{\infty} \theta^k \left[\Lambda_{t,t+k} \left(\frac{1}{P_{t+k}}\right) Y_{t+k|t} \left(\tilde{P}_{H,t} - \mathcal{M} MC_{t+k|t}^n\right)\right] = 0, \tag{15}$$

where $\mathcal{M} \equiv \frac{\epsilon}{\epsilon-1}$ denotes the constant (desired) price markup, $Y_{t+k|t} \equiv Y_t \left(\frac{\tilde{P}_{H,t}}{P_{H,t+k}}\right)^{-\epsilon}$ is output in period $t+k$ for a firm that last reset its price in period t, $\tilde{P}_{H,t}$ is the price set in period t by firms reoptimizing their price in that period, $MC_{t+k|t}^n$ is the nominal marginal cost in period $t+k$ for a firm that last reset its price in period t, and $MC_t^n \equiv W_t \left(\frac{N_t^\alpha}{1-\alpha}\right)$ is the nominal marginal cost.

3.4 Demand for Exports and Global Shocks

The demand for exports of domestic good j is given by:

$$EX_t(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\epsilon} EX_t, \tag{16}$$

where EX_t denotes an aggregate export index.

Following Gali and Monacelli[16], the aggregate export is assumed to be given by:

$$EX_t = \nu \mathcal{S}_t Y_t^*, \tag{17}$$

where Y_t^* denotes (per capita) world output.

3.5 Government

Like Gali[13], we assume the government (consisting of fiscal and monetary authorities acting in a coordinated way) finances its expenditures through lump-sum taxes and the issuance of a riskless nominal one-period bond with a nominal interest rate and (non-interest-bearing) money. Thus, the consolidated budget constraint is given by:

$$P_{H,t}G_t + B_{t-1}(1 + i_{t-1}) = P_t TR_t + B_t + \Delta M_t, \tag{18}$$

where Δ is the difference operator, and $G_t \equiv \left(\int_0^1 G_t(j)^{\frac{\epsilon-1}{\epsilon}} dj\right)^{\frac{\epsilon}{\epsilon-1}}$ denotes the (real) government expenditure index.

Like Eq. (2), the optimal allocation of government expenditure is given by:

$$G_t(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\epsilon} G_t. \tag{19}$$

We assume that government expenditure is fully allocated to domestically produced goods.

Dividing both sides of Eq. (18) by the CPI P_t yields:

$$\mathcal{S}_t^{-\nu} G_t + \mathcal{B}_{t-1}\mathcal{R}_{t-1} = TR_t + \mathcal{B}_t + \frac{\Delta M_t}{P_t}, \tag{20}$$

where $\frac{\Delta M_t}{P_t}$ represents the seignorage in period t, that is, the purchasing power of newly issued money. Similar to Gali[13], the analysis below focuses on equilibrium near a steady state with zero inflation, no trend growth, and constant government expenditure, taxes, and government debt. Constancy of real balances requires that $\Delta M = 0$ and hence zero seignorage in the steady state. Note that any variables without time scripts are the steady-state values of those variables.

3.6 The Market-clearing Condition

The market-clearing condition is given by:

$$Y_t(j) = C_{H,t}(j) + EX_t(j) + G_t(j).$$

Plugging Eqs. (2), (3), (16), (17), and (19) into the previous expression, we have:

$$Y_t = (1-\nu)\mathcal{S}_t^{\nu} C_t + \nu \mathcal{S}_t Y_t^* + G_t, \tag{21}$$

where we use the optimal allocation of the output $Y_t(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\epsilon} Y_t$. Note that we assume $Y_t^* = C_t^*$ where C_t^* denotes (per capita) world consumption.

3.7 Trade Balance

Similar to Gali and Monaceli[15], we define the real trade balance as follows:

$$\frac{NX_t}{P_{H,t}} \equiv Y_t - \mathcal{S}_t^{\nu} C_t - G_t, \tag{22}$$

with NX_t being the nominal trade balance.

4 The Steady State, Equilibrium Dynamics, and Fiscal and Monetary Policies

The analysis below considers equilibrium in the neighborhood of a steady state with zero inflation and zero government expenditure, that is, $\Pi = 1$ and $G = 0$, similar to Gali[13]. In addition, $Z = Z^* = 1$ is assumed.

4.1 The Steady State

Eqs. (5), (7), (8), and (9) imply in the steady state as follows:

$$\begin{aligned} i &= i^*, \\ i &= \rho, \\ (1-\alpha) U_c &= V_n N^{\alpha} \mathcal{M}, \\ h\left(\frac{L}{C}\right) &= \frac{\rho}{1+\rho}, \end{aligned}$$

where the former two equalities imply no changes in the nominal exchange rate in the steady state, while the latter two equalities are identical to the steady-state conditions in Gali[13]. In this steady state, world output equals world consumption, $Y^* = C^*$ and $C = C^* = Y$ are applicable. In addition, $\mathcal{S} = 1$ is applicable. That is, the TOT (and the real exchange rate) is pinned down uniquely and is unity in the perfect foresight steady state. We obtain the condition $\mathcal{S} = 1$ by applying the technique in Gali and Monacelli[14].[6] This feature of the steady state, which implies that purchasing power parity (PPP) $\mathcal{Q}_t = 1$ is applicable in the long run, is important for considering our equilibrium dynamics.

4.2 Equilibrium Dynamics

Letting $\hat{y}_t \equiv \log\left(\frac{Y_t}{Y}\right)$, $\hat{c}_t \equiv \log\left(\frac{C_t}{C}\right)$, $\zeta_t = -\log\left(\frac{Z_t^*}{Z_t}\right)$, $s_t \equiv \log\mathcal{S}_t$, $\hat{y}_t^* \equiv \log\left(\frac{Y_t^*}{Y^*}\right)$, $\hat{g}_t \equiv \frac{G_t}{Y}$, $\xi_t \equiv \log\left(\frac{U_{c,t}}{U_c}\right)$, $\pi_t \equiv \log\Pi_t$, $\hat{\rho}_t = -\log\left(\frac{Z_{t+1}}{Z_t}\right)$, $\hat{i}_t \equiv \log\left(\frac{1+i_t}{1+\rho}\right)$, $m_t \equiv \log M_t$, $\hat{l}_t \equiv \log\left(\frac{L_t}{L}\right)$, $\hat{b}_t \equiv \frac{\mathcal{B}_t - B}{Y}$, $\widehat{tr}_t \equiv \frac{TR_t - TR}{Y}$, $\widehat{nx}_t \equiv \log\left(\frac{\frac{NX_t}{P_{H,t}}}{\frac{NX}{P_H}}\right)$, $p_{H,t} \equiv \log P_{H,t}$, $p_{F,t} \equiv \log P_{F,t}$, $p_t^* \equiv \log P_t^*$ and $e_t \equiv \log\mathcal{E}_t$. Then, the equilibrium around the steady state can be approximated as follows (ignoring the zero lower bound (ZLB) constraint at this point):

$$\hat{c}_t = \hat{y}_t^* + \frac{1-\nu}{\sigma} s_t + \frac{1}{\sigma}\zeta_t, \tag{23}$$

[6] Additionally, to obtain this condition with certainty, we assume that the steady-state wedge between the marginal rate of substitution from consumption to leisure and the marginal product of labor is common throughout the world. See Benigno and Woodford[2] for details.

$$\hat{y}_t = \nu(2-\nu)s_t + (1-\nu)\hat{c}_t + \nu\hat{y}_t^* + \hat{g}_t, \tag{24}$$

$$\hat{\xi}_t = \hat{\xi}_{t+1} + \left(\hat{i}_t - \pi_{t+1} - \hat{\rho}_t\right), \tag{25}$$

$$\hat{\xi}_t = -\sigma\hat{c}_t + \upsilon\hat{l}_t, \tag{26}$$

$$\pi_{H,t} = \beta\pi_{H,t+1} - \kappa\hat{\mu}_t, \tag{27}$$

$$\hat{\mu}_t = \hat{\xi}_t - \frac{\alpha+\varphi}{1-\alpha}\hat{y}_t - \nu s_t, \tag{28}$$

$$\hat{l}_t = \hat{c}_t - \eta\hat{i}_t, \tag{29}$$

$$\hat{l}_{t-1} = \hat{l}_t + \pi_t - \Delta m_t, \tag{30}$$

$$\hat{b}_t = (1+\rho)\hat{b}_{t-1} + (1+\rho)b\hat{i}_{t-1} - (1+\rho)b\pi_t + \hat{g}_t - \widehat{tr}_t - \chi\Delta m_t, \tag{31}$$

$$\pi_t = \pi_{H,t} + \nu(s_t - s_{t-1}), \tag{32}$$

$$\widehat{nx}_t = \hat{y}_t - \nu s_t - \hat{c}_t - \hat{g}_t, \tag{33}$$

$$s_t = e_t + p_t^* - p_{H,t}, \tag{34}$$

$$\pi_{H,t} = p_{H,t} - p_{H,t-1}, \tag{35}$$

$$\pi_{F,t} = p_{F,t} - p_{F,t-1}, \tag{36}$$

$$\pi_{F,t} = s_t - s_{t-1} + \pi_{H,t}, \tag{37}$$

with $\kappa \equiv \frac{(1-\theta\beta)(1-\theta)\Theta}{\theta}$, $\Theta \equiv \frac{1-\alpha}{(1-\alpha)+\alpha\epsilon}$, $\mu \equiv \log\mathcal{M}$, $\varphi \equiv \frac{V_{nn}N}{V_n}$, $\upsilon \equiv \frac{U_{cl}L}{U_c}$, $\sigma \equiv -\frac{U_{cc}C}{U_c}$ where $\eta \equiv \frac{\epsilon_{lc}}{\rho}$ with $\epsilon_{lc} \equiv \frac{1}{\sigma_l+\upsilon}$ and $\sigma_l \equiv \frac{U_{ll}L}{U_l}$ denotes the elasticity of substitution between consumption and real balances, $\chi \equiv \frac{L}{Y}$ is the inverse income velocity of money, $b \equiv \frac{\mathcal{B}}{Y}$ denotes the steady-state share of government debt to output, $\pi_{H,t} \equiv \log\Pi_{H,t}$ with $\Pi_{H,t} \equiv \frac{P_{H,t}}{P_{H,t-1}}$ denotes (gross) domestic goods inflation, $\pi_{F,t} \equiv \log\Pi_{F,t}$ with $\Pi_{F,t} \equiv \frac{P_{F,t}}{P_{F,t-1}}$ is (gross) import goods inflation and $\hat{\mu}_t \equiv \mu_t - \mu$ with $\mu_t \equiv -\log MC_t$ being the logarithmic average markup, $MC_t \equiv \frac{MC_t^n}{P_{H,t}}$ being the real marginal cost, is the price markup gap. Note that we assume $Z_t^* = 1$ and $Z_{t+1} = Z_t^{\varrho}$ with $\varrho = 0$. Thus, $\hat{\rho}_t = \log Z_t = \zeta_t$.

Eqs. (23) to (25), (27), (29), (31), and (33) are derived by log-linearizing Eqs. (13), (21), (7), (15), (9), (20), and (22), respectively. Eqs. (26), (28), (30), and (32) are derived by the log-linearized marginal utility of consumption, a combination of the log-linearized Eq. (8) and the definition of the marginal utility, the logarithmic first differential of the definition of the real money balance, and a combination of the logarithmic first differential of the definition of the CPI and the TOT. Eqs. (34) to (37) are derived by log-linearizing the definitions of the TOT, the domestic goods inflation, the import goods inflation, and combining the (logarithmic) definition of the TOT with the (logarithmic) definition of domestic and import goods inflation. Although Eqs. (34) to (37) do not have an essential role in deciding the dynamic paths, they are necessary to calculate the nominal exchange rate and the import goods inflation. Note that a logarithmic definition of the LOOP $p_{F,t} = e_t + p_{F,t}^*$ is used to derive Eq.(34). Plugging this into Eq.(34), Eq.(34) becomes $s_t = p_{F,t} - p_{H,t}$, which is the logarithmic definition of the TOT.

4.3 Fiscal and Monetary Policies

4.3.1 Government Budget Constraint and Financing Regime

As in Gali[13], we assume a simple tax rule throughout the analysis as follows:

$$\widehat{tr}_t = \psi_b\hat{b}_{t-1} + \hat{\varsigma}_t, \tag{38}$$

which shows that tax variations have two components. One is $\psi_b \hat{b}_{t-1}$, which is endogenous and varies in response to deviations in the debt ratio from its long-run target where ψ_b is a tax adjustment parameter. The other is $\hat{\varsigma}_t$, which is independent of the debt ratio and should be interpreted as the exogenous component of the tax rule.

Plugging Eq. (38) into Eq. (31) yields:

$$\hat{b}_t = (1 + \rho - \psi_b)\, \hat{b}_{t-1} + (1 + \rho)\, b\hat{i}_{t-1} - (1 + \rho)\, b\pi_t + \hat{g}_t - \hat{\varsigma}_t - \chi \Delta m_t, \tag{39}$$

which replaces Eq. (31) in the equilibrium. Note that $\psi_b > \rho$ guarantees that $\lim_{k\to\infty} \mathrm{E}_t\,(b_{t+k}) = 0$, that is, the debt ratio converges to its long-run target. Accordingly, the government's transversality condition $\lim_{k\to\infty} \Lambda_{t,t+k} \mathcal{B}_{t+k} = 0$ will be satisfied for any price level path, as long as the discount factor $\Lambda_{t,t+k}$ converges to zero as $k \to \infty$, which is the case in all the experiments considered below. The previous property, often referred to in the literature as Ricardian (or passive) fiscal policy (e.g., Leeper[19]), is assumed in standard specifications of the New Keynesian model, and must be combined with an active monetary policy (as implicitly assumed below) to guarantee a local unique equilibrium.

4.3.2 Experiments

Below we analyze two stylized fiscal interventions, using the basic New Keynesian model with a small open economy setting as a reference framework. The first intervention consists of an exogenous tax cut while the second takes the form of an exogenous increase in government expenditure. Both interventions are announced in period zero and implemented from that period onward, similar to Gali[13]. For concreteness, in the case of a tax cut we assume as follows:

$$\hat{\varsigma}_t = -\delta^t < 0,$$

for $t = 0, 1, 2, ...$ where $\delta \in [0, 1)$ measures the persistence of the exogenous fiscal stimulus. Symmetrically, in the case of an increase in the government expenditure, we assume as follows:

$$\hat{g}_t = \delta^t > 0,$$

for $t = 0, 1, 2, ...$ Note that in both cases the size of the stimulus is normalized to correspond to 1% of steady-state output in period zero. The effects of each type of fiscal intervention are analyzed under two alternative regimes, which jointly describe how the fiscal stimulus is financed and how monetary policy is conducted. The first scheme, which we refer to as a money-financed (*MF*) scheme is the main focus of the present paper, similar to Gali[13]. We define this regime as one in which seignorage is adjusted every period to keep real debt $\mathcal{B}_t$ unchanged. Plugging $\hat{b}_t = 0$ into Eq. (39), we have:

$$\Delta m_t = \frac{1}{\chi} \left[\hat{g}_t - \hat{\varsigma}_t + (1 + \rho)\, b \left(\hat{i}_{t-1} - \pi_t \right) \right], \tag{40}$$

for $t = 0, 1, 2, ...$ Note that the previous assumptions, combined with Eq. (38), imply that under the *MF* regime taxes need not be adjusted as a result of an increase in government expenditure, either in the short or long run, relative to their initial level. Alternatively, in the case of a tax cut, taxes are temporarily lowered by an amount δ^t. In other words, under the *MF* regime, neither taxes nor debt need to be raised in response to the fiscal interventions considered here. In both cases, however, monetary policy must give up control of the nominal interest rate, instead adjusting the money supply to meet the government's financing needs.

Under the second financing scheme considered, which we refer to as a debt-financed (DF) scheme, the fiscal authority issues debt to finance the fiscal stimulus, eventually adjusting the path of taxes to attain the long-run debt target $\mathcal{B}$, as implied by the tax rule Eq. (38). The monetary authority for its part, is assumed to pursue an independent price stability mandate. For concreteness, we assume that if feasible, it conducts policy so that:

$$\pi_t = 0, \tag{41}$$

for all t. The money supply and as a result seignorage then adjust endogenously to bring about the interest rate required to stabilize prices, as well as the regime generally assumed in the New Keynesian literature on the effects of fiscal policy.

4.4 Calibration

Our parameterization is consistent with Gali[13] except for parameters that are peculiar to an open economy, namely, the openness ν, or that are not clarified in Gali[13], namely, the relative risk aversion σ (Table 2). Both parameters are set following Monacelli[20]. Our setting on the subjective discount factor implies a steady state (annualized) real interest rate of about 2% and the index of decreasing returns to labor corresponds to the average labor income share across Greece, Italy, Portugal, and Spain from 1999 to 2014 (see Gali and Monacelli[16]). For the remaining parameters, the elasticity between goods corresponds to a 12.5% steady-state markup, the velocity is the inverse of the middle of the average (quarterly) M0 income velocity in the US and the Euro area (see Gali[13]), the interest semielasticity of money demand follows Ireland[17] (see Ireland[17] and Gali[13]), the target debt ratio is consistent with the 60% reference value specified in EU agreements (see Gali[13] and Ferrero[11]), and the openness roughly corresponds to the ratio of imports to GDP in Canada.

Both our implied assumptions of perfect substitution between domestic and import goods and our benchmark parameterization on relative risk aversion attain balanced trade, that is, $\widehat{nx}_t = 0$ for all t if the demand shock $\hat{\rho}_t$ does not hit the economy.

5 The Effects of Fiscal Stimulus in Normal Times

We first show the effects of fiscal stimulus in normal times and how the assumption of a small open economy changes the responses from those in the closed economy assumed by Gali[13]. Figures 1 and 2 illustrate the responses under a MF fiscal stimulus to a tax cut and to an increase in government expenditure, respectively, while Figures 3 and 4 depict the responses under a DF fiscal stimulus to a tax cut and an increase in government expenditure, respectively. The red line with circles and the blue line with diamonds are responses in a small open economy in which ν is set to 0.4, that is, our benchmark, and in a closed economy in which openness ν is set to zero, respectively. Note that we only plot the quarterly rate-based responses that differ from those in Gali[13] to conserve space.

5.1 The *MF* Fiscal Stimulus

5.1.1 To a Tax Cut

Output increases after a tax cut in a small open economy, as in a closed economy (Panel 1, Figure 1). However, in contrast to a closed economy, the increase in consumption and the decrease in the real consumption interest rate, defined as $\hat{r}_t \equiv \hat{i}_t - \pi_{t+1}$, are limited (Panels 3 and 4, Figure 1). In addition, CPI inflation rises more than twice as much as it does in a closed economy, the nominal interest rate increases, and money growth declines. In a closed economy, the nominal interest rate decreases and money growth increases (Panels 5, 8, and 10, Figure 1). Under the *MF* fiscal stimulus, the tax cut is financed by an increase in money growth, even though money growth is reduced, which is contrary to intuition.

To understand the reason money growth is reduced, we focus on the (logarithmic) government budget constraint Eq. (31) or Eq. (39), which imply that an increase in CPI inflation mitigates the burden of redeeming government debt through a decrease in the real consumption interest rate. In other words, an increase in CPI inflation brings revenue of the so-called inflation tax to the government. Then, we define $\widehat{sp}_t \equiv \widehat{tr}_t - \hat{g}_t + (1+\rho)\, b\pi_t$ as the fiscal surplus with the inflation tax and Eq. (31) can be rewritten as follows:

$$\widehat{sp}_t \quad = \quad -\left[\hat{b}_t - (1+\rho)\,\hat{b}_{t-1} - (1+\rho)\, b\hat{i}_{t-1}\right] - \chi\Delta m_t,$$

which implies that an increase in CPI inflation applies pressure to reduce newly issued government debt $\hat{b}_t - \left[(1+\rho)\,\hat{b}_{t-1} + (1+\rho)\, b\hat{i}_{t-1}\right]$ and money growth through an increase in the fiscal surplus with the inflation tax. Under the *MF* fiscal stimulus, the balance of the real government debt is unchanged. Thus, an increase in CPI inflation provides an incentive to reduce the money growth because of the increase in the fiscal surplus with the inflation tax (Panel 15, Figure 1). This phenomenon that a decrease in money growth accompanies the increase in CPI inflation appears even in a liquidity trap and causes the increase in the nominal interest rate while the adverse demand shock is hitting the small open economy. As shown in the real money balance schedule Eq. (29), a decrease in the real money balance accompanies the increase in the nominal interest rate (Panel 8, Figure 1). Because of the increase in the nominal interest rate, the decrease in the real consumption interest rate is limited and the increase in consumption is not so vigorous, unlike consumption in a closed economy (Panels 3 and 4, Figure 1).

Now, we consider the reason why CPI inflation rises so much in a small open economy. Throughout our analysis, the UIP Eq. (5) applies. By log-linearizing Eq. (5), we have:

$$\hat{i}_t = \hat{i}^*_t + (e_{t+1} - e_t)\,, \tag{42}$$

with $\hat{i}^*_t \equiv \log\left(\frac{1+i^*_t}{1+i^*}\right)$. Combining Eq. (42) with Eq. (34) yields:

$$s_t = \left(\hat{i}^*_t - \pi^*_{t+1}\right) - \left(\hat{i}_t - \pi_{H,t+1}\right) + s_{t+1}, \tag{43}$$

with $\pi^*_t \equiv \log\Pi^*_t$. The TOT (and the real exchange rate) is pinned down uniquely and is unity in the perfect foresight steady state. This implies that PPP is applicable in the long run; i.e., $\lim_{k\to\infty} s_{t+k} = 0$. Hence, we can solve Eq. (43) as follows:

$$s_t = \sum_{k=0}^{\infty} \left(\hat{i}^*_{t+k} - \pi^*_{t+k+1}\right) - \left(\hat{i}_{t+k} - \pi_{H,t+k+1}\right), \tag{44}$$

which shows that the TOT is a function of the difference between the current and anticipated foreign real interest rate and the small open economy's real domestic interest rate, which is defined as $\hat{r}_{H,t} \equiv \hat{\imath}_t - \pi_{H,t+1}$. A tax cut applies pressure to increase CPI inflation through the government budget constraint Eq. (31) or Eq. (39) and this increases domestic goods inflation and the TOT through Eq. (32) (Panel 13, Figure 1). This increase (i.e., worsening) in the TOT causes a further increase in domestic goods inflation through a decrease in the average markup (i.e., an increase in the marginal cost) as shown in Eqs. (27) and (28) (Panel 7, Figure 1). While the nominal interest rate rises, this increase in domestic goods inflation makes the real domestic interest rate negative (Panel 14, Figure 1). An increase (i.e., worsening) in the TOT coincides with an increase in import goods inflation and this increase is higher than an increase in domestic goods inflation (Panels 7 and 11, Figure 1). As a result, CPI inflation rises more strongly than in a closed economy, as noted. That is, a large increase in import goods inflation causes a large increase in CPI inflation. In fact, the (logarithmic) definition of CPI inflation is given by $\pi_t = (1 - \nu)\,\pi_{H,t} + \nu\pi_{F,t}$ and this implies that CPI inflation increases with import goods inflation.

The large increase in import goods inflation is caused by a large increase (i.e., depreciation) in the nominal exchange rate. In a small open economy, there is repercussion between domestic goods inflation and the nominal exchange rate. A successive increase in domestic goods inflation then increases (i.e., depreciates) the nominal exchange rate through an increase in the CPI (level). Although the PPP is not applicable in every period in a small open economy, the PPP is applicable in the long run in our model similar to a small open economy model in Gali and Monacelli[15]. Thus, this increase in the CPI (level) shifts the long-run nominal exchange rate upward (i.e., the long-run nominal exchange rate depreciates) and the current nominal exchange rate increases (i.e., depreciates). The TOT increases (i.e., worsens) because the import goods price only depends on the nominal exchange rate.

Further, this increase (i.e., worsening) in the TOT affects output. An increase (i.e., worsening) in the TOT increases output through the market-clearing condition again and increases the domestic inflation again through a decrease in the average markup (i.e., an increase in the marginal cost). Further changes result and that causes a large increase in CPI inflation. This 'positive' repercussion between domestic goods inflation and the nominal exchange rate works well, even in a liquidity trap under *MF* fiscal stimulus, to stabilize both output and the CPI inflation.

As shown, unlike in a closed economy, the increase in consumption in the small open economy is smaller and does not seem to contribute much to boosting output. Instead, it is the increase (i.e., worsening) in the TOT that contributes to increasing output. While the demand for foreign goods weakens due to the increase (i.e., worsening) in the TOT stemming from an increase in the import goods price, the demand for domestic goods increases and the production of domestic goods then contributes to increasing output. That is, the expenditure-switching effect contributes to increasing output. Thus, cumulative output is not lower rather higher than in a closed economy and so *MF* fiscal stimulus is more effective in a small open economy than in a closed economy (Panel 2, Figure 1).

Given the increase (i.e., worsening) in the TOT stemming from the increase in the import goods price, the production of domestic goods becomes vigorous while households must pay more for foreign goods. Under our parameterization, namely $\sigma = 1$ together with the assumption of perfect substitution between foreign and domestic goods, the increase in the production of domestic goods is canceled out by the increase in the purchase amount of foreign goods. Balanced trade is then attained. Note that balanced trade is applicable throughout our analysis if the demand shock does not affect the small open economy.

5.1.2 To an Increase in Government Expenditure

Next, we discuss the dynamic responses to an increase in government expenditure, which are similar to a tax cut under a *MF* fiscal stimulus. Output rises as in a closed economy, but with the increase in consumption being smaller than in a closed economy and the increase in CPI inflation being higher than in a closed economy (Panels 1, 3, and 5, Figure 2). The increase in CPI inflation is accompanied by an increase (i.e., depreciation) in the nominal exchange rate due to the 'positive' repercussion between domestic goods inflation and the nominal exchange rate, and import goods inflation increases (Panels 11 and 12, Figure 2). This increase in import goods inflation coincides with an increase (i.e., worsening) in the TOT (Panel 13, Figure 2). This increase (i.e., worsening) in the TOT increases output and cumulative output is then higher than in a closed economy (Panel 2, Figure 2). Thus, in terms of the effect on cumulative output to a tax cut under *MF* fiscal stimulus, *MF* fiscal stimulus is more effective in a small open economy than in a closed economy, regardless if the means of stimulus is a tax cut or an increase in government expenditure.

It is noteworthy that money growth is reduced in period zero, similar to the tax cut under the *MF* scheme in a small open economy (Panel 10, Figure 2). The large increase in CPI inflation finances the increase in government expenditure through an increase in the fiscal surplus with the inflation tax, rather than money growth (Panel 15, Figure 2). As discussed, this phenomenon appears even in a liquidity trap. The decrease in money growth causes a larger increase in the nominal interest rate and this hampers the decrease in the real consumption interest rate, unlike the experience in a closed economy (Panels 4 and 8, Figure 2).

5.2 The *DF* Fiscal Stimulus

5.2.1 To a Tax Cut

Similar to a closed economy, a tax cut under the *DF* scheme has no effect on any variables except for fiscal variables such as the real government debt, the fiscal surplus with inflation tax, and the taxes (Figure 3). As Gali[13] notes, this neutrality result is well known and a consequence of Ricardian equivalence given any assumption of lump-sum taxes and Ricardian fiscal policy. Any short-run tax reduction is matched by future tax increases, leaving their present discounted value unchanged and the households' intertemporal budget constraint unaffected. As no other equilibrium condition is affected by the tax cut and the increase in government debt, all variables other than the fiscal variables remain unchanged in response to a tax cut under the *DF* scheme.

5.2.2 To an Increase in Government Expenditure

We first review the responses to an increase in government expenditure under the *DF* scheme in the closed economy assumed by Gali[13]. In a closed economy, output increases (Panel 1, Figure 4). This increase in output decreases the average markup (i.e., increases the marginal cost) through Eq. (28) (the TOT is not involved given the closed economy; i.e., $\nu = 0$). This decrease in average markup applies pressure to increase CPI inflation and decrease future CPI inflation through Eq. (27) (note that there is no distinction between CPI inflation and domestic goods inflation in a closed economy because of zero openness; i.e., $\nu = 0$). However, the policy authority must retain

CPI inflation targeting Eq. (41) under the *DF* fiscal stimulus such that the authority attempts to increase the marginal utility of consumption. To increase the marginal utility of consumption, a decrease in consumption is required, as shown in Eq. (26). As shown in Eqs. (29) and (30), the decrease in money growth decreases consumption through a decrease in the real money balance. Thus, the nominal interest rate is hiked to reduce money growth (Panels 8 and 10, Figure 4). The real consumption interest rate increases and consumption decreases, that is, a crowding-out of consumption takes place (Panels 3, and 4, Figure 4).

The responses in a small open economy are quite different from those in a closed economy. There is still crowding-out in consumption in response to the increase in government expenditure, but the decrease in the consumption in the small open economy is smaller than in the closed economy (Panel 3, Figure 4). Given this smaller decrease in consumption, the increase in output in the small open economy is about 5.88 times larger than in a closed economy in period zero and cumulative output is higher than in a closed economy (Panels 1 and 2, Figure 4). In a small open economy, we must be aware that CPI inflation and domestic goods inflation differ and that changes in the TOT as well as the nominal exchange rate are involved in any responses. While domestic goods inflation is stable in the closed economy, because it is identical to CPI inflation in a closed economy, domestic goods inflation increases in the small open economy, and import goods inflation decreases to cancel out any increase in domestic inflation (Panels 5 and 11, Figure 4). As a result, CPI inflation is completely stabilized, even in a small open economy.

In a closed economy, with an increase in government expenditure the average markup (i.e., the marginal cost) must be stabilized to stabilize domestic goods inflation which is identical to CPI inflation. To stabilize the average markup, the pressure to increase output in response to an increase in government expenditure is canceled by the substantial decrease in consumption. Thus, the nominal interest rate is hiked and money growth falls. However, in a small open economy, an increase in domestic goods inflation is allowed and this increase decreases (i.e., improves) the TOT with little increase in the nominal interest rate, as shown in Eq. (44) (Panels 8 and 13, Figure 4). This decrease (i.e., an improvement) in the TOT coincides with a decline in import goods inflation (Panel 11, Figure 4) and this serves to completely stabilize CPI inflation. Glaring changes in the nominal interest rate as well as money growth are not necessary in a small open economy, unlike in a closed economy (Panels 8 and 10, Figure 4). Thus, the crowding-out of consumption is smaller in a closed economy and so output increases more.

Together with less crowded-out consumption, there is another notable feature of a small open economy in its responses to the increase in government expenditure under the *DF* scheme. In a small open economy, there is an increase in both government debt and taxes but less than in a closed economy. Further, while money growth decreases in a closed economy, the response of money growth in a small open economy is small. Thus, increases in government debt and taxes are suppressed in a small open economy, unlike the experience in a closed economy (Panels 9 and 17, Figure 4).

Gali[13] highlights the effectiveness of *MF* fiscal stimulus, especially a tax cut under the *MF* scheme, to boost output and argues that the increase in government expenditure under the *DF* scheme is strongly subdued in comparison with the increase in government expenditure under the *MF* fiscal stimulus. However, in a small open economy, given the smaller crowding-out in consumption, the increase in government expenditure under the *DF* scheme is not meaningless; rather, it is effective as it can boost output sufficiently. *MF* fiscal stimulus is then not necessarily essential for boosting output in normal times in a small open economy.

5.3 Sensitivity Analysis

We now discuss the sensitivity of some of the above qualitative findings regarding the effectiveness of fiscal policies. The focus is the parameter measuring the degree of openness ν, which is peculiar to a small open economy and does not appear in a closed economy instead of focusing on the degree of price stickiness θ and the persistence of the shock δ which are focused upon by Gali[13]. Focusing on openness is important to understand how the assumption of a small open economy affects the effectiveness.

Following Gali[13], we define the cumulative output multiplier $(1-\delta)\sum_{t=0}^{\infty}\hat{y}_t$. Figure 5 depicts the cumulative output multipliers for a tax cut and an increase in government expenditure as a function of openness ν. The multipliers are on the vertical axis and the level of openness on the horizontal axis. The red line with circles and the blue line with diamonds are the multipliers under the *MF* and the *DF* fiscal stimulus, respectively. Panel 1 depicts the response of the fiscal multipliers to a tax cut while Panel 2 plots these for an increase in government expenditure. On the left in each figure, openness is zero; that is, the multipliers shown on the extreme left correspond to those in a closed economy, that is, $\nu = 0$, as assumed by Gali[13].

5.3.1 Fiscal Multipliers and a Tax Cut

We first discuss the multipliers for a tax cut as a function of openness ν. The multipliers are zero regardless of the changes in ν under the *DF* scheme (Panel 1, Figure 5). Ricardian equivalence holds and there are no fiscal effects in the first instance.

In contrast, under the *MF* scheme, the multipliers strongly increase with a tax cut as ν increases (Panel 1, Figure 5). As discussed in Section 5.1.1, there is a 'positive' repercussion between domestic inflation and the nominal exchange rate. A tax cut (as well as an increase in government expenditure) increases domestic goods inflation through a decrease in the average markup (i.e., an increase in the marginal cost) and CPI inflation increases. The increase in CPI inflation implies that the CPI (level) shifts upward in the long run. Because PPP holds in the long run, the nominal exchange rate in the long run increases (i.e., depreciates). This increase (i.e., depreciation) leads to an increase in import goods inflation as well as the increase in the import goods price. As the definition of the (logarithmic) CPI (level) $p_t = (1-\nu)\,p_{H,t} + \nu p_{F,t}$ suggests, as openness increases, the share of import goods price increases and the CPI (level) increases. That is, the greater the openness, the higher the CPI (level) and the higher (more depreciating) the nominal exchange rate because of PPP in the long run. Because an increase (i.e., depreciation) in the nominal exchange rate causes an increase (i.e., worsening) in the TOT, the TOT increases (i.e., worsens) with openness. In other words, the 'positive' repercussion between the domestic inflation and the nominal exchange rate becomes stronger, as openness increases.

This increase in the TOT accompanies a (relative) increase in domestic output. Combining Eqs. (23) and (24) yields:

$$s_t = (\hat{y}_t - \hat{y}_t^*) - \hat{g}_t - (1-\nu)\,\zeta_t, \tag{45}$$

where $\sigma = 1$. Eq. (45) shows that a (relative) increase in domestic output accompanies an increase (i.e., worsening) in the TOT due to the expenditure-switching effect together with the effect of an increase in consumption through the risk-sharing transfer of resources (see Eq. (10)). Thus, the more open the economy, the higher its output because of the higher TOT. The multiplier continues to increase with openness ν.

5.3.2 Fiscal Multipliers for an Increase in Government Expenditure

Similar to the multipliers for a tax cut under the *MF* scheme, the multipliers for an increase in government expenditure under the *MF* scheme increase with openness ν (Panel 2, Figure 5). The reason is that the TOT increases (i.e., worsens) as openness ν increases, as described in Section 5.3.1. An increase in government expenditure instead of a tax cut then ignites the 'positive' repercussion, which becomes stronger with openness.

Finally, we discuss the relationship between the multipliers and openness regarding an increase in government expenditure under the *DF* scheme in which the CPI inflation targeting Eq. (41) that is, $\pi_t = 0$, is applicable. To offset the pressure to increase CPI inflation stemming from an increase in domestic goods inflation brought about by an increase in government expenditure, import goods inflation must decrease through a decrease (i.e., an appreciation) in the nominal exchange rate. As openness increases, the share of domestic goods inflation in CPI inflation decreases and the pressure to increase CPI inflation brought about by an increase in the domestic goods inflation is mitigated. Thus, as openness increases, the pressure to decrease import goods inflation must be mitigated. That is, a decrease (i.e., an improvement) in the TOT must be mitigated as openness increases. As shown in Eq. (44), the TOT relates negatively to current and anticipated nominal interest rates. Thus, an increase in the nominal interest rate, which is identical to the real consumption interest rate under a *DF* fiscal stimulus, is reduced as openness increases, such that the decrease in consumption from openness is mitigated. In other words, the crowding-out of consumption is alleviated as openness increases. As a result, the multipliers increase with openness (Panel 2, Figure 5). This is consistent with the result in Section 5.2.2 in which we noted that the *MF* fiscal stimulus is not necessarily essential for boosting output in normal times in a small open economy. In a small open economy, the increase in government expenditure under *DF* is effective in increasing output as long as openness is sufficiently high. Under our benchmark parameterization, openness ν is set to 0.4 and the corresponding fiscal multiplier is 0.44, which is about 3.50 times higher than that in a closed economy (i.e., $\nu = 0$) and only 30.0% smaller than under *MF* fiscal stimulus with a tax cut in a closed economy.

However, in a liquidity trap, the increase in government expenditure under *DF* scheme is no longer effective in stabilizing output and CPI inflation. This result for the increase in government expenditure under *DF* fiscal stimulus is opposite to that derived by Gali[13]. Our results for an increase in government expenditure under *DF* fiscal stimulus in a liquidity trap in a small open economy are detailed in Section 6.3.2.

6 The Effects of Fiscal Stimulus in a Liquidity Trap

This section explores the effectiveness of *MF* fiscal stimulus in stabilizing the economy in the face of a temporary adverse demand shock, similar to Gali[13]. We assume the latter to be large enough to prevent the central bank fully stabilizing output and CPI inflation given the ZLB constraint on the nominal interest rate. We compare the *MF* fiscal stimulus with two alternatives, the *DF* fiscal stimulus and *no response* in which there is no fiscal stimulus such as a tax cut or an increase in government expenditure to the adverse demand shock.

Similar to Gali[13], the ZLB constraint takes the form $\hat{i}_t \geq \log\beta$ and the experiment assumes that $\hat{\rho}_t = -\gamma < \log\beta$ for $t = 0, 1, 2, ...T$ and $\hat{\rho}_t = 0$ for $t = T+1, T+2, ...$ In other words, this describes a temporary adverse demand shock that brings the natural interest rate into negative territory up to period T. After period T, the shock vanishes, and the natural interest rate returns to its initial level. The shock is assumed to be fully unanticipated, but once realized, the trajectory of $\{\hat{\rho}_t\}$ and the corresponding policy responses are known with certainty.

The ZLB constraint can be incorporated formally in the set of equilibrium conditions above by replacing Eq. (29) with a complementary slackness condition:

$$\left(\hat{i}_t - \log\beta\right)\left(\hat{l}_t - \hat{c}_t + \eta\hat{i}_t\right) = 0, \tag{46}$$

for all t, where

$$\hat{i}_t \geq \log\beta, \tag{47}$$

is the ZLB constraint and

$$\hat{l}_t \geq \hat{c}_t - \eta\hat{i}_t, \tag{48}$$

represents the demand for real money balances.

In addition to the previous changes, under the *DF* fiscal stimulus, as well as the *no response* benchmark, Eq. (41) must be replaced with

$$\left(\hat{i}_t - \log\beta\right)\pi_t = 0, \tag{49}$$

for all t, together with

$$\pi_t = 0, \tag{50}$$

which is the zero-inflation target and is applicable for the period when the ZLB constraint on the nominal interest rate is unavailable. By contrast, in the case of the *MF* fiscal stimulus, Eq. (40) determines the money supply for all t. If the nominal interest rate is positive, Eq. (48) holds with equality (but with inequality once the nominal interest rate reaches the ZLB and the real money balances overshoot their satiation level).

Further, similar to Gali[13], we assume $\gamma = -0.01$ and $T = 5$. Thus, and given $\beta = 0.995$, the experiment considered corresponds to an unanticipated fall in the natural interest rate to – 2% (in annual terms) for six quarters and a subsequent revision back to the initial value of 2% (in annual terms).

Figures 6 to 10 depict the responses in the case of *no response*, to a tax cut under the *MF* scheme, to an increase in government expenditure under the *MF* scheme, to a tax cut under the *DF* scheme, and to an increase in government expenditure under the *DF* scheme, respectively. In all figures, the blue line with diamonds shows the responses in a closed economy, that is, $\nu = 0$, while the red line with circles shows the responses in a small open economy, that is, $\nu = 0.4$, being our benchmark parameterization. In the case of *no response* to the shock, that is, $\hat{g}_t = \hat{\varsigma} = 0$, for $t = 1, 2, 3\ldots$, and with monetary policy described by Eqs. (49) and (50), which is a familiar scenario that is taken by Gali[13] as a benchmark. The scenario for the tax cut is that there is a 1% tax cut lasting for the duration of the adverse shock ($\hat{\varsigma}_t = -0.01$, for $t = 0, 1, \ldots, 5$) under the *MF* and the *DF* fiscal stimulus, respectively, similar to Gali[13]. The scenario for the increase in government expenditure is that there is a 1% increase in the steady-state ratio to output in response to the adverse demand shock and that this lasts for the duration of the adverse shock ($\hat{g}_t = 0.01$, for $t = 0, 1, \ldots, 5$) under the *MF* and the *DF* fiscal stimulus, respectively, again similar to Gali[13].

6.1 *No Response*

We start by considering the case of *no response* to the adverse demand shock as the benchmark. When the adverse demand shock hits a small open economy, output decreases and the decrease in output is larger than in a closed economy (Panel 1, Figure 6). The adverse demand shock then decreases domestic goods inflation through the Euler equation Eq. (25) and the FONC for domestic firms Eq. (27) (Panel 7, Figure 6). The fall in domestic goods inflation applies pressure on the nominal exchange rate to decrease (i.e., appreciate) because of PPP in the long run (Panel 12, Figure 6). This decrease (i.e., appreciation) in the nominal exchange rate decreases import goods inflation (Panel 11, Figure 6), and this results in a decrease (i.e., improvement) in the TOT (Panel 13, Figure 8). This serves to decrease domestic goods inflation again because it also decreases consumption and output and increases the average markup (i.e., decreases the marginal cost). We refer to this decrease in domestic goods inflation and appreciation in the nominal exchange rate as a 'negative' repercussion, in contrast to the 'positive' repercussion between the increase in domestic goods inflation and depreciation in the nominal exchange rate shown in Section 5. Thus, the pressure to decrease domestic goods inflation in a small open economy is more severe in a small open economy than in a closed economy and the resulting fall in CPI inflation is correspondingly more severe (Panel 5, Figure 6).

This severe decrease in CPI inflation amplifies the burden of redeeming government debt; in other words, it makes the revenue from the inflation tax negative. Thus, government debt is higher than in a closed economy (Panel 9, Figure 6). Given our simple tax rule, Eq. (38), the larger balance of real government debt brings about higher tax revenue in a small open economy (Panel 17, Figure 6).

While there has been a decrease in CPI inflation, import goods inflation has no nominal rigidity and the pace of recovery in CPI inflation to its steady-state value in a small open economy is fast. This rapid recovery in CPI inflation as well as the higher tax revenue completes fiscal reconstruction in period four (Panel 15, Figure 6). Then, seignorage is no longer necessary. The nominal interest rate is hiked and money growth decreases (Panels 8 and 10, Figure 6). However, this increase in the nominal interest rate overshoots and occurs in period five when the adverse demand shock is still affecting the small open economy. This response of the nominal interest rate is reminiscent of the lifting of the zero-interest-rate policy in August 2000 in Japan. This move was based on the outlook of many members of the Bank of Japan policy board that deflationary concerns were dispelled, even though the lifting was viewed as a premature decision by at least one government official. This increase, more precisely, overshooting, in the nominal interest rate results in an increase in the real consumption interest rate and so the recovery in consumption is weaker than what it would be in a closed economy (Panels 3 and 4, Figure 6). As a result, there is a delay in the recovery of output such that the decrease in (cumulative) output in a small open economy is 1.77 times larger than what it would be in a closed economy (Panels 1 and 2, Figure 6). That is, the presence of a nominal exchange rate affecting CPI inflation amplifies the effect brought about by the adverse demand shock. This exemplifies the problems of no fiscal response in a small open economy during a liquidity trap.

6.2 The *MF* Fiscal Stimulus

6.2.1 To a Tax Cut

A tax cut under the *MF* scheme in a small open economy stabilizes output and CPI inflation and is at least as effective as that in a closed economy (Panels 1, 2, and 5, Figure 7). However, the decrease in CPI inflation is larger than in a closed economy because of the 'negative' repercussion between domestic goods inflation and the nominal exchange rate. However, this larger decrease in CPI inflation provides for an increase in money growth that is larger than in a closed economy because the larger drop in CPI inflation provides an incentive to finance the budgetary deficit (Panels 5 and 10, Figure 7). Indeed, in a small open economy, the fiscal surplus with an inflation tax is –4.49% in period zero, but only –3.03% in a closed economy. This higher money growth mitigates the decline in consumption as well as that for domestic goods inflation. The 'negative' repercussion between domestic goods inflation and the nominal exchange rate is mitigated and the decrease (i.e., appreciation) in the nominal exchange rate is smaller than in the case of *no response* (Panel 12, Figures 6 and 7). Because the nominal exchange rate positively links with import goods inflation, the decrease in import goods inflation is moderated in comparison with the case of *no response* (Panel 11, Figures 6 and 7). Thus, recovery in CPI inflation in a small open economy is faster than in a closed economy and this depresses the real consumption interest rate such that it is lower than that in a closed economy until period four (Panel 4, Figure 7).

This faster recovery in CPI inflation also provides for a more rapid recovery in the fiscal surplus with the inflation tax than in a closed economy (Panels 5, 6, and 15, Figure 7). As discussed in Section 6.1, a rapid recovery in CPI inflation is problematic because it completes fiscal reconstruction and provides an incentive for the government to reduce money growth. However, under the *MF* fiscal stimulus, the balance of real government debt is unchanged and obtaining seignorage is an important way to finance a tax cut. In addition, the increase in government expenditure and money growth is not reduced in period five, even if the fiscal surplus with inflation tax is larger than in a closed economy (Panels 10 and 15, Figure 7). Thus, the nominal interest rate neither starts to increase before the adverse demand shock nor overshoots it in period five and the real consumption interest rate is still negative in period five, even though it is positive in the case of *no response* (Panels 4 and 8, Figures 6 and 7).

Although the recovery in consumption in a small open economy is not necessarily more vigorous than in a closed economy (Panel 3, Figure 7), the recovery in output in a small open economy is more vigorous because of the response of the TOT (Panels 1 and 13, Figure 7). As noted, the decrease in import goods inflation is eased with a tax cut compared with the case of *no response*. This smaller decrease in import goods inflation reduces the decrease (i.e., improvement) in the TOT compared with the case of *no response* (Panel 13, Figures 6 and 7). As shown in Eq. (45), output increases with the TOT via the expenditure-switching effect and the smaller decrease (i.e., improvement) in the TOT contributes to suppressing the decrease in output. Thus, *MF* fiscal stimulus is either as effective or more effective in a small open economy as in a closed economy.

In addition, this result—that the effect of monetary easing under the ZLB in a small open economy is stronger than that in a closed economy—is consistent with the result in section 5.3.1 that the higher the openness, the higher the fiscal multipliers in the *MF* scheme. Mitigated decrease in the import goods inflation contributes to mitigating a decrease (i.e., improvement) in the TOT and to making recovery in the output stronger.

6.2.2 To an Increase in Government Expenditure

An increase in government expenditure is more effective in stabilizing output than in a closed economy, similar to a tax cut in a small open economy (Panels 1 and 2, Figure 8). Although the decrease in CPI inflation is more severe in a small open economy, money growth is hiked more than being in a closed economy, similar to a tax cut under the *MF* scheme in a small open economy (Panels 5 and 10, Figure 8). Like a tax cut, recovery in CPI inflation in a small open economy is faster than in a closed economy. Thus, a fall in the consumption real interest rate is larger than in a closed economy, like a tax cut (Panel 4, Figure 8).

This more rapid recovery in CPI inflation makes recovery in the fiscal surplus with inflation faster than in a closed economy, like a tax cut (Panels 5, 6, and 15, Figure 8). As discussed in Section 6.2.1, under the *MF* fiscal stimulus, obtaining seignorage is the most important way to finance an increase in government expenditure and money growth is not reduced in period five, even if the fiscal surplus with inflation tax is larger than in a closed economy (Panels 10 and 15, Figure 8). Thus, the nominal interest rate neither starts to increase before the adverse demand shock nor overshoots it in period five and the real consumption interest rate remains negative in period five, similar to a tax cut in a small open economy under the *MF* scheme.

6.3 The *DF* Fiscal Stimulus

6.3.1 To a Tax Cut

We do not compare the responses in a closed economy and a small open economy with those in the case of *no responses* here. However, the responses to a tax cut under the *DF* scheme are identical to those in the case of *no response* except for taxes, real government debt, and the fiscal surplus with inflation tax (i.e., the blue line with diamonds and red line with circles in Figure 6 is identical to those in Figure 9). Ricardian equivalence is attained and there are no effects on any variables except for these variables.

6.3.2 To an Increase in Government Expenditure

Unlike the *MF* fiscal stimulus, an increase in government expenditure under the *DF* scheme in a small open economy is not effective for stabilizing output and CPI inflation when compared with a closed economy (Panels 1, 2 and 5, Figure 10). Gali[13] suggests that the increase in government expenditure under the *DF* scheme has a strong impact on output and CPI inflation and that their responses are quite similar to those for an increase in government expenditure under the *MF* scheme. Thus, our finding concerning an increase in government expenditure under the *DF* scheme is quite different from that derived by Gali[13].

To confirm how an increase in government expenditure is effective in comparison with *no response*, Figure 11 plots the dynamic responses to an increase in government expenditure under the *MF* scheme, those under the *DF* scheme, and those in the case of *no response* in a small open economy with openness ν set to 0.4. In Figure 11, the red line with circles is the response under the *MF* scheme, the blue line with diamonds is the response under the *DF* scheme, and the black line with crosses is the response in the case of *no response*. (These three lines are identical to the red lines with circles in Figures 8, 10, and 6, respectively.) If the blue line with diamonds is closer to the red line with circles, we can say that an increase in government expenditure under the *DF* scheme is effective, even in a small open economy, and that Gali[13]'s finding concerning an increase in government expenditure in a closed economy is also applicable in a small open economy. However, we find that the blue line with diamonds in each panel is closer to the black line with crosses in each panel (Figure 11). This suggests that the additional effects brought about by the increase in government expenditure under the *DF* scheme are quite small and that an increase in government expenditure under the *DF* scheme is not effective in a small open economy. Clearly, our finding differs from that in Gali[13].

The reason why the increase in government expenditure under the *DF* scheme is no longer effective is the severe decrease (i.e., appreciation) in the nominal exchange rate (Panel 12, Figures 10 and 11). There is the 'negative' repercussion between domestic inflation and the nominal exchange rate which causes a severe decrease in CPI inflation. This severe decrease in CPI inflation amplifies the burden of redeeming government debt; in other words, it makes the inflation tax revenue negative. Thus, the balance of real government debt is higher than in a closed economy (Panel 9, Figure 10). This larger amount of real government debt brings with it more taxes because of our simple tax rule Eq. (38), as it implies that tax revenue increases with government debt. While the recovery in CPI inflation in a small open economy is not necessarily faster than in a closed economy and there is a successive increase in government expenditure, the higher tax revenue completes fiscal reconstruction more quickly (Panels 5, 6, and 15, Figure 10). This quick fiscal reconstruction is even observed in the case of *no response* and is problematic, as mentioned in Section 6.1 (Panel 15, Figure 11). Then, the seignorage is no longer necessary. The nominal interest rate is hiked, and it overshoots, money growth decreases, and the real consumption interest rate increases (Panels 4, 8, and 10, Figure 10). Thus, there is a delay in the recovery of consumption.

The severe decrease (i.e., appreciation) in the nominal exchange rate also causes a decrease (i.e., improvement) in the TOT (Panel 13, Figure 10). Because the import goods price (level) only depends on the nominal exchange rate, the import goods price (level) severely decreases. Given this severe decrease in the import goods price (level) as well as import goods inflation, the TOT decreases (i.e., improves) severely. This severe decrease (i.e., improvement) in the TOT then decreases output through the expenditure-switching effect despite the increase in government expenditure applying pressure to increase output. Thus, an increase in government expenditure under the *DF* scheme is no longer effective in a small open economy.

6.4 Comparing the Effects of the *MF* Fiscal Stimulus with the Case of *No Response* in a Liquidity Trap

Figure 12 compares the *MF* fiscal stimulus with the case of *no response* in a small open economy (i.e., $\nu = 0.4$). In Figure 12, the red line with circles, the blue line with diamonds, and the black line with crosses are the responses to an increase in government expenditure under the *MF* scheme, a tax cut under the *MF* fiscal stimulus, and the case of *no response*, respectively, to the adverse demand shock. (These three lines are identical to the red lines with circles in Figures 8, 7, and 6, respectively.)

To each fiscal stimulus, a decrease in CPI inflation and a decrease (i.e., improvement) in the TOT is mitigated in comparison to the case of *no response* and this result implies that the 'negative' repercussion is mitigated by a 'positive' repercussion brought about by the *MF* fiscal stimulus (Panels 5 and 13, Figure 12). In fact, a decrease (i.e., appreciation) in the nominal exchange rate is surprisingly mitigated (Panel 12, Figure 12). Thus, the effectiveness of stabilizing both output and CPI inflation under the *MF* fiscal stimulus is obvious. A tax cut and an increase in government expenditure reduce the decrease in cumulative output by 94.3% and 87.0%, respectively, compared with the case of *no response* (Panel 2, Figure 12). Similarly, for an increase in government expenditure, the decrease in CPI inflation in period zero is 86.7% smaller than in the case of *no response* (Panel 5, Figure 12). For a tax cut, the decrease in CPI inflation in period zero is 85.3% smaller than in the case of *no response*.

As noted, *MF* fiscal stimulus in a small open economy is more effective than in a closed economy. In addition, the *MF* fiscal stimulus is certainly more effective in stabilizing both output and CPI inflation compared with the case of *no response* in a small open economy. It is also essential in a small open economy because the *DF* fiscal stimulus (especially as an increase in government expenditure under the *DF* scheme viewed as effective in Gali[13]) is no longer effective in our model, as discussed in Section 6.3.2.

6.5 Additional Analysis: The Effects of an Increase in Government Expenditure in a Liquidity Trap with Imperfect Pass-through

Monacelli[20] argues that there is a limitation on assuming complete pass-through because of two well-established empirical facts, namely, overwhelming failure of the LOOP for tradables and more rapid exchange rate pass-through on wholesale import prices. Thus, in this section, we introduce foreign retailers with pricing-to-market behavior and generate an incomplete pass-through environment and calculate only the responses to an increase in government expenditure for the sake of brevity. This allows us to consider whether an increase in government expenditure under the *DF* scheme viewed as effective in a liquidity trap in Gali[13] is effective in an imperfect pass-through environment.

6.5.1 The Model and the Equilibrium Dynamics in an Imperfect Pass-through Environment

We modify our model to generate an imperfect pass-through environment (see Appendix A for details). To generate an imperfect pass-through environment, we must pay attention to the fact that the LOOP is no longer available. Let $\Psi_t \equiv \frac{\mathcal{E}_t P_t^*}{P_{F,t}}$ denote the LOOP gap. Under the perfect pass-through environment assumed in the previous analysis, $\Psi_t = 1$ for all t is applied because of $\mathcal{E}_t P_{F,t}^* = P_{F,t}$ for all t. However, under imperfect pass-through, $\Psi_t = 1$ as well as $\mathcal{E}_t P_{F,t}^* = P_{F,t}$ for all t is not applicable (Note that the definition of the TOT holds independently of the degree of pass-through; that is, $\mathcal{S}_t \equiv \frac{P_{F,t}}{P_{H,t}}$ is still applicable).

Similar to Monacelli[20], we introduce retailers selling import goods in a monopolistic competitive market in a small open economy following Calvo pricing, similar to domestic producers in a small open economy. While Monacelli[20] assumes those retailers are domestic retailers, that is, they are residents in a small open economy, we assume that they are foreigners and thus they maximize their profits in units of foreign currency and are subsidized by the foreign government. If we were to adopt local retailers similar to Monacelli[20], they could not be subsidized by the domestic government because we do not assume domestic government subsidization, similar to Gali[13], and the steady-state conditions in the imperfect pass-through environment would become more complicated because of monopolistic competitive power hampering the LOOP in the steady state, that is, $\Psi = 1$. (The purpose of that subsidy is to cancel monopolistic competitive power.)

Thus, we assume foreign retailers subsidized by the foreign government. Owing to this assumption, not only $\Psi = 1$ but even $\mathcal{S} = 1$ is applicable and PPP applies in the long run (see Appendix B for details).[7]

Eqs. (23), (24), (34), and (37) are no longer available as the equilibrium dynamics in an imperfect pass-through environment in which the LOOP (i.e., $\Psi_t = 1$ for all t) is not applicable. Eqs. (23), (24), (37), and (34) are replaced by:

$$\hat{c}_t = \hat{y}_t^* + \frac{1}{\sigma}\psi_t + \frac{1-\nu}{\sigma}s_t + \frac{1}{\sigma}\zeta_t, \tag{51}$$

$$\hat{y}_t = \nu(2-\nu)s_t + (1-\nu)\hat{c}_t + \nu\psi_t + \nu\hat{y}_t^* + \hat{g}_t, \tag{52}$$

$$\pi_{F,t} = \beta\pi_{F,t+1} + \kappa_F\psi_t - \frac{1-\theta_F}{\theta_F}(e_{t+1} - e_t), \tag{53}$$

$$s_t = e_t + p_t^* - \psi_t - p_{H,t}, \tag{54}$$

respectively, with $\kappa_F \equiv \frac{(1-\theta_F)(1-\theta_F\beta)}{\theta_F}$ and $\psi_t \equiv \log\Psi_t$ where θ_F denotes the Calvo index of price rigidities for import goods. In addition,

$$e_t = \psi_t + p_{F,t} - p_t^*, \tag{55}$$

is necessary to calculate the (logarithmic) LOOP gap. Therefore, the equilibrium dynamics in an imperfect pass-through environment consist of Eqs. (25) to (33), (35), (36), and (51) to (55). Note that plugging Eq.(55) into Eq.(54) yields $s_t = p_{F,t} - p_{H,t}$, which is the logarithmic definition of the TOT.

Note that Eq. (53) is derived by combining the Calvo pricing rule and

$$\tilde{p}_{F,t} - e_t = (1-\theta_F\beta)\sum_{k=0}^{\infty}(\theta_F\beta)^k\left[\psi_{t+k} + (p_{F,t+k} - e_{t+k})\right],$$

which is the log-linearized FONC for foreign retailers. In the previous expression on the right-hand side, there are terms for the LOOP gap, the import goods price (level), and the nominal exchange rate (which are logarithmic). Because the numerator of the LOOP gap is the (nominal) buying price, which is identical to the foreign price, while the denominator is the (nominal) selling price, which is identical to the import goods price (level) in units of domestic currency, the LOOP gap corresponds to something like the real marginal cost. However, foreign retailers are foreigners whose profit is evaluated in units of foreign currency. Thus, something like the real marginal cost must be nominalized by multiplying the import goods price (level) in units of domestic currency and dividing by the nominal exchange rate, which is the price of one unit of the foreign currency in units of domestic currency. That is, the (logarithmic) variables within a square bracket are (logarithmically) something like the nominal marginal cost in units of foreign currency. Thus, the right-hand side is the discounted sum of (logarithmically) something like the nominal marginal cost considered nominal rigidity. The left-hand side of the previous expression is the (logarithmic) price set in period t by foreign retailers reoptimizing their price in that period $\tilde{p}_{F,t}$ in units of foreign currency. Thus, the previous expression merely shows that foreign retailers choose the optimal price that corresponds to the discounted sum of the marginal cost considered nominal rigidity and the previous expression is not strange. Their profit is evaluated in units of foreign currency so that the term for the changes in the nominal exchange rate appears on the right-hand side in Eq. (53).

[7] All the features in the perfect foresight steady state in a perfect pass-through environment in Section 4.1 appear in the perfect foresight steady state in the imperfect pass-through environment, as discussed in this section. Additionally, in the perfect foresight steady state in an imperfect pass-through environment, there is a condition $\Psi = 1$, which is derived by the FONC for foreign retailers subsidized by the foreign government.

6.5.2 The Effects of an Increase in Government Expenditure in a Liquidity Trap with Imperfect Pass-through

Figure 13 illustrates the effects of an increase in government expenditure in the imperfect pass-through environment in a liquidity trap. The red line with circles is the response under the *MF* scheme, the blue line with diamonds is the response under the *DF* scheme, the black line with crosses is the response in the case of *no response* in an imperfect pass-through environment, and the magenta line with pluses is the response in the case of *no response* in the perfect pass-through environment, which corresponds to the red line with circles in Figure 6. Note that we set $\theta_F = \frac{3}{4}$, similar to the Calvo index of price rigidities for domestic goods θ. The other parameters are set following Table 2.

In the case of *no response* with imperfect pass-through, in period zero when the adverse shock starts to affect the small open economy, CPI inflation decreases similarly to the case of *no response* in a perfect pass-through environment. However, the decrease in CPI inflation in the case of *no response* in period zero in an imperfect pass-through environment is obviously small (Panel 5, Figure 13). The decrease in CPI inflation in period zero in the case of *no response* in the imperfect pass-through environment is 77.8% smaller than in the perfect pass-through environment. The reason the decrease in CPI inflation is so much smaller is obvious. In an imperfect pass-through environment, the 'negative' repercussion between domestic goods inflation and the nominal exchange rate is mitigated because import goods inflation has nominal rigidity. Indeed, the decrease in import goods inflation in period zero in the case of *no response* is 88.4% smaller than in the perfect pass-through environment and this smaller decrease in import goods inflation contributes to mitigating the decrease in CPI inflation (Panel 11, Figure 13). As repeatedly noted, PPP applies in the long run and the decrease in CPI inflation causes a decrease in money growth as long as *MF* fiscal stimulus is not conducted (except for the *MF* scheme, there is no vigorous incentive to increase money growth). However, in an imperfect pass-through environment, the decrease in money growth is limited, even in the case of *no response* (Panel 10, Figure 13). Thus, the decrease in output in the case of *no response* in an imperfect pass-through environment is limited (Panel 1, Figure 13).

Interestingly, the TOT increases (i.e., worsens) after the adverse demand shock hits the small open economy with an imperfect pass-through environment in the case of *no response*, unlike in the case of *no response* in a perfect pass-through environment as discussed in Section 6.1. This is because import goods inflation has nominal rigidity in an imperfect pass-through environment, which prevents a further decrease (i.e., improvement) in the TOT (Panel 13, Figure 13). This increase (i.e., worsening) in the TOT following the adverse demand shock in a small open economy is consistent with the result that the decrease in output is mitigated with imperfect pass-through in the case of *no response* because the increase (i.e., worsening) of the TOT increases output through the expenditure-switching effect.

Even in an imperfect pass-through environment, the *MF* fiscal stimulus is effective in stabilizing both output and CPI inflation (Panels 1, 2, and 5, Figure 13). In period zero when the adverse shock starts to affect the small open economy, money growth is hiked and this contributes to stabilizing both output and CPI inflation (Panel 10, Figure 13). However, the *DF* fiscal stimulus is not so effective. The blue line with diamonds, which shows the response of output and CPI inflation to an increase in government expenditure under the *DF* scheme, is close to the black line with crosses, which shows the response of output and CPI inflation in the case of *no response*

(Panels 1, 2, and 5, Figure 13). Thus, an increase in government expenditure under the *DF* scheme is not effective, even with imperfect pass-through.

Intuitively, if import goods inflation has nominal rigidity like domestic goods inflation, the response of CPI inflation is close to that in a closed economy. In a closed economy, the decrease in CPI inflation in period zero is less and this mitigates the decrease in money growth and output (Panels 5 and 10, Figure 9). This follows Gali[13]'s policy implication that an increase in government expenditure under the *DF* scheme is available, even in a small open economy, as long as there is imperfect pass-through. However, our result differs from Gali[13]'s policy implication. The increase in government expenditure directly increases output and this decreases the average markup (i.e., increases the marginal cost) and domestic goods inflation increases, while the increase in output does not change the import goods inflation directly. Thus, the TOT decreases (i.e., improves), differently from the case of *no response* in the imperfect pass-through environment (Panel 13, Figure 13). This decrease (i.e., improvement) in the TOT applies pressure to reduce output through the expenditure-switching effect. Thus, Gali[13]'s policy implication is not applicable, even in an imperfect pass-through environment. The existence of the TOT, which is peculiar to an open economy, interferes with the materialization of Gali's policy implication in a closed economy.

7 Conclusion

We analyze the effects of *MF* fiscal stimulus compared with those resulting from conventional *DF* fiscal stimulus with and without the ZLB on the nominal interest rate in a small open economy. In normal times when the ZLB is not applicable, the *MF* fiscal stimulus is effective in increasing output, similar to a closed economy. With a liquidity trap, and in the case of *no response*, the decrease in both CPI inflation and output is more severe than in a closed economy. Even with a liquidity trap, *MF* fiscal stimulus is effective in stabilizing both output and CPI inflation, similar to a closed economy. We provide additional analysis relating to imperfect pass-through in a liquidity trap. In the case of *no response*, the decrease in both output and CPI inflation with *MF* fiscal stimulus is mitigated and an increase in government expenditure under the *MF* scheme can stabilize both.

We provide policy implications concerning an increase in government expenditure under *DF* fiscal stimulus that differ or are opposite to those derived by Gali[13]. In normal times, we find an increase in government expenditure is more effective in a small open economy than in a closed economy whereas Gali[13] argues that the effectiveness of that policy is strongly subdued. In a liquidity trap, the increase in government expenditure under the *DF* scheme is less effective, although its effectiveness is highlighted by Gali[13] assuming a closed economy. Even with an imperfect pass-through environment, an increase in government expenditure under the *DF* scheme is not effective.

Regarding our results, it can be said that, in normal times in a small open economy, *MF* fiscal stimulus is not necessarily essential for boosting output. An increase in government expenditure under the *DF* scheme, which is a conventional form of fiscal stimulus, is still effective in normal times in a small open economy. However, *MF* fiscal stimulus is more important than what Gali[13] suggests in a small open economy in a liquidity trap because the absence of a fiscal response is more threatening, especially with perfect pass-through and an increase in government expenditure under *DF* fiscal stimulus is no longer effective, irrespective of nominal exchange rate pass-through, that is, pricing behavior.

Appendix

A Modification of the Model to Generate Imperfect Pass-through

As Eq. (11) is not necessarily available in an imperfect pass-through environment, Eq. (12) is replaced by:

$$\mathcal{Q}_t = \Psi_t \mathcal{S}_t^{1-\nu}. \tag{A.1}$$

Eq. (13) is replaced as follows:

$$U_{c,t}^{-1} = \vartheta \left(U_{c,t}^*\right)^{-1} \Psi_t \mathcal{S}_t^{1-\nu} \frac{Z_t}{Z_t^*}, \tag{A.2}$$

which is obtained by plugging Eq. (A.1) into Eq. (10). By log-linearizing Eq. (A.2), we have Eq. (51).

Foreign retailers face a maximization problem as follows:

$$\max_{\tilde{P}_{F,t}} \sum_{k=0}^{\infty} \theta_F^k \left\{ \Lambda_{t,t+k}^* \left(\frac{1}{P_{t+k}^*} \right) \left[\frac{\tilde{P}_{F,t}}{\mathcal{E}_t} - P_{F,t+k}^* \left(1 - \tau_F\right) \right] C_{F,t+k|t} \right\},$$

s.t.

$$C_{F,t+k|t} \equiv \left(\frac{\tilde{P}_{F,t}}{P_{F,t+k}} \right)^{-\epsilon} C_{F,t+k},$$

with $\tilde{P}_{F,t}$ being the price set in period t by foreign retailers reoptimizing their price in that period, $\Lambda_{t,t+k}^*$ being the foreign discount factor where τ_F denotes the export subsidiary rate. The export subsidiary is paid by the foreign government. The role of this subsidiary is analogous to the role of the employment subsidiary in Gali and Monacelli[15].

The FONC for foreign retailers is given by:

$$\sum_{k=0}^{\infty} \theta_F^k \left\{ \Lambda_{t,t+k}^* \left(\frac{1}{P_{t+k}^*} \right) \left[\frac{\tilde{P}_{F,t}}{\mathcal{E}_t} - \mathcal{M} \left(1 - \tau_F\right) P_{F,t+k}^* \right] C_{F,t+k|t} \right\} = 0. \tag{A.3}$$

By log-linearizing Eq. (A.3), we have the log-linearized FONC for foreign retailers in Section 6.5.1.

Under imperfect pass-through, Eq. (21) is not available and is replaced by the following expression:

$$EX_t = \nu \mathcal{S}_t \Psi_t Y_t^*. \tag{A.4}$$

Eq. (21) is replaced by:

$$Y_t = \left(1 - \nu\right) \mathcal{S}_t^{\nu} C_t + \nu \mathcal{S}_t \Psi_t Y_t^* + G_t, \tag{A.5}$$

because Eq. (A.4) replaces Eq. (17). Log-linearization of Eq. (A.5) yields Eq. (52).

B The Steady State in an Imperfect Pass-through Environment

The FONC for foreign retailers Eq. (A.3) suggests as follows:

$$\Psi = [\mathcal{M}(1-\tau_F)]^{-1},$$

which implies that as long as $\tau_F = \frac{1}{\epsilon}$ is chosen:

$$\Psi = 1, \tag{B.1}$$

is applicable. Eq. (B.1) implies that the LOOP is applicable in the steady state. If there is no subsidiary, that is, $\tau_F = 0$, Eq. (B.1) is replaced by $\Psi = \mathcal{M}^{-1} < 1$ implying that monopolistic competitive power remains in the steady state and the LOOP is no longer applicable, even in the steady state.

Eq. (B.1) is essential to form the steady-state relationship $C = C^*$. In fact, Eq. (16) implies in the steady state:

$$U_c^{-1} = \vartheta\,(U_c^*)^{-1}\,\Psi.$$

The previous expression implies that the steady-state relationship $C = C^*$ is no longer applicable if $\Psi = 1$ is not applicable. To attain $C = C^*$ in the steady state with imperfect pass-through, the assumption $\tau_F = \frac{1}{\epsilon}$ is essential. Thus, we adopt foreign retailers instead of local retailers in Monacelli[20].[8] Owing to assumption $\tau_F = \frac{1}{\epsilon}$, all of the steady state conditions shown in Section 4.1 are inherited in the steady state in an imperfect pass-through environment as well as $\Psi = 1$. Note that our strategy is not novel, having been already developed by Gali and Monacelli[14].

In addition, $\mathcal{S} = 1$ remains applicable. That is, the TOT (along with the real exchange rate) is pinned down uniquely and is unity in the perfect foresight steady state, even with imperfect pass-through.

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Table 1: Share of GDP to GWP and Overnight Interest Rate

Country	Share of GDP to GWP (%, 2019)	Overnight Interest Rate (%)				
		2020				2021
		Q1	Q2	Q3	Q4	Q1
Australia	1.59	1.21	–0.05	0.12	–0.175	0
Canada	1.99	0.7406	0.2327	0.2181	0.1741	0.1529
Chile	0.32	1.64	0.49	0.38	0.3	0.3
Colombia	0.37	4.25	2.73	2	1.76	1.7
Costa Rica	0.07	2.09	1.0633	0.7167	0.78	0.7267
Czech Republic	0.29	0.24	0.11	0.04	0.03	0.03
Eurozone	15.30	0.02	0.02	0.02	0.02	0.01
Hungary	0.19	0.83	0.62	0.41	0.55	0.65
Iceland	0.03	1.925	0.675	0.675	0.475	0.475
Israel	0.45	0.25	0.1	0.1	0.1	0.1
Japan	5.89	–0.068	–0.055	–0.051	–0.027	–0.018
South Korea	1.89	1.16	0.64	0.49	0.5	0.49
Mexico	1.45	6.5	5	4.25	4.25	4
New Zealand	0.24	0.65	0	0.2	0	0
Norway	0.46	2	1	1	1	1
Poland	0.68	1.26	0.05	0.02	0.02	0.02
Sweden	0.61	0.081	–0.036	–0.058	–0.058	–0.054
Switzerland	0.84	–0.91	-0.71	–0.71	–0.71	–0.57
Turkey	0.87	10.5086	7.4305	11.1654	15.17	17.4249
United Kingdom	3.24	0.2135	0.0636	0.0546	0.0487	0.0494
United States	24.54	0.656	0.143	0.14	0.138	0.023
Source	World Economic Outlook Database April, 2021	DataStream				

Notes: Member countries of the OECD except for member countries of the European Monetary Union (EMU) and Denmark, and a total of 19 member countries of the EMU.

Figure 1: Dynamic Response to a Tax Cut under the *MF* Scheme

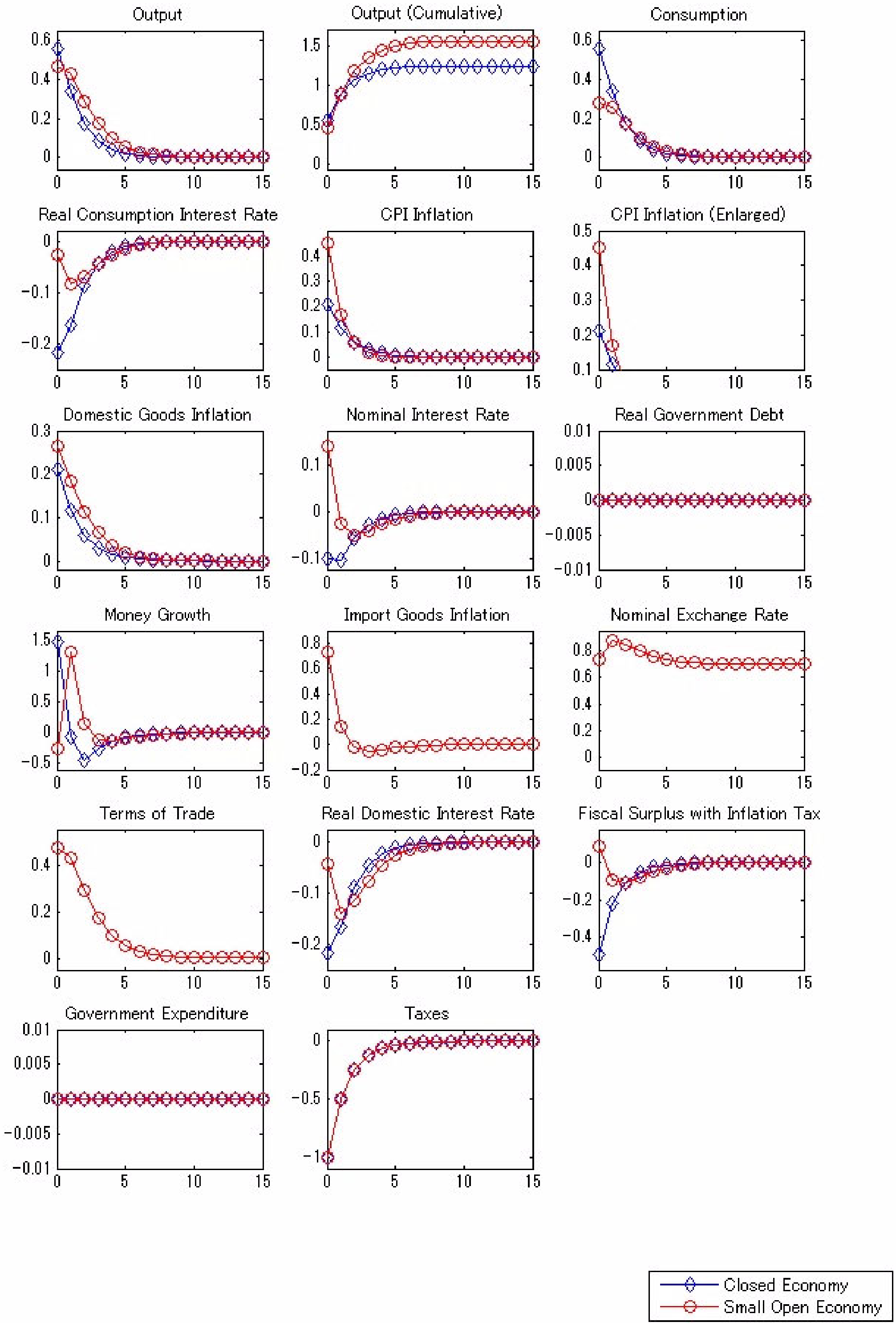


Figure 2: Dynamic Response to an Increase in Government Expenditure under the *MF* Scheme

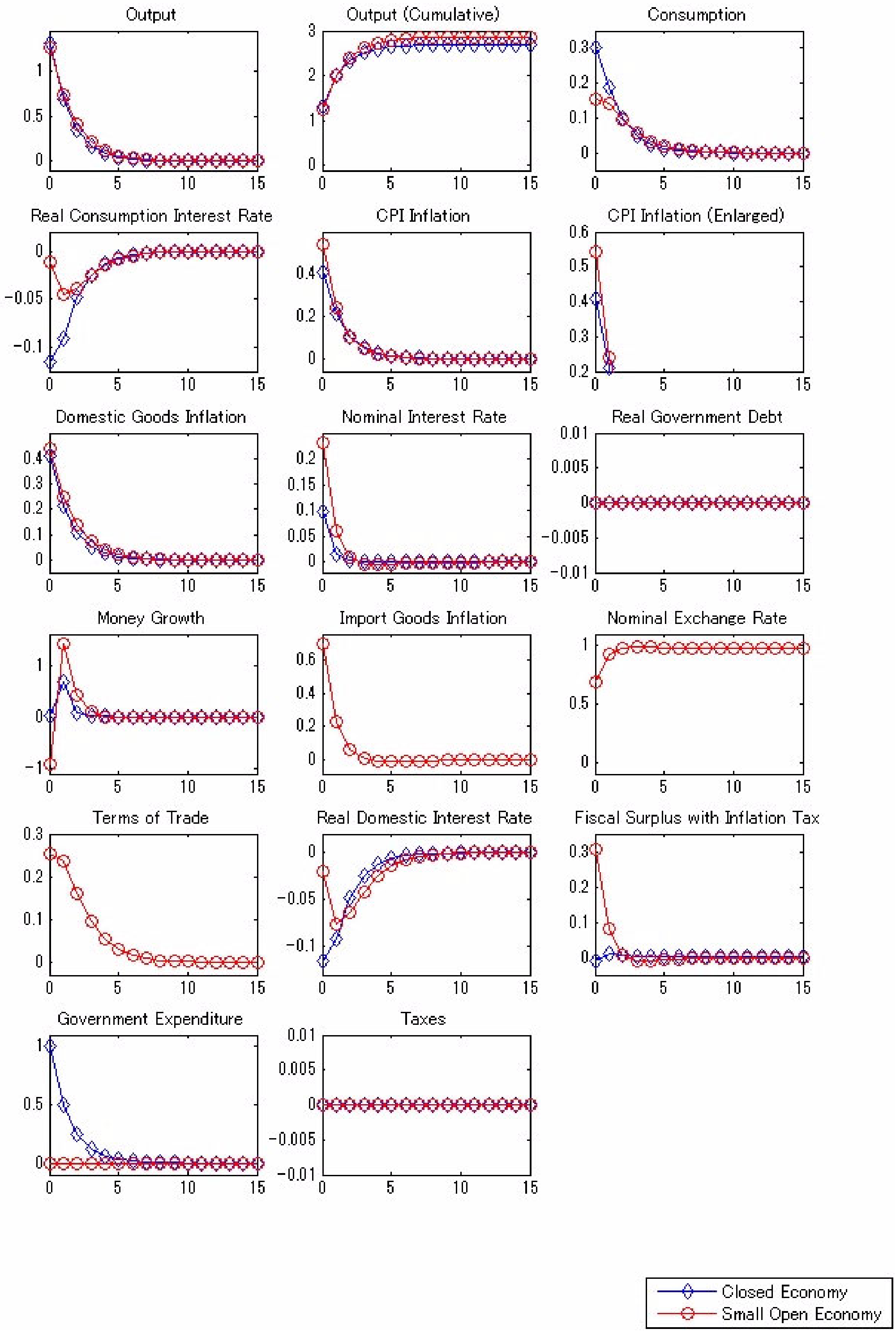


Figure 3: Dynamic Response to a Tax Cut under the *DF* Scheme

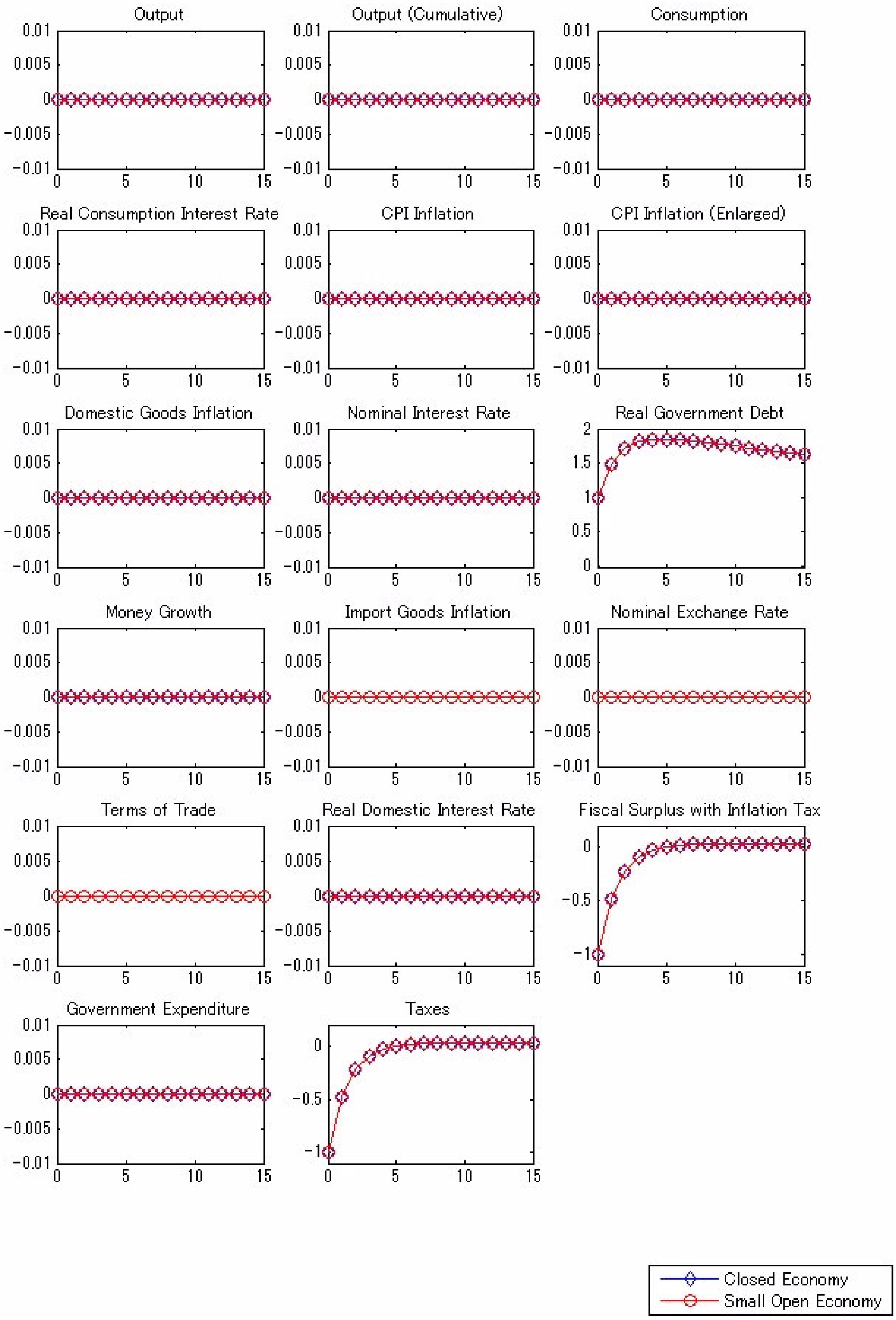


Figure 4: Dynamic Response to an Increase in Government Expenditure under the *DF* Scheme

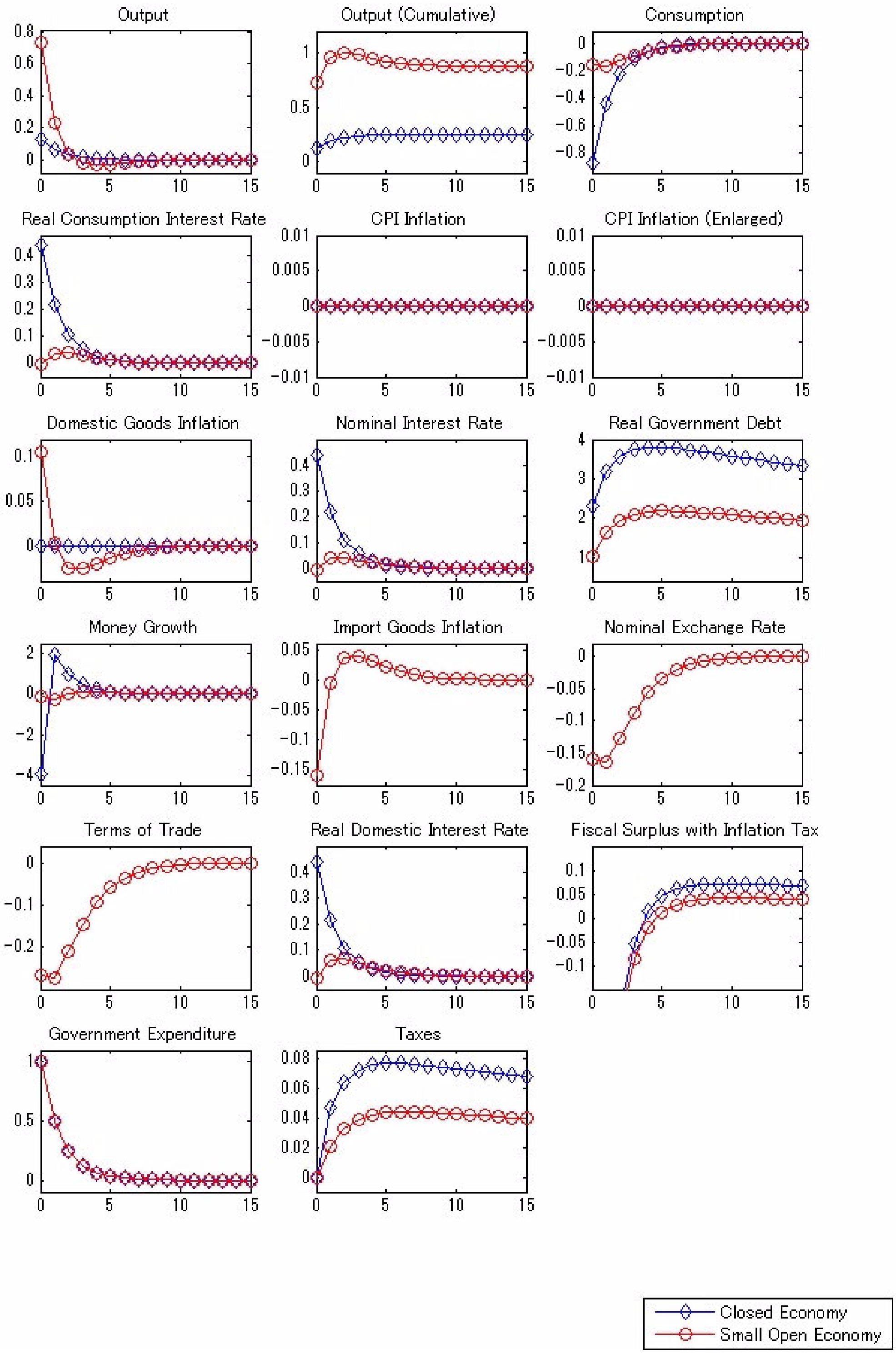


Table 2: Parameterization

Parameter	Description	Value	Source
σ	Relative Risk Aversion	1	Monacelli[20]
ν	Openness	0.4	
β	Discount Factor	0.995	
φ	Curvature of Labor Disutility	5	
α	Index of Decreasing Returns to Labor	0.25	
ϵ	Elasticity of Substitution among Goods	9	
θ	Calvo Index of Price Rigidities	$\frac{3}{4}$	
χ	The Steady State Inverse Velocity	$\frac{1}{3}$	Gali[13]
η	Semielasticity of Money Demand	7	
υ	Separability of Real Balances	0	
ψ_b	Tax Adjustment	0.02	
b	Target Debt Ratio	2.4	
δ	Persistence	0.5	

Figure 5: Fiscal Multipliers for an Increase in Government Expenditure

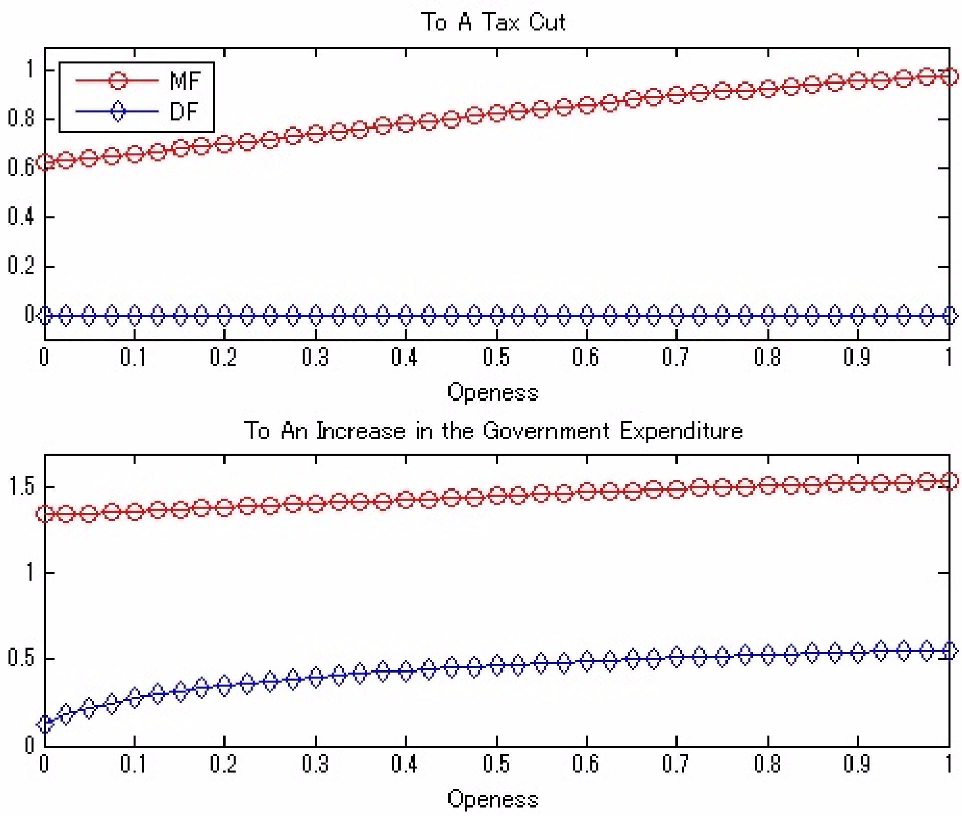


Figure 6: Responses in the Case of *No Response* in a Liquidity Trap

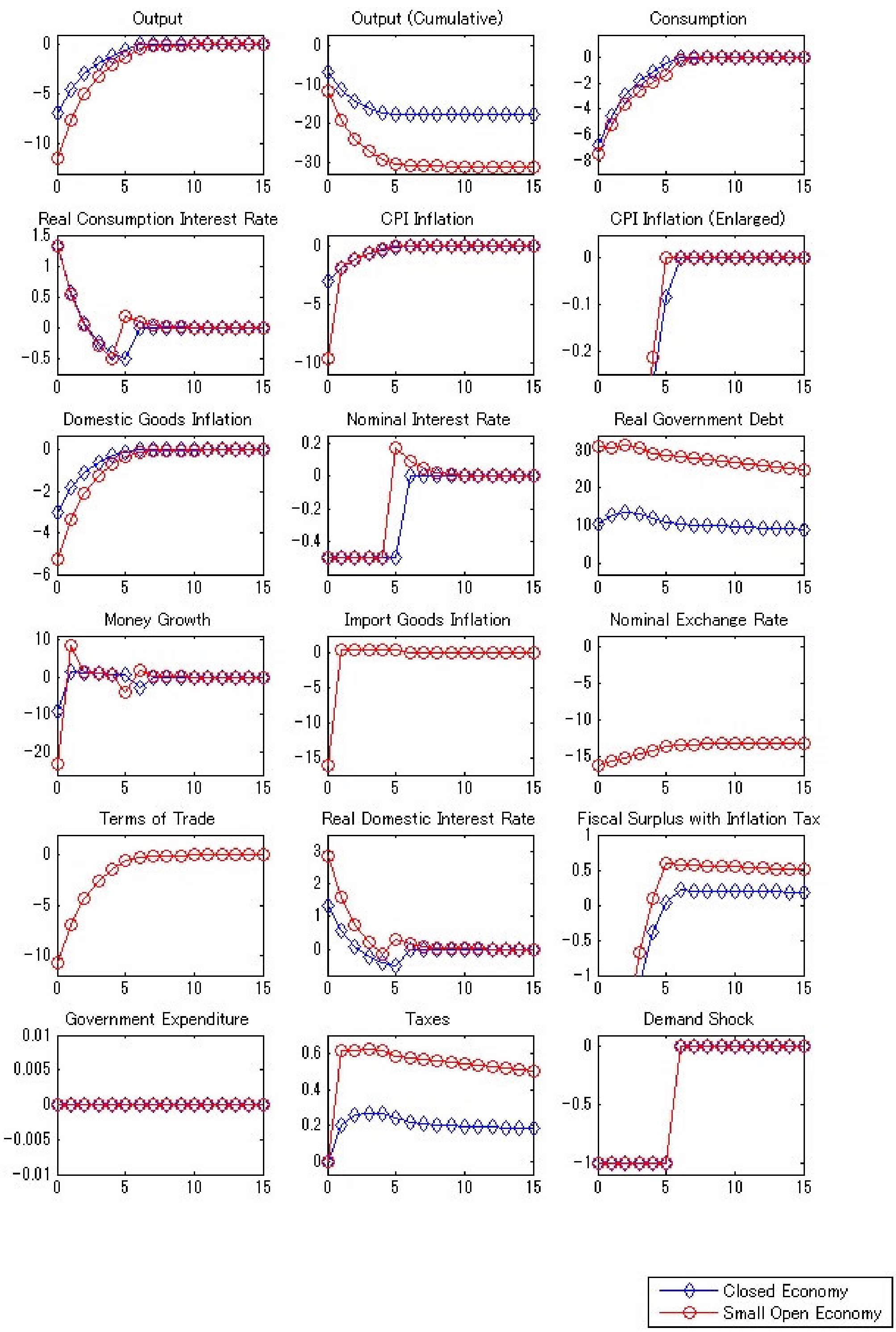


Figure 7: Dynamic Effects of a Tax Cut under *MF* Fiscal Stimulus in a Liquidity Trap

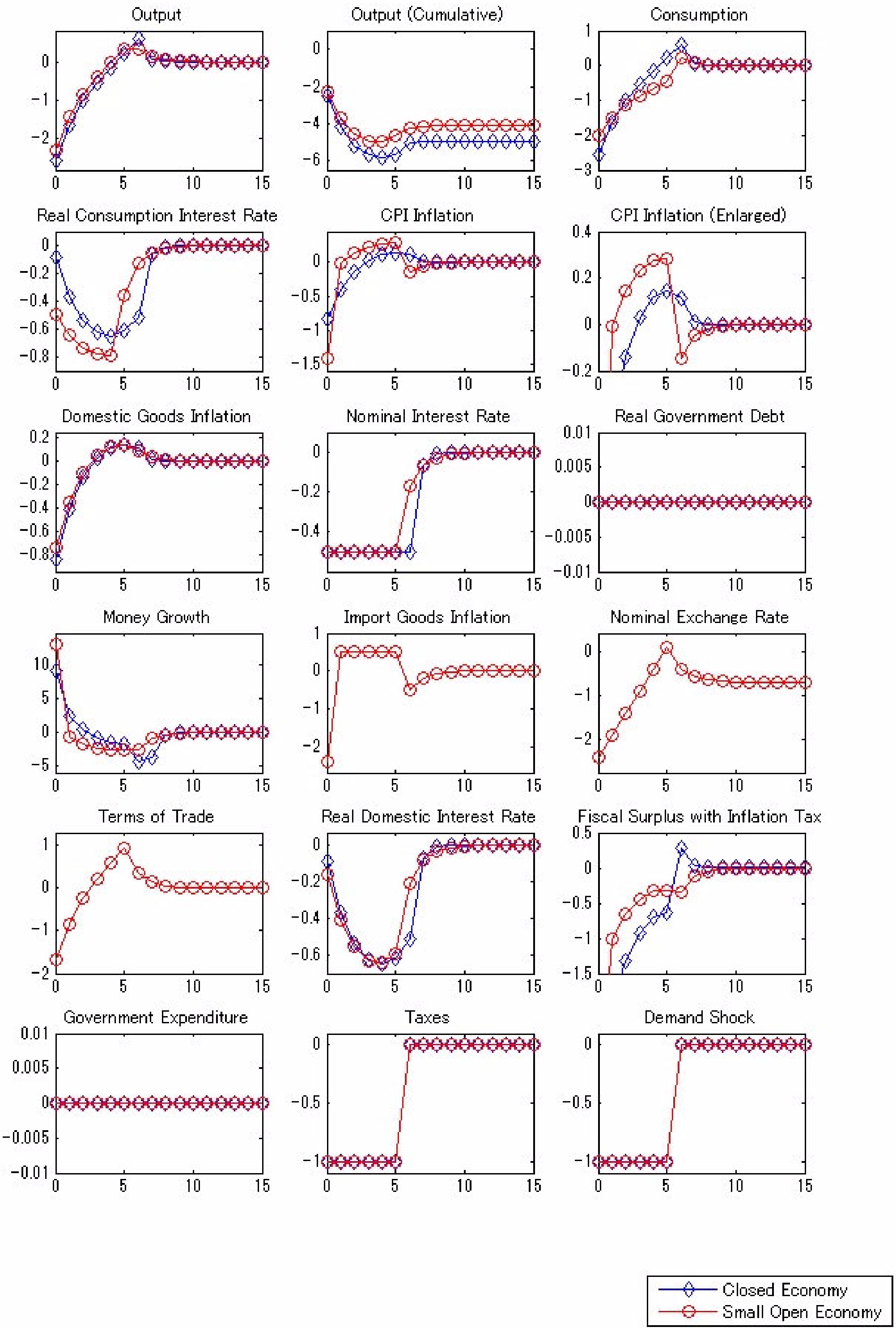


Figure 8: Dynamic Effects of an Increase in Government Expenditure under *MF* Fiscal Stimulus in a Liquidity Trap

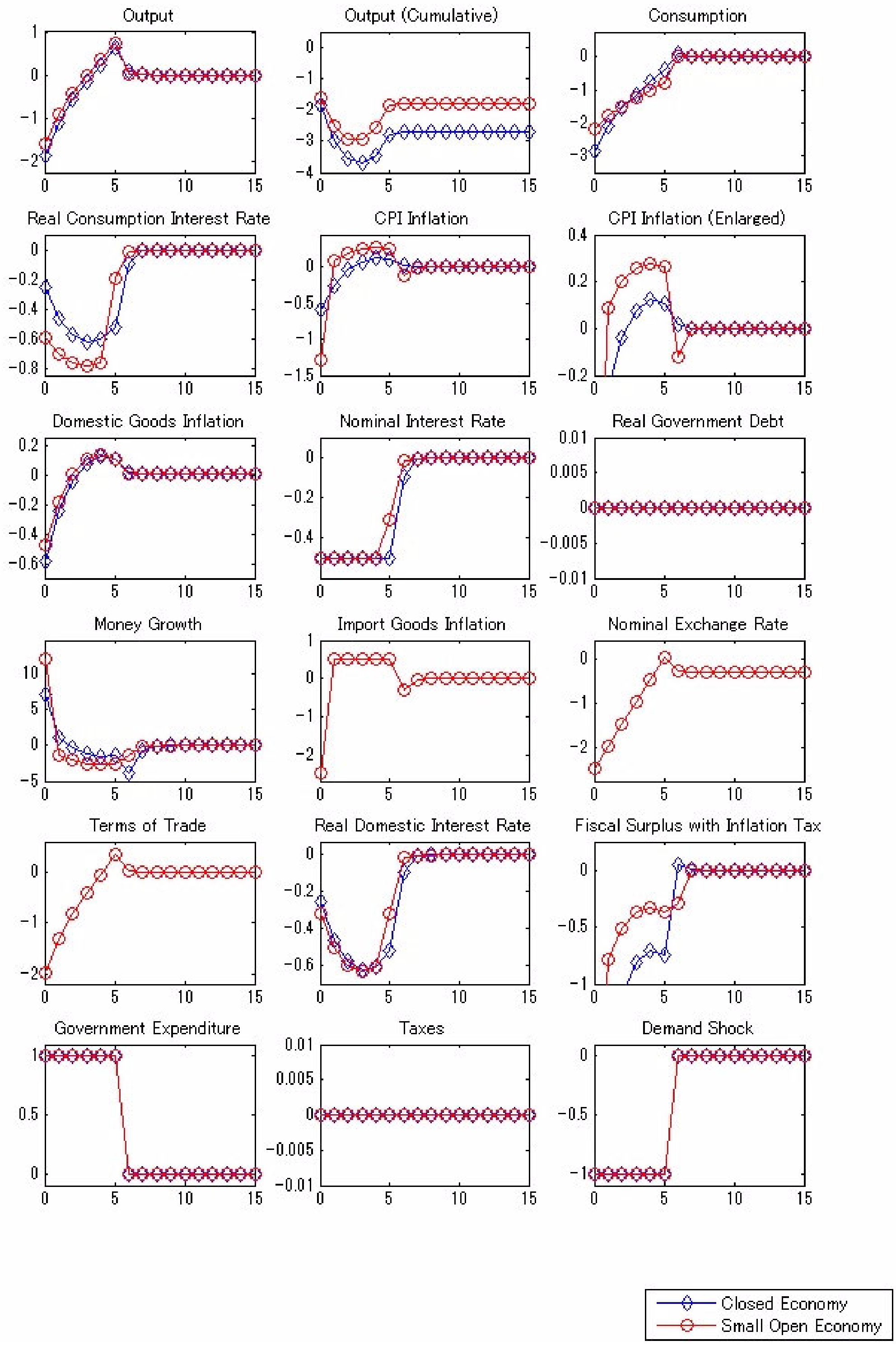


Figure 9: Dynamic Effects of a Tax Cut under the *DF* Scheme in a Liquidity Trap

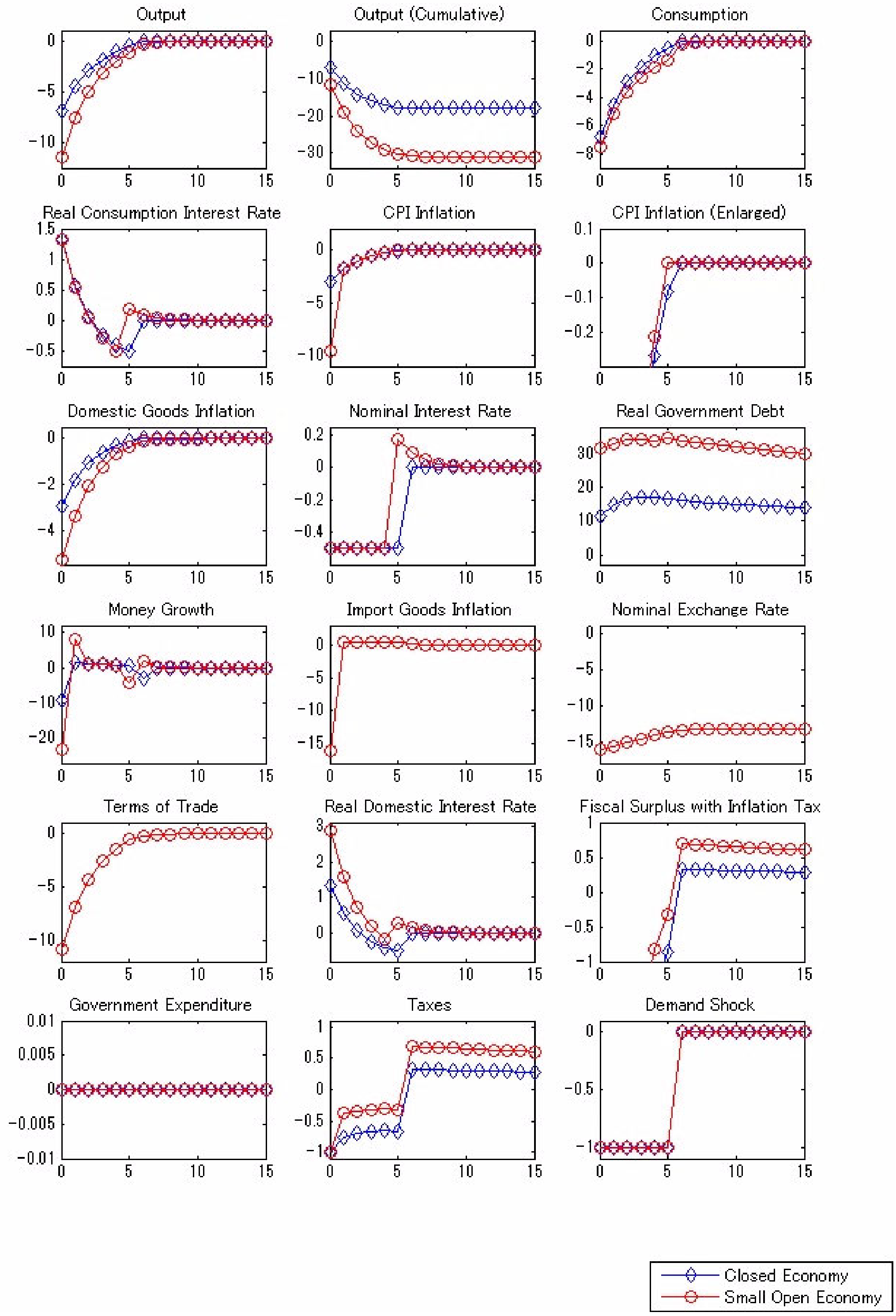


Figure 10: Dynamic Effects of an Increase in Government Expenditure under the *DF* Scheme in a Liquidity Trap

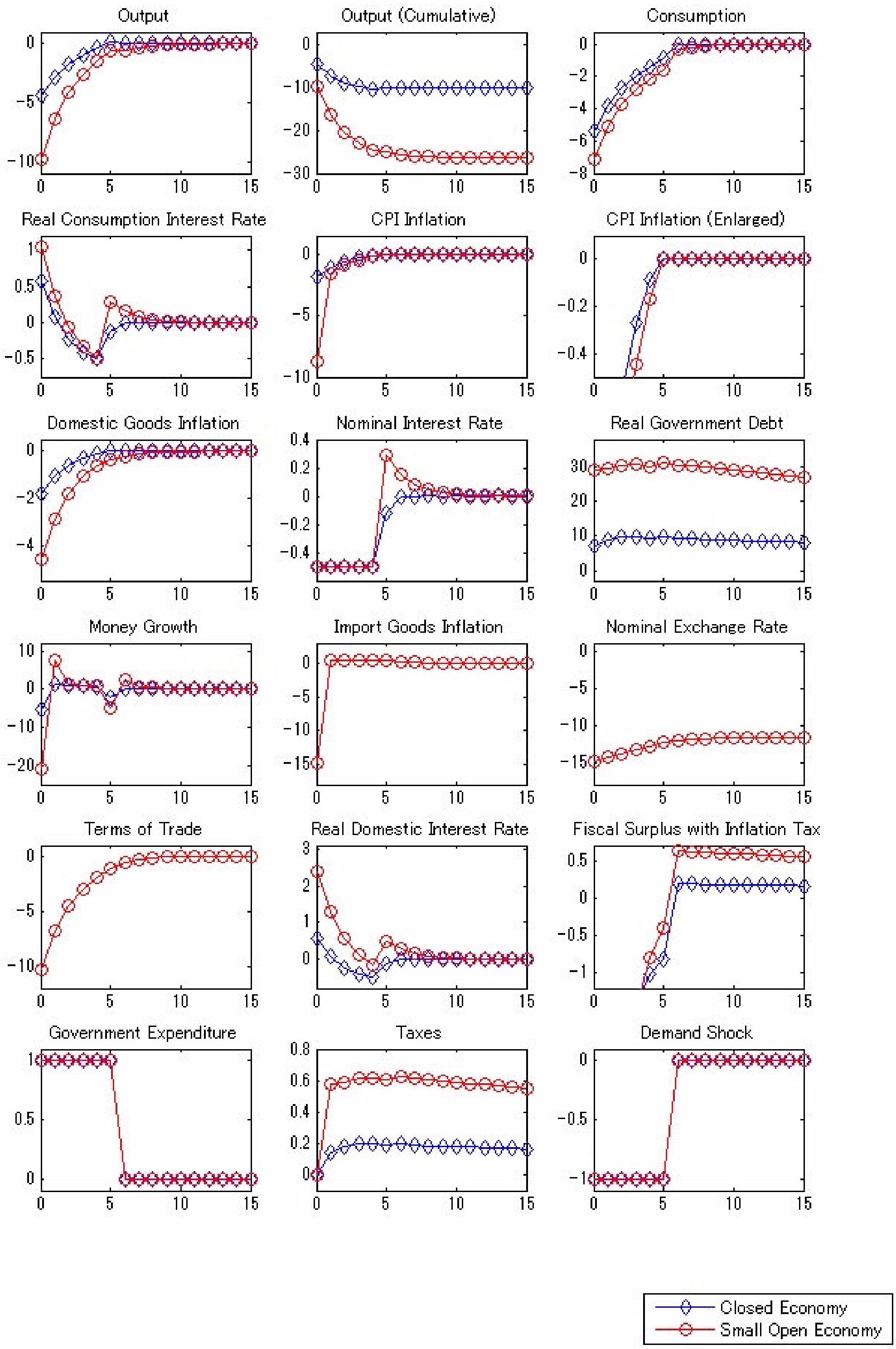


Figure 11: Dynamic Effects of an Increase in Government Expenditure under the *DF* Scheme in a Liquidity Trap: Comparison of the *MF* Scheme, the *DF* Scheme, and the Case of *No Response* in a Small Open Economy

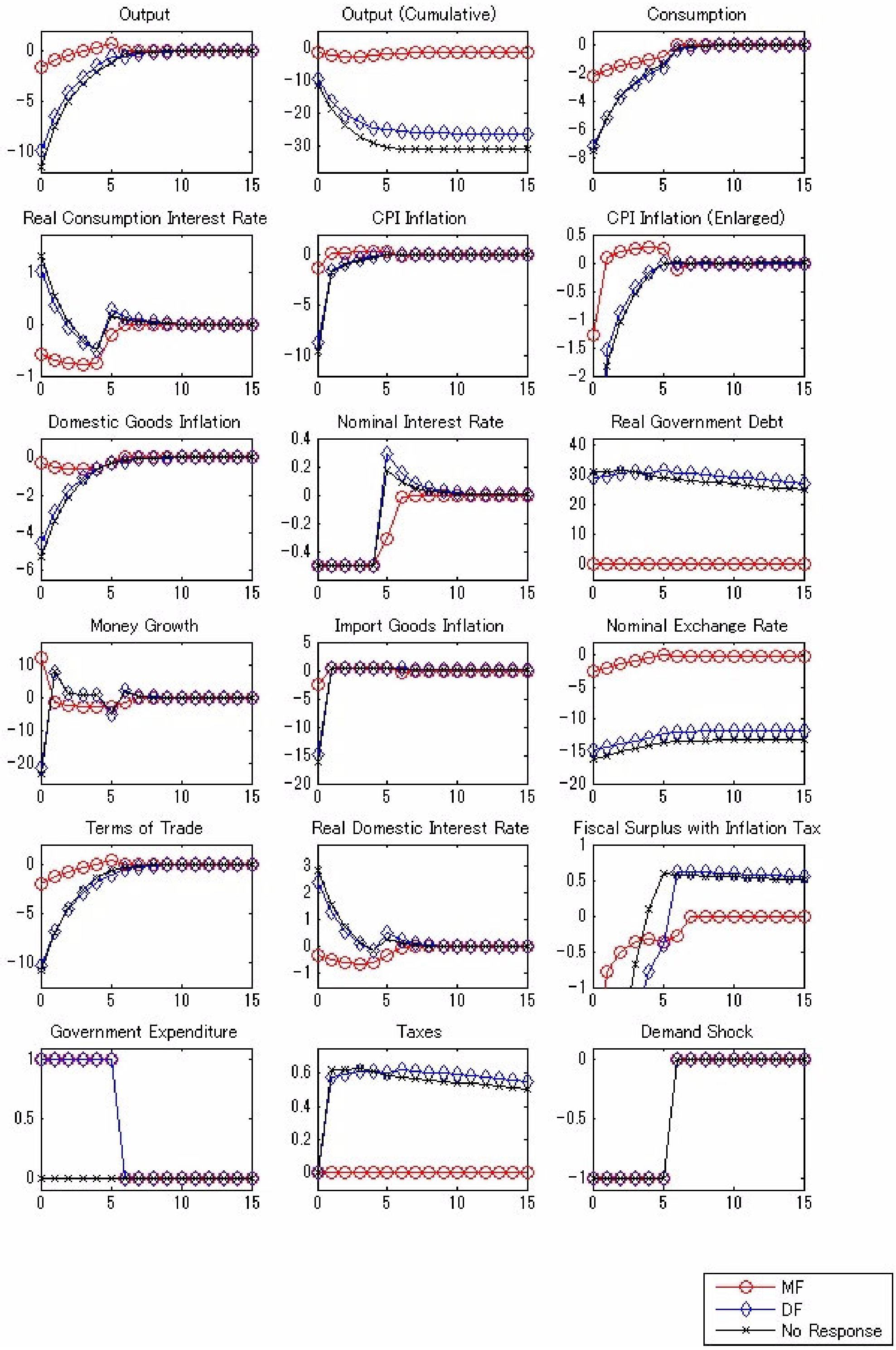


Figure 12: Dynamic Effects of an Increase in Government Expenditure under the *DF* Scheme in a Liquidity Trap: Comparison of the *MF* Scheme with the Case of No Response in a Small Open Economy

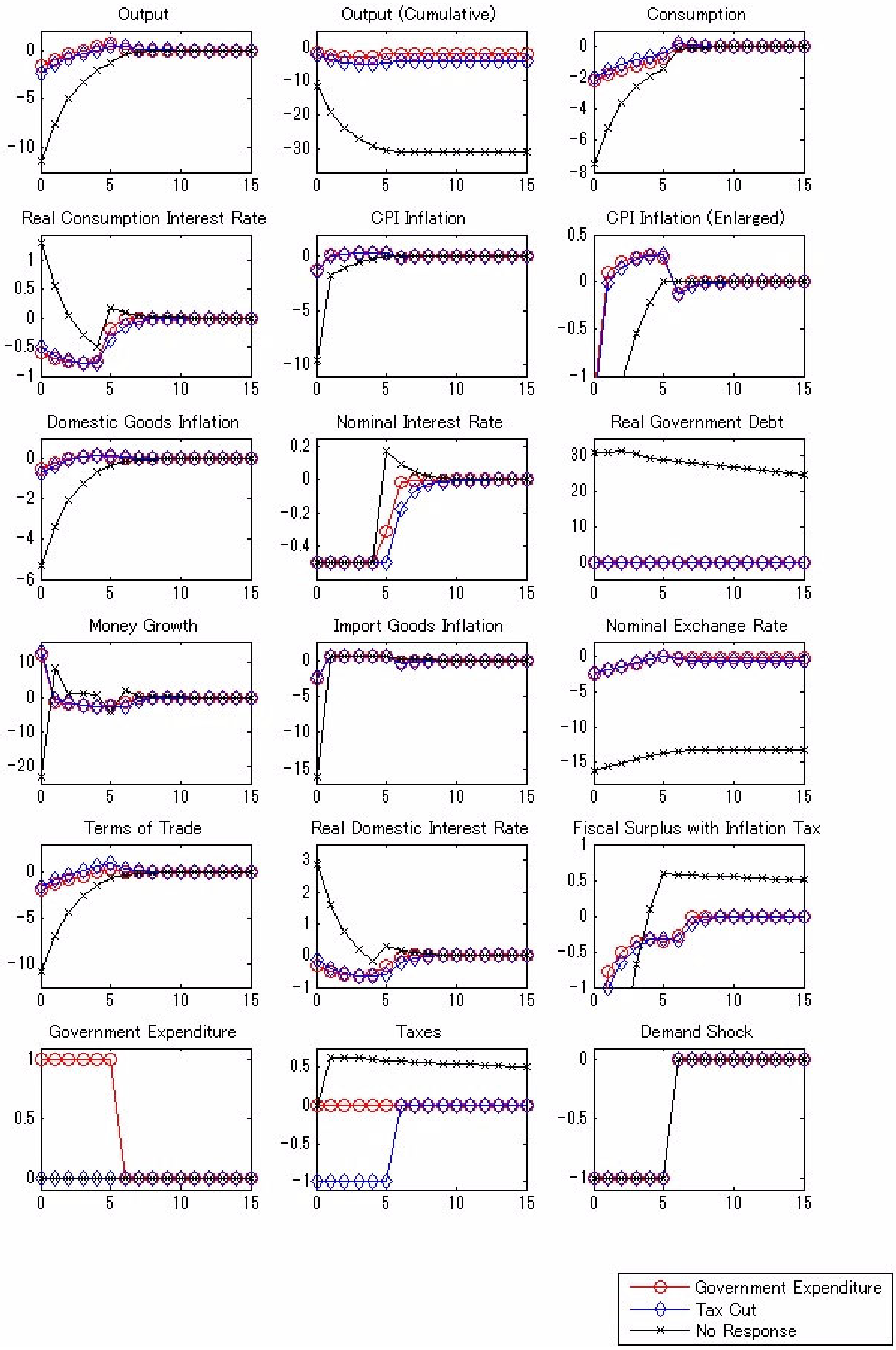
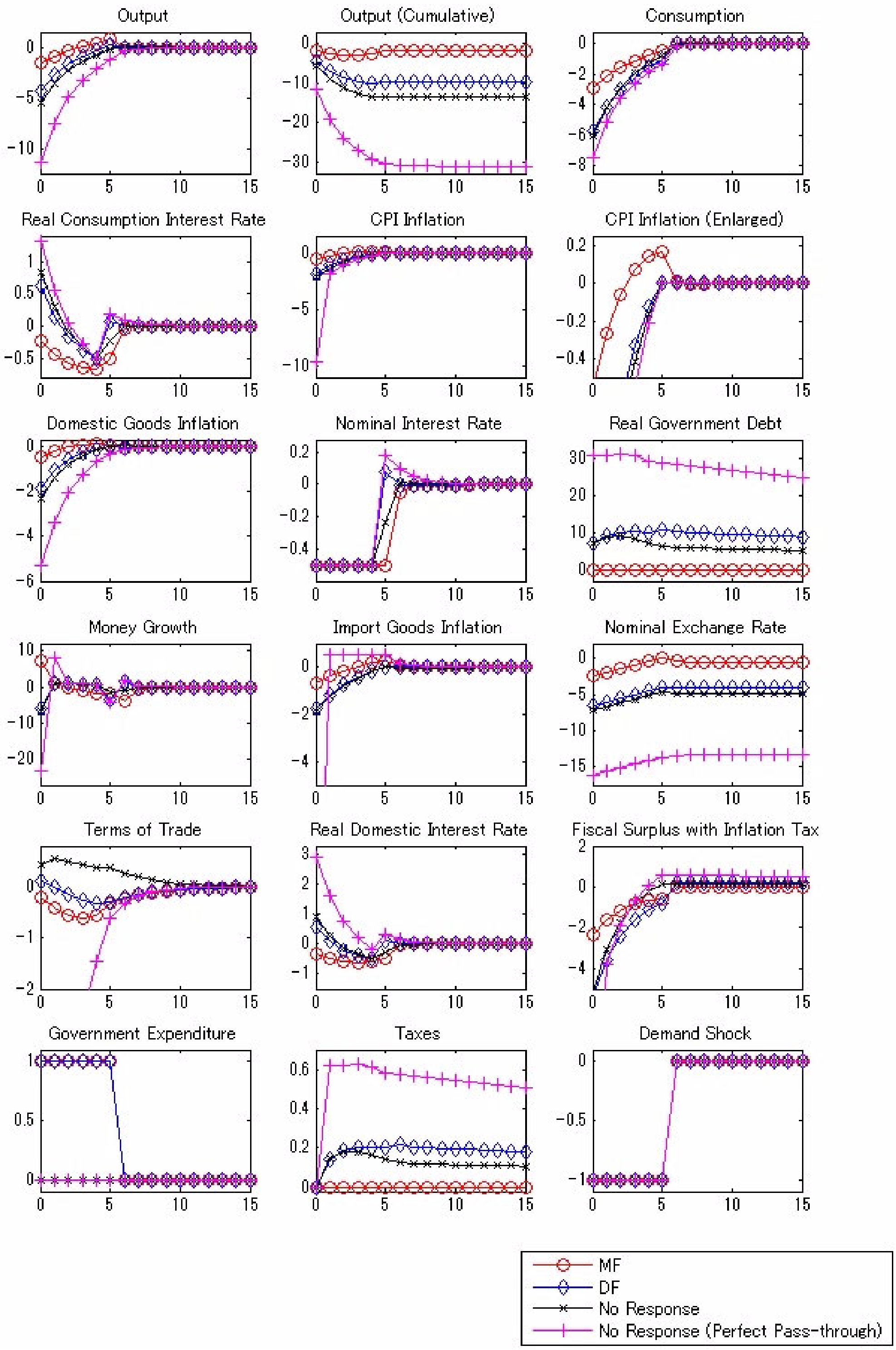


Figure 13: Dynamic Effects of an Increase in Government Expenditure in an Imperfect Pass-through Environment in a Liquidity Trap



Technical Appendix to ``The Effects of A Money-financed Fiscal Stimulus in A Small Open-economy’’

(Not for Publication)

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1 The Model

1.1 Households

Households are line-up [0,1] and openness is v.
(Nominal) Households' budget constraint is given by:

$$P_tC_t + B_t + B_t^*E_t + M_t = B_{t-1}(1+i_{t-1}) + B_{t-1}^*(1+i_{t-1}^*)E_t + M_{t-1} + W_tN_t + D_t - P_tTR_t \, .$$

Dividing both sides of the previous expression by P_t yields:

$$C_t + \mathrm{B}_t + \frac{E_tP_t^*}{P_t}\mathrm{B}_t^* + L_t = \frac{P_{t-1}}{P_t}\mathrm{B}_{t-1}(1+i_{t-1}) + \frac{E_tP_t^*}{P_t}\frac{P_{t-1}^*}{P_t^*}\mathrm{B}_{t-1}^*(1+i_{t-1}^*) + \frac{P_{t-1}}{P_t}L_{t-1}$$
$$+\frac{W_t}{P}N_t + \frac{D_t}{P_t} - TR_t \, ,$$

which can be rewritten as:

$$C_t + \mathrm{B}_t + Q_t\mathrm{B}_t^* + L_t = \Pi_t^{-1}\mathrm{B}_{t-1}(1+i_{t-1}) + Q_t\left(\Pi_t^*\right)^{-1}\mathrm{B}_{t-1}^*(1+i_{t-1}^*) + \Pi_t^{-1}L_{t-1}$$
$$+\frac{W_t}{P}N_t + \frac{PR_t}{P_t} - TR_t \, ,$$

which is identical with Eq.(4) in the text.

with $\mathrm{B}_t \equiv \frac{B_t}{P_t}$, $\mathrm{B}_t^* \equiv \frac{B_t^*}{P_t^*}$ and $Q_t \equiv \frac{E_tP_t^*}{P_t}$.

Alternatively, we have:

$$C_t + \mathrm{B}_t + \frac{E_tP_t^*}{P_t}\mathrm{B}_t^* + L_t = \frac{P_{t-1}}{P_t}\mathrm{B}_{t-1}(1+i_{t-1}) + \frac{E_t}{E_{t-1}}\frac{P_{t-1}}{P_t}\frac{E_{t-1}P_{t-1}^*}{P_{t-1}}\mathrm{B}_{t-1}^*(1+i_{t-1}^*) + \frac{P_{t-1}}{P_t}L_{t-1}$$
$$+\frac{W_t}{P}N_t + \frac{D_t}{P_t} - TR_t \, ,$$

which can be rewritten as:

$$C_t + \mathrm{B}_t + Q_t\mathrm{B}_t^* + L_t = \Pi_t^{-1}\mathrm{B}_{t-1}(1+i_{t-1}) + \Pi_t^{-1}\frac{E_t}{E_{t-1}}Q_{t-1}\mathrm{B}_{t-1}^*(1+i_{t-1}^*) + \Pi_t^{-1}L_{t-1} + \frac{W_t}{P_t}N_t$$
$$+\frac{D_t}{P_t} - TR_t \, .$$

Further, the previous expression can be rewritten as:

$$C_t + \frac{1}{1+i_t}\left[(1+i_t)\mathrm{B}_t + (1+i_t)Q_t\mathrm{B}_t^* + (1+i_t)L_t\right]$$
$$= \left[\mathrm{B}_{t-1}(1+i_{t-1}) + \frac{E_t}{E_{t-1}}Q_{t-1}\mathrm{B}_{t-1}^*(1+i_{t-1}^*) + L_{t-1}\right]\Pi_t^{-1} + \frac{W_t}{P_t}N_t + \frac{D_t}{P_t} - TR_t \, .$$

Further, the previous expression can be rewritten as:

$$C_t + \frac{1}{1+i_t}\left[(1+i_t)\mathrm{B}_t + (1+i_t)Q_t\mathrm{B}_t^* + L_t\right]\Pi_{t+1}^{-1}\Pi_{t+1} + L_t\left(1-\frac{1}{1+i_t}\right)$$
$$= \left[\mathrm{B}_{t-1}(1+i_{t-1}) + \frac{E_t}{E_{t-1}}Q_{t-1}\mathrm{B}_{t-1}^*(1+i_{t-1}^*) + L_{t-1}\right]\Pi_t^{-1} + \frac{W_t}{P_t}N_t + \frac{D_t}{P_t} - TR_t .$$

Let define $\mathrm{A}_t \equiv \left[(1+i_{t-1})\mathrm{B}_{t-1} + Q_{t-1}\mathrm{B}_{t-1}^*\frac{E_t}{E_{t-1}}(1+i_{t-1}^*) + L_{t-1}\right]\Pi_t^{-1}$. Then, the previous expression can be rewritten as:

$$C_t + \frac{1}{1+i_t}\mathrm{A}_{t+1}\Pi_{t+1} + L_t\left(1-\frac{1}{1+i_t}\right) = \mathrm{A}_t + \frac{W_t}{P_t}N_t + \frac{D_t}{P_t} - TR_t ,$$

where we assume that the UIP. The previous expression is identical with Eq.(6) in the text.

Households' optimization problem is given by:

$$\max_{C_t, C_{t+1}, N_t, \mathrm{A}_{t+1}, L_t} \sum_{t=0}^{\infty}\beta^t U_t ,$$

s.t.

$$U_t \equiv \left[U(C_t, L_t) - V(N_t)\right]Z_t ,$$

$$C_t + \frac{1}{1+i_t}\mathrm{A}_{t+1}\Pi_{t+1} + L_t\left(1-\frac{1}{1+i_t}\right) = \mathrm{A}_t + \frac{W_t}{P_t}N_t + \frac{D_t}{P_t} - TR_t .$$

The Lagrangean is given by:

$$L \equiv \beta^t\left(U(C_t, L_t) - V(N_t)\right)Z_t + \beta^{t+1}\left(U(C_{t+1}, L_{t+1}) - V(N_{t+1})\right)Z_{t+1} + \cdots$$
$$+\lambda_t\beta^t\left[\mathrm{A}_t + \frac{W_t}{P_t}N_t + \frac{PR_t}{P_t} - T_t - C_t - \frac{1}{1+i_t}\mathrm{A}_{t+1}\Pi_{t+1} - L_t\left(1-\frac{1}{1+i_t}\right)\right]$$
$$+\lambda_{t+1}\beta^{t+1}\left[\mathrm{A}_{t+1} + \frac{W_{t+1}}{P_{t+1}}N_t + \frac{PR_{t+1}}{P_{t+1}} - T_{t+1} - C_{t+1} - \frac{1}{1+i_{t+1}}\mathrm{A}_{t+2}\Pi_{t+2} - L_{t+1}\left(1-\frac{1}{1+i_{t+1}}\right)\right] .$$
$$+\cdots$$

FONCs are given by:

$$\frac{\partial L}{\partial C_t} = \beta^t U_{c,t}Z_t - \beta^t\lambda_t = 0 ,$$

$$\frac{\partial L}{\partial C_{t+1}} = \beta^{t+1} U_{c,t+1} Z_{t+1} - \beta^{t+1} \lambda_{t+1} = 0,$$

$$\frac{\partial L}{\partial N_t} = \beta^t \left(-V_{n,t}\right) Z_t + \lambda_t \beta^t \frac{W_t}{P_t} = 0,$$

$$\frac{\partial L}{\partial \mathrm{A}_{t+1}} = -\lambda_t \beta^t \frac{1}{1+i_t} \Pi_{t+1} + \lambda_{t+1} \beta^{t+1} = 0,$$

$$\frac{\partial L}{\partial L_t} = \beta^t U_{l,t} Z_t - \beta^t \lambda_t \left(1 - \frac{1}{1+i_t}\right) = 0,$$

which can be rewritten as:

$\lambda_t = U_{c,t} Z_t$, (1-1)

$\lambda_{t+1} = U_{c,t+1} Z_{t+1}$, (1-2)

$\lambda_t = \left(\frac{W_t}{P_t}\right)^{-1} V_{n,t} Z_t$, (1-3)

$\lambda_t = \beta \lambda_{t+1} \Pi_{t+1}^{-1} \left(1+i_t\right)$, (1-4)

$\lambda_t = U_{l,t} \left(\frac{i_t}{1+i_t}\right)^{-1} Z_t$, (1-5)

where λ_t denotes Lagrange multiplier.

Combining Eqs.(1-1), (1-2) and (1-4) yields:

$U_{c,t} Z_t = \beta \left(1+i_t\right) \Pi_{t+1}^{-1} U_{c,t+1} Z_{t+1}$. (1-6)

Combing Eqs.(1-1) and (1-3) yields:

$\frac{W_t}{P_t} = \frac{V_{n,t}}{U_{c,t}}$. (1-7)

Combing Eqs.(1-1) and (1-5) yields:

$\frac{U_{l,t}}{U_{c,t}} = \frac{i_t}{1+i_t}$. (1-8)

Note that Eqs.(1-6)—(1-8) are identical with Eqs.(7)—(9) in the text.

1.2 International Risk sharing Condition

Under the assumption of complete markets for securities traded internationally, a condition analogous to Eq.(1-6) must also hold for the representative household in the foreign country:

$U_{c,t}^{*} Z_{t}^{*} = \beta \left(1 + i_{t}^{*}\right) \left(\Pi_{t+1}^{*}\right)^{-1} U_{c,t+1}^{*} Z_{t+1}^{*}$. (1-9)

Eqs.(1-6) and (1-9) can be rewritten as:

$$\beta^{-1} = \left(1 + i_{t}\right) \Pi_{t+1}^{-1} \frac{U_{c,t+1}}{U_{c,t}} \frac{Z_{t+1}}{Z_{t}},$$

$$\beta^{-1} = \left(1 + i_{t}^{*}\right) \left(\Pi_{t+1}^{*}\right)^{-1} \frac{U_{c,t+1}^{*}}{U_{c,t}^{*}} \frac{Z_{t+1}^{*}}{Z_{t}^{*}}.$$

Combining the previous expressions each other yields:

$$\left(1 + i_{t}\right) \Pi_{t+1}^{-1} \frac{U_{c,t+1}}{U_{c,t}} \frac{Z_{t+1}}{Z_{t}} = \left(1 + i_{t}^{*}\right) \left(\Pi_{t+1}^{*}\right)^{-1} \frac{U_{c,t+1}^{*}}{U_{c,t}^{*}} \frac{Z_{t+1}^{*}}{Z_{t}^{*}}.$$

Dividing both sides of the previous expression by $\left(1 + i_{t}\right) \Pi_{t+1}^{-1}$ yields:

$$\frac{U_{c,t+1}}{U_{c,t}} \frac{Z_{t+1}}{Z_{t}} = \frac{1 + i_{t}^{*}}{1 + i_{t}} \frac{P_{t+1}}{P_{t}} \frac{P_{t}^{*}}{P_{t+1}^{*}} \frac{U_{c,t+1}^{*}}{U_{c,t}^{*}} \frac{Z_{t+1}^{*}}{Z_{t}^{*}}.$$

Plugging the UIP $\frac{1 + i_{t}}{1 + i_{t}^{*}} = \frac{E_{t+1}}{E_{t}}$ into the previous expression yields:

$$\frac{U_{c,t+1}}{U_{c,t}} \frac{Z_{t+1}}{Z_{t}} = \frac{Q_{t}}{Q_{t+1}} \frac{U_{c,t+1}^{*}}{U_{c,t}^{*}} \frac{Z_{t+1}^{*}}{Z_{t}^{*}}.$$

In the period -1, the previous expression is given by:

$$\frac{U_{c,0}}{U_{c,-1}} \frac{Z_{0}}{Z_{-1}} = \frac{Q_{-1}}{Q_{0}} \frac{U_{c,0}^{*}}{U_{c,-1}^{*}} \frac{Z_{0}^{*}}{Z_{-1}^{*}},$$

which can be rewritten as:

$$U_{c,0}^{-1} = \frac{U_{c,-1}^{*}}{U_{c,-1}} \frac{Z_{-1}^{*}}{Z_{-1}} \frac{1}{Q_{-1}} \left(U_{c,0}^{*}\right)^{-1} Q_{0} \frac{Z_{0}}{Z_{0}^{*}}.$$

Let define $\vartheta \equiv \frac{U_{c,-1}^{*}}{U_{c,-1}} \frac{Z_{-1}^{*}}{Z_{-1}} \frac{1}{Q_{-1}}$ as an initial condition. Then the previous expression can be generalized as follows:

$$U_{c,t}^{-1} = \vartheta\left(U_{c,t}^{*}\right)^{-1} Q_t \frac{Z_t}{Z_t^{*}}, \text{ (1-10)}$$

which is identical with Eq.(10) in the text.

1.3 Optimal Allocation of Goods

Let define consumption indices as follows:

$$C_t \equiv \frac{1}{(1-\nu)^{1-\nu}\nu^{\nu}} C_{H,t}^{1-\nu} C_{F,t}^{\nu}, \text{ (1-11)}$$

with $C_{H,t} \equiv \left[\int_0^1 C_{H,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}}$ and $C_{F,t} \equiv \left[\int_0^1 C_{F,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}}$.

By solving cost-minimization problems for households, we have optimal allocation of expenditures as follows:

$$C_{H,t}(j) = \left(\frac{P_t(j)}{P_{H,t}}\right)^{-\varepsilon} C_{H,t}, \text{ (1-12)}$$

and

$$C_{F,t}(j) = \left(\frac{P_{F,t}(j)}{P_{F,t}}\right)^{-\varepsilon} C_{F,t}, \text{ (1-13)}$$

with:

$$P_{H,t} \equiv \left[\int_0^1 P_{H,t}(j)^{1-\varepsilon} dj\right]^{\frac{1}{1-\varepsilon}} \text{ and } P_{F,t} \equiv \left[\int_0^1 P_{F,t}(j)^{1-\varepsilon} dj\right]^{\frac{1}{1-\varepsilon}}.$$

Now, we get total demand for goods produced in the SOE and the ROW. Optimization problem is given by:

$$\max_{C_{H,t}, C_{F,t}} C_t,$$

s.t.

Eq.(1-11) and $X_t - \left(P_{H,t}C_{H,t} + P_{F,t}C_{F,t}\right) = 0$.

The Lagrangean is given by

$$L \equiv \frac{1}{(1-\nu)^{1-\nu}\nu^{\nu}} C_{H,t}^{1-\nu} C_{F,t}^{\nu} + \lambda\left(X_t - P_{H,t}C_{H,t} - P_{F,t}C_{F,t}\right).$$

The FONCs is given by:

$$\frac{\partial L}{\partial C_{H,t}} = \frac{1}{(1-\nu)^{1-\nu}\nu^{\nu}}(1-\nu)C_{H,t}^{-\nu}C_{F,t}^{\nu} - \lambda P_{H,t}$$
$$= (1-\nu)^{\nu}\nu^{-\nu}C_{H,t}^{-\nu}C_{F,t}^{\nu} - \lambda P_{H,t}$$
$$= 0$$
$$\frac{\partial L}{\partial C_{F,t}} = (1-\nu)^{-(1-\nu)}\nu^{1-\nu}C_{H,t}^{1-\nu}C_{F,t}^{-(1-\nu)} - \lambda P_{F,t}$$
$$= 0$$

These previous expressions can be rewritten as:

$$(1-\nu)^{\nu}\nu^{-\nu}C_{H,t}^{-\nu}C_{F,t}^{\nu} = \lambda P_{H,t},$$

$$(1-\nu)^{-(1-\nu)}\nu^{1-\nu}C_{H,t}^{1-\nu}C_{F,t}^{-(1-\nu)} = \lambda P_{F,t}.$$

Combining these expression yields:

$$(1-\nu)\nu^{-1}C_{H,t}^{-1}C_{F,t} = \frac{P_{H,t}}{P_{F,t}},$$

which can be rewritten as:

$$C_{F,t} = \frac{\nu}{1-\nu}\left(\frac{P_{H,t}}{P_{F,t}}\right)C_{H,t} \quad . \text{(1-14)}$$
$$= \frac{\nu}{1-\nu}S_t^{-1}C_{H,t}$$

Plugging Eq.(1-14) into Eq.(1-11) yields:

$$C_t = \frac{1}{(1-\nu)^{1-\nu}\nu^{\nu}}C_{H,t}^{1-\nu}\left(\frac{\nu}{1-\nu}S_t^{-1}C_{H,t}\right)^{\nu}$$
$$= \frac{\nu^{\nu}}{(1-\nu)^{1-\nu}\nu^{\nu}}(1-\nu)^{-\nu}S_t^{-\nu}C_{H,t} \qquad . \text{(1-15)}$$
$$= \frac{1}{1-\nu}S_t^{-\nu}C_{H,t}$$

The definition of the PPI in the SOE is given by:

$$P_t \equiv P_{H,t}^{1-\nu}P_{F,t}^{\nu} \text{ . (1-16)}$$

Eq.(1-15) can be rewritten as:

$$C_{H,t} = (1-\nu)S_t^{\nu}C_t \text{ . (1-17) ,}$$

Which is identical with the LHS in Eq.(3) in the text.

Eq.(1-14) can be rewritten as:

$$C_{H,t} = \frac{1-\nu}{\nu} S_t C_{F,t} .$$

Plugging the previous expression into Eq.(1-15) yields:

$$C_t \equiv \frac{1}{(1-\nu)^{1-\nu} \nu^{\nu}} \left(\frac{1-\nu}{\nu} S_t C_{F,t} \right)^{1-\nu} C_{F,t}^{\nu}$$

$$= \frac{1}{\nu} S_t^{1-\nu} C_{F,t}$$

which can be rewritten as:

$$C_{F,t} = \nu S_t^{-(1-\nu)} C_t , \quad (1\text{-}18)$$

which is identical with the RHS in Eq.(3) in the text.

1.4 Domestic Producers

Production function is given by:

$$Y_t(j) = N_t(j)^{1-\alpha} . \quad (1\text{-}19)$$

Combining Eq.(1-19) and $Y_t \equiv \left[\int_0^1 Y_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}$ with the definitions of the PPI indices yields:

$$Y_t(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\varepsilon} Y_t , \quad (1\text{-}20)$$

Plugging Eq.(1-19) into Eq.(1-20) yields:

$$N_t(j)^{1-\alpha} = \left(\frac{P_t(j)}{P_{H,t}} \right)^{-\varepsilon} Y_t ,$$

which can be rewritten as:

$$N_t(j) = \left(\frac{P_t(j)}{P_{H,t}} \right)^{-\frac{\varepsilon}{1-\alpha}} Y_t^{\frac{1}{1-\alpha}}$$

Let define $N_t \equiv \int_0^1 N_t(j) dj$. Plugging the previous expression into the definition yields:

$$N_t = \int_0^1 N_t(j)dj$$

$$= \int_0^1 \left(\frac{P_t(j)}{P_{H,t}}\right)^{-\frac{\varepsilon}{1-\alpha}} Y_t^{\frac{1}{1-\alpha}} dj\ ,$$

$$= \int_0^1 \left(\frac{P_t(j)}{P_{H,t}}\right)^{-\frac{\varepsilon}{1-\alpha}} dj Y_t^{\frac{1}{1-\alpha}}$$

The previous expression can be rewritten as:

$N_t^{1-\alpha} = Y_t \Omega^{1-\alpha}$, (1-21)

with $\Omega = \int_0^1 \left(\frac{P_t(j)}{P_{H,t}}\right)^{-\frac{\varepsilon}{1-\alpha}} dj$. Eq.(1-21) is identical with Eq.(14) in the text.

Now we consider firms' maximization problem following Gali (2015). The firms' maximization problem is given by:

$$\max_{\tilde{P}_{H,t}} \sum_{k=0}^{\infty} \theta^k \mathrm{E}_t \left\{ \Lambda_{t,t+k} \left(\frac{1}{P_{t+k}}\right) \left[\tilde{P}_{H,t} Y_{t+k|t} - \mathbb{C}_{t+k}\left(Y_{t+k|t}\right) \right] \right\},$$

with $Y_{t+k|t} \equiv \left(\frac{\tilde{P}_{H,t}}{P_{H,t+k}}\right)^{-\varepsilon} Y_{t+k}$ and $\Lambda_{t,t+k} \equiv \mathrm{Q}_{t,t+k}\left(\frac{P_{t+k}}{P_t}\right) = \beta^k \left(\frac{U_{c,t}^{-1}}{U_{c,t+k}^{-1}} \frac{Z_{t+1}}{Z_t}\right)$ being the real stochastic discount factor where $\mathrm{Q}_{t,t+k}$ denotes the price of a one period discount bond paying off one unit of domestic currency. The previous expression can be rewritten as :

$$\max_{\tilde{P}_{H,t}} \left\{ \begin{array}{l} \Lambda_{t,t}\left(\frac{1}{P_t}\right)\left[\tilde{P}_{H,t}\left(\frac{\tilde{P}_{H,t}}{P_{H,t}}\right)^{-\varepsilon} Y_t - \mathbb{C}_t\left(\left(\frac{\tilde{P}_{H,t}}{P_{H,t}}\right)^{-\varepsilon} Y_t\right)\right] + \theta \Lambda_{t,t+1}\left(\frac{1}{P_{t+1}}\right)\left[\tilde{P}_{H,t}\left(\frac{\tilde{P}_{H,t}}{P_{H,t+1}}\right)^{-\varepsilon} Y_{t+1} - \mathbb{C}_{t+1}\left(\left(\frac{\tilde{P}_{H,t}}{P_{H,t+1}}\right)^{-\varepsilon} Y_{t+1}\right)\right] \\ +\theta^2 \Lambda_{t,t+2}\left(\frac{1}{P_{t+2}}\right)\left[\tilde{P}_{H,t}\left(\frac{\tilde{P}_{H,t}}{P_{H,t+2}}\right)^{-\varepsilon} Y_{t+2} - \mathbb{C}_{t+2}\left(\left(\frac{\tilde{P}_{H,t}}{P_{H,t+2}}\right)^{-\varepsilon} Y_{t+2}\right)\right] + \cdots \end{array} \right\}$$

The FONC for firms is given by:

$$\Lambda_{t,t}\left(\frac{1}{P_t}\right)\left[(1-\varepsilon)\tilde{P}_{H,t}^{-\varepsilon}P_{H,t}^{\varepsilon}Y_t - MC_{t|t}^n(-\varepsilon)\tilde{P}_{H,t}^{-\varepsilon-1}P_{H,t}^{\varepsilon}Y_t\right]$$
$$+\theta\Lambda_{t,t+1}\left(\frac{1}{P_{t+1}}\right)\left[(1-\varepsilon)\tilde{P}_{H,t}^{-\varepsilon}P_{H,t+1}^{\varepsilon}Y_{t+1} - MC_{t+1|t}^n(-\varepsilon)\tilde{P}_{H,t}^{-\varepsilon-1}P_{H,t+1}^{\varepsilon}Y_{t+1}\right] \quad ,$$
$$+\theta^2\Lambda_{t,t+2}\left(\frac{1}{P_{t+2}}\right)\left[(1-\varepsilon)\tilde{P}_{H,t}^{-\varepsilon}P_{H,t+2}^{\varepsilon}Y_{t+2} - MC_{t+2|t}^n(-\varepsilon)\tilde{P}_{H,t}^{-\varepsilon-1}P_{H,t+2}^{\varepsilon}Y_{t+2}\right]+\cdots=0$$

which can be rewritten as:

$$\Lambda_{t,t}\left(\frac{1}{P_t}\right)\left[\tilde{P}_{H,t}\left(\frac{\tilde{P}_{H,t}}{P_{H,t}}\right)^{-\varepsilon}Y_t - \frac{\varepsilon}{\varepsilon-1}MC_{t|t}^n\left(\frac{\tilde{P}_{H,t}}{P_{H,t}}\right)^{-\varepsilon}Y_t\right]$$
$$+\theta\Lambda_{t,t+1}\left(\frac{1}{P_{t+1}}\right)\left[\tilde{P}_{H,t}\left(\frac{\tilde{P}_{H,t}}{P_{H,t+1}}\right)^{-\varepsilon}Y_{t+1} - \frac{\varepsilon}{\varepsilon-1}MC_{t+1|t}^n\left(\frac{\tilde{P}_{H,t}}{P_{H,t+1}}\right)^{-\varepsilon}Y_{t+1}\right] \quad ,$$
$$+\theta^2\Lambda_{t,t+2}\left(\frac{1}{P_{t+2}}\right)\left[\tilde{P}_{H,t}\left(\frac{\tilde{P}_{H,t}}{P_{H,t+2}}\right)^{-\varepsilon}Y_{t+2} - \frac{\varepsilon}{\varepsilon-1}MC_{t+2|t}^n\left(\frac{\tilde{P}_{H,t}}{P_{H,t+2}}\right)^{-\varepsilon}Y_{t+2}\right]+\cdots=0$$

with $MC_{t+k|t}^n \equiv \mathbb{C}'_{t+k}\left(Y_{t+k|t}\right)$ being the nominal marginal cost.

By using the definition $Y_{t+k|t} \equiv \left(\frac{\tilde{P}_{H,t}}{P_{H,t+k}}\right)^{-\varepsilon} Y_{t+k}$, the previous expression can be rewritten as:

$$\Lambda_{t,t}\left(\frac{1}{P_t}\right)\left[\tilde{P}_{H,t}Y_{t|t} - \frac{\varepsilon}{\varepsilon-1}MC_{t|t}^n Y_{t|t}\right]$$
$$+\theta\Lambda_{t,t+1}\left(\frac{1}{P_{t+1}}\right)\left[\tilde{P}_{H,t}Y_{t+1|t} - \frac{\varepsilon}{\varepsilon-1}MC_{t+1|t}^n Y_{t+1|t}\right] \quad ,$$
$$+\theta^2\Lambda_{t,t+2}\left(\frac{1}{P_{t+2}}\right)\left[\tilde{P}_{H,t}Y_{t+2|t} - \frac{\varepsilon}{\varepsilon-1}MC_{t+2|t}^n Y_{t+2|t}\right]+\cdots=0$$

which can be rewritten as:

$$
\begin{aligned}
&\Lambda_{t,t}\left(\frac{1}{P_t}\right)Y_{t|t}\left(\tilde{P}_{H,t}-\frac{\varepsilon}{\varepsilon-1}MC^n_{t|t}\right)\\
&+\theta\Lambda_{t,t+1}\left(\frac{1}{P_{t+1}}\right)Y_{t+1|t}\left(\tilde{P}_{H,t}-\frac{\varepsilon}{\varepsilon-1}MC^n_{t+1|t}\right)\\
&+\theta^2\Lambda_{t,t+2}\left(\frac{1}{P_{t+2}}\right)Y_{t+2|t}\left(\tilde{P}_{H,t}-\frac{\varepsilon}{\varepsilon-1}MC^n_{t+2|t}\right)+\cdots=0
\end{aligned}
.
$$

The previous expression can be compact expression as:

$$
\sum_{k=0}^{\infty}\theta^k \mathrm{E}_t\left[\Lambda_{t,t+k}\left(\frac{1}{P_{t+k}}\right)Y_{t+k|t}\left(\tilde{P}_{H,t}-\frac{\varepsilon}{\varepsilon-1}MC^n_{t+k|t}\right)\right]=0\text{, (1-22)}
$$

which is identical with Eq.(15) in the text.

Nominal marginal cost is given by:

$$
MC^n_{t+k|t}=\frac{W_{t+k}}{MPN_{t+k|t}},
$$

with $MPN_{t+k}\equiv\frac{\partial Y_{t+k}}{\partial N_{t+k}}$ and $MPN_{t+k|t}\equiv\frac{\partial Y_{t+k}}{\partial N_{t+k|t}}$.

Plugging the definition of the real marginal cost into Eq.(1-22) yields:

$$
\sum_{k=0}^{\infty}(\theta\beta)^k \mathrm{E}_t\left[\left(\frac{U^{-1}_{c,t}}{U^{-1}_{c,t+k}}\frac{Z_{t+1}}{Z_t}\right)\left(\frac{1}{P_{t+k}}\right)Y_{t+k|t}\left(\tilde{P}_{H,t}-\frac{\varepsilon}{\varepsilon-1}MC^n_{t+k|t}\right)\right]=0\,.
$$

By multiplying $U_{c,t}Z_t$ both sides of the previous expression yields:

$$
\sum_{k=0}^{\infty}(\theta\beta)^k \mathrm{E}_t\left[\left(\frac{1}{P_{t+k}U^{-1}_{c,t+k}}\right)Z_{t+1}Y_{t+k|t}\left(\tilde{P}_{H,t}-\frac{\varepsilon}{\varepsilon-1}MC^n_{t+k|t}\right)\right]=0\,,
$$

which can be rewritten as:

$$
\sum_{k=0}^{\infty}(\theta\beta)^k \mathrm{E}_t\left[\left(\frac{1}{P_{t+k}U^{-1}_{c,t+k}}\right)Z_{t+1}Y_{t+k|t}\left(\frac{\tilde{P}_{H,t}}{P_{H,t-1}}-\frac{\varepsilon}{\varepsilon-1}\frac{MC^n_{t+k|t}}{P_{H,t+k}}\frac{P_{H,t+k}}{P_{H,t-1}}\right)\right]=0\,.
$$

Let define $MC_{t+k|t}\equiv\frac{MC^n_{t+k|t}}{P_{H,t+k}}$ and $\Pi_{H,t-1,t+k}\equiv\frac{P_{H,t+k}}{P_{H,t-1}}$. Then, the previous expression can be rewritten as:

$$
\sum_{k=0}^{\infty}(\theta\beta)^k \mathrm{E}_t\left[\left(\frac{1}{P_{t+k}U^{-1}_{c,t+k}}\right)Z_{t+1}Y_{t+k|t}\left(\frac{\tilde{P}_{H,t}}{P_{H,t-1}}-\frac{\varepsilon}{\varepsilon-1}MC_{t+k|t}\Pi_{H,t-1,t+k}\right)\right]=0\,.
$$

Let define $\tilde{X}_{H,t} \equiv \frac{\tilde{P}_{H,t}}{P_{H,t-1}}$. Then the previous expression can be rewritten as:

$\sum_{k=0}^{\infty} (\theta\beta)^k \mathrm{E}_t \left\{ \left[\left(U_{c,t+k}^{-1} \right)^{-1} Z_{t+k} \right] Y_{t+k|t} \frac{P_{H,t-1}}{P_{t+k}} \left(\tilde{X}_{H,t} - \frac{\varepsilon}{\varepsilon-1} \Pi_{H,t-1,t+k} MC_{t+k|t} \right) \right\} = 0$. Note that $P_{H,t-1}$ is multiplied on both sides of the previous expression. The previous expression can be rewritten as:
which can be rewritten as:

$$\begin{aligned}
&\left[\left(U_{c,t}^{-1} \right)^{-1} Z_t \right] Y_{t|t} \frac{P_{H,t}}{P_t} \frac{P_{H,t-1}}{P_{H,t}} \left(\tilde{X}_{H,t} - \frac{\varepsilon}{\varepsilon-1} \frac{P_{H,t}}{P_{H,t-1}} MC_{t|t} \right) \\
&+\theta\beta \left[\left(U_{c,t+1}^{-1} \right)^{-1} Z_{t+1} \right] Y_{t+1|t} \frac{P_{H,t+1}}{P_{t+1}} \frac{P_{H,t}}{P_{H,t+1}} \frac{P_{H,t-1}}{P_{H,t}} \left(\tilde{X}_{H,t} - \frac{\varepsilon}{\varepsilon-1} \frac{P_{H,t+1}}{P_{H,t}} \frac{P_{H,t}}{P_{H,t-1}} MC_{t+1|t} \right) \\
&+(\theta\beta)^2 \left[\left(U_{c,t+2}^{-1} \right)^{-1} Z_{t+2} \right] Y_{t+2|t} \frac{P_{H,t+2}}{P_{t+2}} \frac{P_{H,t+1}}{P_{H,t+2}} \frac{P_{H,t}}{P_{H,t+1}} \frac{P_{H,t-1}}{P_{H,t}} \left(\tilde{X}_{H,t} - \frac{\varepsilon}{\varepsilon-1} \frac{P_{H,t+2}}{P_{H,t+1}} \frac{P_{H,t+1}}{P_{H,t}} \frac{P_{H,t}}{P_{H,t-1}} MC_{t+2|t} \right) \\
&+\cdots = 0
\end{aligned},$$

which can be rewritten as:

$$\begin{aligned}
&\left[\left(U_{c,t}^{-1} \right)^{-1} Z_t \right] Y_{t|t} g(S_t)^{-1} \Pi_{H,t}^{-1} \left(\tilde{X}_{H,t} - \frac{\varepsilon}{\varepsilon-1} \Pi_{H,t} MC_{t|t} \right) \\
&+\theta\beta \left[\left(U_{c,t+1}^{-1} \right)^{-1} Z_{t+1} \right] Y_{t+1|t} g(S_{t+1})^{-1} \Pi_{H,t+1}^{-1} \Pi_{H,t}^{-1} \left(\tilde{X}_{H,t} - \frac{\varepsilon}{\varepsilon-1} \Pi_{H,t+1} \Pi_{H,t} MC_{t+1|t} \right) \\
&+(\theta\beta)^2 \left[\left(U_{c,t+2}^{-1} \right)^{-1} Z_{t+2} \right] Y_{t+2|t} g(S_{t+2})^{-1} \Pi_{H,t+2}^{-1} \Pi_{H,t+1}^{-1} \Pi_{H,t}^{-1} \begin{pmatrix} \tilde{X}_{H,t} \\ -\frac{\varepsilon}{\varepsilon-1} \Pi_{H,t+2} \Pi_{H,t+1} \Pi_{H,t} MC_{t+2|t} \end{pmatrix} \\
&+\cdots = 0
\end{aligned},$$

with $g(S_t) \equiv \frac{P_t}{P_{H,t}} = \frac{P_{H,t}^{1-\nu} P_{F,t}^{\nu}}{P_{H,t}} = \left(\frac{P_{F,t}}{P_{H,t}} \right)^{\nu} = S_t^{\nu}$. The previous expression can be rewritten as:

$$
\begin{aligned}
&\left[\left(U_{c,t}^{-1}\right)^{-1} Z_t\right] Y_{t|t} S_t^{-\nu} \Pi_{H,t}^{-1} \tilde{X}_{H,t} - \frac{\varepsilon}{\varepsilon-1}\left[\left(U_{c,t}^{-1}\right)^{-1} Z_t\right] Y_{t|t} S_t^{-\nu} MC_{t|t} \\
&+\theta\beta\left[\left(U_{c,t+1}^{-1}\right)^{-1} Z_{t+1}\right] Y_{t+1|t} S_{t+1}^{-\nu} \Pi_{H,t+1}^{-1} \Pi_{H,t}^{-1} \tilde{X}_{H,t} \\
&-\frac{\varepsilon}{\varepsilon-1}\theta\beta\left[\left(U_{c,t+1}^{-1}\right)^{-1} Z_{t+1}\right] Y_{t+1|t} S_{t+1}^{-\nu} MC_{t+1|t} \\
&+(\theta\beta)^2\left[\left(U_{c,t+2}^{-1}\right)^{-1} Z_{t+2}\right] Y_{t+2|t} S_{t+2}^{-\nu} \Pi_{H,t+2}^{-1} \Pi_{H,t+1}^{-1} \Pi_{H,t}^{-1} \tilde{X}_{H,t} \\
&-\frac{\varepsilon}{\varepsilon-1}(\theta\beta)^2\left[\left(U_{c,t+2}^{-1}\right)^{-1} Z_{t+2}\right] Y_{t+2|t} S_{t+2}^{-\nu} MC_{t+2|t} \\
&+\cdots = 0
\end{aligned}
.
$$

By moving the terms related to the marginal cost to the RHS yields:

$$
\begin{aligned}
&\left[\left(U_{c,t}^{-1}\right)^{-1} Z_t\right] Y_{t|t} S_t^{-\nu} \Pi_{H,t}^{-1} \tilde{X}_{H,t} \\
&+\theta\beta\left[\left(U_{c,t+1}^{-1}\right)^{-1} Z_{t+1}\right] Y_{t+1|t} S_{t+1}^{-\nu} \Pi_{H,t+1}^{-1} \Pi_{H,t}^{-1} \tilde{X}_{H,t} \\
&+(\theta\beta)^2\left[\left(U_{c,t+2}^{-1}\right)^{-1} Z_{t+2}\right] Y_{t+2|t} S_{t+2}^{-\nu} \Pi_{H,t+2}^{-1} \Pi_{H,t+1}^{-1} \Pi_{H,t}^{-1} \tilde{X}_{H,t} \\
&+\cdots \\
= \; &\frac{\varepsilon}{\varepsilon-1}\left[\left(U_{c,t}^{-1}\right)^{-1} Z_t\right] Y_{t|t} S_t^{-\nu} MC_{t|t} \\
&+\frac{\varepsilon}{\varepsilon-1}\theta\beta\left[\left(U_{c,t+1}^{-1}\right)^{-1} Z_{t+1}\right] Y_{t+1|t} S_{t+1}^{-\nu} MC_{t+1|t} \\
&+\frac{\varepsilon}{\varepsilon-1}(\theta\beta)^2\left[\left(U_{c,t+2}^{-1}\right)^{-1} Z_{t+2}\right] Y_{t+2|t} S_{t+2}^{-\nu} MC_{t+2|t} \\
&+\cdots
\end{aligned}
$$

which can be simplified as follows:

$$
\begin{aligned}
&\tilde{X}_{H,t}\left\{
\begin{aligned}
&\left[\left(U_{c,t}^{-1}\right)^{-1} Z_t\right] Y_{t|t} S_t^{-\nu} \Pi_{H,t}^{-1} \\
&+\theta\beta\left[\left(U_{c,t+1}^{-1}\right)^{-1} Z_{t+1}\right] Y_{t+1|t} S_{t+1}^{-\nu} \Pi_{H,t+1}^{-1} \Pi_{H,t}^{-1} \\
&+(\theta\beta)^2\left[\left(U_{c,t+2}^{-1}\right)^{-1} Z_{t+2}\right] Y_{t+2|t} S_{t+2}^{-\nu} \Pi_{H,t+2}^{-1} \Pi_{H,t+1}^{-1} \Pi_{H,t}^{-1} + \cdots
\end{aligned}
\right\} \\
&= \frac{\varepsilon}{\varepsilon-1}\left\{
\begin{aligned}
&\left[\left(U_{c,t}^{-1}\right)^{-1} Z_t\right] Y_{t|t} S_t^{-\nu} MC_{t|t} \\
&+\theta\beta\left[\left(U_{c,t+1}^{-1}\right)^{-1} Z_{t+1}\right] Y_{t+1|t} S_{t+1}^{-\nu} MC_{t+1|t} \\
&+(\theta\beta)^2\left[\left(U_{c,t+2}^{-1}\right)^{-1} Z_{t+2}\right] Y_{t+2|t} S_{t+2}^{-\nu} MC_{t+2|t} + \cdots
\end{aligned}
\right\}
\end{aligned}
.
$$

Then, we have:

$$\tilde{X}_{H,t}\sum_{k=0}^{\infty}(\theta\beta)^k\left[\left(U_{c,t+k}^{-1}\right)^{-1}Z_{t+k}\right]Y_{t+k|t}S_{t+k}^{-\nu}\prod_{h=0}^{k}\Pi_{H,t+h}^{-1}=\frac{\varepsilon}{\varepsilon-1}\sum_{k=0}^{\infty}(\theta\beta)^k\left[\left(U_{c,t+k}^{-1}\right)^{-1}Z_{t+k}\right]Y_{t+k|t}S_{t+k}^{-\nu}MC_{t+k|t},$$

or:

$$\tilde{X}_{H,t}=\frac{\frac{\varepsilon}{\varepsilon-1}\sum_{k=0}^{\infty}(\theta\beta)^k\left[\left(U_{c,t+k}^{-1}\right)^{-1}Z_{t+k}\right]Y_{t+k|t}g\left(S_{t+k}\right)^{-1}MC_{t+k|t}}{\sum_{k=0}^{\infty}(\theta\beta)^k\left[\left(U_{c,t+k}^{-1}\right)^{-1}Z_{t+k}\right]Y_{t+k|t}g\left(S_{t+k}\right)^{-1}\prod_{h=0}^{k}\Pi_{H,t+h}^{-1}}. \text{ (1-23)}$$

Plugging

$$\begin{aligned}MC_{t+k|t}^{n}&=\frac{W_{t+k}}{MPN_{t+k|t}}\\&=W_{t+k}\left(\frac{\partial Y_{t+k}}{\partial N_{t+k|t}}\right)^{-1}\\&=W_{t+k}\left[(1-\alpha)N_{t+k|t}^{-\alpha}\right]^{-1}\\&=\frac{W_{t+k}}{1-\alpha}N_{t+k|t}^{\alpha}\end{aligned},$$

into the definition of the marginal cost yields:

$$MC_{t+k}\equiv\frac{W_{t+k}}{P_{H,t+k}(1-\alpha)}N_{t+k}^{\alpha}. \text{ (1-24)}$$

1.5 Market Clearing Condition

The market clearing conditions in the SOE and the ROW are given by:

$$Y_t(j)=C_{H,t}(j)+EX_t(j)+G_t(j), \text{ (1-25)}$$

where $EX_t(j)$ is export demand for the good produced by firm *j* and $G_t(j)$ denotes the government purchase for the good produced by firm *j*, respectively.

Combining $G_t\equiv\left[\int_0^1 G_t(j)^{\frac{\varepsilon-1}{\varepsilon}}dj\right]^{\frac{\varepsilon}{\varepsilon-1}}$ with the definitions of the PPIs yields:

$$G_t(j)=\left(\frac{P_t(j)}{P_{H,t}}\right)^{-\varepsilon}G_{H,t}. \text{ (1-26)}$$

Similar to Eq.(1-12), we assume that the demands for $C_{H,t}^{*}(j)$ follows as:

$$EX_t(j) = \left(\frac{P^*_{H,t}(j)}{P^*_{H,t}}\right)^{-\varepsilon} EX_t \text{ , (1-27)}$$

Analogous to Eq.(1-18), demands for domestic goods in foreign country is assumed as:

$$\begin{aligned} EX_t &= \nu \left(\frac{P^*_{H,t}}{P^*_t}\right)^{-1} Y^*_t \\ &= \nu \left(\frac{P^*_{H,t}}{P^*_{F,t}}\right)^{-1} Y^*_t \text{ , (1-28)} \\ &= \nu S_t Y^*_t \end{aligned}$$

which is identical with Eq.(17) in the text.

Plugging Eqs. (1-12), (1-20), (1-26) and (1-27) into Eq.(1-25) yields:

$$\begin{aligned} \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} Y_t &= \left(\frac{P_t(j)}{P_{H,t}}\right)^{-\varepsilon} C_{H,t} + \left(\frac{P^*_{H,t}(j)}{P^*_{H,t}}\right)^{-\varepsilon} EX_t + \left(\frac{P_t(j)}{P_{H,t}}\right)^{-\varepsilon} G_t \\ &= \left(\frac{P_t(j)}{P_{H,t}}\right)^{-\varepsilon} \left(C_{H,t} + EX_t + G_t\right) \end{aligned},$$

where we use the LOOP implying that $P^*_{H,t}(j) = \frac{P_{H,t}(j)}{E_t}$ and $P^*_{H,t} = \frac{P_{H,t}}{E_t}$. By dividing both sides of the previous expression $\left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon}$ yields as follows:

$$Y_t = C_{H,t} + EX_t + G_t.$$

Plugging Eqs.(1-17) and (1-28) into the previous expression yields:

$$Y_t = (1-\nu) S_t^{\nu} C_t + \nu S_t Y^*_t + G_t \text{ , (1-29)}$$

where we apply the definition of the TOT $S_t \equiv \frac{P_{F,t}}{P_{H,t}}$ and $Y^*_t = C^*_t$. Eq.(1-29) is identical with Eq(21) in the text.

1.6 Government Budget Constraint

The government budget constraint is given by:

$$P_{H,t} G_t + B_{t-1}(1 + i_{t-1}) = P_t TR_t + B_t + \Delta M_t \text{ , (1-30)}$$

which is identical with Eq.(18) in the text.

Dividing both side of Eq.(1-30) by P_t yields:

$$\frac{P_{H,t}}{P_t}G_t + \mathrm{B}_{t-1}\left(1+i_{t-1}\right)\frac{P_{t-1}}{P_t} = TR_t + \mathrm{B}_t + \frac{\Delta M_t}{P_t}.$$

Let define the (ex-post) real interest rate $\mathrm{P}_t \equiv \left(1+i_t\right)\frac{P_t}{P_{t+1}}$. Then the previous expression can be rewritten as:

$$S_t^{-\nu}G_t + \mathrm{B}_{t-1}\mathrm{P}_{t-1} = T_t + \mathrm{B}_t + \frac{\Delta M_t}{P_t}, \text{ (1-31)}$$

where we use $\frac{P_{H,t}}{P_t} = \frac{P_{H,t}}{P_{H,t}^{1-\nu}P_{F,t}^{\nu}} = S_t^{-\nu}$. Eq(3-1) is identical with Eq.(20) in the text.

The level of seignorage, expressed as a fraction of steady state output can be approximated as:

$$\begin{aligned}\frac{\Delta M_t}{P_t}\frac{1}{Y} &= \frac{\Delta M_t}{P_t}\frac{M_{t-1}}{M_{t-1}}\frac{P_{t-1}}{P_{t-1}}\frac{1}{Y} \\ &= \frac{\Delta M_t}{M_{t-1}}\frac{M_{t-1}}{P_{t-1}}\frac{P_{t-1}}{P_t}\frac{1}{Y}. \text{ (1-32)} \\ &= \frac{\Delta M_t}{M_{t-1}}\frac{P_{t-1}}{P_t}L_{t-1}\frac{1}{Y}\end{aligned}$$

Quantity theory of money implies as follows:

$MV = PY$,

which can be rewritten as:

$$V^{-1} = \frac{L}{Y}.$$

Plugging the previous expression into Eq.(1-32) yields:

$$\frac{\Delta M_t}{P_t}\frac{1}{Y} = \chi \Delta m_t, \text{ (1-33)}$$

with $\chi \equiv V^{-1}$ being the inverse of income velocity of money. Note that Eq.(1-33) ignore changes in the inflation and the deviation of the real money balance from its steady state.

If we do not ignore them, we have:

$$\begin{aligned}\frac{\Delta M_t}{P_t}\frac{1}{Y} &= \frac{\Delta M_t}{M_{t-1}}\frac{P_{t-1}}{P_t}L_{t-1}\frac{1}{Y} \\ &= \ln\left(\frac{M_t}{M_{t-1}}\right)\Pi_{t-1}^{-1}\frac{L_{t-1}}{L}\frac{L}{Y} \\ &= \chi \ln\left(\frac{M_t}{M_{t-1}}\right)\Pi_{t-1}^{-1}\frac{L_{t-1}}{L}\end{aligned}$$

1.7 Trade Balance

Similar to Gali and Monaceli (2005, RES), We define the real trade balance as follows:

$$\begin{aligned}\frac{NX_t}{P_{H,t}} &= Y_t - g(S_t)C_t - G_t \\ &= Y_t - S_t^{\nu} C_t - G_t\end{aligned}, \text{ (1-34)}$$

with NX_t being the (nominal) trade balance. Eq.(34) is identical with Eq.(22) in the text.

Note that:

$$g(S_t) = \frac{P_t}{P_{H,t}} = \frac{P_{H,t}^{1-\nu} P_{F,t}^{\nu}}{P_{H,t}} = S_t^{\nu}.$$

2 The Steady State

We focus on equilibria where the state variables follow paths that are close to a deterministic stationary equilibrium, in which $\Pi_{H,t} = \Pi_t = 1$. Further, we assume $Z_t = Z_t^* = 1$ and $G_t = 0$.

Eqs.(1-6) and (1-9) implies as follows:

$$\begin{aligned}\beta &= \frac{1}{1+i} \\ &= \frac{1}{1+i^*}\end{aligned}. \text{ (2-1)}$$

Eq.(1-7) implies that:

$$\frac{W}{P} = \frac{V_n}{U_c}. \text{ (2-2)}$$

Eq.(1-8) implies as follows:

$$\frac{U_l}{U_c} = \beta i. \text{ (2-3)}$$

Eq.(1-23) implies:

$$1 = \frac{\frac{\varepsilon}{\varepsilon-1}\left[1+\theta\beta+(\theta\beta)^2+\cdots\right]\left[\left(U_c^{-1}\right)^{-1}\right] Y g(S)^{-1} MC}{\left[1+\theta\beta+(\theta\beta)^2+\cdots\right]\left[\left(U_c^{-1}\right)^{-1}\right] Y g(S)^{-1}},$$

which can be rewritten as:

$MC = \mathrm{M}^{-1}$, (2-4)

with $\mathrm{M} \equiv \frac{\varepsilon}{\varepsilon - 1}$ being the constant markup.

Eq.(1-24) implies:

$$MC = \frac{1}{1-\alpha}\frac{W}{P_H}N^{\alpha}. \text{ (2-5).}$$

Eq.(1-16) can be rewritten as:

$$\frac{V_n}{U_c} = \frac{W}{P_H}\frac{P_H}{P}. \text{ (2-6)}$$

Plugging Eq.(2-5) into Eq.(2-6) yields:

$$\frac{V_n}{U_c} = \frac{1-\alpha}{N^{\alpha}\mathrm{M}}\frac{P_H}{P}. \text{ (2-7)}$$

Let define:

$$g(S) \equiv \frac{P}{P_H} = S^{\nu}. \text{ (2-8)}$$

Plugging Eq.(2-8) into Eq.(2-7) yields:

$$\frac{V_n}{U_c} = \frac{1-\alpha}{N^{\alpha}\mathrm{M}S^{\nu}},$$

which can be written as:

$$V_n = \frac{1-\alpha}{N^{\alpha}\mathrm{M}S^{\nu}}U_c. \text{ (2-9)}$$

Eq.(1-10) implies:

$$U_c^{-1} = \vartheta\left(U_c^*\right)^{-1} q(S), \text{ (2-10)}$$

with $q(S) \equiv Q$. Note that:

$$Q = \frac{EP^*}{P} = \frac{EP_F^*}{P_H^{1-\nu}P_F^{\nu}} = \frac{P_F}{P_H^{1-\nu}P_F^{\nu}} = \left(\frac{P_F}{P_H}\right)^{1-\nu} = S^{1-\nu}. \text{ (2-11)}$$

Eq.(2-10) can be rewritten as:

$$U_c^{-1} = \vartheta\left(U_c^*\right)^{-1} S^{1-\nu},$$

where we use Eq.(2-11). The previous expression can be rewritten as:

$$S^{\nu} = \vartheta\left(U_c^*\right)^{-1} SU_c.$$

Plugging the previous expression into Eq.(2-9) yields:

$$V_n = \frac{1-\alpha}{N^{\alpha}} \frac{\mathrm{M}^{-1}}{S\vartheta\left(U_c^*\right)^{-1}}. \text{ (2-12)}$$

Let define $H\left(S,U_c^*\right) \equiv V_n N^{\alpha}$. Plugging this definition into Eq.(2-12) yields:

$$H\left(S,U_c^*\right) \equiv (1-\alpha) \frac{\mathrm{M}^{-1}}{S\vartheta\left(U_c^*\right)^{-1}}.$$

Notice that $H_S < 0$, $\lim_{S\to 0} H\left(S,U_c^*\right) = +\infty$ and $\lim_{S\to\infty} H\left(S,U_c^*\right) = 0$.

On the other hand, the market clearing Eq.(1-29) implies:

$$Y = (1-\nu) S^{\nu} C + \nu S Y^*. \text{ (2-13)}$$

Because of $C = F\left(U_c^{-1}\right)$ and Eq.(2-10), we have:

$$\begin{aligned} C &= F\left[\vartheta\left(U_c^*\right)^{-1} q(S)\right] \\ &= F\left[\vartheta\left(U_c^*\right)^{-1} S^{1-\nu}\right], \end{aligned}$$

with *F* being the operator of function.

Plugging the previous expression into Eq.(2-13) yields:

$$Y = (1-\nu) S^{\nu} F\left[\vartheta\left(U_c^*\right)^{-1} S^{1-\nu}\right] + \nu S C^*. \text{ (2-14)}$$

Let define $J\left(S,C^*\right) \equiv (1-\nu) S^{\nu} F\left[\vartheta\left(U_c^*\right)^{-1} S^{1-\nu}\right] + \nu S Y^*$. Note that $J_S > 0$, $\lim_{S\to 0} J\left(S,C^*\right) = 0$ and $\lim_{S\to\infty} J\left(S,C^*\right) = +\infty$.

Hence, given a value for C^*, ϑ and Y^*, Eqs.(2-12) and (2-14), jointly determine the steady state value for *S* and *q*(*S*), i.e., the steady state value of the TOT and the real exchange rate (Figure TA-1). This way to show how the TOT as well as the real exchange rate is pinned down in the steady state is almost same as Gali and Monacelli (2002, NBER-WP).

Dividing both sides of Eq.(2-13) by C^* yields:

$$\frac{Y}{C^*} = (1-\nu) S^{\nu} \frac{C}{C^*} + \nu S.$$

For convenience, and without loss of generality, we can assume that initial conditions (i.e., initial distribution of wealth) are such that $\vartheta = 1$ which implies that $Q = \frac{C}{C^*}$.

Plugging this condition into the previous expression yields:

$$\frac{Y}{C^*} = (1-\nu)S^{\nu}Q + \nu S$$
$$= (1-\nu)S^{\nu}S^{1-\nu} + \nu S,$$
$$= S$$

which can be rewritten as:

$Y = SY^*$, (2-15)

by using $Y^* = C^*$ which is the steady state market clearing condition in the foreign country.

Let assume symmetric labor market in the foreign country. Then, following condition is applicable:

$$\frac{V_n^*}{U_c^*} = \frac{1-\alpha}{(N^*)^{\alpha}\,\mathrm{M}}, \text{ (2-16)}$$

similar to Eq.(2-7).

Dividing Eq.(2-16) by Eq.(2-9) yields:

$$\frac{V_n^*}{V_n}\left(\frac{N^*}{N}\right)^{\alpha} = \frac{U_c^*}{U_c}S^{\nu}. \text{ (2-17)}$$

Combining Eq.(2-10) and the initial condition yields:

$$\frac{U_c^*}{U_c} = S^{1-\nu}, \text{ (2-18)}$$

where we use $Q = S^{1-\nu}$. Plugging Eq.(2-18) into Eq.(2-17) yields:

$$\frac{V_n^*}{V_n}\left(\frac{N^*}{N}\right)^{\alpha} = S. \text{ (2-19)}$$

In the steady state, Eq.(14) in the text implies as follows:

$N^{1-\alpha} = Y$.

Plugging the previous expression into Eq.(2-19) yields:

$$\frac{V_n^*}{V_n}\left(\frac{Y^*}{Y}\right)^{\frac{\alpha}{1-\alpha}} = S,$$

where we use the foreign country has production technology identical to Eq.(14).

Plugging Eq.(2-15) into the previous expression yields:

$$\frac{V_n^*}{V_n} = S^{\frac{1}{1-\alpha}}.$$

Let multiply $\left(U_c^*\right)^{-1}$ on both sides of the previous expression. Then, we have:

$$\frac{V_n^*}{U_c^*} = S^{\frac{1-(1-\alpha)(1-\nu)}{1-\alpha}} \frac{V_n}{U_c}, \text{ (2-20)}$$

where we use Eq.(2-18). Let define $\frac{V_n}{U_c} \equiv 1-\Phi$ where Φ denotes the steady-state wedge between the marginal rate of substitution between consumption and leisure and the marginal product of labor (See Benigno and Woodford, 2005). We assume this steady state wedge is common throughout the world, i.e., $\frac{V_n}{U_c} \equiv 1-\Phi = \frac{V_n^*}{U_c^*}$.

Then, Eq.(2-20) boils down to:

$$S = 1, \text{ (2-21)}$$

which implies that the PPP is applicable in the long run.

Plugging Eq.(2-21) into Eq.(2-15) yields:

$$Y = Y^*. \text{ (2-22)}$$

Plugging Eq.(2-21) into the initial condition yields:

$$C = C^*.$$

Combining the previous expression, the steady state market clearing condition in the foreign country $Y^* = C^*$ and Eq.(2-22) yields:

$$Y = C.$$

Finally, Eq.(1-22) implies:

$$P = \frac{\varepsilon}{\varepsilon-1} MC^n, \text{ (2-16)}$$

which can be rewritten as:

$$\left(\frac{\varepsilon}{\varepsilon-1}\right)^{-1} = MC,$$

which corresponds to Eq.(2-4).

3 Log-linearization of the Model

3.1 Log-linearizing the International Risk Sharing Condition

By raising both sides of Eq.(1-10) to the power of -1, we get:

$$U_{c,t} = U^*_{c,t} Q_t^{-1} \vartheta^{-1} \frac{Z^*_t}{Z_t},$$

which can be rewritten as:

$$Q_t = U_{c,t}^{-1} U^*_{c,t} \vartheta^{-1} \frac{Z^*_t}{Z_t}.$$

Total derivative of the previous expression yields:

$$\begin{aligned} dQ_t &= -\vartheta^{-1} U^*_c U_c^{-2} U_{cc} dC_t + \vartheta^{-1} U_c^{-1} U^*_{cc} dC^*_t + U_c^{-1} U^*_c \vartheta^{-1} dZ^*_t + U_c^{-1} U^*_c \vartheta^{-1} (-1) dZ_t \\ &= -\vartheta^{-1} \frac{U^*_c}{U_c} \frac{U_{cc}}{U_c} C \frac{dC_t}{C} + \vartheta^{-1} \frac{U^*_c}{U_c} \frac{U^*_{cc}}{U^*_c} C^* \frac{dC^*_t}{C^*} + \vartheta^{-1} \frac{U^*_c}{U_c} dZ^*_t - \vartheta^{-1} \frac{U^*_c}{U_c} dZ_t \\ &= \vartheta^{-1} \frac{U^*_c}{U_c} \left(-\frac{U_{cc}}{U_c} C \right) \frac{dC_t}{C} - \vartheta^{-1} \frac{U^*_c}{U_c} \left(-\frac{U^*_{cc}}{U^*_c} C^* \right) \frac{dC^*_t}{C^*} + \vartheta^{-1} \frac{U^*_c}{U_c} dZ^*_t - \vartheta^{-1} \frac{U^*_c}{U_c} dZ_t \end{aligned}.$$

Let define $\sigma \equiv -\frac{U_{cc}}{U_c} C$. Then:

$$\log Q_t = \sigma \log\left(\frac{C_t}{C}\right) - \sigma \log\left(\frac{C^*_t}{C^*}\right) - \left[-\log\left(\frac{Z^*_t}{Z_t}\right) \right],$$

where we use the fact that $C = C^*$. The previous expression can be rewritten as:

$q_t = \sigma \hat{c}_t - \sigma \hat{c}^*_t - \zeta_t$, (3-1)

where $q_t \equiv \log Q_t$, $\hat{c}_t \equiv \log\left(\frac{C_t}{C}\right)$, $\hat{c}^*_t \equiv \log\left(\frac{C^*_t}{C^*}\right)$ and $\zeta_t \equiv -\log\left(\frac{Z^*_t}{Z_t}\right)$.

Total derivative of the definition of the real exchange rate is given by:

$$dQ_t = \frac{dP_{F,t}}{P_F} - \frac{dP_t}{P},$$

which can be rewritten as:

$q_t = p_{F,t} - p_t$ (3-2).

Dividing both sides of the previous expression by P yields:

$$\log\left(\frac{P_t}{P}\right)=(1-\nu)\log\left(\frac{P_{H,t}}{P_H}\right)+\nu\log\left(\frac{P_{F,t}}{P_F}\right),$$

which can be rewritten as:

$p_t=(1-\nu)p_{H,t}+\nu p_{F,t}$. (3-3)

Plugging Eq.(3-3) into Eq.(3-2) yields:

$$\begin{aligned} q_t &= p_{F,t}-\left[(1-\nu)p_{H,t}+\nu p_{F,t}\right] \\ &=(1-\nu)\left(p_{F,t}-p_{H,t}\right) \end{aligned}. \text{ (3-4)}$$

Total derivative of the definition of the TOT is given by:

$$\begin{aligned} dS_t &= \frac{dP_{F,t}}{P_H}-P_F P_H^{-2}dP_{H,t} \\ &=\frac{dP_{F,t}}{P_F}-\frac{dP_{H,t}}{P_H} \end{aligned},$$

which can be rewritten as:

$$\log S_t=\log\left(\frac{P_{F,t}}{P_F}\right)-\log\left(\frac{P_{H,t}}{P_H}\right).$$

Then, we have:

$s_t=p_{F,t}-p_{H,t}$. (3-5)

Plugging Eq.(3-5) into Eq.(3-4) yields:

$q_t=(1-\nu)s_t$. (3-6)

Plugging Eq.(3-6) into Eq.(3-1) yields:

$$\hat{c}_t=\hat{c}_t^*+\frac{1-\nu}{\sigma}s_t+\frac{1}{\sigma}\zeta_t,$$

which is log-linearized international risk sharing condition. Plugging the log-linearized market clearing in the foreign country $\hat{y}_t^*=\hat{c}_t^*$ into the previous expression yields:

$\hat{c}_t=\hat{y}_t^*+\frac{1-\nu}{\sigma}s_t+\frac{1}{\sigma}\zeta_t$, (3-7)

which is identical with Eq.(23) in the text.

3.2 Log-linearizing the Market Clearing Condition

Eq.(1-29) can be rewritten as:

$Y_t=(1-\nu)S_t^{\nu}C_t+\nu S_t C_t^*+G_t$, (3-8)

Total derivative of Eq.(3-8) yields:

$dY_t = \left[(1-\nu)C\nu + \nu C^*\right]dS_t + (1-\nu)dC_t + \nu dC_t^* + dG_t$. (3-9)

By dividing both sides of Eq.(3-9) by Y yields:

$$\log\left(\frac{Y_t}{Y}\right) = \left[(1-\nu)\nu + \nu\right]\log S_t + (1-\nu)\log\left(\frac{C_t}{C}\right) + \nu\log\left(\frac{C_t^*}{C}\right) + \log\left(\frac{G_t}{Y}\right)$$
$$= \nu(2-\nu)\log S_t + (1-\nu)\log\left(\frac{C_t}{C}\right) + \nu\log\left(\frac{C_t^*}{C}\right) + \log\left(\frac{G_t}{Y}\right),$$

where we use the fact that $Y = C = Y^* = C^*$. The previous expression can be rewritten as:

$\hat{y}_t = \nu(2-\nu)s_t + (1-\nu)\hat{c}_t + \nu\hat{y}_t^* + \hat{g}_t$, (3-10)

where we use Eq.(3-9) to derive Eq.(3-10). Eq.(3-10) is identical with Eq.(24) in the text.

3.3 Log-linearizing Euler Equation

Total derivative of Eq.(1-6) is given by:

$$dU_{c,t} = U_c\beta d(1+i_t) + U_c(-1)d\Pi_{t+1} + dU_{c,t+1} - U_c dZ_t + U_c dZ_{t+1}.$$

Note that $\beta = (1+i)^{-1}$ and $i=\rho$. Thus:

$$dU_{c,t} = U_c\beta\frac{d(1+i_t)}{1+\rho} + U_c(-1)d\Pi_{t+1} + dU_{c,t+1} - U_c dZ_t + U_c dZ_{t+1}.$$

Dividing both sides of the previous expression by U_c yields:

$$\frac{dU_{c,t}}{U_c} = \frac{d(1+i_t)}{1+\rho} - d\Pi_{t+1} + \frac{dU_{c,t+1}}{U_c} - \left[-(dZ_{t+1} - dZ_t)\right],$$

which can be rewritten as:

$$\log\left(\frac{U_{c,t}}{U_c}\right) = \log\left(\frac{1+i_t}{1+\rho}\right) - \log\Pi_{t+1} + \log\left(\frac{U_{c,t+1}}{U_c}\right) - \left[-\log\left(\frac{Z_{t+1}}{Z_t}\right)\right].$$

Let define $\hat{\xi}_t \equiv \log\left(\frac{U_{c,t}}{U_c}\right)$, $\hat{i}_t \equiv \log\left(\frac{1+i_t}{1+\rho}\right)$, $\pi_t \equiv \log\Pi_t$ and $\hat{\rho}_t \equiv -\log\left(\frac{Z_{t+1}}{Z_t}\right)$. Then, the previous expression can be rewritten as:

$\hat{\xi}_t = \hat{\xi}_{t+1} + \hat{i}_t - \pi_{t+1} - \hat{\rho}_t$, (3-11)

which is a class of log-linearized Euler equation. Eq. (3-11) is identical with Eq(6-5) in the text.

3.4 Log-linearizing Marginal Utility of Consumption

Marginal utility of consumption can be depicted as:

$U_{c,t} = U_{l,t}\left(\frac{U_{l,t}}{U_{c,t}}\right)^{-1}$. (3-12)

Total derivative of Eq.(3-12) is given by:

$$
\begin{aligned}
dU_{c,t} &= \left\{U_{ll}\left(\frac{U_l}{U_c}\right)^{-1} + U_l(-1)\left(\frac{U_l}{U_c}\right)^{-2}\left[\frac{1}{U_c}U_{ll} + U_l(-1)U_c^{-2}\frac{\partial U_c}{\partial L}\right]\right\}dL_t \\
&\quad + \left\{U_{lc}\left(\frac{U_l}{U_c}\right)^{-1} + U_l(-1)\left(\frac{U_l}{U_c}\right)^{-2}\left[\frac{1}{U_c}U_{lc} + U_l(-1)U_c^{-2}\frac{\partial U_c}{\partial C}\right]\right\}dC_t \\
&= U_l^2\left(\frac{U_c}{U_l}\right)^2\frac{1}{U_c^2}U_{cl}L\frac{dL_t}{L} + \left[U_{lc}\frac{U_c}{U_l}C - U_l\left(\frac{U_c}{U_l}\right)^2\left(\frac{U_{lc}}{U_c}C - \frac{U_l}{U_c}\frac{U_{cc}}{U_c}C\right)\right]\frac{dC_t}{C} \\
&= U_{cl}L\frac{dL_t}{L} + \left[\frac{U_{lc}}{U_l}U_cC - U_c\left(\frac{U_{lc}}{U_l}C - \frac{U_{cc}}{U_c}C\right)\right]\frac{dC_t}{C}
\end{aligned}
$$

Dividing both sides of the previous expression by U_c yields:

$$
\begin{aligned}
\frac{dU_{c,t}}{U_c} &= \frac{U_{cl}}{U_c}L\frac{dL_t}{L} + \left(\frac{U_{lc}}{U_l}C - \frac{U_{lc}}{U_l}C + \frac{U_{cc}}{U_c}C\right)\frac{dC_t}{C} \\
&= \frac{U_{cl}}{U_c}L\frac{dL_t}{L} + \frac{U_{cc}}{U_c}C\frac{dC_t}{C}
\end{aligned},
$$

which can be rewritten as:

$\hat{\xi}_t = \upsilon\hat{l}_t - \sigma\hat{c}_t$,

which is identical with Eq(26) in the text.

3.5 Deriving the FONC for Domestic Producers

Total derivative of Eq.(1-23) yields:

$$
\begin{aligned}
d\tilde{X}_{H,t} &= \frac{\varepsilon}{\varepsilon-1}\left[1+\theta\beta+(\theta\beta)^2+\cdots\right]^{-1}\begin{bmatrix} dMC_{t|t} + \theta\beta dMC_{t+1|t} \\ +(\theta\beta)^2 dMC_{t+2|t} + \cdots \end{bmatrix} \\
&\quad + \frac{\varepsilon}{\varepsilon-1}(MC)\left[d\Pi_{H,t} + \theta\beta d\Pi_{H,t+1} + (\theta\beta)^2 d\Pi_{H,t+2} + \cdots\right]
\end{aligned}
$$

Note that $1+\theta\beta+(\theta\beta)^2+\cdots=\frac{1}{1-\theta\beta}$ and $MC=\frac{\varepsilon-1}{\varepsilon}$. Then, the previous expression can be rewritten as:

$$d\tilde{X}_{H,t}=(1-\theta\beta)\left[\frac{dMC_{t|t}}{MC}+\theta\beta\frac{dMC_{t+1|t}}{MC}+(\theta\beta)^2\frac{dMC_{t+2|t}}{MC}+\cdots\right],$$
$$+\left[d\Pi_{H,t}+\theta\beta d\mathrm{E}_t\left(\Pi_{H,t+1}\right)+(\theta\beta)^2 d\mathrm{E}_t\left(\Pi_{H,t+2}\right)+\cdots\right]$$

which can be rewritten as:

$$\tilde{p}_{H,t}-p_{H,t-1}=(1-\theta\beta)\left[\widehat{mc}_{t|t}+\theta\beta\widehat{mc}_{t+1|t}+(\theta\beta)^2\widehat{mc}_{t+2|t}+\cdots\right],$$
$$+\left[\pi_{H,t}+\theta\beta\pi_{H,t+1}+(\theta\beta)^2\pi_{H,t+2}+\cdots\right]$$

with $\widehat{mc}_{t+k|t}\equiv\log\left(\frac{MC_{t+k|t}}{MC}\right)=mc_{t+k|t}-mc$, $mc_t\equiv\log MC_t$ and $mc\equiv\log MC=-\log\left(\frac{\varepsilon-1}{\varepsilon}\right)$.

Previous expression can be rewritten as:

$$\begin{aligned}
\tilde{p}_{H,t}-p_{H,t-1}&=(1-\theta\beta)\left[\widehat{mc}_{t|t}+\theta\beta\widehat{mc}_{t+1|t}+(\theta\beta)^2\widehat{mc}_{t+2|t}+\cdots\right]\\
&\quad+\left[p_{H,t}-p_{H,t-1}+\theta\beta\left(p_{H,t+1}-p_{H,t}\right)+(\theta\beta)^2\left(p_{H,t+2}-p_{H,t+1}\right)+\cdots\right]\\
&=(1-\theta\beta)\left[\widehat{mc}_{t|t}+\theta\beta\widehat{mc}_{t+1|t}+(\theta\beta)^2\widehat{mc}_{t+2|t}+\cdots\right]\\
&\quad+\left[(1-\theta\beta)p_{H,t}+\theta\beta(1-\theta\beta)p_{H,t+1}+(\theta\beta)^2(1-\theta\beta)p_{H,t+2}+\cdots\right]-p_{H,t-1}\\
&=(1-\theta\beta)\left(\widehat{mc}_{t|t}+p_{H,t}\right)+\theta\beta(1-\theta\beta)\left(\widehat{mc}_{t+1|t}+p_{H,t+1}\right)\\
&\quad+(\theta\beta)^2(1-\theta\beta)\left(\widehat{mc}_{t+2|t}+p_{H,t+2}\right)+\cdots-p_{H,t-1}
\end{aligned}.$$

(3-13)

Note that:

$$\begin{aligned}
\widehat{mc}_{t+k|t}+p_{H,t+k}&=mc_{t+k|t}+p_{H,t+k}-\log MC\\
&=mc^n_{t+k|t}-\log MC\\
&=mc^n_{t+k|t}-\log\left(\frac{\varepsilon}{\varepsilon-1}\right)^{-1}\\
&=mc^n_{t+k|t}+\log\left(\frac{\varepsilon}{\varepsilon-1}\right)\\
&=mc^n_{t+k|t}+\mu
\end{aligned} \quad . \text{ (3-14)}$$

Plugging Eq.(3-14) into Eq.(3-13) yields:

$$\begin{aligned}\tilde{p}_{H,t}-p_{H,t-1}&=(1-\theta\beta)\left(mc^{n}_{t|t}+\mu\right)+\theta\beta(1-\theta\beta)\left(mc^{n}_{t+1|t}+\mu\right)\\&\quad+(\theta\beta)^{2}(1-\theta\beta)\left(mc^{n}_{t+2|t}+\mu\right)+\cdots-p_{H,t-1}\\&=(1-\theta\beta)\left[mc^{n}_{t|t}+\theta\beta mc^{n}_{t+1|t}+(\theta\beta)^{2}mc^{n}_{t+2|t}+\cdots\right]-p_{H,t-1}\\&\quad+(1-\theta\beta)\left[1+\theta\beta+(\theta\beta)^{2}+\cdots\right]\mu\\&=\mu+(1-\theta\beta)\left[mc^{n}_{t|t}+\theta\beta mc^{n}_{t+1|t}+(\theta\beta)^{2}mc^{n}_{t+2|t}+\cdots\right]-p_{H,t-1}\end{aligned},$$

which can be rewritten as:

$$\tilde{p}_{H,t}=\mu+(1-\theta\beta)\sum_{k=0}^{\infty}(\theta\beta)^{k}mc^{n}_{t+k|t}\,. \quad (3\text{-}15)$$

(Corresponding to Eq.11 in Chap. 3, Gali, 2015)

Eq.(3-13) can be rewritten as:

$$\begin{aligned}\tilde{p}_{H,t}-p_{H,t-1}&=(1-\theta\beta)\left[\widehat{mc}_{t|t}+\theta\beta\widehat{mc}_{t+1|t}+(\theta\beta)^{2}\widehat{mc}_{t+2|t}+\cdots\right]\\&\quad+\left[p_{H,t}-p_{H,t-1}+\theta\beta\left(p_{H,t+1}-p_{H,t}\right)+(\theta\beta)^{2}\left(p_{H,t+2}-p_{H,t+1}\right)+\cdots\right]\\&=(1-\theta\beta)\left[\widehat{mc}_{t|t}+\theta\beta\widehat{mc}_{t+1|t}+(\theta\beta)^{2}\widehat{mc}_{t+2|t}+\cdots\right]\\&\quad+\left[(1-\theta\beta)p_{H,t}+\theta\beta(1-\theta\beta)p_{H,t+1}+(\theta\beta)^{2}(1-\theta\beta)p_{H,t+2}+\cdots\right]-p_{H,t-1}\\&=(1-\theta\beta)\left(\widehat{mc}_{t|t}+p_{H,t}\right)+\theta\beta(1-\theta\beta)\left(\widehat{mc}_{t+1|t}+p_{H,t+1}\right)\\&\quad+(\theta\beta)^{2}(1-\theta\beta)\left(\widehat{mc}_{t+2|t}+p_{H,t+2}\right)+\cdots-p_{H,t-1}\end{aligned}$$

Eq.(3-13) can be rewritten as:

$$\tilde{p}_{H,t}-p_{H,t-1}=(1-\theta\beta)\sum_{k=0}^{\infty}(\theta\beta)^{k}\widehat{mc}_{t+k|t}+\sum_{k=0}^{\infty}(\theta\beta)^{k}\pi_{H,t+k}\,. \quad (3\text{-}16)$$

Eq.(3-16) can be rewritten as:

$$\tilde{p}_{H,t}-p_{H,t-1}=(1-\theta\beta)\widehat{mc}_{t|t}+\pi_{H,t}+(1-\theta\beta)\sum_{k=1}^{\infty}(\theta\beta)^{k}\widehat{mc}_{t+k|t}+\sum_{k=1}^{\infty}(\theta\beta)^{k}\pi_{H,t+k}\,. \quad (3\text{-}17)$$

Forwarding Eq.(3-17) one period yields:

$$\tilde{p}_{H,t+1}-p_{H,t}=\frac{1-\theta\beta}{\theta\beta}\sum_{k=1}^{\infty}(\theta\beta)^{k}\widehat{mc}_{t+k|t}+\frac{1}{\theta\beta}\sum_{k=1}^{\infty}(\theta\beta)^{k}\pi_{H,t+k}\,.$$

Multiplying $\theta\beta$ on the both sides of the previous expression yields:

$$\theta\beta\left(\tilde{p}_{H,t+1}-p_{H,t}\right)=(1-\theta\beta)\sum_{k=1}^{\infty}(\theta\beta)^{k}\widehat{mc}_{t+k|t}+\sum_{k=1}^{\infty}(\theta\beta)^{k}\pi_{H,t+k}\,.$$

Plugging the previous expression into Eq.(3-17) yields:

$$\tilde{p}_{H,t}-p_{H,t-1}=\theta\beta\left(\tilde{p}_{H,t+1}-p_{H,t}\right)+(1-\theta\beta)\widehat{mc}_{t|t}+\pi_{H,t}\,. \quad (3\text{-}18)$$

Calvo-pricing's transitory equation is given by:

$$P_{H,t} = \left[\theta P_{H,t-1}^{1-\varepsilon} + (1-\theta)\tilde{P}_{H,t}^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$$

Log-linearizing the previous expression around the steady state yields:

$$p_{H,t} = \theta p_{H,t-1} + (1-\theta)\tilde{p}_{H,t}.$$

Subtracting $p_{H,t-1}$ from the both sides of the previous expression yields:

$$\pi_{H,t} = (1-\theta)\left(\tilde{p}_{H,t} - p_{H,t-1}\right). \text{ (3-19)}$$

Plugging Eq.(3-19) into Eq.(3-18) yields:

$$\frac{1}{1-\theta}\pi_{H,t} = \theta\beta\frac{1}{1-\theta}\pi_{H,t+1} + (1-\theta\beta)\widehat{mc}_{t|t} + \pi_{H,t},$$

which can be rewritten as:

$$\frac{\theta}{1-\theta}\pi_{H,t} = \theta\beta\frac{1}{1-\theta}\pi_{H,t+1} + (1-\theta\beta)\widehat{mc}_{t|t}.$$

Multiplying both sides of the previous expression by $\frac{1-\theta}{\theta}$ yields:

$$\pi_{H,t} = \beta\pi_{H,t+1} + \frac{(1-\theta\beta)(1-\theta)}{\theta}\widehat{mc}_{t|t}.$$

Let assume $Y_{t+k} = N_{t+k|t}^{1-\alpha}\Omega_{t+k}^{-(1-\alpha)}$. Then, the (nominal) marginal cost for an individual firm that last set its price is given by:

$$\begin{aligned} MC_{t+k|t}^{n} &= W_{t+k}\frac{\partial N_{t+k|t}}{\partial Y_{t+k}} \\ &= W_{t+k}MPN_{t+k|t}^{-1} \\ &= W_{t+k}\left(\frac{\partial Y_{t+k}}{\partial N_{t+k|t}}\right)^{-1} \\ &= W_{t+k}\left(\frac{\partial N_{t+k|t}^{1-\alpha}\Omega_{t+k}^{-(1-\alpha)}}{\partial N_{t+k|t}}\right)^{-1} \\ &= W_{t+k}\left[(1-\alpha)N_{t+k|t}^{-\alpha}\right]^{-1}\Omega_{t+k}^{1-\alpha} \end{aligned}.$$

Note that the (nominal) average marginal cost is given by:

$$
\begin{aligned}
MC^n_{t+k} &= W_{t+k}\frac{\partial N_{t+k}}{\partial Y_{t+k}} \\
&= W_{t+k}\left(\frac{\partial Y_{t+k}}{\partial N_{t+k}}\right)^{-1} \\
&= W_{t+k}\left[(1-\alpha)N^{-\alpha}_{t+k}\right]^{-1}\Omega^{1-\alpha}_{t+k}
\end{aligned}
$$

Total derivative of the (nominal) marginal cost for an individual firm that last set its price is given by:

$$
\begin{aligned}
dMC^n_{t+k|t} &= \frac{N^\alpha}{1-\alpha}dW_{t+k} + W(-1)\left[(1-\alpha)N^{-\alpha}\right]^{-2}(1-\alpha)(-\alpha)N^{-\alpha-1}dN_{t+k|t} \\
&= \frac{WN^\alpha}{1-\alpha}\frac{dW_{t+k}}{W} + W\frac{1}{(1-\alpha)N^{-\alpha}}\frac{(1-\alpha)\alpha N^{-\alpha}}{(1-\alpha)N^{-\alpha}}\frac{dN_{t+k|t}}{N} \\
&= \frac{WN^\alpha}{1-\alpha}\frac{dW_{t+k}}{W} + \frac{WN^\alpha}{1-\alpha}\alpha\frac{dN_{t+k|t}}{N}
\end{aligned}
.
$$

Dividing both sides of the previous expression by MC^n yields:

$$
\begin{aligned}
\frac{dMC^n_{t+k|t}}{MC^n} &= \frac{1-\alpha}{WN^\alpha}\left(\frac{WN^\alpha}{1-\alpha}\frac{dW_{t+k}}{W} + \frac{WN^\alpha}{1-\alpha}\alpha\frac{dN_{t+k|t}}{N}\right) \\
&= \frac{dW_{t+k}}{W} + \alpha\frac{dN_{t+k|t}}{N}
\end{aligned}
,
$$

which can be rewritten as:

$mc^n_{t+k|t} = w_{t+k} + \alpha\hat{n}_{t+k|t}$. (3-20)

Log-linearization of the average (log) marginal cost is given by:

$mc^n_{t+k} = w_{t+k} + \alpha\hat{n}_{t+k}$. (3-21)

Subtracting Eq.(3-21) from Eq.(3-20) yields:

$$mc^n_{t+k|t} - mc^n_{t+k} = \alpha\left(\hat{n}_{t+k|t} - \hat{n}_{t+k}\right),$$

which can be rewritten as:

$$mc^n_{t+k|t} = mc^n_{t+k} + \alpha\left(\hat{n}_{t+k|t} - \hat{n}_{t+k}\right).$$

Plugging the logarithmic production function $\hat{y}_{t+k|t} = (1-\alpha)\hat{n}_{t+k|t}$ and $\hat{y}_t = (1-\alpha)\hat{n}_t$ into the previous expression yields:

$$mc^n_{t+k|t} = mc^n_{t+k} + \frac{\alpha}{1-\alpha}\left(\hat{y}_{t+k|t} - \hat{y}_{t+k}\right).$$

Plugging logarithmic demand function of $Y_{t+k|t} \equiv \left(\frac{\tilde{P}_{H,t}}{P_{H,t+k}} \right)^{-\varepsilon} Y_{t+k}$ which is given by $\hat{y}_{t+k|t} = -\varepsilon\left(\tilde{p}_{H,t} - p_{H,t+k}\right) + \hat{y}_{t+k}$ into the previous expression yields:

$$mc^n_{t+k|t} = mc^n_{t+k} - \frac{\alpha\varepsilon}{1-\alpha}\left(\tilde{p}_{H,t} - p_{H,t+k}\right). \quad (3\text{-}22)$$

Plugging Eq.(3-22) into Eq.(3-15) yields:

$$\begin{aligned}\tilde{p}_{H,t} &= \mu + (1-\theta\beta)\sum_{k=0}^{\infty}(\theta\beta)^k\left[mc^n_{t+k} - \frac{\alpha\varepsilon}{1-\alpha}\left(\tilde{p}_{H,t} - p_{H,t+k}\right)\right] \\ &= \mu + (1-\theta\beta)\sum_{k=0}^{\infty}(\theta\beta)^k\left(mc^n_{t+k} + \frac{\alpha\varepsilon}{1-\alpha}p_{H,t+k}\right) - \frac{\alpha\varepsilon}{1-\alpha}\tilde{p}_{H,t}\end{aligned},$$

which can be rewritten as:

$$\begin{aligned}\frac{(1-\alpha)+\alpha\varepsilon}{1-\alpha}\tilde{p}_{H,t} &= \mu + (1-\theta\beta)\sum_{k=0}^{\infty}(\theta\beta)^k\left(mc^n_{t+k} + \frac{\alpha\varepsilon}{1-\alpha}p_{H,t+k}\right) \\ &= \mu + (1-\theta\beta)\sum_{k=0}^{\infty}(\theta\beta)^k\left(p_{H,t+k} - \mu_{t+k} + \frac{\alpha\varepsilon}{1-\alpha}p_{H,t+k}\right) \\ &= \mu + (1-\theta\beta)\sum_{k=0}^{\infty}(\theta\beta)^k\left(-\mu_{t+k} + \frac{(1-\alpha)+\alpha\varepsilon}{1-\alpha}p_{H,t+k}\right) \\ &= (1-\theta\beta)\sum_{k=0}^{\infty}(\theta\beta)^k\left(\mu - \mu_{t+k} + \frac{(1-\alpha)+\alpha\varepsilon}{1-\alpha}p_{H,t+k}\right)\end{aligned},$$

where we use the definition of the (log) desired markup $\mu_t \equiv -\left(mc^n_t - p_{H,t}\right)$ which is (log) inverse of the real marginal cost.

Let define $\hat{\mu}_t \equiv \mu_t - \mu$ being the deviation between the average and desired marginal cost. Plugging the definition into the previous expression yields:

$$\tilde{p}_{H,t} = (1-\theta\beta)\sum_{k=0}^{\infty}(\theta\beta)^k\left[p_{H,t+k} - \frac{1-\alpha}{(1-\alpha)+\alpha\varepsilon}\hat{\mu}_{t+k}\right]. \quad (3\text{-}23)$$

with $\Theta \equiv \frac{1-\alpha}{(1-\alpha)+\alpha\varepsilon}$.

Eq.(3-23) can be rewritten as:

$$
\begin{aligned}
\tilde{p}_{H,t} - p_{H,t-1} &= (1-\theta\beta)\sum_{k=0}^{\infty}(\theta\beta)^k \left(p_{H,t+k} - \Theta\hat{\mu}_{t+k}\right) - p_{H,t-1} \\
&= (1-\theta\beta)\left[p_{H,t} + \theta\beta p_{H,t+1} + (\theta\beta)^2 p_{H,t+2} + \cdots\right] - p_{H,t-1} - (1-\theta\beta)\sum_{k=0}^{\infty}(\theta\beta)^k \Theta\hat{\mu}_{t+k} \\
&= -p_{H,t-1} + \left[p_{H,t} + \theta\beta p_{H,t+1} + (\theta\beta)^2 p_{H,t+2} + \cdots\right] - \theta\beta\left[p_{H,t} + \theta\beta p_{H,t+1} + (\theta\beta)^2 p_{H,t+2} + \cdots\right] \\
&\quad -(1-\theta\beta)\sum_{k=0}^{\infty}(\theta\beta)^k \Theta\hat{\mu}_{t+k} \\
&= p_{H,t} - p_{H,t-1} + \theta\beta\left(p_{H,t+1} - p_{H,t}\right) + (\theta\beta)^2\left(p_{H,t+2} - p_{H,t+1}\right) + (\theta\beta)^3\left(p_{H,t+3} - p_{H,t+2}\right) + \cdots \\
&\quad -(1-\theta\beta)\sum_{k=0}^{\infty}(\theta\beta)^k \Theta\hat{\mu}_{t+k} \\
&= \pi_{H,t} + \theta\beta\pi_{H,t+1} + (\theta\beta)^2 \pi_{H,t+2} + (\theta\beta)^3 \pi_{H,t+3} + \cdots - (1-\theta\beta)\sum_{k=0}^{\infty}(\theta\beta)^k \Theta\hat{\mu}_{t+k} \\
&= \sum_{k=0}^{\infty}(\theta\beta)^k \pi_{H,t+k} - (1-\theta\beta)\sum_{k=0}^{\infty}(\theta\beta)^k \Theta\hat{\mu}_{t+k}
\end{aligned}
$$

, (3-24)

Forwarding Eq.(3-24) one period yields:

$$
\begin{aligned}
\tilde{p}_{H,t+1} - p_{H,t} &= \pi_{H,t+1} + \theta\beta\pi_{H,t+2} + (\theta\beta)^2 \pi_{H,t+3} + (\theta\beta)^3 \pi_{H,t+4} + \cdots \\
&\quad -(1-\theta\beta)\Theta\left\{\hat{\mu}_{t+1} + \theta\beta\hat{\mu}_{t+2} + (\theta\beta)^2 \hat{\mu}_{t+3} + (\theta\beta)^3 \hat{\mu}_{t+4} + \cdots\right\} \\
&= \frac{1}{\theta\beta}\sum_{k=1}^{\infty}(\theta\beta)^k \pi_{H,t+k} - \frac{(1-\theta\beta)\Theta}{\theta\beta}\sum_{k=1}^{\infty}(\theta\beta)^k \hat{\mu}_{t+k}
\end{aligned}
$$

Multiplying $\theta\beta$ on the both sides of the previous expression yields:

$$
\theta\beta\left(\tilde{p}_{H,t+1} - p_{H,t}\right) = \sum_{k=1}^{\infty}(\theta\beta)^k \pi_{H,t+k} - (1-\theta\beta)\sum_{k=1}^{\infty}(\theta\beta)^k \Theta\hat{\mu}_{t+k} .
$$

Plugging the previous expression into Eq.(3-24) yields:

$$
\begin{aligned}
\tilde{p}_{H,t} - p_{H,t-1} &= \pi_{H,t} + \theta\beta\pi_{H,t+1} + (\theta\beta)^2 \pi_{H,t+2} + (\theta\beta)^3 \pi_{H,t+3} + \cdots \\
&\quad -(1-\theta\beta)\left[\Theta\hat{\mu}_t + \theta\beta\Theta\hat{\mu}_{t+1} + (\theta\beta)^2 \hat{\mu}_{t+2} + (\theta\beta)^3 \hat{\mu}_{t+3}\right] \\
&= \pi_{H,t} - (1-\theta\beta)\Theta\hat{\mu}_t + \sum_{k=1}^{\infty}(\theta\beta)^k \pi_{H,t+k} - (1-\theta\beta)\sum_{k=1}^{\infty}(\theta\beta)^k \Theta\hat{\mu}_{t+k} . \\
&= \pi_{H,t} - (1-\theta\beta)\Theta\hat{\mu}_t + \theta\beta\left(\tilde{p}_{H,t+1} - p_{H,t}\right)
\end{aligned}
$$

Plugging Eq.(3-19) into the previous expression yields:

$$
\frac{1}{1-\theta}\pi_{H,t} = \pi_{H,t} - (1-\theta\beta)\Theta\hat{\mu}_t + \theta\beta\frac{1}{1-\theta}\pi_{H,t+1},
$$

which can be rewritten as:

$$\pi_{H,t}=\frac{1-\theta}{\theta}\left[-(1-\theta\beta)\Theta\hat{\mu}_t+\theta\beta\frac{1}{1-\theta}\pi_{H,t+1}\right].$$

Then, we have:

$\pi_{H,t}=\beta\pi_{H,t+1}-\kappa\hat{\mu}_t$, (3-25)

with $\kappa\equiv\frac{(1-\theta\beta)(1-\theta)}{\theta}\Theta$. Eq.(3-25) is identical with Eq.(27) in the text.

3.6 Log-linearization of Intra-temporal Optimality Condition

Dividing both sides of Eq.(1-7) by $P_{H,t}/P_{H,t}$ yields:

$$\frac{W_t}{P_{H,t}}=\frac{V_{n,t}}{U_{c,t}}S_t^{\nu}.$$

Plugging the previous expression into Eq.(1-24) yields:

$$MC_t=\frac{V_{n,t}}{U_{c,t}}S_t^{\nu}\frac{N_t^{\alpha}}{(1-\alpha)}.$$

Total derivative of the previous expression is given by:

$$\begin{aligned}dMC_t&=\left(\frac{1}{U_c}\frac{N^{\alpha}}{1-\alpha}\frac{\partial V_n}{\partial N}+\frac{V_n}{U_c}\frac{\alpha}{1-\alpha}\frac{N^{\alpha}}{N}\right)dN_t-U_c^{-2}V_n\frac{N^{\alpha}}{1-\alpha}dU_{c,t}+\frac{V_n}{U_c}\frac{N^{\alpha}}{1-\alpha}\nu dS_t\\&=\frac{V_n}{U_c}\frac{N^{\alpha}}{1-\alpha}\left(\frac{V_{nn}N}{V_n}+\alpha\right)\frac{dN_t}{N}-\frac{V_n}{U_c}\frac{N^{\alpha}}{1-\alpha}\frac{dU_{c,t}}{U_c}+\frac{V_n}{U_c}\frac{N^{\alpha}}{1-\alpha}\nu dS_t\end{aligned}.$$

Plugging Eq.(2-9) into the previous expression yields:

$$\begin{aligned}dMC_t&=\frac{1-\alpha}{N^{\alpha}\mu S^{\nu}}\frac{N^{\alpha}}{1-\alpha}\left(\frac{V_{nn}N}{V_n}+\alpha\right)\frac{dN_t}{N}-\frac{1-\alpha}{N^{\alpha}\mu S^{\nu}}\frac{N^{\alpha}}{1-\alpha}\frac{dU_{c,t}}{U_c}\\&\quad+\frac{1-\alpha}{N^{\alpha}\mu S^{\nu}}\frac{N^{\alpha}}{1-\alpha}\nu dS_t\\&=\mu^{-1}\left(\frac{V_{nn}N}{V_n}+\alpha\right)\frac{dN_t}{N}-\mu^{-1}\frac{dU_{c,t}}{U_c}+\mu^{-1}\nu dS_t\end{aligned}.$$

Multiplying both sides of the previous expression by μ yields:

$$\frac{dMC_t}{MC}=\left(\frac{V_{nn}N}{V_n}+\alpha\right)\frac{dN_t}{N}-\frac{dU_{c,t}}{U_c}+\nu dS_t,$$

which can be rewritten as:

$$\log\left(\frac{MC_t}{MC}\right)=\left(\frac{V_{nn}N}{V_n}+\alpha\right)\log\left(\frac{N_t}{N}\right)-\log\left(\frac{U_{c,t}}{U_c}\right)+\nu\log S_t .$$

By using the definition of $\widehat{mc}_t\equiv\log\left(\frac{MC_t}{MC}\right)$, $\hat{n}_t\equiv\log\left(\frac{N_t}{N}\right)$, $\hat{\xi}_t\equiv\log\left(\frac{U_{c,t}}{U_c}\right)$ and $\varphi\equiv\frac{V_{nn}N}{V_n}$, the previous expression can be rewritten as:

$\widehat{mc}_t=(\varphi+\alpha)\hat{n}_t-\hat{\xi}_t+\nu s_t$, (3-26)

where we use Eq.(3-9).

Eq.(3-14) implies as follows:

$$\begin{aligned}\widehat{mc}_t+p_{H,t}&=mc_t^n+\mu\\&=-\mu_t+p_{H,t}+\mu,\\&=-\hat{\mu}_t+p_{H,t}\end{aligned}$$

or $\widehat{mc}_t=-\hat{\mu}_t$,

where we use $mc_t^n=-\mu_t+p_{H,t}$ which is derived by the definition of the desired markup. Plugging the previous expression into Eq.(3-26) yields:

$$\hat{\mu}_t=\hat{\xi}_t-(\varphi+\alpha)\hat{n}_t-\nu s_t .$$

Plugging the (log) production function $\hat{y}_t=(1-\alpha)\hat{n}_t$ derived from Eq.(1-21) into the previous expression yields:

$$\hat{\mu}_t=\hat{\xi}_t-\frac{\varphi+\alpha}{1-\alpha}\hat{y}_t-\nu s_t ,$$

which is identical with Eq.(28) in the text.

3.7 Deriving the LM Equation

Eq.(1-8) can be rewritten as:

$$\frac{U_{l,t}}{U_{c,t}}=\frac{i_t}{1+i_t} .$$

Multiplying —1 on both sides of the previous expression yields:

$$-\frac{U_{l,t}}{U_{c,t}}=-\frac{i_t}{1+i_t}$$

Summing 1 both sides of the previous expression yields:

$$
\begin{aligned}
1-\frac{U_{l,t}}{U_{c,t}} &= 1-\frac{i_t}{1+i_t} \\
&= \frac{1+i_t}{1+i_t}-\frac{i_t}{1+i_t} . \\
&= \frac{1}{1+i_t}
\end{aligned}
$$

Raise both sides of the previous expression to power −1 yields:

$$
1+i_t=\left(1-\frac{U_{l,t}}{U_{c,t}}\right)^{-1} .
$$

Total derivative of the previous expression yields:

$$
\begin{aligned}
d(1+i_t) &= -\left(1-\frac{U_l}{U_c}\right)^{-2}\left[-U_l(-1)U_c^{-2}U_{cc}-\frac{1}{U_c}U_{lc}\right]dC_t+\left[-\frac{1}{U_c}U_{ll}-U_l(-1)U_c^{-2}U_{cl}\right]dL_t \\
&= -(1+\rho)^{-2}\left[\left(\frac{U_l}{U_c}\frac{U_{cc}}{U_c}-\frac{U_{lc}}{U_c}\right)dC_t+\left(-\frac{U_l}{U_c}\frac{U_{ll}}{U_l}+\frac{U_l}{U_c}\frac{U_{lc}}{U_c}\right)dL_t\right] \\
&= -(1+\rho)^{-2}\frac{U_l}{U_c}\left[\left(\frac{U_{cc}}{U_c}C-\frac{U_c}{U_l}\frac{U_{lc}}{U_c}C\right)\frac{dC_t}{C}+\left(-\frac{U_{ll}}{U_l}L+\frac{U_{lc}}{U_c}L\right)\frac{dL_t}{L}\right] \\
&= -(1+\rho)\rho\left[\left(\frac{U_{cc}}{U_c}C-\frac{U_c}{U_l}\frac{U_{lc}}{U_c}C\right)\frac{dC_t}{C}+\left(-\frac{U_{ll}}{U_l}L+\frac{U_{lc}}{U_c}L\right)\frac{dL_t}{L}\right]
\end{aligned}
$$

Dividing both sides of the previous expression by $1+\rho$ yields:

$$
\begin{aligned}
\frac{d(1+i_t)}{1+\rho} &= -\rho\left[\left(\frac{U_{cc}}{U_c}C-\frac{U_c}{U_l}\frac{U_{lc}}{U_c}C\right)\frac{dC_t}{C}+\left(-\frac{U_{ll}}{U_l}L+\frac{U_{lc}}{U_c}L\right)\frac{dL_t}{L}\right] \\
&= -\rho\left(\frac{U_{cc}}{U_c}C-\frac{U_c}{U_l}\frac{U_{lc}}{U_c}C\right)\frac{dC_t}{C}-\rho\left(-\frac{U_{ll}}{U_l}L+\frac{U_{lc}}{U_c}L\right)\frac{dL_t}{L}
\end{aligned},
$$

which can be rewritten as:

$$
\left(-\frac{U_{ll}}{U_l}L+\frac{U_{lc}}{U_c}L\right)\frac{dL_t}{L}=-\left(\frac{U_{cc}}{U_c}C-\frac{U_c}{U_l}\frac{U_{lc}}{U_c}C\right)\frac{dC_t}{C}-\frac{1}{\rho}\frac{d(1+i_t)}{1+\rho} . \text{ (3-27)}
$$

Iff $-\frac{U_{ll}}{U_l}L+\frac{U_{lc}}{U_c}L-\left[-\left(\frac{U_{cc}}{U_c}C-\frac{U_c}{U_l}\frac{U_{lc}}{U_c}C\right)\right]=0$, (3-28)

$$-\frac{U_{ll}}{U_l}L+\frac{U_{lc}}{U_c}L=-\left(\frac{U_{cc}}{U_c}C-\frac{U_c}{U_l}\frac{U_{lc}}{U_c}C\right), \text{(3-29)}$$

is applicable.

Let assume $U(C,L)=\frac{1}{1-\nu}\left(C^{1-\vartheta}L^{\vartheta}\right)^{1-\nu}$. Then, we have:

$$U_c=(1-\vartheta)h'\left(\frac{L}{C}\right)^{\vartheta},$$

$$U_{cc}=(1-\vartheta)\left[-\sigma(1-\vartheta)-\vartheta\right]h'\left(\frac{L}{C}\right)^{\vartheta}C^{-1},$$

$$U_{cl}=\vartheta(1-\vartheta)(1-\nu)h'\left(\frac{L}{C}\right)^{\vartheta}L^{-1}, \qquad \text{(3-30)}$$

$$U_l=\vartheta h'\left(\frac{L}{C}\right)^{\vartheta-1},$$

$$U_{ll}=\vartheta\left[-\sigma\vartheta-(1-\vartheta)\right]h'\left(\frac{L}{C}\right)^{\vartheta-1}L^{-1},$$

with $h'\equiv\left(C^{1-\vartheta}L^{\vartheta}\right)^{-\nu}$.

Plugging Eq.(3-30) into the RHS of Eq.(3-29) yields:

$$-\left(\frac{U_{cc}}{U_c}C-\frac{U_c}{U_l}\frac{U_{lc}}{U_c}C\right)=-\left[-\sigma(1-\vartheta)-\vartheta-(1-\vartheta)(1-\nu)\right] \text{. (3-31)}$$
$$=1$$

Plugging Eq.(3-30) into the LHS of Eq.(3-29) yields:

$$-\frac{U_{ll}}{U_l}L+\frac{U_{lc}}{U_c}L=-\left[-\nu\vartheta-(1-\vartheta)\right]+\vartheta(1-\nu) \text{. (3-32)}$$
$$=1$$

Plugging Eqs.(3-31) and (3-32) into the LHS of Eq.(3-28) yields:

1—1＝0.

Thus, Eq.(3-29) is applicable. Plugging Eq.(3-29) into Eq.(3-27) yields:

$$\left(-\frac{U_{ll}}{U_l}L+\frac{U_{lc}}{U_c}L\right)\log\left(\frac{L_t}{L}\right)=-\left(-\frac{U_{ll}}{U_l}L+\frac{U_{lc}}{U_c}L\right)\log\left(\frac{C_t}{C}\right)-\frac{1}{\rho}\log\left(\frac{1+i_t}{1+\rho}\right),$$

where we use the fact that $\frac{dL_t}{L}=\log\left(\frac{L_t}{L}\right)$, $\log\left(\frac{C_t}{C}\right)$ and $\frac{d(1+i_t)}{1+\rho}=\log\left(\frac{1+i_t}{1+\rho}\right)$.

By using definitions $\sigma_l \equiv -\frac{U_{ll}}{U_l}L$ and $\upsilon \equiv \frac{U_{lc}}{U_c}L$, the previous expression can be rewritten as:

$$(\sigma_l + \upsilon)\hat{l}_t = (\sigma_l + \upsilon)\hat{c}_t - \frac{1}{\rho}\hat{i}_t,$$

which can be rewritten as:

$$\hat{l}_t = \hat{c}_t - \frac{1}{\rho(\sigma_l + \upsilon)}\hat{i}_t.$$

By using the definition $\varepsilon_{lc} \equiv \frac{1}{\sigma_l + \upsilon}$, the previous expression can be rewritten as:

$$\hat{l}_t = \hat{c}_t - \eta\hat{i}_t, \text{ (3-33)}$$

with $\eta \equiv \frac{\varepsilon_{lc}}{\rho}$. Eq.(3-33) is identical with Eq(29) in the text.

3.8 Relationship between Changes in the Real Money Balance and Inflation

Total derivative of the definition of the real money balance $L_t \equiv \frac{M_t}{P_t}$ is given by:

$$\begin{aligned} dL_t &= \frac{dM_t}{P} + (-1)P^{-2}MdP_t \\ &= \frac{M}{P}\frac{dM_t}{M} - \frac{M}{P}\frac{dP}{M} \\ &= L\frac{dM_t}{M} - L\frac{dP}{M} \end{aligned} .$$

Dividing both sides of the previous expression yields:

$$\hat{l}_t = \log\left(\frac{M_t}{M}\right) - \log\left(\frac{P_t}{M}\right). \text{ (3-34)}$$

First differential equation of Eq.(3-34) is given by:

$$\begin{aligned} \hat{l}_t - \hat{l}_{t-1} &= \log\left(\frac{M_t}{M}\right) - \log\left(\frac{M_{t-1}}{M}\right) - \left[\log\left(\frac{P_t}{M}\right) - \log\left(\frac{P_{t-1}}{M}\right)\right] \\ &= \log M_t - \log M_{t-1} - \left(\log P_t - \log P_{t-1}\right) \\ &= -\pi_t + \Delta m_t \end{aligned} ,$$

which can be rewritten as:

$\hat{l}_{t-1} = \hat{l}_t + \pi_t - \Delta m_t$. (3-35)

Eq.(3-35) is identical with Eq.(30) in the text.

3.9 Log-linearization of the Consolidated Government Budget Constraint

By using the definition $g(S_t) \equiv \frac{P_t}{P_{H,t}} = S_t^{\nu}$ and $\mathrm{P}_t \equiv (1+i_t)\frac{P_t}{P_{t+1}}$, Eq(3-1) can be rewritten as:

$$g(S_t)^{-1} G_t + \mathrm{B}_{t-1}(1+i_{t-1})\Pi_t^{-1} = T_t + \mathrm{B}_t + \frac{\Delta M_t}{P_t}$$

with $\Pi_t \equiv \frac{P_t}{P_{t-1}}$. The previous expression can be rewritten as:

$$\mathrm{B}_t = S_t^{-\nu} G_t + \mathrm{B}_{t-1}(1+i_{t-1})\Pi_t^{-1} - T_t - \frac{\Delta M_t}{P_t}.$$

Total derivative of the previous expression yields:

$$\begin{aligned} d\mathrm{B}_t &= G(-\nu)dS_t + dG_t + (1+\rho)d\mathrm{B}_{t-1} + \mathrm{B}d(1+i_{t-1}) + \mathrm{B}(1+\rho)(-1)d\Pi_t \\ &\quad -dT_t - d(\Delta M_t/P_t) \end{aligned}.$$

Dividing both sides of the previous expression by *Y* yields:

$$\begin{aligned} \frac{d\mathrm{B}_t}{Y} &= -\frac{G}{Y}\nu dS_t + \frac{dG_t}{Y} + (1+\rho)\frac{d\mathrm{B}_{t-1}}{Y} + \frac{\mathrm{B}}{Y}d(1+i_{t-1}) - \frac{\mathrm{B}}{Y}(1+\rho)d\Pi_t \\ &\quad -\frac{dT_t}{Y} - \frac{d(\Delta M_t/P_t)}{Y} \\ &= \frac{dG_t}{Y} + (1+\rho)\frac{d\mathrm{B}_{t-1}}{Y} + (1+\rho)b\frac{d(1+i_{t-1})}{1+\rho} - b(1+\rho)d\Pi_t \\ &\quad -\frac{dT_t}{Y} - \frac{d(\Delta M_t/P_t)}{Y} \end{aligned}, \quad (3\text{-}36)$$

where we use the definition $b \equiv \frac{\mathrm{B}}{Y}$ and the fact that $G = 0$.

Seignorage can be rewritten as:

$$\begin{aligned} \frac{\Delta M_t}{P_t} &= \frac{\Delta M_t}{M_{t-1}}\frac{P_{t-1}}{P_t}L_{t-1} \\ &= \frac{\Delta M_t}{M_{t-1}}\Pi_t^{-1}L_{t-1} \end{aligned}. \quad (3\text{-}37)$$

Total derivative of Eq.(3-37) yields:

$$
\begin{aligned}
d\left(\Delta M_t / P_t\right) &= \frac{L}{M} d\Delta M_t - M^{-2}\Delta M L dM_{t-1} + \frac{\Delta M}{M} L(-1) d\Pi_t + \frac{\Delta M}{M} dL_{t-1} \\
&= \frac{L}{M} d\Delta M_t
\end{aligned},
$$

where we use the fact that $\Delta M = 0$. Dividing both sides of the previous expression yields by Y yields:

$$\frac{d\left(\Delta M_t / P_t\right)}{Y} = \frac{L}{Y}\frac{d\Delta M_t}{M}, \text{ (3-38)}$$

with $\chi \equiv \frac{L}{Y}$.

Plugging Eqs.(3-9) and (3-38) into Eq.(3-36) yields:

$$\hat{b}_t = \hat{g}_t + (1+\rho)\hat{b}_{t-1} + (1+\rho)b\hat{i}_{t-1} - b(1+\rho)\pi_t - \widehat{tr}_t - \chi\Delta m_t, \text{ (3-39)}$$

with $\hat{b}_t \equiv \frac{d\mathrm{B}_t}{Y}$, $\hat{g}_t \equiv \frac{dG_t}{Y}$ and $\widehat{tr}_t \equiv \frac{TR_t - TR}{Y}$. Eq.(3-39) is identical with Eq.(31) in the text.

A simple tax rule is given by:

$$\widehat{tr}_t = \psi_b \hat{b}_{t-1} + \hat{\varsigma}_t, \text{ (3-40)}$$

which is identical with Eq.(38) in the text.

Plugging Eq.(3-40) into Eq.(3-39) yields:

$$\hat{b}_t = (1+\rho-\psi_b)\hat{b}_{t-1} + (1+\rho)b\hat{i}_{t-1} - b(1+\rho)\pi_t + \hat{g}_t - \hat{\varsigma}_t - \chi\Delta m_t, \text{ (3-41)}$$

which is identical with Eq.(39) in the text.

3.10 Relationship between the CPI Inflation and GDP Inflation

Eq.(3-3) can be rewritten as:

$$
\begin{aligned}
p_t &= (1-\nu)p_{H,t} + \nu p_{F,t} \\
&= p_{H,t} + \nu s_t
\end{aligned}.
$$

First order differential equation of the previous expression is given by:

$$\pi_t = \pi_{H,t} + \nu\left(s_t - s_{t-1}\right),$$

which is identical with Eq.(32) in the text.

3.11 Trade balance

Total derivative of Eq.(1-34) is given by:

$$d\left(NX_t/P_{H,t}\right)=dY_t-\nu CdS_t-dC_t-dG_t\,.$$

By dividing both sides of the previous expression by *Y* yields:

$$\frac{d\left(NX_t/P_{H,t}\right)}{Y}=\frac{dY_t}{Y}-\nu dS_t-\frac{dC_t}{C}-\frac{dG_t}{Y},$$

which can be rewritten as:

$$\log\left[\frac{\left(NX_t/P_{H,t}\right)}{Y}\right]=\log\left(\frac{Y_t}{Y}\right)-\log g\left(S_t\right)-\log\left(\frac{C_t}{C}\right)-\log\left(\frac{G_t}{Y}\right).$$

Let define $\widehat{nx}_t\equiv\log\left[\frac{\left(NX_t/P_{H,t}\right)}{Y}\right]$, which is the ratio of trade balance to the GDP. Then the previous expression can be rewritten as:

$$\widehat{nx}_t=\hat{y}_t-\nu s_t-\hat{c}_t-\hat{g}_t\,, \quad (3\text{-}42)$$

which is identical with Eq.(33) in the text.

Plugging Eqs.(3-7) and (3-10) into Eq.(3-42) yields:

$$\widehat{nx}_t=\frac{\nu\left(1-\nu\right)\left(\sigma-1\right)}{\sigma}s_t-\frac{\nu}{\sigma}\hat{\rho}_t\,,$$

where we use $\zeta_t=\hat{\rho}_t$. Plugging $\sigma=1$ into the previous expression yields:

$$\widehat{nx}_t=-\nu\hat{\rho}_t\,,$$

which implies that just the demand shock affects the trade balance under our benchmark parameterization. As long as the demand shock hit does not hit the economy, balanced trade attains (See Section 4.4 in the text).

4 Policy Regimes

Plugging $\hat{b}_t=0$ for all *t* into Eq.(3-41) yields:

$$\Delta m_t=\frac{1}{\chi}(1+\rho)b\hat{i}_{t-1}-\frac{b}{\chi}(1+\rho)\pi_t+\frac{1}{\chi}\hat{g}_t-\frac{1}{\chi}\hat{\varsigma}_t\,,$$

which is identical with Eq.(40) in the text.

5 Some Entities

The domestic and the imported goods inflation is given by:

$$\begin{aligned}\pi_{H,t} &= p_{H,t} - p_{H,t-1} \\ \pi_{F,t} &= p_{F,t} - p_{F,t-1}\end{aligned}, \text{ (5-1)}$$

which are Eqs(35) and(36) in the text, respectively.
The nominal exchange rate is calculated by:

$$s_t = e_t + p_t^* - p_{H,t} \,,$$

which is identical with Eq.(34) in the text.
Subtracting the first equality in Eq.(5-1) from the second equality in Eq.(5-1) yields:

$$\begin{aligned}\pi_{F,t} - \pi_{H,t} &= p_{F,t} - p_{H,t} - \left(p_{F,t-1} - p_{H,t-1}\right) \\ &= s_t - s_{t-1}\end{aligned},$$

which can be rewritten as:
$\pi_{F,t} = s_t - s_{t-1} + \pi_{H,t}$. (5-2)
Eq.(5-2) is identical with Eq(37) in the text.

6 Introducing Imperfect Pass-through

6.1 International Risk Sharing Condition

Note that $Q_t \equiv \frac{E_t P_t^*}{P_t}$ can be rewritten as in the imperfect pass-through environment as follows:

$$\begin{aligned}Q_t &\equiv \frac{E_t P_t^*}{P_t} \\ &= \frac{E_t P_{F,t}^*}{P_t^{1-\nu} P_{F,t}^{\nu}} \\ &= \frac{P_{F,t}}{P_t^{1-\nu} P_{F,t}^{\nu}} \frac{E_t P_{F,t}^*}{P_{F,t}} \\ &= S_t^{1-\nu} \Psi_t\end{aligned},$$

which is identical with Eq.(A.1) in Appendix A. Plugging the previous expression into

Eq.(1-10) yields:

$$U_{c,t}^{-1}=\vartheta\left(U_{c,t}^{*}\right)^{-1}S_t^{1-\nu}\Psi_t\frac{Z_t}{Z_t^{*}}, \text{ (6-1)}$$

which is identical with Eq.(A.2) in Appendix A.

6.2 Foreign Retailers

Consider a foreign exporter exporting good j at a cost (i.e., price paid in the world market) $E_t P_{F,t}^{*}(j)$. Like local producers, the same exporter faces a downward sloping demand for such goods and therefore chooses a price $\tilde{P}_{F,t}(j)$, expressed in units of domestic currency, to maximize:

$$\max_{\tilde{P}_{F,t}(j)}\sum_{k=0}^{\infty}\theta_F^k \mathrm{E}_t\left\{\Lambda_{t,t+k}^{*}\left(\frac{1}{P_{t+k}^{*}}\right)\left[\frac{\tilde{P}_{F,t}(j)}{E_t}-P_{F,t+k}^{*}(j)(1-\tau_F)\right]C_{F,t+k}(j)\right\},$$

with $C_{F,t+k}(j)\equiv\left(\frac{\tilde{P}_{F,t}(j)}{P_{F,t+k}}\right)^{-\varepsilon}C_{F,t+k}$ where $\Lambda_{t,t+k}^{*}\equiv \mathrm{Q}_{t,t+k}^{*}\left(\frac{P_{t+k}^{*}}{P_t^{*}}\right)=\beta^k\left[\frac{\left(U_{c,t}^{*}\right)^{-1}}{\left(U_{c,t+k}^{*}\right)^{-1}}\frac{Z_{t+k}^{*}}{Z_t^{*}}\right]$

denotes the discount factor, $\mathrm{Q}_{t,t+k}^{*}$ denotes the price of a one period discount bond paying off one unit of foreign currency and τ_F denotes an export subsidiary. The previous expression can be rewritten as :

$$\max_{\tilde{P}_{F,t}(j)}\left\{\begin{array}{l}\Lambda_{t,t}^{*}\left(\frac{1}{P_t^{*}}\right)\left[\frac{\tilde{P}_{F,t}(j)}{E_t}\left(\frac{\tilde{P}_{F,t}(j)}{P_{F,t}}\right)^{-\varepsilon}C_{F,t}-P_{F,t}^{*}(j)(1-\tau_F)\left(\frac{\tilde{P}_{F,t}(j)}{P_{F,t}}\right)^{-\varepsilon}C_{F,t}\right]+\\ \theta_F\Lambda_{t,t+1}^{*}\left(\frac{1}{P_{t+1}^{*}}\right)\left[\frac{\tilde{P}_{F,t}(j)}{E_t}\left(\frac{\tilde{P}_{F,t}(j)}{P_{F,t+1}}\right)^{-\varepsilon}C_{F,t+1}-P_{F,t+1}^{*}(j)(1-\tau_F)\left(\frac{\tilde{P}_{F,t}(j)}{P_{F,t+1}}\right)^{-\varepsilon}C_{F,t+1}\right]\\ +\theta^2\Lambda_{t,t+2}^{*}\left(\frac{1}{P_{t+2}^{*}}\right)\left[\frac{\tilde{P}_{F,t}(j)}{E_t}\left(\frac{\tilde{P}_{F,t}(j)}{P_{F,t+2}}\right)^{-\varepsilon}C_{F,t+2}-P_{F,t+1}^{*}(j)(1-\tau_F)\left(\frac{\tilde{P}_{F,t}(j)}{P_{F,t+2}}\right)^{-\varepsilon}C_{F,t+2}\right]+\cdots\end{array}\right\}$$

The FONC for firms is given by:

$$\Lambda_{t,t}^{*}\left(\frac{1}{P_{t}^{*}}\right)\left[(1-\varepsilon)\frac{\tilde{P}_{F,t}}{E_{t}}(j)^{-\varepsilon}P_{F,t}^{\varepsilon}C_{F,t}-P_{F,t}^{*}(j)(1-\tau_{F})(-\varepsilon)\tilde{P}_{F,t}(j)^{-\varepsilon-1}P_{F,t}^{\varepsilon}C_{F,t}\right]$$

$$+\theta\Lambda_{t,t+1}^{*}\left(\frac{1}{P_{t+1}^{*}}\right)\left[\begin{array}{l}(1-\varepsilon)\frac{\tilde{P}_{F,t}}{E_{t}}(j)^{-\varepsilon}P_{F,t+1}^{\varepsilon}C_{F,t+1}\\-P_{F,t+1}^{*}(j)(1-\tau_{F})(-\varepsilon)\tilde{P}_{F,t}(j)^{-\varepsilon-1}P_{F,t+1}^{\varepsilon}C_{F,t+1}\end{array}\right],$$

$$+\theta^{2}\Lambda_{t,t+2}^{*}\left(\frac{1}{P_{t+2}^{*}}\right)\left[\begin{array}{l}(1-\varepsilon)\frac{\tilde{P}_{F,t}}{E_{t}}(j)^{-\varepsilon}P_{F,t+2}^{\varepsilon}C_{F,t+2}\\-P_{F,t+2}^{*}(j)(1-\tau_{F})(-\varepsilon)\tilde{P}_{F,t}(j)^{-\varepsilon}P_{F,t+2}^{\varepsilon}C_{F,t+2}\end{array}\right]+\cdots=0$$

which can be rewritten as:

$$\Lambda_{t,t}^{*}\left(\frac{1}{P_{t}^{*}}\right)\left[\frac{\tilde{P}_{F,t}(j)}{E_{t}}\left(\frac{\tilde{P}_{F,t}(j)}{P_{F,t}}\right)^{-\varepsilon}C_{F,t}-\frac{\varepsilon}{\varepsilon-1}(1-\tau_{F})P_{F,t}^{*}(j)\left(\frac{\tilde{P}_{F,t}(j)}{P_{F,t}}\right)^{-\varepsilon}C_{F,t}\right]$$

$$+\theta\Lambda_{t,t+1}^{*}\left(\frac{1}{P_{t+1}^{*}}\right)\left[\frac{\tilde{P}_{F,t}(j)}{E_{t}}\left(\frac{\tilde{P}_{F,t}(j)}{P_{F,t+1}}\right)^{-\varepsilon}C_{F,t+1}-\frac{\varepsilon}{\varepsilon-1}(1-\tau_{F})P_{F,t+1}^{*}(j)\left(\frac{\tilde{P}_{F,t}(j)}{P_{F,t+1}}\right)^{-\varepsilon}C_{F,t+1}\right].$$

$$+\theta^{2}\Lambda_{t,t+2}^{*}\left(\frac{1}{P_{t+2}^{*}}\right)\left[\frac{\tilde{P}_{F,t}(j)}{E_{t}}\left(\frac{\tilde{P}_{F,t}(j)}{P_{F,t+2}}\right)^{-\varepsilon}C_{F,t+2}-\frac{\varepsilon}{\varepsilon-1}(1-\tau_{F})P_{F,t+1}^{*}(j)\left(\frac{\tilde{P}_{F,t}(j)}{P_{F,t+2}}\right)^{-\varepsilon}C_{F,t+2}\right]+\cdots=0$$

By using the definition $C_{F,t+k|t}\equiv\left(\frac{\tilde{P}_{F,t}(j)}{P_{F,t+k}}\right)^{-\varepsilon}C_{F,t+k}$, the previous expression can be rewritten as:

$$\Lambda_{t,t}^{*}\left(\frac{1}{P_{t}^{*}}\right)\left[\frac{\tilde{P}_{F,t}}{E_{t}}(j)C_{F,t|t}-\frac{\varepsilon}{\varepsilon-1}(1-\tau_{F})P_{F,t}^{*}(j)C_{F,t|t}\right]$$

$$+\theta\Lambda_{t,t+1}^{*}\left(\frac{1}{P_{t+1}^{*}}\right)\left[\frac{\tilde{P}_{F,t}}{E_{t}}(j)C_{F,t+1|t}-\frac{\varepsilon}{\varepsilon-1}(1-\tau_{F})P_{F,t+1}^{*}(j)C_{F,t+1|t}\right],$$

$$+\theta^{2}\Lambda_{t,t+2}^{*}\left(\frac{1}{P_{t+2}^{*}}\right)\left[\frac{\tilde{P}_{F,t}}{E_{t}}(j)C_{F,t+2|t}-\frac{\varepsilon}{\varepsilon-1}(1-\tau_{F})P_{F,t+1}^{*}(j)C_{F,t+2|t}\right]+\cdots=0$$

which can be rewritten as:

$$
\begin{aligned}
&\Lambda^*_{t,t}\left(\frac{1}{P^*_t}\right)C_{F,t|t}\left[\frac{\tilde{P}_{F,t}(j)}{E_t}-\frac{\varepsilon}{\varepsilon-1}(1-\tau_F)P^*_{F,t}(j)\right]\\
&+\theta\Lambda^*_{t,t+1}\left(\frac{1}{P^*_{t+1}}\right)C_{F,t+1|t}\left[\frac{\tilde{P}_{F,t}(j)}{E_t}-\frac{\varepsilon}{\varepsilon-1}(1-\tau_F)P^*_{F,t+1}(j)\right] \quad .\\
&+\theta^2\Lambda^*_{t,t+2}\left(\frac{1}{P^*_{t+2}}\right)C_{F,t+2|t}\left[\frac{\tilde{P}_{F,t}(j)}{E_t}-\frac{\varepsilon}{\varepsilon-1}(1-\tau_F)P^*_{F,t+2}(j)\right]+\cdots=0
\end{aligned}
$$

The previous expression can be compact expression as:

$$
\sum_{k=0}^{\infty}\theta^k \mathrm{E}_t\left[\Lambda^*_{t,t+k}\left(\frac{1}{P^*_{t+k}}\right)C_{F,t+k|t}\left(\frac{\tilde{P}_{F,t}}{E_t}-\frac{\varepsilon}{\varepsilon-1}(1-\tau_F)P^*_{F,t+k}\right)\right]=0\text{ , (6-2)}
$$

where we use the fact that $\tilde{P}_{F,t}(j)=\tilde{P}_{F,t}$ and $P^*_{F,t+k}(j)=P^*_{F,t+k}$ in the symmetric equilibrium. Eq.(6-2) is identical with Eq.(A.3) in Appendix A.

Plugging $\Lambda^*_{t,t+k}\equiv \mathrm{Q}^*_{t,t+k}\left(\frac{P^*_{t+k}}{P^*_t}\right)=\beta^k\left[\frac{\left(U^*_{c,t}\right)^{-1}}{\left(U^*_{c,t+k}\right)^{-1}}\frac{Z^*_{t+k}}{Z^*_t}\right]$ into Eq.(6-2) yields:

$$
\sum_{k=0}^{\infty}(\theta\beta)^k \mathrm{E}_t\left[\left[\frac{\left(U^*_{c,t}\right)^{-1}}{\left(U^*_{c,t+k}\right)^{-1}}\frac{Z^*_{t+k}}{Z^*_t}\right]\left(\frac{1}{P^*_{t+k}}\right)C_{F,t+k|t}\left(\frac{\tilde{P}_{F,t}}{E_t}-\frac{\varepsilon}{\varepsilon-1}(1-\tau_F)P^*_{F,t+k}\right)\right]=0\,.
$$

By multiplying $U_{c,t}Z_t$ both sides of the previous expression yields:

$$
\sum_{k=0}^{\infty}(\theta\beta)^k \mathrm{E}_t\left\{\left[\frac{1}{P^*_{t+k}\left(U^*_{c,t+k}\right)^{-1}}\right]Z^*_{t+k}C_{F,t+k|t}\left(\frac{\tilde{P}_{F,t}}{E_t}-\frac{\varepsilon}{\varepsilon-1}(1-\tau_F)P^*_{F,t+k}\right)\right\}=0\,.
$$

Multiplying both sides of the previous expression by $\dfrac{1}{P_{F,t-1}/E_{t-1}}$ yields:

$$
\sum_{k=0}^{\infty}(\theta\beta)^k \mathrm{E}_t\left\{\left[\frac{1}{P^*_{t+k}\left(U^*_{c,t+k}\right)^{-1}}\right]Z^*_{t+k}C_{F,t+k|t}\left[\frac{\tilde{P}_{F,t}/E_t}{P_{F,t-1}/E_{t-1}}-\frac{\varepsilon}{\varepsilon-1}(1-\tau_F)\frac{P^*_{F,t+k}}{P_{F,t+k}}\frac{P_{F,t+k}}{P_{F,t-1}/E_{t-1}}\right]\right\}=0\,,
$$

which can be rewritten as:

$$
\sum_{k=0}^{\infty}(\theta\beta)^k \mathrm{E}_t\left\{\left[\frac{1}{P^*_{t+k}\left(U^*_{c,t+k}\right)^{-1}}\right]Z^*_{t+k}C_{F,t+k|t}\left[\frac{\tilde{P}_{F,t}}{P_{F,t-1}}\frac{E_{t-1}}{E_t}-\frac{\varepsilon}{\varepsilon-1}(1-\tau_F)\frac{E_{t+k}P^*_{F,t+k}}{P_{F,t+k}}\frac{E_{t-1}}{E_{t+k}}\frac{P_{F,t+k}}{P_{F,t-1}}\right]\right\}=0
$$

Let define $\Psi_{t+k}\equiv\dfrac{E_{t+k}P^*_{F,t+k}}{P_{F,t+k}}$ and $\Pi_{F/E,t-1,t+k}\equiv\dfrac{P_{F,t+k}}{P_{F,t-1}}\dfrac{E_{t-1}}{E_{t+k}}$. Then, the previous expression can be rewritten as:

$$\sum_{k=0}^{\infty}(\theta\beta)^k \mathrm{E}_t\left\{\left[\frac{1}{P^*_{t+k}\left(U^*_{c,t+k}\right)^{-1}}\right]Z^*_{t+k}C_{F,t+k|t}\left[\frac{\tilde{P}_{F,t}}{P_{F,t-1}}\frac{E_{t-1}}{E_t}-\frac{\varepsilon}{\varepsilon-1}(1-\tau_F)\Psi_{t+k}\Pi_{F/E,t-1,t+k}\right]\right\}=0$$

Let define $\tilde{X}_{F/E,t}\equiv\frac{\tilde{P}_{F,t}}{P_{F,t-1}}\frac{E_{t-1}}{E_t}$. Then the previous expression can be rewritten as:

$$\sum_{k=0}^{\infty}(\theta\beta)^k \mathrm{E}_t\left\{\left[\frac{1}{P^*_{t+k}\left(U^*_{c,t+k}\right)^{-1}}\right]Z^*_{t+k}C_{F,t+k|t}\left[\tilde{X}_{F/E,t}-\frac{\varepsilon}{\varepsilon-1}(1-\tau_F)\Psi_{t+k}\Pi_{F/E,t-1,t+k}\right]\right\}=0,$$

which can be rewritten as:

$$\sum_{k=0}^{\infty}(\theta\beta)^k \mathrm{E}_t\left\{\left[\frac{1}{\left(U^*_{c,t+k}\right)^{-1}}\right]Z^*_{t+k}C_{F,t+k|t}\frac{P_{F,t+k}}{E_{t+k}P^*_{t+k}}\frac{E_{t+k}}{P_{F,t+k}}\frac{P_{t-1}}{E_{t-1}}\left[\begin{array}{l}\tilde{X}_{F/E,t}\\-\frac{\varepsilon}{\varepsilon-1}(1-\tau_F)\Psi_{t+k}\Pi_{F/E,t-1,t+k}\end{array}\right]\right\}=0.$$

By using the definition $\Psi_{t+k}\equiv\frac{E_{t+k}P^*_{F,t+k}}{P_{F,t+k}}$ and $\Pi_{F/E,t-1,t+k}\equiv\frac{P_{F,t+k}}{P_{F,t-1}}\frac{E_{t-1}}{E_{t+k}}$, the previous expression can be rewritten as:

$$\sum_{k=0}^{\infty}(\theta\beta)^k \mathrm{E}_t\left\{\left[\frac{1}{\left(U^*_{c,t+k}\right)^{-1}}\right]Z^*_{t+k}C_{F,t+k|t}\Psi^{-1}_{t+k}\Pi^{-1}_{F/E,t-1,t+k}\left[\begin{array}{l}\tilde{X}_{F/E,t}\\-\frac{\varepsilon}{\varepsilon-1}(1-\tau_F)\Psi_{t+k}\Pi_{F/E,t-1,t+k}\end{array}\right]\right\}=0.$$

The previous compact form can be rewritten as:

$$\begin{aligned}
&\left[\left(U^*_{c,t}\right)^{-1}\right]^{-1}Z^*_tC_{F,t|t}\Psi^{-1}_t\Pi^{-1}_{F/E,t}\tilde{X}_{F/E,t}\\
&+\theta\beta\left[\left(U^*_{c,t+1}\right)^{-1}\right]^{-1}Z^*_{t+1}C_{F,t+1|t}\Psi^{-1}_{t+1}\Pi^{-1}_{F/E,t+1}\Pi^{-1}_{F/E,t}\tilde{X}_{F,t}\\
&+(\theta\beta)^2\left[\left(U^*_{c,t+2}\right)^{-1}\right]^{-1}Z^*_{t+2}C_{F,t+2|t}\Psi^{-1}_{t+2}\Pi^{-1}_{F/E,t+2}\Pi^{-1}_{F/E,t+1}\Pi^{-1}_{F/E,t}\tilde{X}_{F,t}\\
&+\cdots=\frac{\varepsilon}{\varepsilon-1}(1-\tau_F)\left[\left(U^*_{c,t}\right)^{-1}\right]^{-1}Z^*_tC_{F,t|t}\\
&+\theta\beta\left[\left(U^*_{c,t+1}\right)^{-1}\right]^{-1}Z^*_{t+1}C_{F,t+1|t}\frac{\varepsilon}{\varepsilon-1}(1-\tau_F)\\
&+(\theta\beta)^2\left[\left(U^*_{c,t+2}\right)^{-1}\right]^{-1}Z^*_{t+2}C_{F,t+2|t}\frac{\varepsilon}{\varepsilon-1}(1-\tau_F)+\cdots
\end{aligned},$$

with $\Pi_{F/E,t} \equiv \frac{P_{F,t}/E_t}{P_{F,t-1}/E_{t-1}} = \frac{P_{F,t}}{E_t}\frac{E_{t-1}}{P_{F,t-1}}$. Rearranging the previous expression yields:

$$
\tilde{X}_{F/E,t}\left\{\begin{array}{l}
\left[\left(U^{*}_{c,t}\right)^{-1}\right]^{-1} Z^{*}_{t} C_{F,t|t} \Psi_{t}^{-1} \Pi^{-1}_{F/E,t} \\
+\theta\beta\left[\left(U^{*}_{c,t+1}\right)^{-1}\right]^{-1} Z^{*}_{t+1} C_{F,t+1|t} \Psi_{t+1}^{-1} \Pi^{-1}_{F/E,t+1} \Pi^{-1}_{F/E,t} \\
+(\theta\beta)^{2}\left[\left(U^{*}_{c,t+2}\right)^{-1}\right]^{-1} Z^{*}_{t+2} C_{F,t+2|t} \Psi_{t+2}^{-1} \Pi^{-1}_{F/E,t+2} \Pi^{-1}_{F/E,t+1} \Pi^{-1}_{F/E,t} \\
+\cdots
\end{array}\right\},
$$

$$
= \frac{\varepsilon}{\varepsilon-1}(1-\tau_F)\left\{\begin{array}{l}
\left[\left(U^{*}_{c,t}\right)^{-1}\right]^{-1} Z^{*}_{t} C_{F,t|t} \\
+\theta\beta\left[\left(U^{*}_{c,t+1}\right)^{-1}\right]^{-1} Z^{*}_{t+1} C_{F,t+1|t} \frac{\varepsilon}{\varepsilon-1}(1-\tau_F) \\
+(\theta\beta)^{2}\left[\left(U^{*}_{c,t+2}\right)^{-1}\right]^{-1} Z^{*}_{t+2} C_{F,t+2|t} \frac{\varepsilon}{\varepsilon-1}(1-\tau_F)+\cdots
\end{array}\right\}
$$

which can be rewritten as:

$$
\tilde{X}_{F/E,t} = \frac{\varepsilon}{\varepsilon-1}(1-\tau_F)\left\{\begin{array}{l}
\left[\left(U^{*}_{c,t}\right)^{-1}\right]^{-1} Z^{*}_{t} C_{F,t|t} \\
+\theta_F\beta\left[\left(U^{*}_{c,t+1}\right)^{-1}\right]^{-1} Z^{*}_{t+1} C_{F,t+1|t} \frac{\varepsilon}{\varepsilon-1}(1-\tau_F) \\
+(\theta_F\beta)^{2}\left[\left(U^{*}_{c,t+2}\right)^{-1}\right]^{-1} Z^{*}_{t+2} C_{F,t+2|t} \frac{\varepsilon}{\varepsilon-1}(1-\tau_F)+\cdots
\end{array}\right\}
$$

$$
\times\left\{\begin{array}{l}
\left[\left(U^{*}_{c,t}\right)^{-1}\right]^{-1} Z^{*}_{t} C_{F,t|t} \Psi_{t}^{-1} \Pi^{-1}_{F/E,t} \\
+\theta_F\beta\left[\left(U^{*}_{c,t+1}\right)^{-1}\right]^{-1} Z^{*}_{t+1} C_{F,t+1|t} \Psi_{t+1}^{-1} \Pi^{-1}_{F/E,t+1} \Pi^{-1}_{F/E,t} \\
+(\theta_F\beta)^{2}\left[\left(U^{*}_{c,t+2}\right)^{-1}\right]^{-1} Z^{*}_{t+2} C_{F,t+2|t} \Psi_{t+2}^{-1} \Pi^{-1}_{F/E,t+2} \Pi^{-1}_{F/E,t+1} \Pi^{-1}_{F/E,t} \\
+\cdots
\end{array}\right\}^{-1} \quad \text{(6-3)}
$$

or:

$$
\tilde{X}_{F/E,t} = \frac{\frac{\varepsilon}{\varepsilon-1}(1-\tau_F)\sum_{k=0}^{\infty}(\theta\beta)^{k}\left[\left(U^{*}_{c,t+k}\right)^{-1}\right]^{-1} Z^{*}_{t+k} C_{F,t+k|t}}{\sum_{k=0}^{\infty}(\theta\beta)^{k}\left[\left(U^{*}_{c,t+k}\right)^{-1}\right]^{-1} Z^{*}_{t+k} C_{F,t+k|t} \Psi_{t+k}^{-1} \prod_{h=0}^{k} \Pi^{-1}_{F/E,t+h}}. \quad \text{(6-4)}
$$

6.3 Market Clearing Condition

Demands for export in the PTM environment is given by:

$$
\begin{aligned}
EX_t &= \nu \left(\frac{P_{H,t}^*}{P_t^*} \right)^{-1} C_t^* \\
&= \nu \left(\frac{P_{H,t}^*}{P_{F,t}^*} \right)^{-1} C_t^* \\
&= \nu \left(\frac{P_{H,t}}{E_t P_{F,t}^*} \right)^{-1} C_t^* \qquad , (6\text{-}5) \\
&= \nu \left(\frac{P_{H,t}}{P_{F,t}} \frac{P_{F,t}}{E_t P_{F,t}^*} \right)^{-1} C_t^* \\
&= \nu \left(\frac{P_{F,t}}{P_{H,t}} \right) \left(\frac{E_t P_{F,t}^*}{P_{F,t}} \right) C_t^* \\
&= \nu S_t \Psi_t Y_t^*
\end{aligned}
$$

which is identical with Eq.(A.4) in Appendix A where we use the definition of LOOP gap $\Psi_t \equiv \frac{E_t P_{F,t}^*}{P_{F,t}}$ as well as $C_t^* = Y_t^*$.

Plugging Eqs.(1-12), (1-17), (1-20), (1-27), (1-26) and (6-5) into Eq.(1-25) yields:

$$
\begin{aligned}
\left(\frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\varepsilon} Y_t &= \left(\frac{P_t(j)}{P_{H,t}} \right)^{-\varepsilon} C_{H,t} + \left(\frac{P_{H,t}^*(j)}{P_{H,t}^*} \right)^{-\varepsilon} EX_t + \left(\frac{P_t(j)}{P_{H,t}} \right)^{-\varepsilon} G_t \\
&= (1-\nu) \left(\frac{P_t(j)}{P_{H,t}} \right)^{-\varepsilon} S_t^{\nu} C_t + \nu \left(\frac{P_{H,t}^*(j)}{P_{H,t}^*} \right)^{-\varepsilon} S_t \Psi_t Y_t^* + \left(\frac{P_t(j)}{P_{H,t}} \right)^{-\varepsilon} G_t \\
&= \left(\frac{P_t(j)}{P_{H,t}} \right)^{-\varepsilon} \left[(1-\nu) S_t^{\nu} C_t + \nu S_t \Psi_t Y_t^* + G_t \right] \qquad , \\
&= \left(\frac{P_t(j)}{P_{H,t}} \right)^{-\varepsilon} \left[(1-\nu) S_t^{\nu} C_t + \nu S_t \Psi_t Y_t^* + G_t \right]
\end{aligned}
$$

where we use the LOOP implying that $P_{H,t}^*(j) = \frac{P_{H,t}(j)}{E_t}$ and $P_{H,t}^* = \frac{P_{H,t}}{E_t}$. By dividing both sides of the previous expression by $\left(\frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\varepsilon}$ yields:

$Y_t = (1-\nu) S_t^{\nu} C_t + \nu S_t \Psi_t C_t^* + G_t$, (6-6)

which is identical with Eq.(A.5) in Appendix A.

6.4 The Steady State

We focus on equilibria where the state variables follow paths that are close to a deterministic stationary equilibrium, in which $\Pi_{H,t} = \Pi_t = 1$. Further, we assume $Z_t = Z_t^* = 1$ and $G_t = 0$.

Eqs.(1-6) and (1-9) implies as follows:

$$\beta = \frac{1}{1+i} = \frac{1}{1+i^*},$$

which is identical with Eq.(2-1).

Eq.(1-7) implies that:

$$\frac{W}{P} = \frac{V_n}{U_c},$$

which is identical with Eq.(2-2).

Eq.(1-8) implies as follows:

$$\frac{U_l}{U_c} = \beta i,$$

which is identical with Eq.(2-3).

Eq.(6-4) implies:

$$1 = \frac{\frac{\varepsilon}{\varepsilon-1}(1-\tau_F)\left[1+\theta\beta+(\theta\beta)^2+\cdots\right]\left[\left(U_c^*\right)^{-1}\right]^{-1} C_F}{\left[1+\theta\beta+(\theta\beta)^2+\cdots\right]\left[\left(U_c^*\right)^{-1}\right]^{-1} C_F \Psi^{-1}},$$

which can be rewritten as:

$$\Psi = \left[\mathrm{M}(1-\tau_F)\right]^{-1},$$

with $\mathrm{M} \equiv \frac{\varepsilon}{\varepsilon-1}$ being the constant markup. As long as we assume $\mathrm{M}(1-\tau_F)=1$,

$\Psi = 1$, (6-7)

which implies that $EP_F^* = P_F$ is applicable. Eq.(6-7) is identical with Eq.(B.1) in Appendix B.

Due to Eq.(6-7), $Q = S^{1-\nu}$ is applicable, then the other steady state conditions are identical to those in the perfect pass-through environment, namely, Section 2 in this appendix.

6.5 Log-linearization of the Model

6.5.1 Log-linearizing the International Risk Sharing Condition

Total derivative of the definition of the TOT is given by:

$$
\begin{aligned}
dS_t &= \frac{1}{P_H} dP_{F,t} + P_F(-)P_H^{-2} dP_{H,t} \\
&= \frac{P_F}{P_H}\frac{dP_{F,t}}{P_F} - \frac{P_F}{P_H}\frac{dP_{H,t}}{P_H} \\
&= S\frac{dP_{F,t}}{P_F} - S\frac{dP_{H,t}}{P_H}
\end{aligned}
.
$$

Dividing both sides of the previous expression yields:

$$\frac{dS_t}{S} = \frac{dP_{F,t}}{P_F} - \frac{dP_{H,t}}{P_H},$$

Which can be expressed as:

$$s_t = p_{F,t} - p_{H,t}.$$

Total derivative of the definition of the real exchange rate

$Q_t \equiv \frac{E_t P_t^*}{P_t} = \frac{E_t P_{F,t}^*}{P_{F,t}} \frac{P_{F,t}}{P_{H,t}^{1-\nu} P_{F,t}^{\nu}} = \Psi_t S_t^{1-\nu}$ is given by:

$$
\begin{aligned}
dQ_t &= S^{1-\nu} d\Psi_t + \Psi(1-\nu) S^{-\nu} dS_t \\
&= S^{1-\nu}\frac{d\Psi_t}{\Psi} + (1-\nu) S^{-\nu}\frac{dS_t}{S}
\end{aligned}
,
$$

which can be rewritten as:

$q_t = \psi_t + (1-\nu)s_t$. (6-8)

Plugging Eq.(6-8) into Eq.(3-1) yields:

$$\hat{c}_t = \frac{1}{\sigma}\psi_t + \frac{1-\nu}{\sigma}s_t + \hat{c}_t^* + \frac{1}{\sigma}\zeta_t,$$

with $\zeta_t \equiv -\log\left(\frac{Z_t^*}{Z_t}\right)$ which is log-linearized international risk sharing condition.

Plugging the log-linearized market clearing in the foreign country $\hat{y}_t^* = \hat{c}_t^*$ into the previous expression yields:

$$\hat{c}_t = \hat{y}_t^* + \frac{1}{\sigma}\psi_t + \frac{1-\nu}{\sigma}s_t + \frac{1}{\sigma}\zeta_t,$$

which is identical with Eq.(51) in the text.

6.5.2 Log-linearizing the Market Clearing Condition

Total derivative of Eq.(6-6) is given by:

$$
\begin{aligned}
dY_t &= \left[(1-\nu)\nu C + \nu Y^*\right] dS_t + (1-\nu) dC_t + \nu Y^* d\Psi_t + \nu dY_t^* + dG_t \\
&= \nu\left[(1-\nu)+1\right] Y dS_t + (1-\nu) dC_t + \nu Y^* d\Psi_t + \nu dY_t^* + dG_t \quad 。\\
&= \nu(2-\nu) Y dS_t + (1-\nu) dC_t + \nu Y^* d\Psi_t + \nu dY_t^* + dG_t
\end{aligned}
$$

By dividing both sides of Eq.(2-3-8) by Y, we have:

$$
\frac{dY_t}{Y} = \nu(2-\nu) dS_t + (1-\nu)\frac{dC_t}{C} + \nu d\Psi_t + \nu\frac{dY_t^*}{Y^*} + \frac{dG_t}{Y},
$$

which can be rewritten as:

$$
\log\left(\frac{Y_t}{Y}\right) = \nu(2-\nu)\log S_t + (1-\nu)\log\left(\frac{C_t}{C}\right) + \nu\log\Psi_t + \nu\log\left(\frac{Y_t^*}{Y^*}\right) + \log\left(\frac{G_t}{Y}\right),
$$

The previous expression can be rewritten as:

$$
\hat{y}_t = \nu(2-\nu) s_t + (1-\nu)\hat{c}_t + \nu\psi_t + \nu\hat{y}_t^* + \hat{g}_t,
$$

which is identical with Eq.(52) in the text.

6.5.3 Deriving the Import Goods Inflation Equation in Imperfect Pass-through Environment

$$
\tilde{X}_{F,t}\left\{\begin{aligned}
&\left[\left(U_{c,t}^*\right)^{-1}\right]^{-1} Z_t^* C_{F,t|t}\Psi_t^{-1}\Pi_{F,t}^{-1} \\
&+\theta_F\beta\left[\left(U_{c,t+1}^*\right)^{-1}\right]^{-1} Z_{t+1}^* C_{F,t+1|t}\frac{E_{t+1}}{E_t}\Psi_{t+1}^{-1}\Pi_{F,t+1}^{-1}\Pi_{F,t}^{-1} \\
&+(\theta_F\beta)^2\left[\left(U_{c,t}^*\right)^{-1}\right]^{-1} Z_{t+2}^* C_{F,t+2|t}\frac{E_{t+2}}{E_{t+1}}\frac{E_{t+1}}{E_t}\Psi_{t+2}^{-1}\Pi_{F,t+2}^{-1}\Pi_{F,t+1}^{-1}\Pi_{F,t}^{-1} \\
&+\cdots
\end{aligned}\right\}
$$

$$
= \frac{\varepsilon}{\varepsilon-1}(1-\tau_F)\left\{\begin{aligned}
&\left[\left(U_{c,t}^*\right)^{-1}\right]^{-1} Z_t^* C_{F,t|t} + \theta_F\beta\left[\left(U_{c,t}^*\right)^{-1}\right]^{-1} Z_{t+1}^* C_{F,t+1|t} \\
&+(\theta_F\beta)^2\left[\left(U_{c,t}^*\right)^{-1}\right]^{-1} Z_{t+2}^* C_{F,t+2|t} + \cdots
\end{aligned}\right\}
$$

Total derivative of Eq.(6-3) is given by:

$$d\tilde{X}_{F/E,t}=\left[1+\theta_F\beta+(\theta_F\beta)^2+\cdots\right]U_c^*C_F(-1)\left\{\left[1+\theta_F\beta+(\theta_F\beta)^2+\cdots\right]U_c^*C_F\right\}^{-2}$$

$$\times\begin{Bmatrix}C_FdU_{c,t}^*+\theta_F\beta C_FdU_{c,t+1}^*+(\theta_F\beta)^2C_FdU_{c,t+2}^*+\cdots\\+U_c^*C_FdZ_t^*+\theta_F\beta U_c^*C_FdZ_{t+1}^*+(\theta_F\beta)^2U_c^*C_FdZ_{t+2}^*+\cdots\\+U_c^*dC_{F,t|t}+\theta_F\beta U_c^*dC_{F,t+1|t}+(\theta_F\beta)^2U_c^*dC_{F,t+2|t}+\cdots\\+(-1)U_c^*C_Fd\Psi_t+\theta_F\beta(-1)U_c^*C_Fd\Psi_{t+1}+(\theta_F\beta)^2(-1)U_c^*C_Fd\Psi_{t+2}+\cdots\\+(-1)U_c^*C_F\left[1+\theta_F\beta+(\theta_F\beta)^2+\cdots\right]d\Pi_{F/E,t}\\+(-1)U_c^*C_F\left[\theta_F\beta+(\theta_F\beta)^2+\cdots\right]d\Pi_{F/E,t+1}+(-1)U_c^*C_F\left[(\theta_F\beta)^2+\cdots\right]d\Pi_{F/E,t+2}\\+\cdots\end{Bmatrix}$$

$$+\left\{\left[1+\theta_F\beta+(\theta_F\beta)^2+\cdots\right]U_c^*C_F\right\}^{-1}$$

$$\times\begin{bmatrix}C_FdU_{c,t}^*+\theta_F\beta C_FdU_{c,t+1}^*+(\theta_F\beta)^2C_FdU_{c,t+2}^*+\cdots\\+U_c^*C_FdZ_t^*+\theta_F\beta U_c^*C_FdZ_{t+1}^*+(\theta_F\beta)^2U_c^*C_FdZ_{t+2}^*+\cdots\\+U_c^*dC_{F,t|t}+\theta_F\beta U_c^*dC_{F,t+1|t}+(\theta_F\beta)^2U_c^*dC_{F,t+2|t}+\cdots\end{bmatrix},$$

which can be rewritten as:

$$d\tilde{X}_{F/E,t}=\left\{\left[1+\theta_F\beta+(\theta_F\beta)^2+\cdots\right]U_c^*C_F\right\}^{-1}$$

$$\times\begin{Bmatrix}U_c^*C_Fd\Psi_t+\theta_F\beta(-1)U_c^*C_Fd\Psi_{t+1}+(\theta_F\beta)^2(-1)U_c^*C_Fd\Psi_{t+2}+\cdots\\+U_c^*C_F\left[1+\theta_F\beta+(\theta_F\beta)^2+\cdots\right]d\Pi_{F/E,t}\\+U_c^*C_F\left[\theta_F\beta+(\theta_F\beta)^2+\cdots\right]d\Pi_{F/E,t+1}+U_c^*C_F\left[(\theta_F\beta)^2+\cdots\right]d\Pi_{F/E,t+2}\\+\cdots\end{Bmatrix}.$$

Further:

$$d\tilde{X}_{F/E,t}=\left[1+\theta_F\beta+(\theta_F\beta)^2+\cdots\right]^{-1}\begin{Bmatrix}d\Psi_t+\theta_F\beta d\Psi_{t+1}+(\theta_F\beta)^2d\Psi_{t+2}+\cdots\\+\left[1+\theta_F\beta+(\theta_F\beta)^2+\cdots\right]d\Pi_{F/E,t}\\+\left[\theta_F\beta+(\theta_F\beta)^2+\cdots\right]d\Pi_{F/E,t+1}+\left[(\theta_F\beta)^2+\cdots\right]d\Pi_{F/E,t+2}\\+\cdots\end{Bmatrix}.$$

Note that $1+\theta\beta+(\theta\beta)^2+\cdots=\frac{1}{1-\theta\beta}$. Then, the previous expression can be rewritten as:

$$
\begin{aligned}
d\tilde{X}_{F/E,t} &= (1-\theta_F\beta)\left\{
\begin{aligned}
&d\Psi_t + \theta_F\beta d\Psi_{t+1} + (\theta_F\beta)^2 d\Psi_{t+2} + \cdots \\
&+\frac{1}{1-\theta_F\beta} d\Pi_{F/E,t} + \frac{\theta_F\beta}{1-\theta_F\beta} d\Pi_{F/E,t+1} + \frac{(\theta_F\beta)^2}{1-\theta_F\beta} d\Pi_{F/E,t+2} + \cdots
\end{aligned}
\right\} \\
&= (1-\theta_F\beta)\left[d\Psi_t + \theta_F\beta d\Psi_{t+1} + (\theta_F\beta)^2 d\Psi_{t+2} + \cdots\right] \\
&\quad + d\Pi_{F/E,t} + \theta_F\beta d\Pi_{F/E,t+1} + (\theta_F\beta)^2 d\Pi_{F/E,t+2} + \cdots
\end{aligned} . \quad (6\text{-}9)
$$

The definition of $\tilde{X}_{F/E,t}$ can be log-linearized as:

$$
\begin{aligned}
d\tilde{X}_{F/E,t} &= d\left(\frac{\tilde{P}_{F,t}}{P_{F,t-1}}\right) - d\left(\frac{E_t}{E_{t-1}}\right) \\
&= d\left(\frac{\tilde{P}_{F,t}}{P_F}\frac{P_F}{P_{F,t-1}}\right) - d\left(\frac{E_t}{E}\frac{E}{E_{t-1}}\right) \\
&= d\left(\frac{\tilde{P}_{F,t}}{P_F}\right) - d\left(\frac{P_{F,t-1}}{P_F}\right) - \left[d\left(\frac{E_t}{E}\right) - d\left(\frac{E_{t-1}}{E}\right)\right]
\end{aligned} .
$$

Plugging the previous expression into Eq.(6-9) yields:

$$
\begin{aligned}
\tilde{p}_{F,t} - p_{F,t-1} - (e_t - e_{t-1}) &= (1-\theta_F\beta)\left[\psi_t + \theta_F\beta\psi_{t+1} + (\theta_F\beta)^2\psi_{t+2} + \cdots\right] \\
&\quad + \left[\pi_{F/E,t} + \theta_F\beta\pi_{F/E,t+1} + (\theta_F\beta)^2\pi_{F/E,t+2} + \cdots\right]
\end{aligned}, \quad (6\text{-}10)
$$

with $\pi_{F/E,t} \equiv \log\Pi_{F/E,t}$, $\tilde{p}_{F,t} \equiv \log\left(\frac{\tilde{P}_{F,t}}{P_F}\right)$, $p_{F,t} \equiv \log\left(\frac{P_{F,t}}{P_F}\right)$, $\Delta e_t \equiv e_t - e_{t-1}$ and $e_t \equiv \log E_t$.

Forwarding Eq.(6-10) one period yields:

$$
\begin{aligned}
\tilde{p}_{F,t+1} - p_{F,t} - (e_{t+1} - e_t) &= (1-\theta_F\beta)\left[\psi_{t+1} + \theta_F\beta\psi_{t+2} + (\theta_F\beta)^2\psi_{t+3} + \cdots\right] \\
&\quad + \left[\pi_{F/E,t+1} + \theta_F\beta\pi_{F/E,t+2} + (\theta_F\beta)^2\pi_{F/E,t+3} + \cdots\right]
\end{aligned}.
$$

Multiplying $\theta_F\beta$ on both sides of the previous expression yields:

$$
\begin{aligned}
\theta_F\beta\left[\tilde{p}_{F,t+1} - p_{F,t} - (e_{t+1} - e_t)\right] &= (1-\theta_F\beta)\left[\theta_F\beta\psi_{t+1} + (\theta_F\beta)^2\psi_{t+2} + (\theta_F\beta)^3\psi_{t+3} + \cdots\right] \\
&\quad + \left[\theta_F\beta\pi_{F/E,t+1} + (\theta_F\beta)^2\pi_{F/E,t+2} + (\theta_F\beta)^3\pi_{F/E,t+3} + \cdots\right]
\end{aligned}. \quad (6\text{-}11)
$$

Eq.(6-10) can be rewritten as:

$$\tilde{p}_{F,t} - p_{F,t-1} - (e_t - e_{t-1}) = (1-\theta_F\beta)\psi_t + \pi_{F/E,t} + (1-\theta_F\beta)\left[\theta_F\beta\psi_{t+1} + (\theta_F\beta)^2\psi_{t+2} + \cdots\right] + \left[\theta_F\beta\pi_{F/E,t+1} + (\theta_F\beta)^2\pi_{F/E,t+2} + \cdots\right].$$

Plugging Eq.(6-11) into the previous expression yields:

$$\tilde{p}_{F,t} - p_{F,t-1} - (e_t - e_{t-1}) = (1-\theta_F\beta)\psi_t + \pi_{F/E,t} + \theta_F\beta\left[\tilde{p}_{F,t+1} - p_{F,t} - (e_{t+1} - e_t)\right]. \quad (6\text{-}12)$$

Calvo-pricing's transitory equation is given by:

$$P_{F,t} = \left[\theta_F P_{F,t-1}^{1-\varepsilon} + (1-\theta_F)\tilde{P}_{F,t}^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$$

Log-linearizing the previous expression around the steady state yields:

$$p_{F,t} = \theta_F p_{F,t-1} + (1-\theta_F)\tilde{p}_{F,t}.$$

Subtracting $p_{F,t-1}$ from the both sides of the previous expression yields:

$$\pi_{F,t} = (1-\theta_F)(\tilde{p}_{F,t} - p_{F,t-1}),$$

which can be rewritten as:

$$\tilde{p}_{F,t} - p_{F,t-1} = \frac{1}{1-\theta_F}\pi_{F,t}. \quad (6\text{-}13)$$

Plugging Eq.(6-13) into Eq.(6-12) yields:

$$\frac{1}{1-\theta_F}\pi_{F,t} - (e_t - e_{t-1}) = (1-\theta_F\beta)\psi_t + \pi_{F/E,t} + \theta_F\beta\left[\frac{1}{1-\theta_F}\pi_{F,t+1} - (e_{t+1} - e_t)\right]. \quad (6\text{-}14)$$

Log-linearizing the definition of $\Pi_{F/E,t} \equiv \frac{P_{F,t}/E_t}{P_{F,t-1}/E_{t-1}} = \frac{P_{F,t}}{E_t}\frac{E_{t-1}}{P_{F,t-1}} = \Pi_{F,t}\left(\frac{E_t}{E_{t-1}}\right)^{-1}$ yields:

$$\begin{aligned} d\Pi_{F/E,t} &= d\Pi_{F,t} - \frac{dE_t}{E} + (-1)E(-1)E^{-2}dE_{t-1} \\ &= d\Pi_{F,t} - \frac{dE_t}{E} + \frac{dE_{t-1}}{E} \end{aligned}.$$

Thus, Eq.(6-14) can be rewritten as:

$$\frac{1}{1-\theta_F}\pi_{F,t} - (e_t - e_{t-1}) = (1-\theta_F\beta)\psi_t + \pi_{F,t} - (e_t - e_{t-1}) + \theta_F\beta\left[\frac{1}{1-\theta_F}\pi_{F,t+1} - (e_{t+1} - e_t)\right],$$

which can be rewritten as:

$$\left(\frac{1}{1-\theta_F}-1\right)\pi_{F,t}=(1-\theta_F\beta)\psi_t+\theta_F\beta\left[\frac{1}{1-\theta_F}\pi_{F,t+1}-(e_{t+1}-e_t)\right].$$

Note that $\frac{1}{1-\theta_F}-1=\frac{1-(1-\theta_F)}{1-\theta_F}$. Thus, the previous expression can be rewritten as:

$$=\frac{\theta_F}{1-\theta_F}$$

$$\pi_{F,t}=\frac{1-\theta_F}{\theta_F}\left\{(1-\theta_F\beta)\psi_t+\theta_F\beta\left[\frac{1}{1-\theta_F}\pi_{F,t+1}-(e_{t+1}-e_t)\right]\right\}.$$

Finally, the previous expression can be rewritten as:

$$\pi_{F,t}=\beta\pi_{F,t+1}+\frac{(1-\theta_F)(1-\theta_F\beta)}{\theta_F}\psi_t-\frac{\beta(1-\theta_F)}{\theta_F}(e_{t+1}-e_t),$$

which is Eq.(53) in the text.

Plugging $\pi_{F/E,t}=\pi_{F,t}-(e_t-e_{t-1})$ into Eq.(6-10) yields:

$$\begin{aligned}
\tilde{p}_{F,t}-p_{F,t-1}-(e_t-e_{t-1}) &= (1-\theta_F\beta)\left[\psi_t+\theta_F\beta\psi_{t+1}+(\theta_F\beta)^2\psi_{t+2}+\cdots\right]+\pi_{F,t}-(e_t-e_{t-1}) \\
&\quad +\theta_F\beta\left[\pi_{F,t+1}-(e_{t+1}-e_t)\right]+(\theta_F\beta)^2\left[\pi_{F,t+2}-(e_{t+2}-e_{t+1})\right]+\cdots \\
&= (1-\theta_F\beta)\left[\psi_t+\theta_F\beta\psi_{t+1}+(\theta_F\beta)^2\psi_{t+2}+\cdots\right] \\
&\quad +(p_{F,t}-p_{F,t-1})-(e_t-e_{t-1})+\theta_F\beta\left[(p_{F,t+1}-p_{F,t})-(e_{t+1}-e_t)\right] \\
&\quad +(\theta_F\beta)^2\left[(p_{F,t+2}-p_{F,t+1})-(e_{t+2}-e_{t+1})\right]+\cdots \\
&= (1-\theta_F\beta)\left[\psi_t+\theta_F\beta\psi_{t+1}+(\theta_F\beta)^2\psi_{t+2}+\cdots\right] \\
&\quad -p_{F,t-1}+(1-\theta_F\beta)p_{F,t}+(1-\theta_F\beta)\theta_F\beta p_{F,t+1}+(1-\theta_F\beta)(\theta_F\beta)^2 p_{F,t+2} \\
&\quad +\cdots \\
&\quad +e_{t-1}-(1-\theta_F\beta)e_t-(1-\theta_F\beta)\theta_F\beta e_{t+1}-(1-\theta_F\beta)(\theta_F\beta)^2 e_{t+2}-\cdots \\
&= (1-\theta_F\beta)\begin{bmatrix}\psi_t+\theta_F\beta\psi_{t+1}+(\theta_F\beta)^2\psi_{t+2}+\cdots \\ +p_{F,t}+\theta_F\beta p_{F,t+1}+(\theta_F\beta)^2 p_{F,t+2}+\cdots \\ -e_t-\theta_F\beta e_{t+1}-(\theta_F\beta)^2 e_{t+2}-\cdots\end{bmatrix}-p_{F,t-1}+e_{t-1}
\end{aligned},$$

Rearranging the previous expression on $\tilde{p}_{F,t}-e_t$ yields:

$$\tilde{p}_{F,t}-e_t=(1-\theta_F\beta)\sum_{k=0}^{\infty}(\theta_F\beta)^k\left[\psi_{t+k}+(p_{F,t+k}-e_{t+k})\right],$$

which is identical with log-linearized FONC for foreign retailers in Section 6.5.1 in the text.

6.5.4 Some Entities

The nominal exchange rate is calculated by:

$$s_t = e_t + p_t^* - \psi_t - p_{H,t}\,.$$

which is Eq.(54) in the text.

The LOOP gap is calculated via:

$$e_t = \psi_t + p_{F,t} - p_t^*,$$

which is Eq.(55) in the text.

6.5.5 The Steady State in the Case of No Subsidiary

We focus on equilibria where the state variables follow paths that are close to a deterministic stationary equilibrium, in which $\Pi_{H,t} = \Pi_t = 1$. Further, we assume $Z_t = Z_t^* = 1$ and $G_t = 0$.

Eqs.(1-6) and (1-9) implies as follows:

$$\begin{aligned}\beta &= \frac{1}{1+i} \\ &= \frac{1}{1+i^*}\end{aligned},$$

which is identical with Eq.(2-1).

Eq.(1-7) implies that:

$$\frac{W}{P} = \frac{V_n}{U_c},$$

which is identical with Eq.(2-2).

Eq.(1-8) implies as follows:

$$\frac{U_l}{U_c} = \beta i,$$

which is identical with Eq.(2-3).

Eq.(6-4) implies:

$$1 = \frac{\frac{\varepsilon}{\varepsilon - 1}(1-\tau_F)\left[1+\theta\beta+(\theta\beta)^2+\cdots\right]\left[\left(U_c^*\right)^{-1}\right]^{-1} C_F}{\left[1+\theta\beta+(\theta\beta)^2+\cdots\right]\left[\left(U_c^*\right)^{-1}\right]^{-1} C_F \Psi^{-1}},$$

which can be rewritten as:

$$\Psi = \left[\mathrm{M}(1-\tau_F)\right]^{-1},$$

with $\mathrm{M} \equiv \frac{\varepsilon}{\varepsilon - 1}$ being the constant markup. As long as we assume $\tau_F = 0$,

$\Psi = \mathrm{M}^{-1}$. (6-15)

Eq.(1-24) implies:

$$MC = \frac{1}{1-\alpha} \frac{W}{P_H} N^{\alpha},$$

Which is identical with Eq.(2-5).

Eq.(1-16) can be rewritten as:

$$\frac{V_n}{U_c} = \frac{W}{P_H} \frac{P_H}{P},$$

which is identical with Eq.(2-6).

Plugging Eq.(2-5) into Eq.(2-6) yields:

$$\frac{V_n}{U_c} = \frac{1-\alpha}{N^{\alpha} \mathrm{M}} \frac{P_H}{P},$$

which is identical with Eq.(2-7).

Plugging Eq.(2-8) into Eq.(2-7) yields:

$$\frac{V_n}{U_c} = \frac{1-\alpha}{N^{\alpha} \mathrm{M} S^{\nu}},$$

which can be written as:

$$V_n = \frac{1-\alpha}{N^{\alpha} \mathrm{M} S^{\nu}} U_c,$$

which is identical with Eq.(2-9)

Eq.(6-1) implies:

$$U_c^{-1} = \vartheta \left(U_c^{*} \right)^{-1} S^{1-\nu} \Psi .$$

Plugging Eq.(6-15) into the previous expression yields:

$$\begin{aligned} U_c^{-1} &= \vartheta \left(U_c^{*} \right)^{-1} S^{1-\nu} \mathrm{M}^{-1} \\ &= \vartheta \left(U_c^{*} \right)^{-1} \omega(S) \end{aligned} . \text{ (6-16)}$$

Note that:

$$\begin{aligned} \omega(S) &\equiv Q \\ &= \frac{EP^{*}}{P} = \frac{P_F}{P_H^{1-\nu} P_F^{\nu}} \frac{EP_F^{*}}{P_F} = \left(\frac{P_F}{P_H} \right)^{1-\nu} \mathrm{M}^{-1} \\ &= S^{1-\nu} \mathrm{M}^{-1} \end{aligned} . \text{ (6-17)}$$

Eq.(6-16) can be rewritten as:

$$S^{\nu} = \vartheta\left(U_c^*\right)^{-1} S\mathrm{M}^{-1} U_c$$

Plugging the previous expression into Eq.(2-9) yields:

$$\begin{aligned} V_n &= \frac{1-\alpha}{N^{\alpha}} \frac{\mathrm{M}^{-1}}{\vartheta\left(U_c^*\right)^{-1} S\mathrm{M}^{-1}} \\ &= \frac{1-\alpha}{N^{\alpha}} \frac{1}{\vartheta\left(U_c^*\right)^{-1} S} \end{aligned}. \text{ (6-18)}$$

Let define $H\left(S,U_c^*\right) \equiv V_n N^{\alpha}$. Plugging this definition into Eq.(6-18) yields:

$$H\left(S,U_c^*\right) \equiv (1-\alpha) \frac{1}{S\vartheta\left(U_c^*\right)^{-1}}.$$

Notice that $H_S < 0$, $\lim_{S\to 0} H\left(S,U_c^*\right) = +\infty$ and $\lim_{S\to\infty} H\left(S,U_c^*\right) = 0$ ($g_S > 0$).

On the other hand, the market clearing Eq.(6-6) implies:

$Y = (1-\nu) S^{\nu} C + \nu \mathrm{M}^{-1} S Y^*$, (6-19)

where we use $C^* = Y^*$.

Because of $C = F\left(U_c^{-1}\right)$ and Eq.(6-16), we have:

$$\begin{aligned} C &= F\left[\vartheta\left(U_c^*\right)^{-1} \omega(S)\right] \\ &= F\left[\vartheta\left(U_c^*\right)^{-1} S^{1-\nu} \mathrm{M}^{-1}\right] \end{aligned},$$

with *F* being the operator of function.
Plugging the previous expression into Eq.(6-19) yields:

$Y = (1-\nu) S^{\nu} F\left[\vartheta\left(U_c^*\right)^{-1} S^{1-\nu} \mathrm{M}^{-1}\right] + \nu S C^*$. (6-20)

Let define $J\left(S,C^*\right) \equiv (1-\nu) S^{\nu} F\left[\vartheta\left(U_c^*\right)^{-1} S^{1-\nu} \mathrm{M}^{-1}\right] + \nu S Y^*$. Note that $J_S > 0$, $\lim_{S\to 0} J\left(S,C^*\right) = 0$ and $\lim_{S\to\infty} J\left(S,C^*\right) = +\infty$.

Hence, given a value for C^*, ϑ and Y^*, Eqs.(6-18) and (6-20), jointly determine the steady state value for *S* and $\omega(S)$, i.e., the steady state value of the TOT and the real exchange rate.

Dividing both sides of Eq.(6-19) by C^* yields:

$$\frac{Y}{C^*}=(1-\nu)S^{\nu}\frac{C}{C^*}+\nu\mathrm{M}^{-1}S.$$

For convenience, and without loss of generality, we can assume that initial conditions (i.e., initial distribution of wealth) are such that $\vartheta=1$ which implies that $Q=\frac{C}{C^*}$.

Plugging this condition into the previous expression yields:

$$\begin{aligned}\frac{Y}{C^*}&=(1-\nu)S^{\nu}Q+\nu S\\&=(1-\nu)S^{\nu}S^{1-\nu}\Psi+\nu S\\&=[(1-\nu)\Psi+\nu]S\end{aligned}$$

where we use a steady state condition $Q=S^{1-\nu}\Psi$ which stems from Eq.(A.1).
which can be rewritten as:

$Y=[(1-\nu)\Psi+\nu]SY^*$, (6-20)

by using $Y^*=C^*$ which is the steady state market clearing condition in the foreign country. Eq.(2-15) is no longer applicable.

Eqs.(2-17)—(2-20) is still applicable. Thus. Eq.(2-21), i.e., $S=1$ is applicable. However, even if plugging Eq.(2-21) into Eq.(6-20), we cannot obtain Eq.(2-15) because $\Psi=1$ is not applicable.

Plugging Eq.(2-21) into a steady state condition $Q=S^{1-\nu}\Psi$ yields:

$Q=\mathrm{M}^{-1}$, (6-21)

where we use Eq.(6-15). The PPP in the long run is no longer available.

Plugging Eq.(6-21) into the initial condition yields:

$C=C^*\mathrm{M}^{-1}$, (6-22)

That is, $C=C^*$ is no longer available.

Plugging Eq.(6-22) into Eq.(6-20) yields:

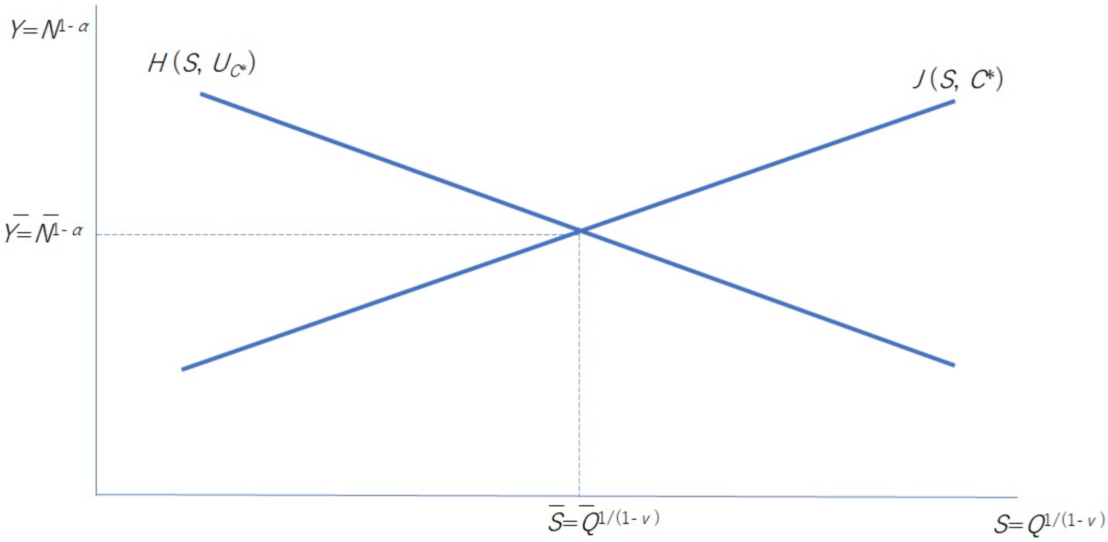
$$\begin{aligned}Y&=[(1-\nu)\mathrm{M}^{-1}+\nu]C\mathrm{M}\\&=(1-\nu)C+\nu C\mathrm{M}\\&=[(1-\nu)+\nu\mathrm{M}]C\end{aligned}.$$

Thus, $Y=C$ is no longer available.

Reference (Not shown in the text only)

Gali, Jordi (2015), ``Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework and Its Application (2nd Eds.),'' *Princeton University Press*, New York.

Figure TA--1



(Relative) PPP in a Small Open Economy in the Steady State