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Central Bank Digital Currency in a Developing Economy: A Dynamic Stochastic General Equilibrium Analysis

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Abstract

Central Bank Digital Currency (CBDC) has been in the center of discussion of many monetary policy research agendas. We explore how the business cycle behavior of a developing economy is affected by the introduction of this type of money as a second monetary policy tool. We emphasize on the characteristic dual formal and informal labor markets that are present in most developing economies, given its relevance on explaining the business cycle dynamics. Our main contribution is the building of a model that encompasses such characteristics and features the relevance of monetary balances to macroeconomic fluctuations. We find that CBDC has the ability to improve the monetary policy effectiveness, and the response of relevant variables may be amplified or dampened, depending on the nature of the shock. Also the magnitude of the new dynamics introduced by CBDC are also profoundly dependant on its structural parameters. The main transmission mechanisms that are affected by CBDC are the dynamics of distortions generated by transaction costs.

1 Introduction

Recently there has been an increased interest in CBDCs at the main monetary policy research agendas. In general, a CBDC is defined as a financial asset that, at the most elemental level, has three properties: i) is a fully digital method of payment, ii) denominated in the national official currency, iii) and its issuance constitutes a liability in the Central Bank's balance sheet. With these fundamental principles, there starts a wide discussion of different aspects on how should such an asset be designed and its implications to economic dynamics. Currently, most research has focused on the implications of a CBDC on financial intermediation and the banking system, which is of main relevance in this subject, from both the positive and normative point of view.[1] However, we want to explore some other aspects of this phenomenon, particularly the interaction of introducing such a novel instrument to the set of monetary policy tools, with labor markets and the ability to control inflation. Taking a specific focus on the characteristics of emerging economies.

Hence, we construct a simple theoretical framework that features the monetary and labor market dynamics, with some of the most common labor market characteristics found on developing economies. Our main aim is to analyze the new business cycle dynamics that come to play with the introduction of a CBDC instrument, using a dynamic stochastic general equilibrium framework. To our best knowledge, a model with such specific characteristics has yet to be presented. Therefore our main contribution relies on the construction of a model that incorporates elements from previous literature's frameworks, but as a whole this model is a new set up to the best of our knowledge.

The kind of CBDC we feature is one that has interest bearing, and has the ability to provide higher liquidity services than cash balances. We focus our attention in the characteristic dual labor markets that are present in developing economies, namely, formal and informal. In the aspect of CBDC, our main reference is Barrdear and Kumhof (2021) who use a thorough banking and financial intermediation NK model to asses the short and long run consequences of CBDC isssuance in the United States. Overall, their findings suggest that CBDC issuance may have beneficial effects on steady state GDP levels, and also could help to improve the central bank's control of the business cycle. The mechanisms through which a higher level of long run GDP is achieved is, among others, the reduction of transaction costs, since CBDC provides greater liquidity services to their holders, compared with other means of exchange. Also, greater business cycle stabilization ability is attained in the case where the substitutability between bank deposits and CBDC is low and most shocks impact the money market.

We argue it is relevant to investigate this monetary-labor, and to a greater extent the monetary-dual labor relationship, since this link has proven to be of main interest for policymakers and researchers, both for policy relevant issues, and for the explanatory dynamics that labor markets have over macroeconomic and monetary outcomes. Not exhaustively, some examples of works that have treated this link between labor, inflation and monetary policy, are the ones of Walsh (2005), Sala et al. (2008), Ravenna and Walsh (2008), Gertler et al. (2008), Thomas (2008), Trigari (2009), among many

[1]Some of the works that have explored this issues are Barrdear and Kumhof (2021), Meaning et al. (2018), Fernández-Villaverde et al. (2020), Ferrari et al. (2020), Agur et al. (2021).

others. Most of these studies use variations of labor search frictions embedded in the New Keynesian framework, to account for real and nominal rigidities. Common findings are that the real rigidities introduced by labor frictions help to explain better the inflation and output dynamics in the business cycle data.[2]

One of the main focus of these studies have been in the context of developed countries. In this context, when investigating labor markets in developing economies, there has to be an special focus to the dual nature of these labor markets, namely the formal and informal sectors. In that sense, in order to study the dynamics of formal and informal labor the labor search framework has been expanded to include such characteristics. For instance, Albrecht et al. (2009) introduce into the Mortensen and Pissarides (1994) framework an heterogeneity in workers characteristics, to model the differences between formal and informal workforce in macroeconomic dynamics. Later to that paper, many studies featured similar theoretical frameworks to assess different relevant questions regarding the impact that introducing informal labor has on the business cycle dynamics of other relevant variables. Some of these works are the ones by Ulyssea (2010), Bosch and Esteban-Pretel (2012), Restrepo-Echavarria (2014), Bosch and Esteban-Pretel (2015), Shapiro (2015), Fernández and Meza (2015), Colombo et al. (2019). On the main findings of those works, is that the introduction of informal labor to the theoretical framework improves the models' fit to business cycle data in emerging economies, and account for the differences observed between developing and developed economies' business cycle behavior.

Continuing with this topic, the research on the interactions of dual labor markets with inflation and monetary policy is scarce, and most of its advancements are relatively new. To our best knowledge, the first paper that addressed this problem is Castillo and Montoro (2010), who embedded heterogeneity in labor types, and search rigidities into a New Keynesian monetary policy model. Later to that work, others in a similar line are the ones of Gómez Ospina (2013) and Alberola and Urrutia (2020). The latter develops a model with formal and informal labor, that features working capital requirements in the formal sector, along with the NK nominal rigidities and rule-based monetary policy. Their main findings are that informality acts as a buffer to shocks, since informality provides great flexibility to the labor market. Concretely, characteristic inflationary acceleration in response to positive demand shocks are dampened in the presence of informality, yet in response to negative supply shocks the inflation acceleration is bigger. Furthermore, despite the buffer effect with demand shocks, the monetary policy is found to be less effective in the presence of informality.

We build a model on the grounds of the framework of Alberola and Urrutia (2020), omitting working capital requirements, which we expand to feature an active role for money balances, through transaction costs as modeled in Schmitt-Grohé and Uribe

[2]One study that contrasts with this affirmation is Krause et al. (2008).

(2004) and Barrdear and Kumhof (2021). Our model has the ability to mimic key business cycle unconditional moments observed for Colombia. The main results we present suggest that the presence of CBDC impacts the magnitude of endogenous responses to shocks, moreover the direction of such changes rely heavily on the structural parameters related to CBDC. The main mechanisms through which CBDC acts, is by changing the response of the distortions generated by transaction costs. The rest of the paper is organized as follows: Section 2 presents the theoretical model, Section 3 shows the calibration procedure and its results, Section 4 presents quantitative experiments and results, finally Section 5 concludes.

2 The economy model

The main aim of this paper is evaluating the effect an novel CBDC co-existing with traditional money, and its interactions with the labor market, featuring the main rigidites present in most developing economies. For that purpose, and on account of simplicity and clarity of results, we abstract from financial intermediation and its role in money supply, thus money is assumed to be created in full by the government, where households demand money since it facilitates transactions. Moreover, in the labor market part, we use the model of Alberola and Urrutia (2020) without working capital requirements, since we want to emphasize the role of money as an asset, and its connection with monetary policy tools. The two types of labor markets are formal and informal, both with labor search rigidities as proposed by Mortensen and Pissarides (1994), and calibrated to match regularly observed differences between them. The demand for money comes from the presence of real transaction costs that money help to reduce, where there are two types of monies when CBDC is introduced. Also, the household chooses optimally between four types of assets, in which each one has its own opportunity costs of holding.

2.1 Households

Following the usual abstraction of a big family with perfect income insurance among family members of Merz (1995) and others, the representative household has associated a felicity function with adjustable wealth effect of the form proposed by Jaimovich and Rebelo (2009). This felicity depends contemporaneously on consumption c_t, the total labor effort $l_t^F + l_t^I$, a convex utility cost of searching for a job while unemployed u_t, and a deterministic shifter $X_t = c_t^{\gamma} X_{t-1}^{1-\gamma}$ that turns the wealth effects on ($\gamma \to 1$) and off ($\gamma \to 0$), with $0 \leq \gamma \leq 1$, this specification allows for more flexibility of the dynamic behavior of the model and consequently a better fit to the data. Therefore, the representative household seeks to maximize lifetime utility

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\ln \left[c_t - \psi X_t \frac{\left(l_t^I + l_t^F\right)^{1+\chi}}{1+\chi} \right] - \frac{\zeta}{2} u_t^2 \right) \tag{1}$$

where $0 < \beta < 1$ is the subjective deterministic discount factor. The household faces the real-terms sequence of budget constraint,

$$\begin{aligned} c_t(1+s_t^c)(1+\tau_t^c) + \Omega_t(1+s_t^\Omega) + b_t + m_t^c + m_t^{dc} \leq \\ (1+\pi_t)^{-1}\left[(1+i_{t-1})b_{t-1} + m_{t-1}^c + (1+i_{t-1}^{dc})m_{t-1}^{dc}\right] + \\ w_t^F l_t^F + w_t^I l_t^I + (1-\tau_t^r) r_t k_t + \Pi_t - \tau_t^{ls} \end{aligned} \tag{2}$$

with consumption c_t and investment Ω_t goods having the same price P_t and being the numeraire, letting $\pi_t = P_t/P_{t-1} - 1$ as the net inflation rate of the economy. Riskless real bonds holdings b_{t-1} from $t-1$ to t yield a nominal gross return of $1 + i_{t-1}$. Labor income comes from supplying formal l_t^F and informal l_t^I labor at bargained real wages w_t^F and w_t^I respectively, then the aggregate labor supply is $l_t = l_t^F + l_t^I$.

Traditional real money holdings, denoted by m_{t-1}^c, yield no nominal return, in contrast to the digital currency real holdings m_{t-1}^{dc} that have a gross nominal return of $1 + i_{t-1}^{dc}$, where i_t^{dc} which is set with a spread with respect to i_t, where both might be controlled by the monetary authority in case of a price rule for both monies. This set up for money holdings allows for analyzing monetary policy price and quantity rules with the novel monetary instrument, we detail on this on further sections. Let Π_t denote the real profits net of taxes from all the firms in the economy, and r_t the real return on capital k_t, τ_t^r the net capital return tax rate, and τ_t^{ls} real lump-sum taxes.

In the spending side, consumption and investment are subject to transaction costs s_t^c and s_t^Ω, functional forms and a little mathematical detail is discussed further in posterior sections. Also, in the same line as Barrdear and Kumhof (2021) consumption after spending costs is subject to a value added tax rate τ_t^c. The capital law of motion face convex investment adjustment costs as in Christiano et al. (2005)

$$k_{t+1} = (1-\delta)k_t + \left[1 - \frac{\phi_\Omega}{2}\left(\frac{\Omega_t}{\Omega_{t-1}} - 1\right)^2\right]\Omega_t \tag{3}$$

where $0 < \delta \leq 1$ is the capital depreciation rate, and $\phi_\Omega > 0$ is the parameter that governs those adjustment costs. The first-order conditions of the recursive contingent paths for c_t, X_t, Ω_t, k_{t+1}, b_t, m_t^c and m_t^{dc} of our model are:

$$\left[1 + s_t^c + c_t\frac{\partial s_t^c}{\partial c_t}\right](1+\tau_t^c)\,\lambda_t^{bc} = \left[c_t - \psi X_t \frac{l_t^{1+\chi}}{1+\chi}\right]^{-1} + \gamma\frac{X_t}{c_t} \tag{4}$$

$$\lambda_t^X = \beta(1-\gamma)\mathbb{E}_t\lambda_{t+1}^X\left(\frac{X_{t+1}}{X_t}\right) - \psi\frac{l_t^{1+\chi}}{1+\chi}\left[c_t - \psi X_t\frac{l_t^{1+\chi}}{1+\chi}\right]^{-1} \tag{5}$$

$$\lambda_t^\Omega = \beta\mathbb{E}_t\left\{\left(1+\tau_{t+1}^r\right) r_{t+1}\lambda_{t+1}^{bc} + (1-\delta)\,\lambda_{t+1}^\Omega\right\} \tag{6}$$

$$\begin{aligned} \left[1 + s_t^\Omega + \Omega_t\tfrac{\partial s_t^\Omega}{\partial \Omega_t}\right]\lambda_t^{bc} = \lambda_t^\Omega\left[1 - \phi_\Omega\left(\tfrac{\Omega_t}{\Omega_{t-1}}\left(\tfrac{\Omega_t}{\Omega_{t-1}} - 1\right) + \tfrac{1}{2}\left(\tfrac{\Omega_t}{\Omega_{t-1}} - 1\right)^2\right)\right] + \\ \phi_\Omega\mathbb{E}_t\lambda_{t+1}^\Omega\tfrac{\Omega_{t+1}}{\Omega_t}\left(\tfrac{\Omega_{t+1}}{\Omega_t} - 1\right)^2 \end{aligned} \tag{7}$$

$$\beta \mathbb{E}_t \frac{\lambda_{t+1}^{bc}}{\lambda_t^{bc}} = \mathbb{E}_t \frac{1+\pi_{t+1}}{1+i_t} \tag{8}$$

$$c_t \left(1+\tau_t^c\right) \frac{\partial s_t^c}{\partial m_t^c} + \Omega_t \frac{\partial s_t^{\Omega}}{\partial m_t^c} = \frac{i_t}{1+i_t} \tag{9}$$

$$c_t \left(1+\tau_t^c\right) \frac{\partial s_t^c}{\partial m_t^{dc}} + \Omega_t \frac{\partial s_t^{\Omega}}{\partial m_t^{dc}} = \frac{i_t - i_t^{dc}}{1+i_t} \tag{10}$$

with λ_t^{bc}, λ_t^{Ω}, λ_t^{X} being the real-terms Lagrange multipliers associated with (2), (3), and $X_t = c_t^{\gamma} X_{t-1}^{1-\gamma}$ respectively.

As the careful reader may notice, this set of first-order conditions jointly represent a sub-system of equations that, to our best knowledge, a model that has not been exactly described in other works.

We talk about more transaction costs and the distortions introduced by them, as well as the money demand functions in further sections.

2.1.1 Labor market categories and measurements

The representative household has a normalized-to-one labor endowment $\bar{l} = 1$. This labor endowment can be thought as the mass of population that is able to work, but may or may not choose to do so, in other words the labor endowment is a model-equivalent of the working age population. Thus, we have that the working age population can be classified in four labor categories: worker in the formal sector l_t^F, worker in the informal sector l_t^I, worker that is looking for a job (unemployed) u_t, and out of the labor force o_t. Then we get the following identity

$$\bar{l} = l_t^F + l_t^I + u_t + o_t. \tag{11}$$

This is especially suiting, since it enables us to define some rates that are common in the labor market statistics. We then define the unemployment rate $u_t^{rate} := u_t/(l_t + u_t)$, the informality rate $l_t^{I,rate} := l_t^I / l_t$, and the labor force participation rate $l_t^{rate} := \left(l_t + u_t\right)/\bar{l} = l_t + u_t$.

2.1.2 Labor supply dynamics

Following Alberola and Urrutia (2020) it is assumed that each period exogenous fractions, $0 < \eta_F \leq 1$ and $0 < \eta_I \leq 1$, of currently filled jobs are destroyed in the formal and informal sector, respectively. As usual, these separation rates are used to reflect the degree of flexiblity of each labor market.

As a consequence, given the nature of the informal labor market, we suppose beforehand that $\eta_F < \eta_I$, this reasoning is congruent with other studies, such as Maloney (1999), Bosch and Maloney (2008), Ulyssea (2010), Bosch and Esteban-Pretel (2012), Bosch and Esteban-Pretel (2015). Another possible way of seeing it, is through the enforcement of labor regulations that act upon the informal sector, provoking higher job destruction rates than in the formal sector, see for example Shapiro (2015). Workers whose jobs get separated, become either unemployed or out of the labor force. Also, it is specified that workers need to be unemployed in order to get a job in either sector, i.e., there's no on-the-job search or job-to-job transitions.

In the job creation side, in aggregate formal and informal firms post vacancies ν_t^F and ν_t^I respectively. In each sector ($x \in \{F, I\}$) those vacancies are filled with the workers that are currently unemployed through a constant-returns-to-scale matching function $(u_t)^\omega(\nu_t^x)^{1-\omega}$, $0 < \omega < 1$. New filled vacancies are $u_t pr^x(\kappa_t^x)$, being $\kappa_t^x := \nu_t^x/u_t$ the labor market tightness, which is taken as given by agents. Hence, the law of movement of each type of labor is:

$$l_t^F = (1-\eta_F)l_{t-1}^F + u_t pr_t^F(\kappa_t^F) \tag{12}$$

$$l_t^I = (1-\eta_I)l_{t-1}^I + u_t pr_t^I(\kappa_t^I). \tag{13}$$

Where $pr_t^x(\kappa_t^x) = (u_t)^\omega(\nu_t^x)^{1-\omega}/u_t$ is the rate at which unemployed workers find a vacancy to fill. From the first order conditions the value accrued to the representative household from an employment relationship in the formal $V_{H,t}^F$ and informal $V_{H,t}^I$ sector are:

$$V_{H,t}^F = w_t^F - z_t^u \psi X_t \frac{l_t^\chi}{\lambda_t^{bc}}\left[c_t - \psi X_t \frac{l_t^{1+\chi}}{1+\chi}\right]^{-1} + (1-\eta_F)\mathbb{E}_t\Lambda_{t,t+1}V_{H,t+1}^F \tag{14}$$

where $\lambda_t^F = V_{H,t}^F\lambda_t^{bc}$, and λ_t^F is the real-terms Lagrange multiplier associated with the law of motion of formal labor, and $\Lambda_{t,t+1} = \beta\lambda_{t+1}^{bc}/\lambda_t^{bc}$ is the stochastic discount factor of the representative household. Similarly, we have for the informal sector[3]:

$$V_{H,t}^I = w_t^I - z_t^u \psi X_t \frac{l_t^\chi}{\lambda_t^{bc}}\left[c_t - \psi X_t \frac{l_t^{1+\chi}}{1+\chi}\right]^{-1} + (1-\eta_I)\mathbb{E}_t\Lambda_{t,t+1}V_{H,t+1}^I \tag{15}$$

with $\lambda_t^I = V_{H,t}^I\lambda_t^{bc}$, and λ_t^I its particular real-terms Lagrange multiplier associated with (13). This two value equations describe the optimal allocation of labor depending on real wages. The definition of wages will be discussed further below. Additional to this first order conditions, the relationship between unemployment and the marginal utility of working in the formal and informal sectors is given by: $\zeta u_t = pr_t^F\lambda_t^F + pr_t^I\lambda_t^I$.

2.2 Transaction costs and money demand

2.2.1 The expenditure-based velocity of transactions

The willingness of the representative household for demanding real monetary balances come from the fact that transacting for consuming and investing carry costs, it is possi-

[3]For details of this derivation see the original paper of Alberola and Urrutia (2020)

ble to reduce those costs by holding money, since money facilitate these transactions. As already mentioned, these costs are denoted by s_t^c and s_t^{Ω}, which following Schmitt-Grohé and Uribe (2004) are functions of the respective expenditure-based velocity $s_t^c(\upsilon_t^c)$ and $s_t^{\Omega} = s_t^{\Omega}(\upsilon_t^{\Omega})$. The respective velocities are defined as $\upsilon_t^c = (1+\tau_t^c)c_t / \left[LFG\left(m^c, m^{dc}\right)\right]$ and $s_t^{\Omega} = \Omega_t / \left[LFG\left(m^c, m^{dc}\right)\right]$, where the *liquidity generating function* LGF is strictly increasing in both arguments. We borrow the functional form used in Barrdear and Kumhof (2021) for LFG since it satisfies the following assumption.

Assumption 1 *(a)$LGF > 0$, $LGF(m^c, m^{dc})$ is strictly increasing in both arguments, (b) $LGF \in C^2$, (c) $\frac{\partial^2 LGF}{\partial m^c \partial m^{dc}} = \frac{\partial^2 LGF}{\partial m^{dc} \partial m^c} = 0$, (d) $1 < \frac{d\ln(m^c/m^{dc})}{d\ln(LGF_{m^{dc}}/LGF_{m^c})} < \infty$.*

Assumption 1(a) states that the greater the real money holdings the greater liquidity services are disposable for the household. Assumption 1(b) is needed for LGF to be well-behaved. Assumption 1(c) is introduced for simplification purposes, which also intuitively detaches the marginal liquidity services one money provides from the other one, and loosely relates to the next assumption. Assumption 1(d) is based on two reasons, first we want to design a setting in which there is no complementarity between the two types of monies. Second, in order for the household to have strictly positive demand for each type of currency, thus avoiding one of the currencies to drive the other out of the economy, we need to impose that the elasticity of substitution between them is less than infinity. For our setting, this elasticity of substitution is less than, but as close as possible to,[4] infinity. Then, one function that satisfies this assumption and is also shown in Barrdear and Kumhof (2021) is

$$LGF(m_t^c, m_t^{dc}) = (m_t^c)^{\theta} + (Tm_t^{dc})^{\theta}$$

where $0 < \theta < 1$, but a θ that is closer to one is preferred, since $\frac{d\ln(m^c/m^{dc})}{d\ln(LGF_{m^{dc}}/LGF_{m^c})} = 1/(1-\theta)$. The parameter $T > 1$ represents the greater ability of digital money holdings to provide liquidity services than the traditional currency.

2.2.2 Transaction costs

A well-defined transaction costs function in this context is one that go along with the same assumptions of Schmitt-Grohé and Uribe (2004), which correspond to:

$$s_t^c(\upsilon_t^c) = A\upsilon_t^c + \frac{B}{\upsilon_t^c} - 2\sqrt{AB} \tag{16}$$

$$s_t^{\Omega}(\upsilon_t^{\Omega}) = A\upsilon_t^{\Omega} + \frac{B}{\upsilon_t^{\Omega}} - 2\sqrt{AB} \tag{17}$$

with $A > 0, B > 0$.

[4]This means that as mentioned, the most plausible scenario is where the household has the most flexibility in the choosing of the means of payment, but for elasticities of substitution that are too big some numeric problems arise for the non-stochastic equilibrium finding of this model.

From the household's first-order conditions, note that in equations (4) and (7), transaction costs introduce wedges that distort the household's otherwise optimal decision paths. With the outlined functional forms, these wedges take the forms:

$$1 + s_t^c + c_t \frac{\partial s_t^c}{\partial c_t} = 1 + 2\left(A\upsilon_t^c - \sqrt{AB}\right)$$

$$1 + s_t^\Omega + \Omega_t \frac{\partial s_t^\Omega}{\partial \Omega_t} = 1 + 2\left(A\upsilon_t^\Omega - \sqrt{AB}\right).$$

In the velocity satiation point $\sqrt{B/A}$ these wedges vanish, but are incremental in any deviation from this point. Hence, given the mentioned assumptions, the household would optimally want to hold an amount of money that sets the transaction costs slightly away from the satiation point. We account for this distortion with the following variable:

$$s_t^{avg} := \frac{c_t\left(1 + 2\left(A\upsilon_t^c - \sqrt{AB}\right)\right) + \Omega_t\left(1 + 2\left(A\upsilon_t^\Omega - \sqrt{AB}\right)\right)}{c_t + \Omega_t}.$$

Which is equivalent to the average liquidity tax in Barrdear and Kumhof (2021), and reflects the average size of the distortions generated by transaction costs, relative to the levels of consumption and investment.

On the other hand, looking at equations (9) and (10), these reflect the demand for each type of money, with respect to their particular opportunity cost. The (implicit) demand for cash, can be written as:

$$\left[A\left(\upsilon_t^c\right)^2 + A\left(\upsilon_t^\Omega\right)^2 - 2B\right]\frac{\theta}{\left(m_t^c\right)^{1-\theta}} = \frac{i_t}{1+i_t}.$$

This equation relates the expenditure-based velocity with the opportunity cost of holding money balances. This equation states one of the transmission mechanisms of the non-CBDC monetary policy, in which changes in nominal interest i_t, together with price stickiness, change the level of distortion generated by transaction costs, therefore generating changes in real variables. Similarly, the demand for CBDC might be reduced to a substitution relationship of the form:

$$\frac{m_t^c}{m_t^{dc}} = \left[\frac{1}{T^\theta}\left(\frac{i_t - i_t^{dc}}{i_t}\right)\right]^{\frac{1}{1-\theta}}. \tag{18}$$

In which the ratio of the two types of money is determined by the ratio of their respective opportunity costs. Note that the opportunity cost of CBDC might be reduced with an increase in its liquidity coefficient T, or by an increase in its net interest yielding i_t^{dc}, which constitutes the transmission mechanism of a CBDC price rule policy rule. In which a change in i_t^{dc} change the optimal allocation between the two types of money.

2.3 The production side

There are two firms one formal land one informal that hire labor with search frictions. These firms' production is then aggregated by a firm that then supplies to intermediate firms, that use this labor-aggregated product altogether with rented capital by the representative household and produce the intermediate differentiated goods, which supply are supplied to the final good producer, with Calvo's imperfect competition and staggered price setting (Calvo, 1983).

2.3.1 Labor market: demand side

Again, following Alberola and Urrutia (2020), we assume for both formal and informal sectors that there's full specialization in labor and vacant jobs, i.e., currently filled jobs cannot be offered in the decentralized market. In the formal sector side, we have a measure one continuum of formal firms, where a particular firm indexed by $j \in [0,1]$ posts vacancies $\nu_t^F(j)$ at fixed exogenous cost $\xi_F > 0$, it supplies labor-derived production $y_t^F(j)$ at competitive market price p_t^F, which is in relative terms to the numeraire. For producing $y_t^F(j)$, it uses a linear technology of the form $y_t^F(j) = a_t^F n_t^F(j)$, where $\mathbb{E}\left\{a_t^F\right\} = 1\ \forall t$ is the productivity of the sector common to all formal firms, and $n_t^F(j)$ are currently filled jobs. Also $\ln a_t^F = \rho_F \ln a_{t-1}^F + \sigma_F \varepsilon_t^F$, $\varepsilon_t^F \sim N(0,1)$, σ_F being a scaling factor. The firm pays a real wage $w_t^F(j)$ to its currently active workers, which is bargained (explained in detail below) in a decentralized market. Analogous to the dynamics of labor from the supply side, the filling of jobs that are vacant follows a Poisson process with parameter rate $q_t^F(\kappa_t^F)$ $=(u_t)^{\omega}(\nu_t^x)^{1-\omega}/\nu_t^F = (\kappa_t^F)^{-\omega}$ (taken as given by the j formal firm), these currently filled jobs are destroyed at exogenous rate η_F, thus we have:

$$n_t(j) = (1-\eta_F)n_{t-1}(j) + \nu_t^F(j)q_t^F(\kappa_t^F) \tag{19}$$

The formal firm chooses $n_t^F(j)$ for maximizing lifetime flows of real profits, discounting future flows with the household's stochastic discount factor, since it's assumed that they own all the firms in this economy. From this problem's first-order conditions we obtain the value accrued to the formal firm from engaging in an employment relationship with a worker:

$$V_{F,t}^F = p_t^F a^F - (1+\tau_t^w)w_t^F(j) + (1-\eta_F)\mathbb{E}_t\Lambda_{t,t+1}V_{F,t+1}^F$$

with $\xi_F = V_{F,t}^F q_t^F(\kappa_t^F)$. Observe that in this expression the wage is related only to common variables, then we may drop the index j, obtaining the aggregated value acrrued to formal firms:

$$V_{F,t}^F = p_t^F a^F - (1+\tau_t^w)w_t^F + (1-\eta_F)\mathbb{E}_t\Lambda_{t,t+1}V_{F,t+1}^F. \tag{20}$$

In the side of informal firms, the $j \in [0,1]$ informal firm (note that we have recycled the index name) produces labor-derived input $y_t^I(j)$ with a linear technology $y_t^I(j) = a^I n_t^I(j)$ which is supplied in a competitive market at price p_t^I. Where $a^I < a^F \implies 0 < a^I < 1$ is a parameter that reflects the lower relative productivity of the informal sector with respect to the formal one, in accordance with the characteristics of the informal sector, see for example OECD (2004), La Porta and Shleifer (2008), La Porta and Shleifer (2014). Note that a^I is not subject to stochastic shocks, this assumption is made for simplicity purposes.

The j informal firm posts vacancies ν_t^I at fixed exogenous cost $\xi_I > 0$, which is expected to be $\xi_I < \xi_F$, and thus will be reflected on calibration. The vacancy filling also follows a Poisson process with rate $q_t^I(\kappa_t^I) = M(\nu_t^I, u_t)/\nu_t^I = \left(\kappa_t^I\right)^{-\omega}$ which is perfectly observable in t but not influenced by particular informal firms. The firm pays bargained real wage w_t^I to its active workers, which is not subject to payroll taxes. The firm maximizes its real lifetime profits by choosing $n_t^I(j)$, discounting future flows with $\Lambda_{t,t+T}$. The first-order condition yields the aggregate value accrued to an informal firm from engaging in an employment relationship:[5]

$$V_{F,t}^I = p_t^I a^I - w_t^I + (1-\eta_I)\mathbb{E}_t \Lambda_{t,t+1} V_{F,t+1}^I \tag{21}$$

and $\xi_I = V_{F,t}^I q_t^I(\kappa_t^I)$.[6]

2.3.2 Intermediate labor firm

There is a unit mass continuum of intermediate labor firms, taking a typical one indexed by j, it produces a labor input $y_t^L(j)$ from aggregated formal and informal labor-derived products $y_t^F(j)$ and $y_t^I(j)$ respectively, which are demanded at competitive market prices. This firm uses the following technology:

$$y_t^L(j) = \left[\left[y_t^F(j)\right]^{\frac{\epsilon_L - 1}{\epsilon_L}} + \left[y_t^I(j)\right]^{\frac{\epsilon_L - 1}{\epsilon_L}} \right]^{\frac{\epsilon_L}{\epsilon_L - 1}}.$$

where $\epsilon_L > 1$ is the elasticity of substitution between formal and informal labor. This firm supplies its product at competitive market price p_t^L to the retailer firm, and chooses $y_t^F(j)$ and $y_t^I(j)$ for maximizing its real profits. Thus the first-order conditions are given by:

[5]Similarly to formal firms, we are able to aggregate since the wage is the only variable that is particular to each j firm.

[6]Details on these derivations can be found in the paper work of Alberola and Urrutia (2020). Note that two minor differences in the mathematical development can be found: 1) we do not aggregate labor firms beforehand, thus we work with a particular indexed-from-the-mass firm and then aggregate behavior, 2) we assume the existence of an intermediate firm that aggregates labor and provides an aggregate labor input for intermediate firms. Neither of these prevents us to arrive to the same first order conditions, thus for this section the underlying structure is the same as the one presented in that model.

$$y_t^F(j) = \left(\frac{p_t^F}{p_t^L}\right)^{-\epsilon_L} y_t^L(j) \qquad\qquad y_t^I(j) = \left(\frac{p_t^I}{p_t^L}\right)^{-\epsilon_L} y_t^L(j)$$

Note that the ratios $y_t^F(j)/y_t^L(j)$ and $y_t^I(j)/y_t^L(j)$ are common to all j firms, allowing for aggregation.

2.3.3 Retailers and final-good firm

As usual in Calvo's framework, there is a final-good firm that aggregates the product of a unit mass continuum of retailers indexed by $j \in [0, 1]$. Particularly, the final good firm produces the final good of the economy Y_t, from all the retail varieties $y_t(j)$ with a Dixit-Stiglitz aggregator: $Y_t = \left(\int_0^1 [y_t(j)]^{(\epsilon_p-1)/\epsilon_p} dj\right)^{\epsilon_p/(\epsilon_p-1)}$, with elasticity of substitution between varieties of $\epsilon_p > 1$. The price of the final good is the same as the numeraire P_t, and the price of each j retail variety is $p_t(j)$, hereby the demand for each j variety is given by: $y_t(j) = (p_t(j)/P_t)^{-\epsilon_p} Y_t$.

The j retail firm produces an intermediate-good $y_t(j)$, in which its inputs are capital $k_t(j)$ and labor-derived good $y_t^L(j)$, assuming a unit elasticity of substitution technology: $y_t(j) = a_t k_t(j)^\alpha y_t^L(j)^{1-\alpha}$, with $0 < \alpha < 1$ being the elasticity of $y_t(j)$ with respect to $k_t(j)$. And a_t is a stochastic shock to total factor productivity common to all retailers, with $\mathbb{E}\{a_t\} = 1\ \forall t$, and $\ln a_t = \rho_a \ln a_{t-1} + \sigma_a \varepsilon_t^a$, $\varepsilon_t^a \sim N(0, 1)$, σ_a being a scaling factor. The competitive price relative to the numeraire of capital is r_t, and the same for labor input is p_t^L. Each retailer j minimizes its real costs in order to satisfy the demand for its variety, resulting in the following input demands after aggregation:

$$k_t = \alpha mc_t \frac{y_t}{r_t} \qquad\qquad y_t^L = (1-\alpha) mc_t \frac{y_t}{p_t^L}$$

where $mc_t(j)$ is the aggregate real marginal cost.

Each j firm has pricing power derived from monopolistic competition, but due to price stickiness we suppose that it has a probability ϕ_p of not being able to change its price in $t+1$, probability that is independent on the particular firm or time, but decreases geometrically each period ahead. Herewith, the optimal price in t is chosen so it maximizes lifetime, discounted by the household's stochastic discount factor, real profit inflows, then the common optimal price p_t^* in inflation terms, defining $\pi_t^* = p_t^*/P_{t-1} - 1$, is given by:

$$1 + \pi_t^* = \frac{\epsilon_p}{\epsilon_p - 1}(1 + \pi_t)\frac{\Xi_{N,t}}{\Xi_{D,t}}$$

$$\Xi_{N,t} = mc_t Y_t + \phi_p \mathbb{E}_t \left\{ (1+\pi_{t+1})^{\epsilon_p} \Lambda_{t,t+1} \Xi_{N,t+1} \right\}$$
$$\Xi_{D,t} = Y_t + \phi_p \mathbb{E}_t \left\{ (1+\pi_{t+1})^{\epsilon_p - 1} \Lambda_{t,t+1} \Xi_{D,t+1} \right\}$$

where $\Xi_{N,t}$ and $\Xi_{D,t}$ are the typical auxiliary variables used for recursive representation. Using the demand for each j variety in the Dixit-Stiglitz aggregator, and realizing that the continuum of retailers can be broken down into the ones that change prices and the ones that cannot do so, we can write the well-known equation for aggregate prices in inflation terms:

$$(1+\pi_t)^{1-\epsilon_p} = \phi_p + (1-\phi_p)\left(1+\pi_t^*\right)^{1-\epsilon_p}.$$

2.4 Wage determination

As equations (14), (15), (20), (21) describe the marginal value accrued for the representative household and firms in the formal and informal sector from a mutual employment relationship, there is yet to define an equation that defines the real wages in each sector. Hereby, following most of the literature on labor search and matching, the wages are negotiated each period, such that the wage is determined trough the bargaining of workers and firms in each sector. This bargained wage splits the surplus firms and workers receive from their agreement according to the proposed solution by Nash (1953). Formally, for each sector $x \in \{F, I\}$, the agreement is reached with a wage $w_t^{*,x}$ that maximizes the Nash product $\left(V_{F,t}^x\right)^{\mu}\left(V_{H,t}^x\right)^{1-\mu}$, with $0 \leq \mu \leq 1$ being the relative bargaining power of firms. Note that we are assuming that this relative bargaining power is the same for both sectors. The wages that solve the problem for each sector satisfy the next equations:

$$(1-\mu)V_{F,t}^F = \mu V_{H,t}^F \qquad\qquad (1-\mu)V_{F,t}^I = \mu V_{H,t}^I$$

which correspond to:

$$w_t^F = \frac{\mu\, mrs_t + (1-\mu)p_t^F a^F}{1+\tau_t^w(1-\mu)} \quad (22) \qquad\qquad w_t^I = \mu\, mrs_t + (1-\mu)p_t^I a^I \quad (23)$$

where $mrs_t = z_t^u \psi X_t \frac{l_t^{\chi}}{\lambda_t^{bc}}\left[c_t - \psi X_t \frac{l_t^{1+\chi}}{1+\chi}\right]^{-1}$. The payroll tax in the formal sector creates a gap in the optimal bargained wage, this gap is shortened as the relative bargaining power of firms gets closer to one.

2.5 Public sector

2.5.1 Monetary policy

The monetary authority is modeled through a Taylor-type reaction function, that responds to deviations of inflation from its target. Especially, when there's issuance of digital money balances ($m_t^{dc} > 0$), the central bank has access to a second monetary policy tool, where both tools available aim at the same inflation target. The first policy tool, which is interest rate Taylor rule associated with cash balances m_t^{dc} is:

$$1 + i_t = (1 + i) \left(\frac{1 + \pi_t}{1 + \pi} \right)^{\varphi_\pi} \left(\frac{gdp_t}{gdp} \right)^{\varphi_{gdp}} \exp \left(\sigma_i \varepsilon_t^i \right) \tag{24}$$

where $\varphi_\pi > 1$ is necessary in order to have determination in the dynamic equilibrium of the model, i is the steady state nominal interest that allows for steady state inflation to be in its target π. Variable $\varepsilon_t^i \sim N(0, 1)$ is i.i.d. and represents discretionary exogenous shocks to the main policy tool, and σ_i is a scaling factor of those shocks.

The second monetary tool, in the form of a price rule is:

$$i_t^{dc} = (i - i_{sp}) \left(\frac{1 + \pi_t}{1 + \pi} \right)^{-\varphi_\pi^{dc}} \tag{25}$$

with $\varphi_\pi^{dc} > 1$, and $0 < i_{sp} < i$, is the spread at which the CBDC remuneration is with respect to the main policy tool. The latter parameter is key to out analysis, since it allows us to asses how the presence of CBDC affects monetary policy, since as shown in Eq. (18), the relative opportunity cost of holding CBDC depends on i_t^{dc}. and its liquidity coefficient T.

For a quantitative CBDC rule, we have the following:

$$\ln m_t^{dc} = \ln \left(rat_{gdp}^{dc} gdp_t \right) \left(\frac{1 + \pi_t}{1 + \pi} \right)^{-\varphi_\pi^{dc}} \tag{26}$$

with $rat_{gdp}^{dc} > 0$ being the steady state ratio of CBDC to GDP.

2.5.2 Fiscal policy

The government issues real money balances in cash and digital form, m_t^c and m_t^{dc} respectively, from the digital money balances issued in $t-1$ it has to pay the gross real interest $(1+i_{t-1}^{dc})(1+\pi_t)^{-1}$. It issues debt in the form of risk-free bonds d_t, in which the bonds issued at $t-1$ pay a real interest of $(1+i_{t-1})(1+\pi_t)^{-1}$. Moreover, the government has a real inflow from tax collection $\tau_t = \tau_t^c c_t + \tau_t^r r_t k_t + \tau_t^w w_t^F l_t^F + \tau_t^{ls}$. The real government expenditure g_t is an exogenous share g_t^{rate} of real GDP, i.e., $g_t = g_t^{rate} gdp_t$, with $\mathbb{E}\{g_t^{rate}\} = g^{rate}\ \forall t$, and $g_t^{rate} = \rho_g g_{t-1}^{rate} + (1 - \rho_g)\, g^{rate} + \sigma_g \varepsilon_t^g$, $\varepsilon_t^g \sim N(0, 1)$. Thus, the government real-terms budget constraint is:

$$m_t^c + m_t^{dc} + d_t + \tau_t = (1+\pi_t)^{-1}\left(m_{t-1}^c + (1+i_{t-1}^{dc})m_{t-1}^{dc} + (1+i_{t-1})d_{t-1}\right) + g_t \tag{27}$$

As in Schmitt-Grohé and Uribe (2007) we define the issued liabilities at the end of $t-1$ as $\ell_{t-1} = (1+\pi_t)^{-1}\left(m_{t-1}^c + (1+i_{t-1}^{dc})m_{t-1}^{dc} + (1+i_{t-1})d_{t-1}\right)$, and the fiscal rule as:

$$\tau_t - \tau = \ell_{t-1} - \ell \tag{28}$$

where dropping the time subscript denotes the deterministic steady state level of the variables. We allow the government access to lump-sum taxation, in the interest of not having confusion in the dynamic effects.

2.6 Equilibrium and aggregation

Given the retailer's demand and first order conditions, the aggregate production function is:

$$Y_t = \frac{a_t}{\upsilon_t^p} k_t^\alpha (y_t^L)^{1-\alpha}. \tag{29}$$

Where $\upsilon_t^p = \int_0^1 (p_t(j)/P_t)\, dj$ is the usual price dispersion term, with aggregate law of motion: $\upsilon_t^p = (1+\pi_t)^{\epsilon_p}\left((1-\phi_p)(1+\pi_t^*)^{-\epsilon_p} + \phi_p \upsilon_{t-1}^p\right)$, which is obtained from the conventional Calvo aggregation.

The bonds market clears when: $b_t = d_t$. Market clearing for labor-related variables imply: $\int_0^1 n_t^x(j)dj = l_t^x$, $\int_0^1 y_t^x(j)dj = y_t^x$, $x \in \{F, I\}$. As the representative household is the owner of all firms in the model economy, we can combine its budget constraint (2) together with the government's budget constraint (27) and profits from all firms, and get a total output Y_t resource constraint condition:

$$Y_t = c_t\left(1 + (1+\tau_t^c)\, s_t^c\right) + \Omega_t\left(1 + s_t^\Omega\right) + g_t + \xi_F \nu_t^F + \xi_I \nu_t^I. \tag{30}$$

This equation reflects that both transaction costs and labor search frictions require real resources from aggregate production. We further define real gross domestic product as: $gdp_t = c_t + \Omega_t + g_t$.

The competitive equilibrium is encompassed by all agents' described first-order conditions of optimality, and the equations describing the government's fiscal and monetary rules.

3 Data and calibration

We perform a calibration procedure for the non-CBDC version of the model, which corresponds to the described set-up but without condition (10), $m_t^{dc} = 0$, $T = 0$ and $i_t^{dc} = 0$. One group of parameters are chosen according to prior assumptions and previous literature, while for the remaining parameters we use relevant business cycle sample second moments and averages from Colombia to calibrate them, such that the model matches those moments. The time frame we use is the period 2007Q1 to 2019Q4, accordingly each t corresponds to a quarter. We use the non-CBDC version of the model, as the observed moments in data are drawn from an economy without CBDC. For the CBDC models, we keep the calibrated parameters, with the exception of those that target steady state moments, which are ξ_F, ξ_I and ζ.

3.1 Business cycle data

We use quarterly business cycle data for Colombia from 2007Q1 to 2019Q4. Particularly, we use the constant prices time series for the levels of GDP, consumption, investment (gross fixed capital formation), government total expenditure, and the rates of unemployment, informality, labor force participation and inflation. For the non-rate variables we transform to natural logarithms towards getting relative standard deviations. For steady state targets, we compute the sample mean for labor force participation rate, informality rate, unemployment rate and government expenditure as share of GDP.

For obtaining the business cycle unconditional second moments, we apply Hodrick-Prescott filter to all variables with the usual $\lambda = 1600$ parameter value.[7] All the time series used, but the one of the inflation rate, are the ones reported by the Departamento Administrativo Nacional de Estadística (DANE, the Colombia's official statistics department). The inflation rate is the three-month moving average consumer price index growth rate for each quarter, this one is reported by Banco de la República, with data from DANE.

3.1.1 Informality measurements

There are currently two main forms of informality measurements: i) employment in the informal sector, and ii) informal employment (Hussmanns, 2004). In a nutshell, *employment in the informal sector* refers to the size or productivity level of the firm, i.e., informal workers are those who work at small size and low productivity level firms, this is the measurement methodology that DANE uses, in which they include non-government workers affiliated to firms with five or less workers, or workers at non-wage

[7]All the variables were seasonally adjusted prior to any further manipulation.

jobs. On the other hand, *informal employment* measures as informality the workers who do not have access to or comply with social security payment.

Having those two perspectives and following the reasoning of Torres (2020), in the interest of estimating the informality level as accurately as possible, we take as the informality rate in each quarter as the cross-section average of the share of workers employed in the informal sector, and the share of workers that do not have social security payments, both as reported by DANE. In addition to that, the last share is computed as the average of workers that do not comply with healthcare insurance or retirement pension payments.

3.2 Calibration

Parameters calibrated from prior assumptions and previous works findings are reported in table 1. All the autoregresive coefficients are taken to be $\rho_x = 0.9$, for all $x \in \{a, F, u, i\}$. In general β, α, ϵ_p, δ, χ take values that are common in Real Business Cycle (RBC) and New Keynesian (NK) literature. The steady state inflation rate, π, is set at the same target CPI inflation of Banco de la República (Colombia's central bank), of 3% per annum, converted to quarterly terms. The steady state nominal interest rate i is determined according to this inflation target. We calibrate $\eta_I = 1$ giving the informal sector the greater flexibility possible, in line with what was asserted before with previous works.

The relative bargaining power of firms in the labor market μ is set to 0.162, pursuant to González et al. (2012), the volatility of discretional monetary policy shocks, $\sigma_i = 0.001$, and the share of firms that are not able to change prices in $t+1$, ϕ_p=0.202, are also calibrated with their results. The elasticity of the matching function with respect to unemployment ω is assigned 1/2, which is implied in the calibration of Blanchard and Galí (2010) and is, as mentioned by these authors, close to most estimates of matching functions. The parameter that controls for the substitutability between formal and informal labor ϵ_L, is given the value of 8 as in Restrepo-Echavarria (2014). It is worth noting that in the latter model, the parameter is the elasticity of substitution between a formal and an informal consumption good, in contrast to our model in which the parameter is the elasticity of substitution between types of labor-derived goods to an intermediate firm, this is an important difference, but we choose the value reflecting the same reasoning expressed in that study, besides this assumption is also implicit in Alberola and Urrutia (2020).

The elasticity of substitution between the two types of money is set to 20 following Barrdear and Kumhof (2021), which implies for θ a value of 0.95. The parameters of the transaction costs function A and B take the values estimated in Schmitt-Grohé and Uribe (2004), of 0.111 and 0.07524, respectively. The productivity of informal labor is set to $a^I = 0.67$, as in Alberola and Urrutia (2020). For the steady state of g_t^{rate}, we use the sample average of the levels of the government expenditure-to-GDP ratio from the Colombian national accounts described before, giving an average of 0.14379, which is close to the one used by Rincón et al. (2017) of 0.1425. Using the Mendoza et al. (1994) methodology we obtain effective tax rates: $\tau^w = 0.02366$, $\tau^c = 0.03209$ and $\tau^r = 0.03055$. The monetary policy reaction to inflation deviations from target for both types of policy, are $\varphi_\pi = \varphi_\pi^{dc} = 2$, and the reaction parameter for the GDP gap is set to zero, $\varphi_{gdp} = 0$. The latter is done since positive values for this parameter do not allow the model to capture the positive correlation between inflation and GDP in the business cycle, at odds with our findings in data.

3.3 Matching of moments

For calibrating the remaining parameters, we solve the model with a perturbations method using Dynare 4.6.4 (Adjemian et al., 2011), at a second-order Taylor expansion. Our calibration strategy is to minimize the distance between the theoretical moments of the model and their data observed counterparts. For steady state moments, we take advantage of the analytical steady state computation we use for some variables, such that we are able to solve the static model for the parameters ξ_F, ξ_I and ζ, keeping fixed the values of l^{rate}, $l^{I,rate}$ and u^{rate} at their targets, granting for an exact match in those three calibrations.

Additionally, for matching key business cycle second moments, we minimize the weighted sum of squared relative deviations of observed moments and the model's theoretical moments, at the mentioned order of accuracy. Specifically, from the state space representation of the model solution it is possible to calculate the spectral density of this solution, and therefore obtaining HP-filtered second moments without the need of performing simulations, see Uhlig (2001) and Adjemian et al. (2011). We filter those theoretical moments also with $\lambda = 1600$, for the purpose of comparability with data. Formally, for the calibration routine, let $\boldsymbol{m}$ be the column vector of observed unconditional second moments, and $\hat{\boldsymbol{m}}(\boldsymbol{\Theta}, \cdot)$ be the model's theoretical second moments that depend on the parameter vector $\boldsymbol{\Theta}$. We seek to vector $\boldsymbol{\Theta}^*$, such that

$$\boldsymbol{\Theta}^* = \arg\min_{\boldsymbol{\Theta}} \left[(\boldsymbol{m} - \hat{\boldsymbol{m}}(\boldsymbol{\Theta}, \cdot)) \odot \boldsymbol{a}\right]^\intercal \boldsymbol{W} \left[(\boldsymbol{m} - \hat{\boldsymbol{m}}(\boldsymbol{\Theta}, \cdot)) \odot \boldsymbol{a}\right].$$ [8]

Here, $\boldsymbol{W}$ is a weighting matrix, $\boldsymbol{W} := diag(w_1, ..., w_6)$, where each $w_j > 0$ is used to give more relevance to some moments when the algorithm is finding a minimum. And $\boldsymbol{a}$ is just the element-wise vector of multiplicative reciprocals of $\boldsymbol{m}$, i.e.,

[8]Note that the right hand side of the equation is equal to $\arg\min_{\boldsymbol{\Theta}} \sum_{j=1}^{6} w_j \left(\frac{m_j - \hat{m}_j(\boldsymbol{\Theta})}{m_j}\right)^2$

Table 1: Calibrated parameters from prior assumptions an literature

Parameter	Value	Source/motivation
α	1/3	
ϵ_p	5	
δ	$(1.1)^{1/4} - 1$	
β	0.99	
χ	1	
π	$(1.03)^{1/4} - 1$	Banco de la República target inflation
i	$(1+\pi)\,\beta^{-1} - 1$	Congruent with target inflation
η_I	1	
ρ_x	0.9, $x \in \{F, g\}$	
ρ_a	0.9	González et al. (2012)
ω	0.5	Blanchard and Galí (2010)
μ	0.162	González et al. (2012)
ϵ_L	8	Restrepo-Echavarria (2014)
θ	0.95	Barrdear and Kumhof (2021)
σ_i	0.001	González et al. (2012)
σ_F	0.001	
A	0.111	Schmitt-Grohé and Uribe (2004)
B	0.07524	Schmitt-Grohé and Uribe (2004)
ϕ_p	0.202	González et al. (2012)
a^I	0.67	Alberola and Urrutia (2020)
g^{rate}	0.14379	Sample mean of g/gdp ratio for Colombia
τ^w	0.02366	Estimated by the authors using Mendoza et al. (1994) methodology for Colombia
τ^c	0.03209	
τ^r	0.03055	
T	1.5	For all CBDC variants
rat^{dc}_{gdp}	20%	For CBDC quantitative rule
i_{sp}	1.5%	For CBDC price rule
φ_π	2	
φ_{gdp}	0	Allows for theoretical $\rho\,(gdp, \pi) > 0$
φ^{dc}_π	2	

$\boldsymbol{a} := [1/m_1, ..., 1/m_6]^{\intercal}$, this is done in order to accentuate the relevance of moments that are relatively far from their targets to the minimization algorithm.

The results from the calibration procedure are shown in Table 2. In general, the model shows ability to fit the selected moments with a good degree of accuracy, under plausible parameter values. Given the fixed parameters, the calibrated ones follow the key assumptions formulated in the theoretical set-up, e.g., $\xi_F > \xi_I$, $\eta_I > \eta_F$. Also worth noting, is the capability of the model to capture the procyclicality of inflation and countercyclicality of informality, which are observed in data and are key to our analysis.

Table 2: Calibrated parameters to steady state and second moment targets

Parameter	Value	Target	Model moment	Data moment
ξ_F	0.31048	l^{rate}	0.65585	0.65585
ξ_I	0.00456	$l^{I,rate}$	0.43028	0.43028
ζ	33.7422	u^{rate}	0.11249	0.11249
η_F	0.23819	$\rho\left(gdp, l^{I,rate}\right)$	-0.26908	-0.26910
ψ	1.62217	$\rho(l^{I,rate}, l^{I,rate}_{-1})$	0.66420	0.65794
ϕ_Ω	246.460	$\sigma(\Omega)/\sigma(gdp)$	3.05456	3.05148
γ	0.00301	$\sigma(l^{rate})/\sigma(gdp)$	0.37147	0.37076
σ_a	0.00554	$\sigma(gdp)$	1.133%	1.133%
σ_g	0.18841	$\rho(gdp, \pi)$	0.32425	0.32422

Here $\sigma(\cdot)$ denotes the percentage standard deviation, and $\rho(\cdot)$ the correlation of variables. The subscript $_{-1}$ denotes the one-period lagged variable. Data was collected from DANE (2021a), DANE (2021b) y Banco de la República (2021).

4 Business cycle analysis

For the business cycle analysis we compute impulse response functions for the three model variants, with the described calibration. These impulse responses are computed at the first-order approximated solution of the model, for the shocks ε_t^a and ε_t^g, at their calibrated magnitude sizes.

4.1 Analysis of dynamic responses

4.1.1 Response to a technological innovation

In Figure 1 we show dynamic impulse responses to a technological innovation ε_t^a. This innovation generate a reduction of current and expected marginal cost, and increase the demand for inputs given their higher marginal productivity, which spurs job creation, and dampens the unemployment rate. Wages in both sectors increase, causing an expansion in the labor force participation rate, due to higher values accrued from engaging in a working relationship for households. This production expansion generates deflation, given lower marginal cost. Households increase their consumption and investment on account of higher income, and demand for monetary balances increase in order to lower transaction costs. Informality rate increases, since this sector as being the most flexible, assumes most of the job creation. Overall, in all the three model variants the transmission mechanisms and dynamics are observed to be quite similar for this shock, and differences in the magnitude of the shock over most of the variables is very slight.

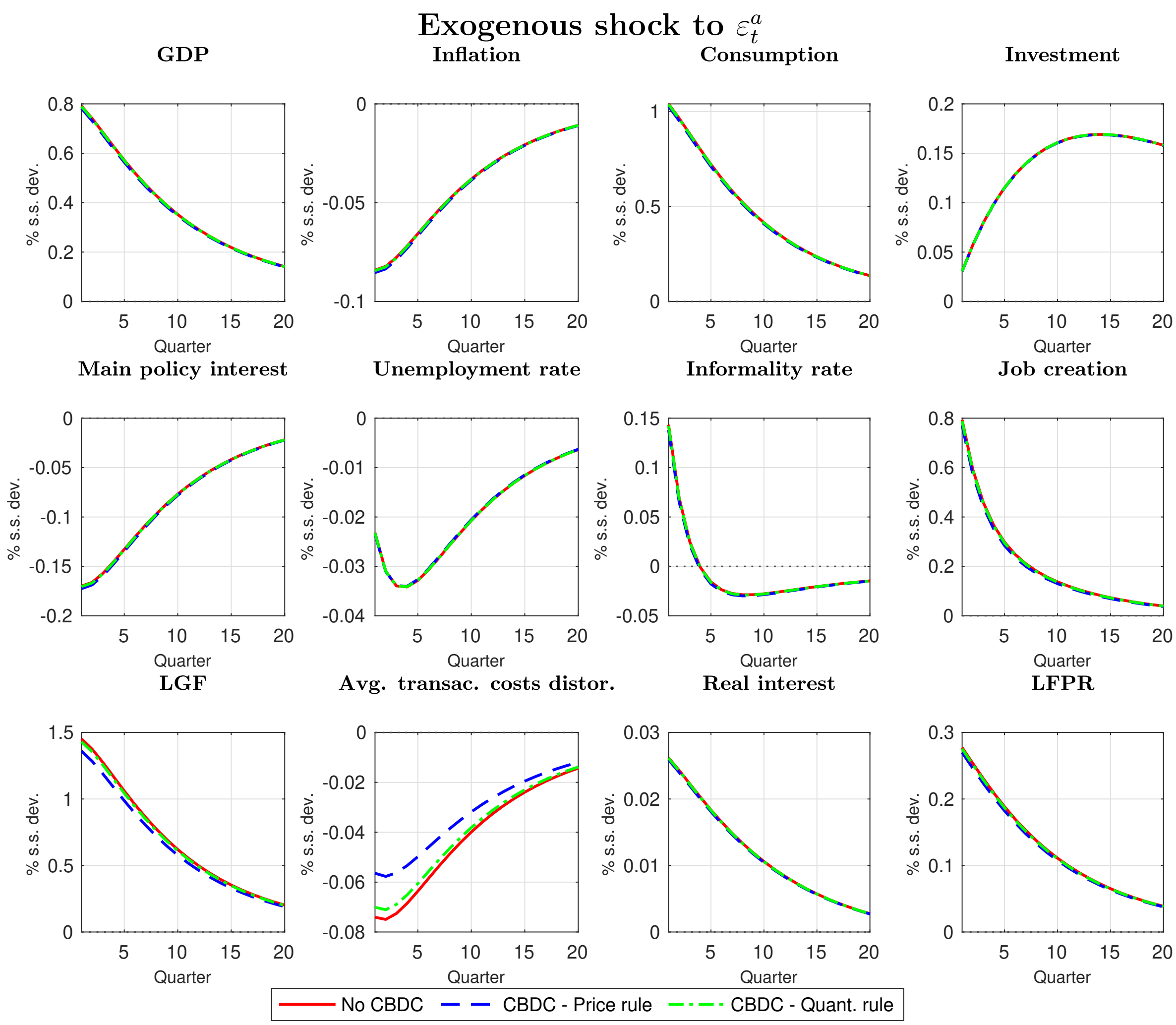

Figure 1: Response to an exogenous one-time increase of a_t through ε_t^a, with baseline values of $T = 1.5$, $i_{sp} = 1.5\%$, and $rat_{gdp}^{dc} = 20\%$.

The main difference can be found in the way households change allocations of monetary balances, in the no CBDC model households increase their cash holdings, as this is the only mean of exchange disposable, but in the CBDC price rule setting, cash demand is heavily lowered, and CBDC demand spurs, as its remuneration increases as a response to deflation. In the quantitative rule, as CBDC supply is reduced instead, the demand for cash does not fall.

Now, we perform the same experiment but increasing the size of the liquidity coefficient T to 5, and reducing the spread of the main policy interest and CBDC interest, to $i_{sp} = 0.15\%$, and we measure the cumulative responses of GDP, inflation and unemployment rate after one year of the shock, relative to its impact on a_t. These results are shown in Table 3.

Table 3: Cumulative responses of variables after one year of the shock to ε_t^a, relative to the cumulative response of a_t.

Variable	No CBDC	CBDC-PR	CBDC-PR	CBDC-PR
		$T = 1.5, i_{sp} = 1.5\%$	$T = 5, i_{sp} = 1.5\%$	$T = 1.5, i_{sp} = 0.15\%$
GDP	1.4927	1.4726	1.4158	1.5255
Inflation	-0.1658	-0.1684	-0.1766	-0.1577
Un. rate	-0.0642	-0.0643	-0.0649	-0.0631

It is noticeable that when the liquidity of CBDC is high, the response of GDP is diminished given a productivity shock, but in turn disinflation is larger, and the unemployment rate does not change as much. The mechanism trough which this occurs, is that with high liquidity of CBDC, transaction costs do not deviate as much from their equilibrium. On the other hand, with a lower interest rate spread, which in other words is a higher steady state CBDC interest rate, the relative cumulative response of GDP is much higher, but lower for disinflation. In summary, when CBDC provides high liquidity services, the impact of technology shocks are diminished with slightly higher inflation, and when its steady state interest remuneration is high, this expansion results accentuated, with slight lower disinflation. Impulse responses are shown in the Figure 5 of Appendix A.

4.1.2 Response to an aggregate demand expansion

Now we simulate the response to a discretional government expenditure increase, the results are shown in Figure 2. Overall, a sudden increase in government expenditure expands GDP, but has a large crowding-out effect on investment and consumption. This expansion is inflationary, given its demand-driven nature. Monetary policy reacts strongly by rising the main nominal interest, lessening the CBDC interest in the price rule set-up, and the supply of CBDC in the quantitative rule one. At this point comes the main differences, first, the increase in the main policy interest rate causes a contraction for the demand of cash in the no-CBDC and the CBDC quantitative rule variants, in contrast the CBDC price rule the demand for cash increases. In both CBDC cases, the demand for CBDC balances is reduced, but with a much larger effect in the price rule case.

This happens since in the price rule setting, as the demand part is the one that determines quantities, meaning that households contract heavily their desired quantity of CBDC, nevertheless the quantity of cash holdings is *increased*, contrasting with the other two cases in which cash holdings drop. This effect is reflected in the response of transaction costs distortions, that have an increase that is very similar in magnitude in the no-CBDC and the CBDC-quantitative rule variants, and smaller the CBDC-price rule model. This causes that a greater amount of the aggregate demand expansion is accounted in GDP categories, rather than being used in the labor-search and transaction costs part, in other words this suggests that under a CBDC price rule, aggregate demand shocks are amplified, as households have greater margin of adjustment of their monetary balances, allowing for shorter distortions of transaction costs.

There is another interesting effect in the labor markets, where due to the higher response of income, and the increase in wages due to the rise of inputs demand, causes that the marginal utility of engaging in employment for the household $(\lambda_t^x, x \in \{F, I\})$ to climb, provoking the labor force participation to soar, nonetheless this mechanism is not strong enough in the no-CBDC and CBDC-quantitative rule economies, and the marginal rate of substitution predominates, causing in turn labor force participation to decline.

In Table 4 cumulative responses relative to g_t^{rate} are shown, performing the same experiment as in the previous case. The introduction of CBDC with the baseline parameter values $(T = 1.5, i_{sp} = 1.5\%)$ amplifies the expansionary effect, and reduces the crowding-out of consumption, whereas the one of investment is slightly bigger.

Inflationary pressures are also amplified, as well as the unemployment rate in a more moderately way. Analyzing the sensitivity of responses to the mentioned parameters, we note that with a CBDC with higher liquidity services, the overall crowding-out effect is reduced considerably, causing a higher relative expansion of GDP, but as well higher inflation acceleration. By contrast, returning to the baseline value for T and increasing the steady state interest rate of CBDC, boosts strikingly the crowding-out effect to the point of generating an overall negative effect on GDP.

This result is due to the fact that at this interest spread, the marginal rate of substitution between the two types of monies is huge, thus provoking extreme substitution effect between them, moreover this is shown to cause a much bigger increase in the distortionary effects of transaction costs, suggesting that under demand shocks, a low opportunity cost of CBDC due to high interest remuneration, causes transaction costs distortions and crowding-out effect to worsen. Impulse responses are shown in the Figure 6 of Appendix A.

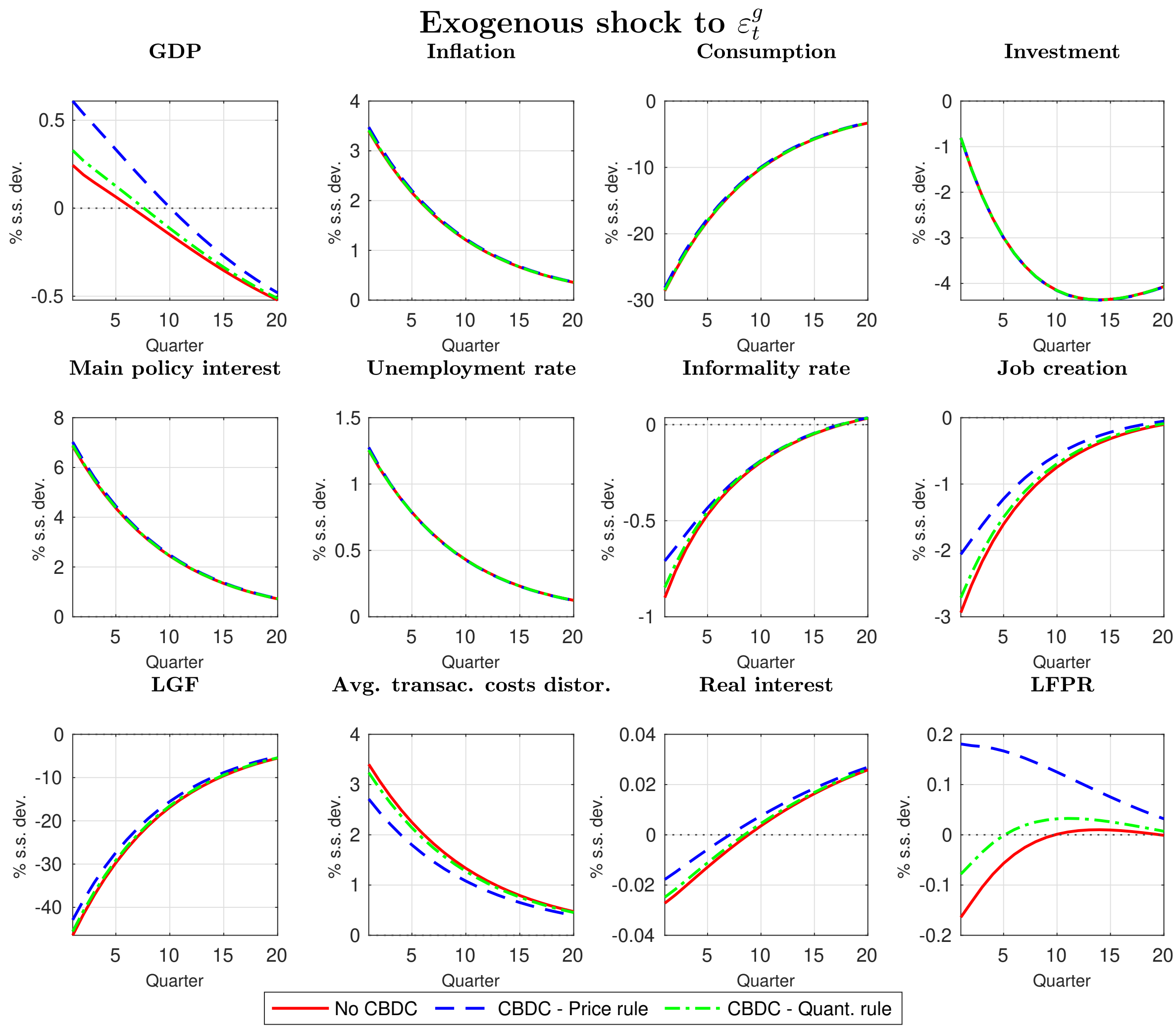

Figure 2: Response to an exogenous one-time increase of g_t^{rate} through ε_t^g, with baseline values of $T = 1.5$, $i_{sp} = 1.5\%$, and $rat_{gdp}^{dc} = 20\%$.

Table 4: Cumulative responses of variables after one year of the shock to ε_t^g, relative to the cumulative response of g_t^{rate}.

Variable	No CBDC	CBDC-PR $T = 1.5, i_{sp} = 1.5\%$	CBDC-PR $T = 5, i_{sp} = 1.5\%$	CBDC-PR $T = 1.5, i_{sp} = 0.15\%$
GDP	0.0106	0.0311	0.0923	−0.0226
Inflation	0.1785	0.1825	0.1948	0.1681
Un. rate	0.0656	0.0662	0.0662	0.0647
Consump.	−1.5000	−1.4732	−1.3928	−1.5464
Invest.	−0.1076	−0.1077	−0.1082	−0.1061

4.2 Effectiveness of monetary policy under dual policy instruments

We now assess the effectiveness of the monetary policy, by performing experiments on how the ability to generate discretionary disinflation and measuring the magnitude of the contraction generated by this. Particularly, we look at the sacrifice ratios of GDP, and compare the performance on each of the model variants. In Figure 3, we show the variation of GDP sacrifice ratio from a one-time contractionary policy aiming disinflation, comparing the no CBDC and the CBDC price rule settings. It is observed that for lower values of the interest rate spread, meaning higher steady state values of CBDC remuneration, the sacrifice ratio is reduced considerably, the mechanism through which this happens, is that the presence of CBDC at a low interest spread, increases the allocation of CBDC balances in static equilibrium, thereby allowing the Central Bank to have more influence over the private sector monetary balances. Also, as the main policy tool is contractive and reduces inflation, the CBDC second tool reacts in a expansionaty way in response to disinflation, causing that the transaction costs distortions to decline, which does not happen in the no-CBDC variant. Moreover, when the interest rate spread gets closer to the value of the nominal steady state interest, i.e., the remuneration of CBDC gets close to zero, the sacrifice ratio is very similar to the variant with no CBDC. This is very meaningful, since it shows that the benefits of CBDC for policy effectiveness do not solely rely on the higher liquidity services that CBDC provide, but this and a higher nominal remuneration of this type of money, is which may deliver benefits to monetary policy.

In summary, so far the presence of CBDC allows for a disinflation monetary policy contraction, accompanied at the same time by a stimulus that acts through the reduction of transaction costs distortions, which is not possible to happen in a no-CBDC economy model. Also worth noting, is that in an economy with low interest spread, the relative disinflation achieved is much lower. Dynamic responses to discretionary disinflation are shown in Figure 7 of Appendix B. This suggests that the introduction of

CBDC, with a secondary price rule policy tool, contributes to the effectiveness of policy objectives. Note that for certain values of the spread, the sacrifice ratio is negative, we do not think this extreme result is plausible, although the model is clear in stating that a second policy tool is more effective than just one, from the disinflation perspective.

Now, looking at the second panel in Figure 3, we note that as the liquidity coefficient of CBDC increases it worsens, although at a small gradient as T increases. This result is in line with the previous one, as CBDC overall increases the liquidity services disposable, transaction costs generate less distortions. Nonetheless, the improvement of the sacrifice ratio is slow as T increases, due to the mentioned interaction that should be between grater liquidity services and remuneration of CBDC holdings. This suggests that if a CBDC were to deliver up to two times the liquidity services of cash solely would not provide greater efficiency of monetary policy, if the interest spread is remain high (as the one in the baseline calibration of 1.5%). In other words, the interest rate a CBDC would delivery is a critical parameter to decide on, given the liquidity level of the digital money.

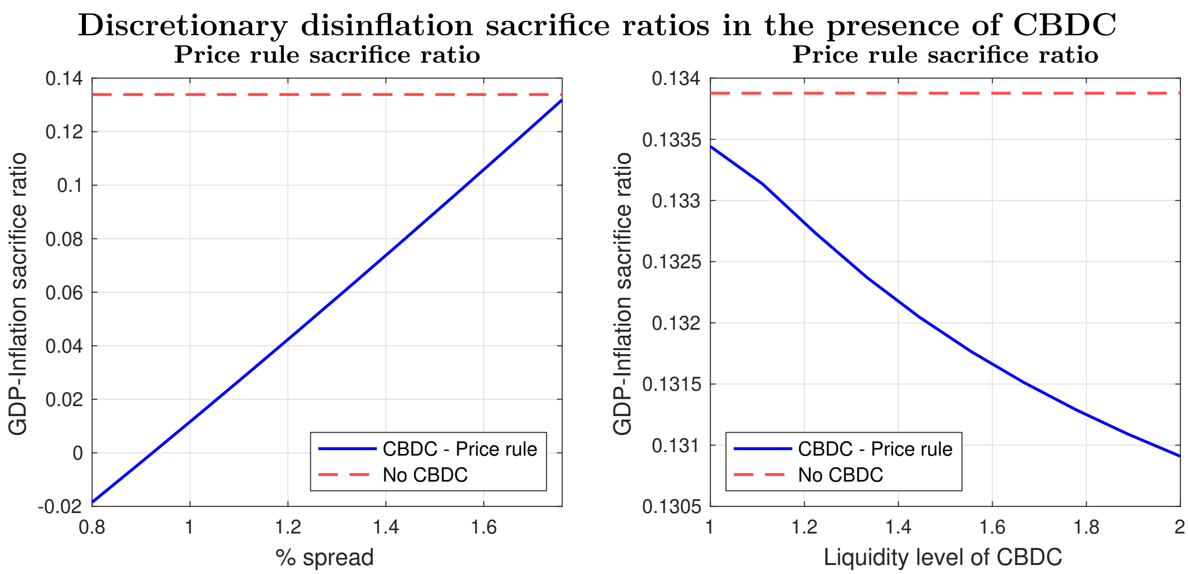

Figure 3: Sacrifice ratios computed as the cumulative response of GDP to a discretionary disinflation monetary policy.

We now turn to study the effects on effectiveness of monetary policy under CBDC quantity rule, we show the results in Figure 4. In this case, the greater the ratio of CBDC to GDP, the worst is the sacrifice ratio. This occurs since when CBDC quantities are given, the household adjust the desired remuneration, but even if the CBDC interest is high due to large supply, the additional benefits of larger liquidity do not offset the diminishing marginal liquidity of CBDC. In other words, when the Central Bank has a large supple of CBDC, the transaction costs distortions are not able to be reduced enough. Note that this is true, but when high demand for CBDC is caused by a high remuneration.

5 Conclusions

We build a simple dynamic stochastic general equilibrium monetary model with CBDC and labor market rigidities common to developing economies. On the interest of interpretability, we abstract from financial intermediation. Our aim is to assess the business cycle dynamics that the introduction of CBDC would generate in such an economy, by comparing the response to supply, demand an monetary shocks in an economy with and without CBDC. We calibrate the model to Colombian data to match business cycle moments, the model matches those moments with a good degree of accuracy, under plausible parameter values.

The main lesson derived from numerical experiments is that the implicit parameters related to CBDC, i.e., its degree of liquidity and its remuneration relative to the nominal interest rate of bonds, are key to consider its dynamic impacts. Also, the key mechanisms through which CBDC influences the economy is the distortionaty effects of transaction costs. Particularly, in the response to positive supply shock is smaller for GDP and slightly higher for disinflation, this effect is stronger if the liquidity coefficient of CBDC is large. Yet, with a higher steady state interest rate of CBDC, the impact of this expansion is amplified for GDP and more muted for disinflation.

With respect to positive demand shocks trough fiscal expansion, in the presence of CBDC with high interest rate spread, amplify the expansion of GDP and the acceleration of inflation. However, with a low interest rate spread the crowding-out effect is magnified, causing an overall contraction of GDP, although inflationary acceleration is dampened, compared with the previous case. Finally, regarding the monetary policy effectiveness under both traditional and CBDC policy tools, under a price rule regime, with high CBDC interest rates, the reduction of transaction costs distortions is significantly reduced, thereby reducing the monetary policy sacrifice ratio, thus improving the effectiveness of monetary policy in the business cycle. On the other hand, a high coefficient of liquidity services of CBDC, also deliver greater effectiveness of monetary policy. Lastly, under a quantitative CBDC setting, the more steady state supply of CBDC, the less effective monetary policy becomes, as the additional liquidity services it provides does not offset the negative real return associated with high supply quantities for households. In summary, the presence of CBDC shows to strengthen the ability of the economy to reduce distortions arising from transaction costs in the business cycle. Nonetheless the degree of usefulness of CBDC stabilizing the business cycle heavily depends on the nature of the shocks, and the structural CBDC parameters.

It is possible to improve the theoretical framework we presented here. Mainly, integrating the role of financial intermediation, and accounting for the differences between the formal and the informal sector regarding their access to credit, may answer some questions of interest. Also, it is arguable that due to the nature of infomality, and depending on which privacy scheme is designed for CBDC, the informal sector would probably be reluctant to use a CBDC, thereby a model that accounts for this assumption would potentially also deliver interesting results. The introduction of wage rigidities in the formal sector could also help to answer relevant questions.

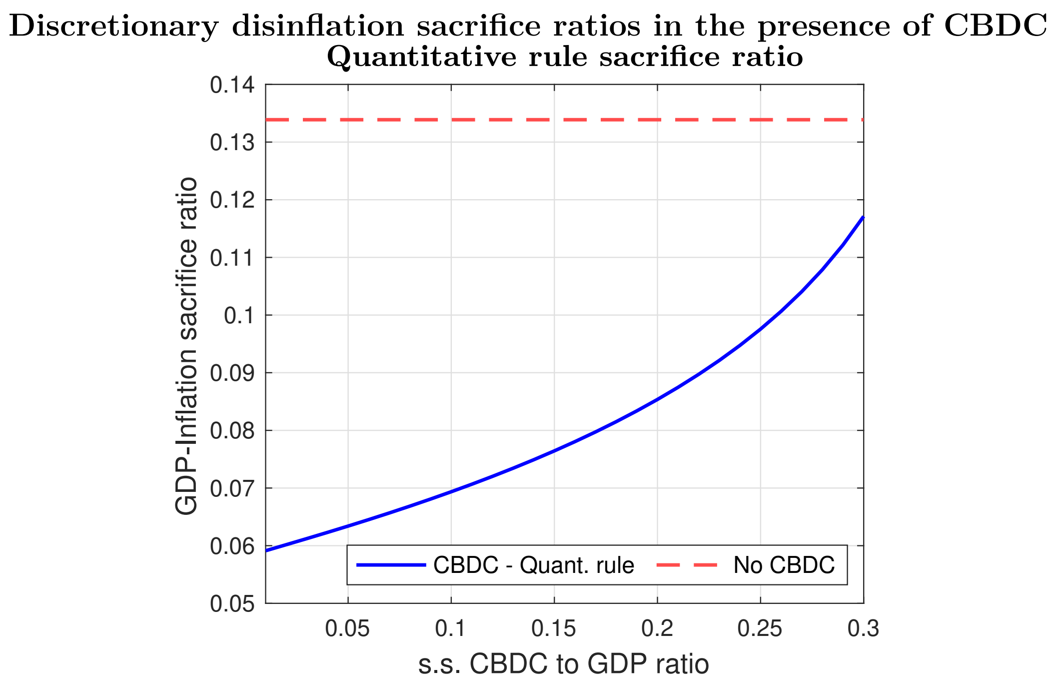

Figure 4: Sacrifice ratios computed as the cumulative response of GDP to a discretionary disinflation monetary policy.

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A Appendix: Impulse response sensitivity to CBDC parameters

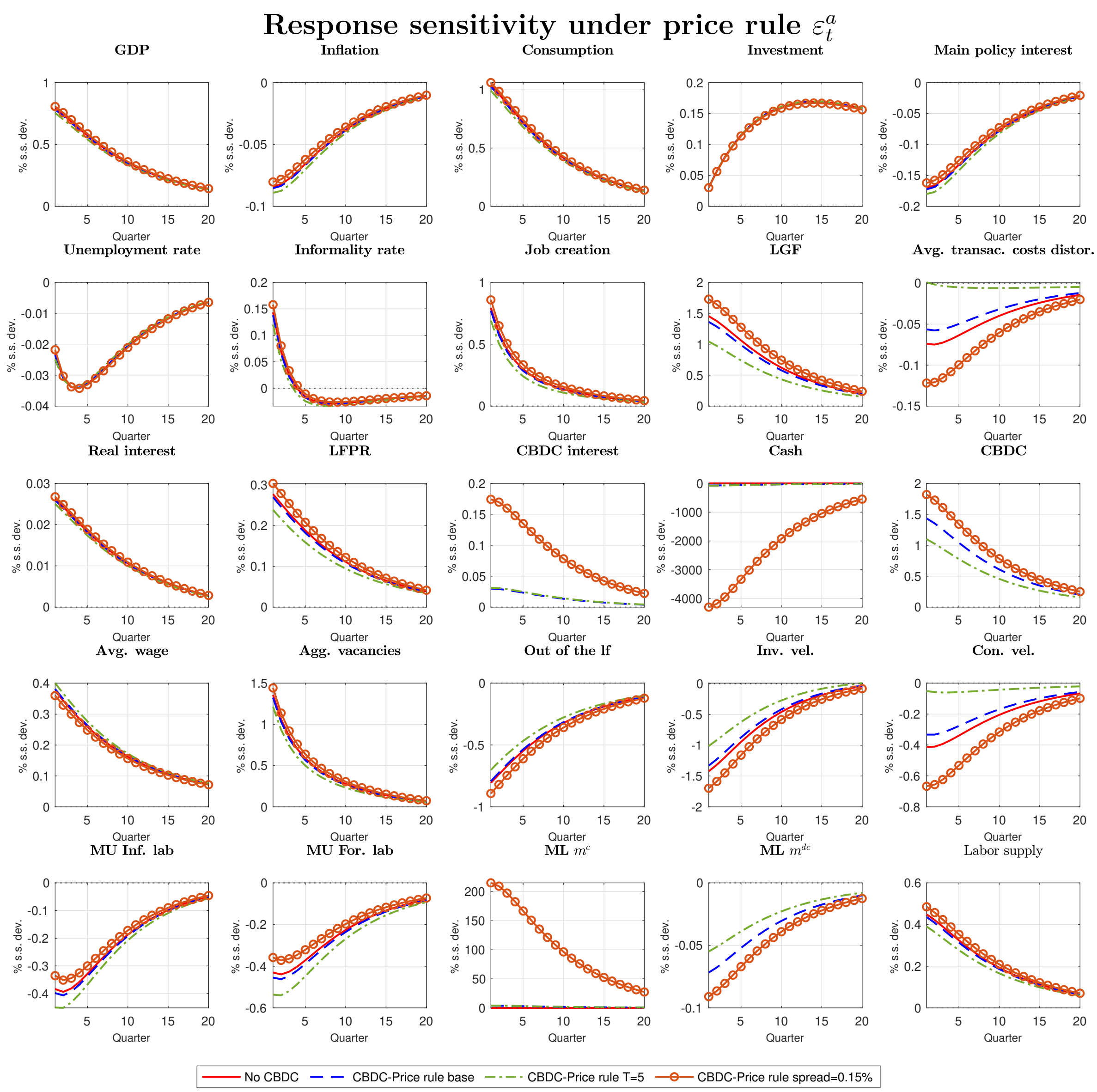

Figure 5: Sensitivity analysis of impulse responses to CBDC parameters *ceteris paribus*.

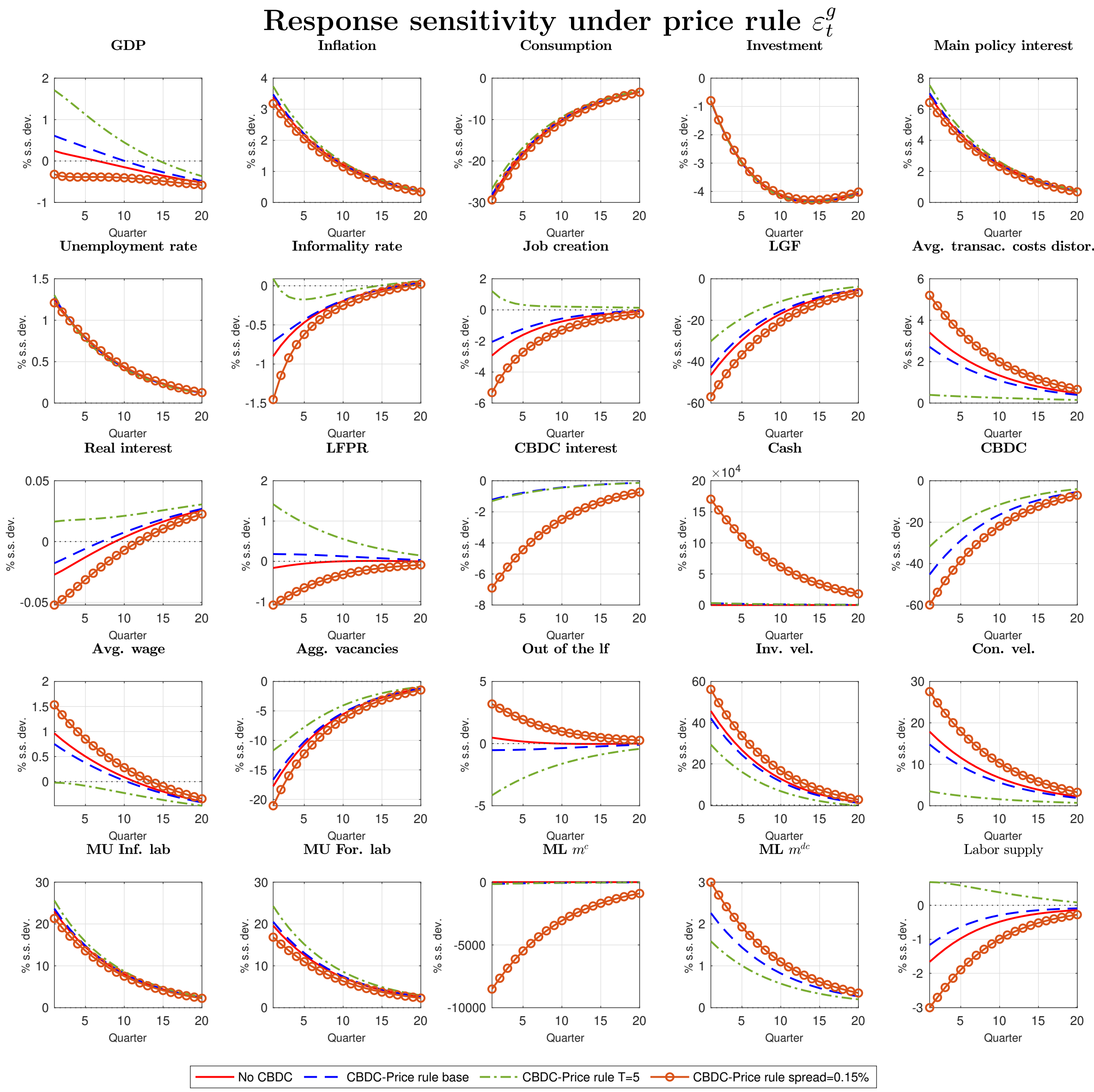

Figure 6: Sensitivity analysis of impulse responses to CBDC parameters *ceteris paribus*.

B Appendix: Dynamic responses to discretionaty disinflation

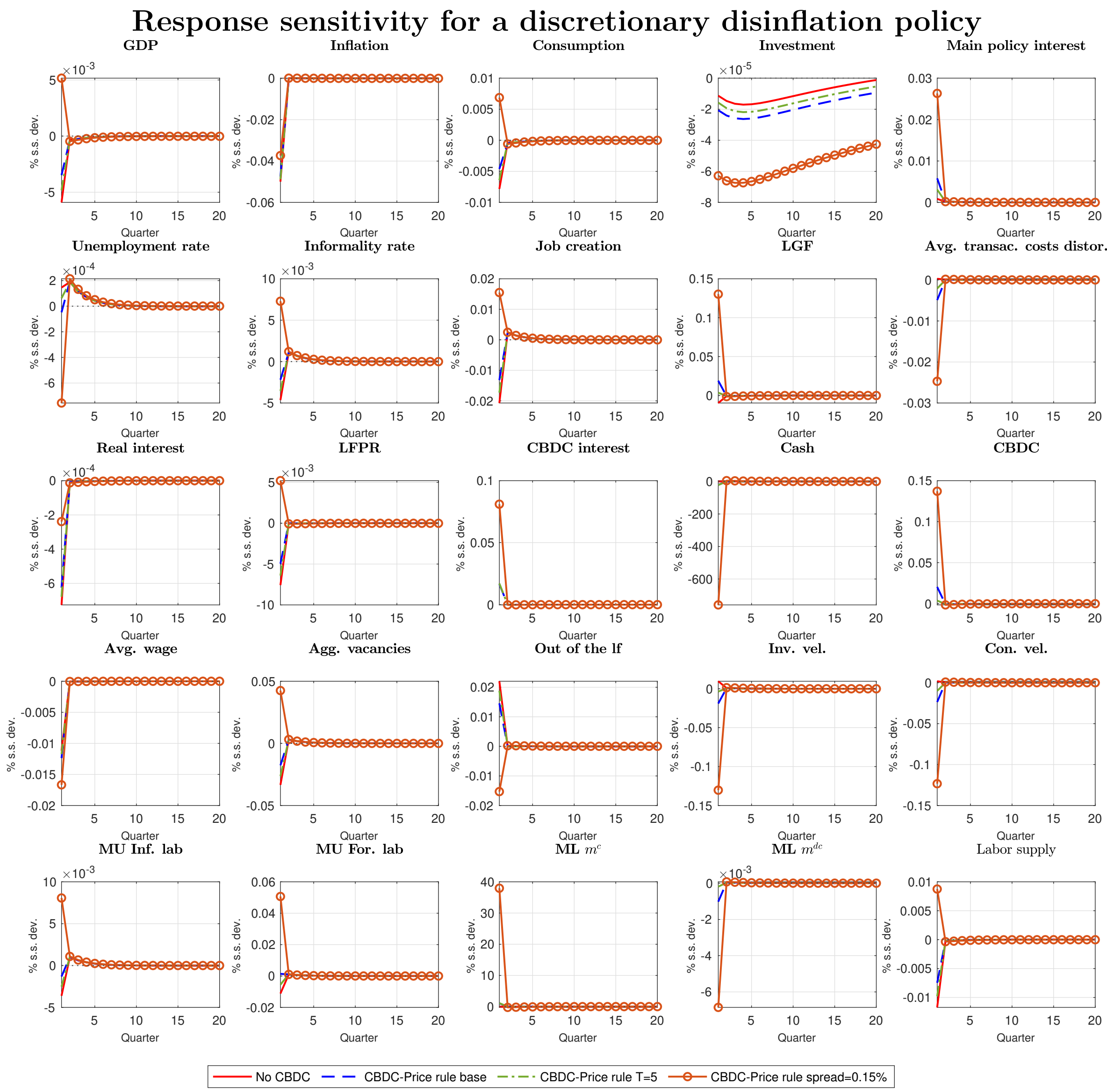

Figure 7: Sensitivity analysis of impulse responses to CBDC parameters *ceteris paribus*.