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Determinacy in Multi-Country DSGE Models: The Role of Pricing Paradigms and Economic Openness*

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Abstract

This paper examines determinacy properties in a multi-country open economy framework, focusing on the impacts of dominant currency pricing (DCP), producer currency pricing (PCP), and local currency pricing (LCP) on monetary policy effectiveness. Utilizing a New Keynesian model with three symmetric economies, each guided by Taylor rules, the study extends the framework of Gopinath et al. (2020) to analyze how these pricing paradigms interact with central bank policies to achieve economic stability. The investigation highlights that higher economic openness amplifies interactions among central banks' policies, complicating the attainment of determinacy. DCP significantly constrains policy parameters ensuring determinacy, particularly in open economies. Conversely, PCP and LCP offer relatively larger determinacy regions, allowing for greater domestic policy control. The findings emphasize the critical role of pricing paradigms and economic openness in formulating effective monetary policies. This study provides essential insights for central banks and policymakers in enhancing global economic stability through tailored policy recommendations based on the chosen pricing paradigm.

Key words: Determinacy, Taylor rule, Three-country new Keynesian model, Pricing paradigms, Openness

JEL Classification: E31, E52, E58, F33, F4

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1 Introduction

In the realm of international macroeconomics, the determination of monetary policy rules and their implications for economic stability has long been a subject of extensive research and debate. One critical aspect of this inquiry revolves around the concept of determinacy—the condition wherein a unique equilibrium path exists for the economy under a specified policy rule. Understanding determinacy is crucial for central banks and policymakers because it illuminates the effectiveness and robustness of monetary policy frameworks in achieving desirable economic outcomes.

This paper embarks on a comprehensive analysis of determinacy within the framework proposed by Gopinath et al. (2020), building upon the seminal contributions of previous studies in the field. The model used in this study, which has become one of the new workhorse models in international macroeconomics, consists of three symmetric, endogenously modeled economies that interact via trade and set their own monetary policies. The core of each economy is similarly modeled to Galí and Monacelli (2005), a model of a small open economy (SOE). Gopinath et al. (2020) introduce three key features: various pricing paradigms including producer currency pricing (PCP), local currency pricing (LCP), and dominant (or Dollar) currency pricing (DCP); the inclusion of intermediate inputs produced domestically and abroad; and strategic complementarity in pricing, enabling variable markups.

Producer currency pricing (PCP) assumes that goods are priced in the currency of the producing country. This implies that exchange rate fluctuations directly affect the prices faced by foreign consumers, making this paradigm sensitive to exchange rate volatility. In contrast, local currency pricing (LCP) entails pricing goods in the currency of the consuming country, insulating consumers from exchange rate fluctuations and potentially stabilizing domestic consumption. Dominant currency pricing (DCP), as discussed by Gopinath et al. (2020), involves pricing goods in a dominant international currency, such as the US dollar, irrespective of the producer or consumer's country. This paradigm reflects the significant role of dominant currencies in global trade and finance, where the exchange rate fluctuations of the dominant currency have pervasive effects on international prices.

In this study, I focus on analyzing how different pricing paradigms—DCP, PCP, and LCP—impact the determinacy of the economic equilibrium. I explore how these pricing regimes interact with central bank policy rules to influence the attainment of determinacy, considering the degree of economic openness as a crucial determinant.

This analysis provides a foundational exploration into a broader research agenda focused on unraveling the complexities of monetary policy determinacy in a globalized economic landscape. By elucidating the intricate dynamics between policy rules, pricing paradigms, and economic openness, I seek to provide valuable insights for central banks and policymakers grappling with the challenges of maintaining stability and fostering growth in an increasingly interconnected world.

The investigation of determinacy in DSGE models has been significantly advanced by several key studies. Blanchard and Kahn (1980) laid the foundational framework for the solution of linear difference models under rational expectations, emphasizing the conditions required for a unique equilibrium. Pesaran (1987) and Farmer (1999) were among the first to use these in various settings of rational expectation models. Further contributions by Bullard and Mitra (2002) highlighted the importance of the Taylor principle in achieving determinacy in closed economy models. Evans and Honkapohja (2001) provided a comprehensive examination of learning and expectations in macroeconomics, illustrating the implications of policy rules on economic stability. Loisel (2022) has recently derived an analytical set of necessary or sufficient conditions for a broad class of models that ensure determinacy. These include how strongly policy instruments react to what lag or lead of a variable. He does not, however, apply these methods to a multi-country model that is also estimated.

The open economy dimension introduces additional complexities, as explored by Galí and Monacelli (2005) and subsequent studies. The role of economic openness and its impact on determinacy have been investigated by several researchers, including Barnett and Eryilmaz (2023), who analyzed monetary policy and determinacy in open economy New Keynesian models, and Karagiannides and Liambas (2019), who examined the implications of trade openness in a semi-New Keynesian framework.

The estimation of DSGE models, particularly in the context of multi-country settings, has also been a focus of significant research. An and Schorfheide (2007) provided a detailed methodology for Bayesian analysis of DSGE models, which has been widely adopted in subsequent studies. Iskrev (2010) and Qu and Tkachenko (2012) further advanced the techniques for local identification in DSGE models, ensuring that parameter estimates are reliable and informative.

This paper builds on these foundational studies by extending the analysis of determinacy to a three-country model with varying degrees of economic openness and different pricing paradigms. The contributions of this study are twofold. Firstly, it provides a comprehensive determinacy analysis in a three-economy setting, which has not been extensively explored in the literature. Secondly, it offers new insights into the interaction between monetary policy rules, pricing paradigms, and economic openness, highlighting the trade-offs and challenges faced by central banks in an interconnected global economy.

The findings of this paper underscore the complexity of achieving determinacy in a multi-country open economy model. Central banks must account for the interactions between their policy parameters and those of other economies, as well as the prevailing pricing paradigm and level of openness. The nuanced trade-offs illustrated by the varying shapes and sizes of the determinacy regions provide valuable insights for policymakers aiming to design robust monetary frameworks in an interconnected global economy.

Comparing the three pricing paradigms, it is evident that DCP leads to the smallest determinacy regions, particularly in open economies. This is due to the dominant role of the currency in international trade, which intensifies the impact of foreign economic conditions on domestic policy effectiveness. In contrast, PCP and LCP provide relatively larger determinacy regions, suggesting that when prices are set in either the producer's or local currency, the central banks retain more control over their domestic economic conditions, even in a highly open setting.

Overall, these analyses reveal that achieving a unique and stable rational expectations equilibrium requires a careful balance between inflation targeting and output gap responsiveness, especially in open economies with significant international trade. The choice of invoicing currency further complicates this balance, highlighting the need for coordinated and well-calibrated policy responses among central banks to ensure global economic stability.

By shedding light on the intricacies of monetary policy determinacy in an open economy context, this paper provides valuable guidance for central banks and policymakers navigating the challenges of maintaining stability and fostering growth in an increasingly interconnected world.

2 The Model

The model I analyze in this study is the one by Gopinath et al. (2020), which has become one of the new workhorse models in international macroeconomics. The model consists of three symmetric, endogenously modeled, economies that interact via trade and set their own monetary policies. The core of each economy is similarly modeled to Galí and Monacelli (2005), which is a model of a small open economy (SOE). On top of that Gopinath et al. (2020) add three main features. The key contribution is the option to analyze various pricing paradigms that include producer currency pricing (PCP), local currency pricing (LCP), and most importantly dominant (or Dollar) currency pricing (DCP). It is shown that the choice of invoicing currency has serious consequences on international trade and spillovers. The second feature is that the production function not only incorporates labor inputs but also intermediate inputs, which are produced domestically and abroad. Hence, the output of each countries' representative firm is sold in the domestic economy and exported to both trading partners. In each of these cases the good is used for final consumption and as an intermediate input for production. The third feature consists of strategic complementarity in pricing, which enables variable markups.

2.1 Households

Each economy j is populated by a continuum of infinitely lived households of measure one. In every period t, each household h consumes a bundle of traded goods $C_{j,t}(h)$ and sets a nominal wage rate $W_{j,t}(h)$ for supplying a differentiated labor service $N_{j,t}(h)$. All domestic firms are owned by the households and thus transfer their profits to them. The households solve the following optimization problem:

$$\max_{C_j, W_j, B_{\$j,t+1}, B_{j,t+1}} E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{1}{1-\sigma_c} C_{j,t}^{1-\sigma_c} - \frac{\kappa}{1+\varphi} N_{j,t}^{1+\varphi} \right), \tag{1}$$

where β denotes the period-by-period discount factor, σ_c is the parameter governing relative risk aversion, φ is the inverse Frisch elasticity of labor supply, and κ is a scale parameter for the disutility of labor. This optimization problem is subject to the per-period budget constraint expressed in home currency:

$$\begin{aligned} P_{j,t} C_{j,t} + e_{\$j,t}(1+i^{\$}_{j,t-1}) B^{\$}_{j,t} + B_{j,t} &= W_{j,t}(h) N_{j,t}(h) \\ &\quad + \Pi_{j,t} + e_{\$j,t} B^{\$}_{j,t+1} + \sum_{s' \in S} Q_{j,t}(s') B_{j,t+1}(s') \end{aligned} \tag{2}$$

In the previous equation, $P_{j,t}$ denotes the price index for domestic consumption aggregator $C_{j,t}$, $e_{\$j,t}$ is the bilateral nominal exchange rate between the originator of the dominant currency, the U.S., and country j such that an increase represents a depreciation of country $j's$ currency versus the Dollar. $B_{\$j,t}$ denotes the dollar debt holdings of a risk-free international bond denominated in Dollars, which pays a nominal interest return of $i^{\$}_{j,t}$. Gopinath et al. (2020) define this interest rate to be country specific, which can be associated with a risk premium. Households also have access to a full set of domestic state-contingent securities in home currency that are traded domestically and in zero net supply. S gives the set of possible states of the world and $Q_{j,t}(s)$ is the price of a security in period t that pays one unit of home currency in $t+1$ and $B_{j,t+1}(s)$ are the holdings. $\Pi_{j,t}$ denotes the profits of the domestic firms that are transferred to its owners, the households.

Aggregate consumption $C_{j,t}$ is implicitly defined as in Kimball (1995) by a homothetic demand aggregator across the domestic and foreign consumption good varieties:

$$\sum_i \frac{1}{|\Omega_i|}\int_{\omega\in\Omega_i} \gamma_{ij}\Upsilon\left(\frac{|\Omega_i|C_{ij,t}(\omega)}{\gamma_{ij}C_{j,t}}\right) d\omega = 1 \tag{3}$$

When taking into account consumption good variety ω in country j that was produced in country i, $C_{ij,t}(\omega)$, households form a home bias γ_{ij}, with $\sum_i \gamma_{ij} = 1$. $|\Omega_i|$ is a measure of varieties produced in country i and the function $\Upsilon(\cdot)$ satisfies $\Upsilon(1) = 1$, $\Upsilon'(\cdot) > 0$, and $\Upsilon''(\cdot) < 0$. The functional form chosen for this demand structure comes from Klenow and Willis (2016) and yields strategic complementarities in pricing and variable markups. Cost minimization gives the demand system:

$$C_{ij,t}(\omega) = \frac{\gamma_{ij}}{|\Omega_i|}\psi\left(\underbrace{\sum_i \int_{\omega\in\Omega_i} \Upsilon'\left(\frac{|\Omega_i|C_{ij,t}(\omega)}{\gamma_{ij}C_{j,t}}\right)\frac{C_{ij,t}(\omega)}{C_{j,t}}}_{D_{j,t}}\frac{P_{ij,t}(\omega)}{P_{j,t}}\right)C_{j,t}, \tag{4}$$

where $P_{ij,t}(\omega)$ denotes the price of variety ω that was produced in country i and sold in country j, in currency j and $D_{j,t}$ is a demand index.[1]

Optimization yields the intertemporal optimality conditions for consumption given by Euler equations:

$$C_{j,t}^{-\sigma_c} = \beta(1+i_{j,t}^{\$})E_t\left[C_{j,t+1}^{-\sigma_c}\frac{1}{\pi_{t+1}}\frac{e_{\$j,t+1}}{e_{\$j,t}}\right], \tag{5}$$

$$C_{j,t}^{-\sigma_c} = \beta(1+i_{j,t})E_t\left[C_{j,t+1}^{-\sigma_c}\frac{1}{\pi_{t+1}}\right], \tag{6}$$

where the nominal interest rate return on domestic bonds is $(1+i_{j,t}) = (\sum_{s'\in S} Q_{j,t}(s'))^{-1}$.

For adjusting their nominal wages, households are subject to a Calvo (1983) friction, which gives them the option to adjust wages in every period with a probability $(1-\delta_w)$, and keep the previous wage otherwise. With a standard downward sloping demand for each labor supply variety, $N_{j,t}(h) = (W_{j,t}(h)/W_{j,t})^{-\vartheta}N_{j,t}$, where ϑ is the elasticity of labor demand and $W_{j,t}$ the aggregate nominal wage in country j at time t. The Phillips curve for optimal wage setting is than given by:

[1] More regarding the derivation of the Kimball aggregator and details about the specific functional form can be found in Appendix B.

$$E_t \sum_{s=t}^{\infty} \delta_w^{s-t} \underbrace{\left[\beta^{s-t} \left(\frac{C_{j,s}^{-\sigma_c}}{C_{j,t}^{-\sigma_c}} \right) \left(\frac{P_{j,t}}{P_{j,s}} \right) \right]}_{\Theta} N_{j,s} W_{j,s}^{\vartheta(1+\psi)}$$
$$\left[\frac{\vartheta}{\vartheta - 1} \kappa P_{j,s} C_{j,s}^{\sigma} N_{j,s}^{\psi} - \frac{\bar{W}_{j,t}(h)^{1+\vartheta\psi}}{W_{j,s}^{\vartheta\psi}} \right] = 0, \tag{7}$$

where Θ is the stochastic discount factor between periods t and s and $\bar{W}_{j,t}(h)$ is the optimal nominal reset wage in period t and country j.

2.2 Producers

Each firm in country j produces a unique variety ω that is being sold in the domestic economy and abroad. It is then used for final consumption and as an intermediate input for production. Hence, the Cobb-Douglas production function uses a combination of labor $L_{j,t}$ and intermediate inputs $X_{j,t}$:

$$Y_{j,t} = A_{j,t} L_{j,t}^{1-\alpha} X_{j,t}^{\alpha}, \tag{8}$$

with α being the share of intermediate inputs in the production process and $A_{j,t}$ denoting an aggregate productivity shock. The labor input $L_{j,t}$ is CES aggregator of individual varieties $L_{j,t}(h)$ and given by

$$L_{j,t} = \left[\int_0^1 L_{j,t}(h)^{(\vartheta-1)/\vartheta} dh \right]^{\vartheta/(\vartheta-1)}.$$

As before with the consumption aggregator, the intermediate input aggregator takes the same functional form:

$$\sum_i \frac{1}{|\Omega_i|} \int_{\omega \in \Omega_i} \gamma_{ij} \Upsilon \left(\frac{|\Omega_i| X_{ij,t}(\omega)}{\gamma_{ij} X_{j,t}} \right) d\omega = 1 \tag{9}$$

In the previous expression, $X_{j,t}(\omega)$ is the demand by firms in country j for variety ω that has been produced in country i used as an intermediate input. This means that the demand system for each variety follows a similar structure to equation 4.

Firms are allowed to set prices for each market and choose the invoicing currency. This yields the per-period nominal profit function for a firm producing variety ω:

$$\Pi_{j,t}(\omega) = \sum_{i,k} e_{kj,t} P^k_{ji,t}(\omega) Y^k_{ji,t}(\omega) - MC_{j,t} Y_{j,t}(\omega), \tag{10}$$

where $P^k_{ji,t}$ is the price of a variety ω produced in country j, sold in country i, and invoiced in currency k. Importantly, $Y^k_{ji,t}(\omega) = C^k_{ji,t}(\omega) + X^k_{ji,t}(\omega)$ is the demand for a variety ω both for consumption and input in production purposes such that $Y_{j,t}(\omega) = \sum_{i,k} Y^k_{ji,t}(\omega)$ is the total demand across all destination markets i and invoicing currencies k. $MC_{j,t}$ gives the nominal marginal cost of a firm in their home currency and is given by:

$$MC_{j,t} = \frac{1}{\alpha^\alpha (1-\alpha)^{1-\alpha}} \frac{W^{1-\alpha}_{j,t} P^\alpha_{j,t}}{A_{j,t}} \tag{11}$$

The final two optimality conditions concern labor hiring and demand for intermediate inputs:

$$(1-\alpha)\frac{Y_{j,t}}{L_{j,t}} = \frac{W_{j,t}}{MC_{j,t}} \tag{12}$$

$$\alpha \frac{Y_{j,t}}{X_{j,t}} = \frac{P_{j,t}}{MC_{j,t}} \tag{13}$$

Similar to nominal wage setting before, firms face a Calvo (1983) price setting rigidity such that firms can only reset their prices each period with probability $(1-\delta_p)$. Allowing for all pricing paradigms, firm can choose to set their prices either in the producer currency j, in the destination currency i, or in the dominant currency \$. θ^k_{ji} thus gives the fraction of exports from country j to country i that are priced in currency k such that $\sum_k \theta^k_{ji} = 1$. This yield that PCP is given when $\theta^j_{ji} = 1$ for every $i \neq j$. LCP is given when $\theta^i_{ji} = 1$ for every $i \neq j$. And finally, DCP is given when $\theta^\$_{ji} = 1$ for every $i \neq j$. All remaining domestic prices are sticky in the home currency such that $\theta^j_{jj} = 1$ for every j. The Phillips curve is than given by:

$$E_t \sum_{s=t}^{\infty} \delta_p^{s-t} \Theta Y_{ji,s|t}(\omega)(\sigma^k_{ji,s}(\omega) - 1)\left[e_{kj,s}\bar{P}^k_{ji,t}(\omega) - \frac{\sigma^k_{ji,s}(\omega)}{\sigma^k_{ji,s}(\omega) - 1} MC_{j,s}\right] = 0, \tag{14}$$

with $Y_{ji,s|t}(\omega)$ being the amount of variety ω sold in country i invoiced in currency k at time s by a firm that last reset prices at time t and $\sigma^k_{ji,s}(\omega)$ is the elasticity of demand.

2.3 Closing the Model

To close the model, the monetary authority sets the nominal interest rate by following a standard Taylor-type rule in targeting inflation and output deviations.

$$i_{i,t} - i^* = \rho_m(i_{i,t-1} - i^*) + (1-\rho_m)(\phi_\pi \pi_{i,t} + \phi_y \tilde{y}_{i,t}) + \varepsilon_{i,t} \tag{15}$$

In the previous equation, ϕ_π denotes how the central bank reacts to changes in consumer price inflation, ϕ_y denotes how the central bank reacts to the output gap, ρ_m denotes the inertia in the policy rate, and i^* is the target and steady state international borrowing rate. In this case, following the setup in Gopinath et al. (2020), the monetary policy shock term is assumed to follow an AR(1) process given by: $\varepsilon_{i,t} = \rho_\varepsilon \varepsilon_{i,t-1} + \epsilon^m_{i,t}$.

All markets clear with $Y_{i,t}(\omega) = \sum_j (C_{ij,t}(\omega) + X_{ij,t}(\omega))$, $N_{i,t=L_{i,t}}$, $B_{i,t}(s') = 0$, $\forall s' \in S$, and $\sum_j B^\$_{j,t} = 0$.

As is usual in open economic models, they are nonstationary in the level of real debt, which also makes other real variables change permanently following transitory shocks. To make the model stationary, I follow the work of Schmitt-Grohé and Uribe (2003) to define the dollar interest rate at which households in G and R borrow internationally. This makes the dollar interest rate an increasing function of its external debt:

$$i^\$_{j,t} = i_{\$,t} + \psi\left(e^{-\bar{B}^\$_j} - 1\right) + \varepsilon^\$_{j,t}, \tag{16}$$

where $\psi > 0$ gives the responsiveness of the dollar rate to the country's real dollar debt holdings $B^\$_{t+1}$ and $\bar{B}^\$_j$ is the steady state real dollar debt.

Furthermore, the (log) UIP conditions for the exchange rates are given by:

$$i_{G,t} - i^{\$}_{G,t} = E_t(e_{\$G,t+1}) - e_{\$G,t} + E_t(\pi^G_{t+1} - \pi^U_{t+1}) + \varepsilon^{UIP_G}_t \tag{17}$$

$$i_{R,t} - i^{\$}_{R,t} = E_t(e_{\$R,t+1}) - e_{\$R,t} + E_t(\pi^R_{t+1} - \pi^U_{t+1}) + \varepsilon^{UIP_R}_t \tag{18}$$

Thus, the exchange rates depend on the differentials between the domestic nominal interest rates $i_{G,t}$ and $i_{R,t}$ and their respective dollar interest rates $i^{\$}_{G,t}$ and $i^{\$}_{R,t}$, with additional, country specific, and autoregressive UIP shocks.

2.4 Calibration

The model described above is calibrated following Gopinath et al. (2020) that implements very standard values for fixed parameters. The value for the discount factor, $\beta = 0.99$, gives a steady state interest rate, i^*, of around 4% in annualized terms. Risk aversion is $\sigma_c = 2$, the inverse of the Frisch elasticity is $\varphi^{-1} = 0.5$, and the disutility of labor, as well as the elasticity of labor demand are given by $\kappa = 1$ and $\vartheta = 4$ respectively. The degree of wage and price rigidity are $\delta_W = 0.85$ and $\delta_P = 0.75$. All of these are standard and are applied in for instance Galí (2008). The share of intermediate inputs in production is $\alpha = 2/3$, which implies a labour share of $(1 - \alpha) = 1/3$. Home bias is set to, $\gamma = 0.7$, which yields steady state shares on imported goods in the consumption and intermediate input bundle equal to 15% from the two other regions each and thus 30% in total. The parameters in the Taylor rule governing the central banks' response to inflation and output gap are calibrated to $\phi_\pi = 1.5$ and $\phi_y = 0.125$ with an inertia term of $\rho_m = 0.5$. The first two of those parameters will be varied in the subsequent sections to analyze determinacy regions. It is also assumed, unless explained otherwise, that all of these parameters are calibrated identically for all three economies.

3 Determinacy Analysis

Determinacy in structural DSGE models revolves around ensuring the existence of a unique and stable rational expectations equilibrium (REE). This equilibrium condition hinges on how central banks formulate their policy rules and respond to key economic variables, particularly inflation and output changes. It has been established that closed economy DSGE models require adherence to the Taylor principle, which dictates that a central bank should raise its nominal policy rate by more than one-to-one in response to inflation, to achieve a unique and stable REE (Bullard and Mitra

Table 1: Calibrated Parameters

Parameter	Description	Value
β	Discount Factor	0.99
σ_c	Risk Aversion	2
φ^{-1}	Frisch Elasticity	0.5
κ	Disutility of Labor	1
ϑ	Elasticity of Labor Demand	4
α	Share of Intermediate Input in Production	2/3
γ	Home Bias	0.7
δ_W	Wage Rigidity	0.85
δ_P	Price Rigidity	0.75
ρ_m	Taylor Rule Inertia	0.5
ϕ_π	Taylor Rule Inflation Sensitivity	1.5
ϕ_y	Taylor Rule Output Gap Sensitivity	0.125
i^*	Steady State Interest Rate	$(1/\beta)$-1

(2002)).

Following Farmer (1999) and Evans and Honkapohja (2001) I consider a system of linear stochastic difference equations, optimality conditions of a DSGE model for instance:

$$x_t = BE_t[x_{t+1}] + Dx_{t-1} + K\nu_t \tag{19}$$

$$\nu_t = R\nu_{t-1} + \xi_t, \tag{20}$$

where x_t is a $n \times 1$ vector of endogenous variables and ν_t is a $k \times 1$ vector of exogenous disturbances, with the matrices $B, D, K,$ and R being of appropriate dimensions. One way to define determinacy is to write the system above, linearized around its steady state as in Blanchard and Kahn (1980), in the following form:

$$E_t[z_{t+1}] = J_1 z_t + J_2 \nu_t, \tag{21}$$

where J_1 and J_2 are functions of the matrices defining the original system and $z_t = [x_t, x_{t-1}]'$. Then, the REE is determinate if the number of stable eigenvalues of J_1 is equal to the number of predetermined variables in z_t.

Traditionally, determinacy requirements in DSGE models are derived analytically, particularly in closed and small-open-economy (SOE) settings, as discussed in Llosa and Tuesta (2008), focusing on key parameters such as those governing the Taylor rule or home bias. However, due to the complex nature of the three-economy setup involving sticky wages, prices, and intermediate goods in production, analytical derivation becomes impractical.

Therefore, I adopt a simulation-based approach, inspired by Araújo (2016), to explore determinacy numerically employing the Dynare toolbox of Adjemian et al. (2024). I start by examining a theoretical scenario where all three central banks implement an identical Taylor rule parametrization, responding uniformly to inflation and output deviations. This exploration allows me to identify combinations of critical monetary policy parameters, ϕ_π and ϕ_y, that lead to a determinant and therefore unique, stable equilibrium.

Subsequently, I use the insights gained from this exercise to investigate a more realistic scenario where central banks maintain a common Taylor rule structure (as in Equation 15) but vary their individual responses. This comparative analysis aims to reveal the impact of nuanced policy adjustments on determinacy within this multi-economy framework.

Following Ida (2023), I define worldwide equilibrium determinacy as the case that the unique rational expectations equilibrium (REE) is achieved in all three economies. Hence, worldwide equilibrium becomes indeterminate if at least one economy faces an indeterminacy problem.

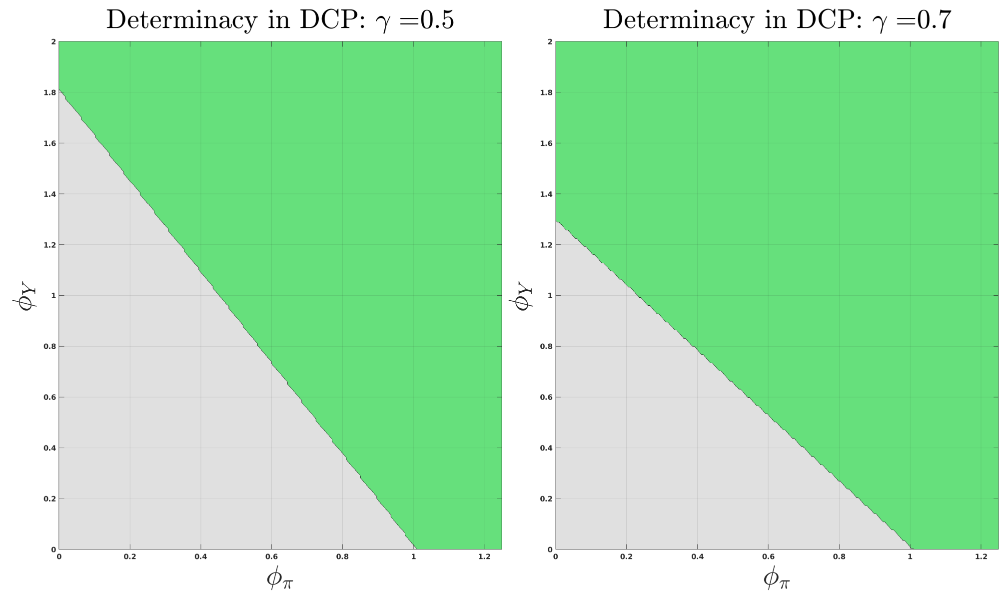
3.1 Same Taylor Rule Calibration

To initiate this analysis, I begin by exploring a theoretical scenario that provides valuable insights into model behavior, despite not perfectly mirroring real-world conditions. In this scenario, all three central banks within the multi-economy framework adopt an identical policy rule calibration as defined in Equation 15. This setup enables me to conduct an assessment that is more comparable to closed or small open economy (SOE) models studied in prior literature, such as Llosa and Tuesta (2008), Araújo (2016), Karagiannides and Liambas (2019), and Barnett and Eryilmaz (2023). It can also be interpreted as a case in which the central banks coordinate on policy responses.

The key determinants of stability in this scenario are the policy rule parameters ϕ_π and ϕ_y, representing the central bank's response to domestic consumer price index (CPI) inflation and output gap, respectively.

Another critical parameter influencing determinacy in international macroeconomics is the degree of economic openness or home bias, denoted by γ. This parameter determines the proportion of consumption and intermediate inputs in production that come from the domestic economy. I investigate determinacy under two levels of home bias: $\gamma = 0.7$, representing a relatively closed

Figure 1: Determinacy in DCP - Same Calibration



Determinacy regions when all three economies follow the same Taylor rule under DCP. The green area shows the Taylor rule parameters consistent with determinacy. The left panel indicates a more open economy, $\gamma = 0.5$, whereas the right panel shows a more close economy, $\gamma = 0.7$. For each panel, the openness parameter is identical in all three regions.

economy with predominantly domestic consumption and production, and $\gamma = 0.5$, representing a more open economy with significant trade and exchange rate effects. Notably, in these models, the home bias parameter remains consistent across all three economies, with the remaining economic openness equally distributed among the other two regions.

For the analysis, I choose parameter ranges of $\phi_\pi \in [0, 1.25]$ and $\phi_y \in [0, 2]$, providing enough scope to explore the critical values that yield determinacy in the model. Unlike studies such as Ida (2023) and Zhao (2022), which introduce specific model features leading to their findings, I focus on the core parameter range to investigate determinacy without encountering the behaviors of indeterminacy observed with large values of ϕ_π and ϕ_y.

Figure 1 illustrates the determinacy regions within the DCP setup, focusing on a scenario where all central banks adhere to the same Taylor rule calibration. The green areas represent parameter combinations of the policy rule that lead to determinacy - a unique and stable rational expectations equilibrium (REE) or worldwide equilibrium - while the gray area signifies regions of indeterminacy. The right panel has a home bias parameter calibrated to be $\gamma = 0.7$, following the standard calibration in Gopinath et al. (2020). Reducing the home bias, i.e., making the economies more open in their choice of consumption and intermediate goods, is shown in the left panel where home bias is lowered to $\gamma = 0.5$.

In the context of this analysis, two key parameters, ϕ_π and ϕ_y, dictate the central banks' responses to inflation and output fluctuations, respectively. Notably, when $\phi_y = 0$, ensuring determinacy requires that $\phi_\pi > 1$ across all economies, irrespective of their level of openness. This result mirrors findings from closed economy models and small open economy (SOE) investigations and is known as the Taylor principle.

However, as central banks begin to react to output deviations, the inflation response may decrease below unity, no longer satisfying the Taylor principle. This effect is depicted by the diagonal line in the plot, separating the green (determinate) and gray (indeterminate) regions—a phenomenon previously demonstrated by Llosa and Tuesta (2008).

The expenditure-switching effect between domestic and foreign goods, driven by exchange rate movements, plays a pivotal role in this dynamic. For instance, a currency appreciation resulting from a domestic policy rate increase can alter export and import prices, influencing consumption patterns. Notably, central banks' CPI-based inflation targeting takes into account foreign goods' prices, underscoring the significance of invoicing currency in shaping determinacy outcomes.

In the right panel of Figure 1, the diagonal line illustrates that if all three central banks solely respond to the output gap without considering inflation, a value of approximately $\phi_y \approx 1.3$ is sufficient to achieve worldwide equilibrium with determinacy. In this setup of a relatively closed economy with DCP pricing in trade, central banks can consistently achieve determinacy by adhering to the Taylor principle. However, as they reduce their inflation responses, they must significantly increase their sensitivity to output fluctuations, ultimately transitioning to a regime focused solely on output stabilization without inflation targeting, as indicated by the intersection of the diagonal line with the y-axis.

The left panel of Figure 1 presents the model analysis under conditions of increased economic openness. This analysis highlights the influence of greater openness on the parameter choices necessary to achieve a unique and stable equilibrium. Still, when central banks emphasize inflation control over output stabilization, the Taylor principle holds. However, the trade-offs between inflation targeting and output stabilization become more significant in more open economies, indicating that central banks need to carefully balance these objectives to maintain economic stability.

These findings diverge from those observed in traditional SOE models, where greater openness typically expands the determinacy region due to heightened expenditure-switching effects towards foreign goods, reducing the need for aggressive policy responses. In contrast, in an endogenously modeled multi-economy environment like this one, increased openness imposes stricter determinacy conditions, as depicted by the steeper diagonal line in Figure 1. This has critical implications for policymakers, as demonstrated by Bullard and Singh (2008), who noted similar trends in a two-country setting.

Furthermore, a central bank operating under incorrect assumptions regarding economic openness risks choosing Taylor rule parameters that are insufficient for achieving determinacy, potentially leading to unstable outcomes.

This phenomenon stems from the observed behavior of consumers and firms within each country, particularly in more open economies, where there is a notable shift towards consuming and utilizing more internationally traded goods over domestically produced ones. Consequently, a smaller proportion of aggregate demand within these economies is satisfied by local production. When a central bank responds to output fluctuations by increasing its nominal interest rate, agents find it easier to switch to imported goods, diminishing the efficacy of this policy measure.

The mechanism behind this effect is straightforward: an increase in the domestic policy rate leads to currency appreciation, making imports cheaper and prompting a shift towards relatively less expensive foreign goods. Moreover, the inflation measure that central banks react to, namely the Consumer Price Index (CPI), incorporates the costs of imported goods, amplifying its significance in more open economies.

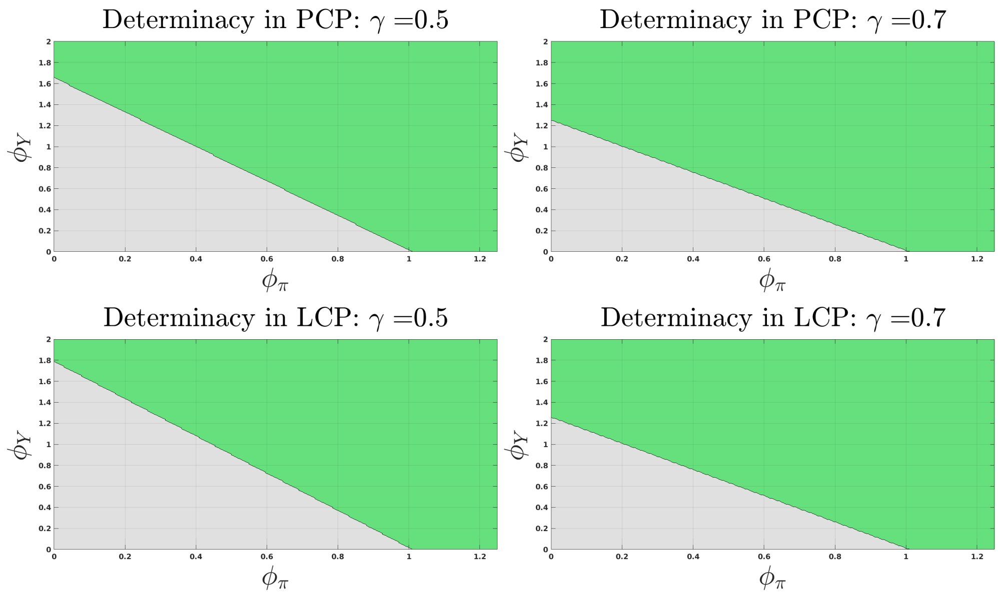
The choice of invoice currency in international trade becomes pivotal in this context, as exchange rate dynamics play a crucial role. In the current scenario, all prices are denominated in the dominant currency, referred to as DCP. This assumption aligns with the findings of Gopinath et al. (2020), indicating its realism and highlighting its impact on exchange rates, export and import prices, and quantities.

However, disentangling these effects solely from Figure 1 is challenging and requires comparison with other economic regimes to fully appreciate the implications of different policy settings and economic conditions.

Therefore, in open economy models, central bank decisions diverge from those in closed economy settings due to the influence of exchange rate movements and subsequent expenditure-switching effects. As illustrated in Figure 1, the determinacy region's steepness increases with a smaller home bias parameter γ, indicative of a more open economy where central banks trade-offs between inflation and output stabilization becomes more difficult.

As highlighted earlier, the dynamics of exchange rates become critical in open economies, underscoring the importance of currency invoicing arrangements in shaping policy trade-offs. This significance holds true even in a simplified scenario where all three central banks adhere to identical Taylor rule

Figure 2: Determinacy in PCP & LCP - Same Calibration



Determinacy regions when all three economies follow the same Taylor rule under PCP and LCP. The green area shows the Taylor rule parameters consistent with determinacy. The first column indicates a more open economy, $\gamma = 0.5$*, whereas the second column shows a more close economy,* $\gamma = 0.7$*. For each of them, the openness parameter is identical in all three regions.*

specifications.

Figure 2 extends this analysis by presenting determinacy region assessments for scenarios implementing Producer Currency Pricing (PCP) and Local Currency Pricing (LCP) paradigms, shedding further light on how different pricing strategies can impact determinacy outcomes.

Observing the determinacy outcomes in relatively closed economies, depicted in the right column of Figures 1 and 2, reveals a notable trend: the choice of pricing paradigm exerts only a marginal influence on policy determinacy. This finding, although specific to this special case of same Taylor rule calibration and relatively closed economies, carries significant implications. Despite potential variations in shock transmission mechanisms emphasized in prior research, Gopinath et al. (2020), this result suggest that, in these contexts, pricing paradigms do not significantly affect the parameter space available to central banks in achieving determinacy. This conclusion underscores the robustness of determinacy outcomes to pricing paradigm variations within relatively closed economic systems. Or also highlights the potential gains by central banks coordination of policies, bringing them closer to this case of identical reaction functions.

However, as economies become more open, the impact of pricing paradigms on determinacy becomes increasingly apparent. Comparing the left column of Figure 2 with the same column in Figure 1, it is evident that the choice of invoicing currency can be crucial in determining whether specific

Taylor rule parameters lead to determinacy. Notably, the cases of DCP and LCP exhibit closer relationships compared to the more conventional PCP scenario. This represents a fundamental finding of this study: determinacy hinges not only on the extent of economic openness, or the level of home bias, but also critically on the chosen pricing paradigm. Consequently, a central bank analyzing its economy must carefully consider both the degree of openness and the appropriate trade pricing to ensure that selected Taylor rule coefficients are sufficient to achieve determinacy, thus averting endogenous volatility as discussed in Bullard and Singh (2008). Given the prevailing use of PCP in open economy modeling literature, researchers and policymakers may inadvertently select Taylor rule parameters that ensure determinacy in PCP scenarios but fail to do so in the more realistic context of DCP.[2]

To understand the intuition behind this result let's assume again from the perspective of the home economy G the effect of a sunspot shock in economy U and how different pricing paradigms influence transmission of shocks. The central bank in economy U increases its monetary policy rate to bring inflation down. If all central banks target their respective domestic inflation exclusively, the Taylor principle holds regardless the invoicing currency. In this case, it is ensured that each price increase, be it domestic or foreign, is taken care of by at least one central bank. It does not matter which currency this traded goods are invoiced in or how these shocks influence exchange rates as the collective central banks will always react strong enough.

This changes as soon as the central banks also target output deviation, as shown above. The increase in the nominal interest rate in U appreciates the Dollar against the home currency in G. This has varying effects depending on the invoice currency of trade. For the case of PCP, exports become more competitive while imports become less competitive. Thus, the term of trade improve and thus output increases. There is also a slight increase in inflation, which endogenously increases the interest rate. As import prices increase, there is an expenditure switching effect toward domestic goods, and goods from R.

In the case of LCP, there is a similar effect at play. However, now the prices of imports remain relatively stable because they are invoiced in the local currency. This stability means that the expenditure-switching effect is less pronounced compared to the PCP scenario. Instead, the main channel through which the economy G is affected is through the change in the competitiveness of its exports. As the home currency in G depreciates relative to the Dollar, exports from G become cheaper and more attractive on the global market. This shift boosts output in G but may also lead to increased inflationary pressures as the demand for domestic goods rises. The central bank in G must then carefully balance its response to these changes, taking into account the

[2] To show that these results do not only hold because of the symmetric calibration of parameters, I present an analysis in Appendix C where the economies are calibrated differently and find the same qualitative results.

dual objectives of stabilizing both output and inflation. This delicate balancing act is crucial for maintaining determinacy and avoiding indeterminate or volatile economic outcomes.

Considering the DCP scenario, where prices are set in the dominant currency, the dynamics change significantly. When the central bank in U raises interest rates, it not only appreciates the Dollar but also directly influences the import prices in all economies, including G. The immediate effect is a decrease in the price competitiveness of goods from U and an increase in import prices for G. Consequently, there is a pronounced expenditure-switching effect as consumers in G shift their consumption towards relatively cheaper domestic and non-U imported goods. This shift can lead to an increase in domestic production and output, potentially offsetting the inflationary impact of higher import prices. However, the overall effect on determinacy hinges on how well the central banks in G and R can adjust their policy rates to stabilize both output and inflation. The interconnectedness of economies under DCP necessitates a more coordinated and nuanced approach to monetary policy to ensure global stability.

These examples illustrate the complexities that central banks face in open economies, especially when dealing with different pricing paradigms and varying degrees of economic openness. They underscore the importance of understanding the specific channels through which monetary policy and exchange rate movements interact to influence domestic economic conditions. Central banks must consider these interactions when designing their policy frameworks to achieve determinacy and avoid undesirable economic fluctuations. The findings from this analysis highlight the need for a tailored approach to monetary policy that accounts for the unique characteristics of each economy's openness and pricing paradigm, ensuring that the chosen Taylor rule parameters are robust and effective in maintaining economic stability.

Furthermore, the role of home bias in these dynamics cannot be overstated. As the degree of home bias increases, the economy becomes less exposed to foreign shocks, allowing central banks to rely more on traditional policy rules that prioritize domestic conditions. In more open economies, however, the central banks must be vigilant about the international transmission of shocks and the resulting implications for their policy settings. The interplay between home bias and pricing paradigms adds another layer of complexity to the determination of appropriate policy responses, making it essential for policymakers to consider these factors in their decision-making processes.

Overall, these insights contribute to a deeper understanding of how different monetary policy frameworks and economic structures influence the determinacy and stability of rational expectations equilibria in multi-economy models. By carefully considering the effects of pricing paradigms, economic openness, and home bias, central banks can better navigate the challenges of maintaining stable and predictable economic environments in an increasingly interconnected global economy.

3.2 Different Taylor Rule Calibration

In this section, I delve into a more interesting scenario where each of the three central banks has the flexibility to choose different parameter values governing their responses to price and output changes, all while adhering to the same form of the monetary policy rule outlined in Equation 15. This adds more complexity as it involves six parameters in total: ϕ_π^j and ϕ_y^j for each central bank denoted by $j \in \{G, U, R\}$.

To effectively analyze this scenario, 2-dimensional plots like those in Figures 1 and 2 are inadequate. Instead, I opt to use three-dimensional plots to visualize the determinacy regions. The axes of these plots represent the parameters governing each central bank's sensitivity to inflation: ϕ_π^U, ϕ_π^G, and ϕ_π^R. Similarly, the parameters governing sensitivity to the output gap (ϕ_y^U, ϕ_y^G, and ϕ_y^R) are varied one-by-one to observe their impact on determinacy.

In this analysis, the inflation sensitivity parameters ϕ_π are explored within the interval $[0, 1.25]$, evenly spaced to form a grid. Additionally, the output gap sensitivity parameters ϕ_y are set to values in the set $\{0,\ 0.5,\ 1\}$ to examine the trade-offs between higher output sensitivity and determinacy under different inflation targeting scenarios.[3]

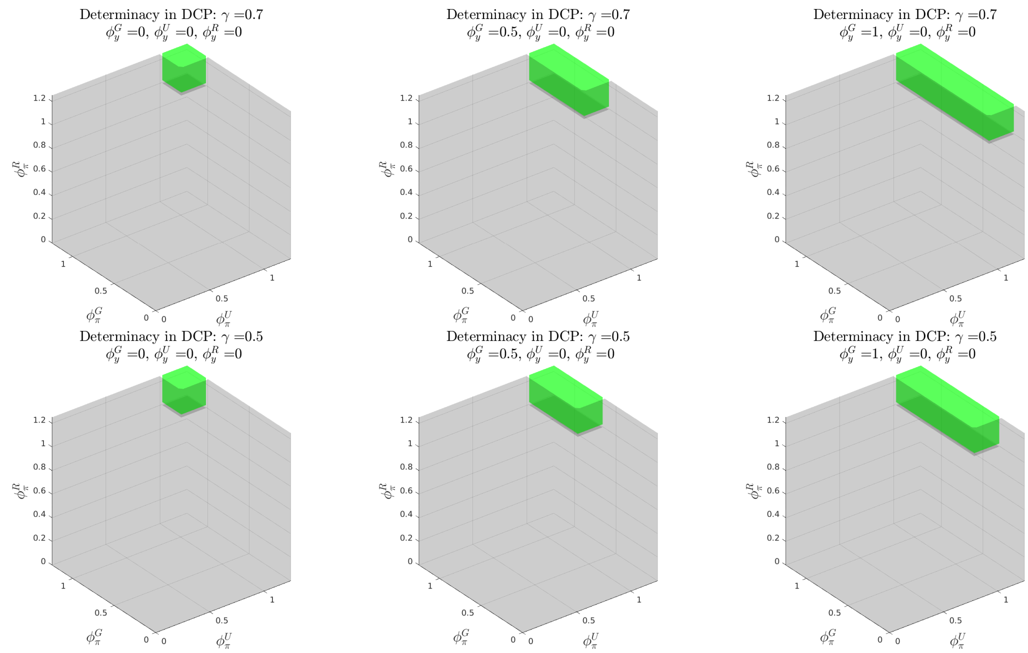
For this first analysis, when varying ϕ_y^G as shown in Figure 3, the output gap sensitivities ϕ_y^U and ϕ_y^R are set to zero. This focused approach allows me to understand how policy choices within one economy influence determinacy without interference from other economies. Subsequent stages of the analysis involve simultaneous variations of all three parameters across economies to capture interdependencies more comprehensively. Furthermore, the degree of openness γ and its influence on determinacy are considered alongside the chosen pricing paradigm, which is also investigated within this section to understand its impact on the determinacy of policy parameters and economic stability.

3.2.1 The Case of DCP

This analysis begins by examining the dominant currency paradigm (DCP), focusing on the scenario where central banks have the flexibility to adjust parameters in their respective Taylor rules. Figure 3 illustrates this scenario by varying one parameter at a time to assess its impact on determinacy. Specifically, I explore the output gap sensitivity parameter (ϕ_y^G) in economy G, which represents

[3] The values for ϕ_y being only in the set $\{0,\ 0.5,\ 1\}$ is of course a simplification. What is crucial at this point is to understand the interaction of output gap targeting and inflation targeting within each country but also across economies. The choice also has computational reasons. The three-dimensional grid for the inflation targeting parameters ϕ_π is of the size $50 \times 50 \times 50$. Thus, for each combination of ϕ_y parameters, the model has to be evaluated a total of $125,000$ times. With the choices for ϕ_y as explained above, this yields a total number of evaluations of over three million.

Figure 3: Determinacy in DCP - Varying ϕ_y^G



Determinacy regions when all three economies have different Taylor rule parametrization in DCP. The green area shows the Taylor rule parameters consistent with determinacy. The parameter that is adjusted between each subplot is ϕ_y^G, while $\phi_y^U = \phi_y^R = 0$. The first row indicates a more closed economy, $\gamma = 0.7$, whereas the second row shows a more open one, $\gamma = 0.5$. For each of them, the openness parameter is identical in all three regions.

how much the central bank in G responds to domestic output fluctuations.

In Figure 3, each column corresponds to a different value of ϕ_y^G, ranging from $\phi_y^G = 0$ (indicating no sensitivity to the domestic output gap) to $\phi_y^G = 1$ (indicating strong sensitivity to output fluctuations). Meanwhile, the other two output sensitivity parameters (ϕ_y^U and ϕ_y^R) are kept constant at $\phi_y^U = \phi_y^R = 0$ to isolate the influence of output targeting within economy G.

The rows in Figure 3 represent varying levels of home bias, γ, serving as a proxy for openness. A lower value of γ implies greater openness, indicating that a smaller proportion of consumption and intermediate goods originate from the domestic economy. Hence, $(1-\gamma)$ is the share of consumption and intermediate goods that come from abroad, which is evenly divided into the two trading partners respectively.

This approach allows for a focused investigation into how output targeting affects determinacy within economy G, providing insights into the relationship between output sensitivity, home bias (or openness), and the stability of monetary policy across interconnected economies. The results shed light on the importance of considering these factors when formulating optimal policy parameters in a multi-economy context.

Beginning with the first element in Figure 3, I set the home bias parameter $\gamma = 0.7$, reflecting a relatively closed economy akin to Gopinath et al. (2020). In this setup, the central bank of the home economy (economy G) does not consider the domestic output gap ($\phi_y^G = 0$) and varies only its sensitivity to inflation. Previously, as illustrated in Figure 1, it was established that if all central banks share the same calibration and do not target the output gap, achieving a determinate equilibrium requires an inflation sensitivity parameter (ϕ_π) exceeding 1, adhering to the Taylor principle. In this current scenario within Figure 3, the same principle holds true. The central bank of the home economy must set an inflation sensitivity parameter of $\phi_\pi^G > 1$ to establish a determinate domestic equilibrium, thereby contributing to a stable worldwide equilibrium. This finding is consistent across the exercise, where central banks without output targeting rely on strong inflation responses to achieve equilibrium, underscoring the importance of inflation control in sustaining stability across interconnected economies.

Moving to the second element in the first row, now the central bank in economy G also targets the domestic output gap with a parameter of $\phi_y^G = 0.5$. Meanwhile, the other two central banks continue to focus solely on inflation targeting. The immediate observation is the extension of the green cuboid in the direction towards $\phi_\pi^G = 0$. This extension signifies a crucial effect: a higher sensitivity to the domestic output gap allows for a reduction in the sensitivity to domestic inflation while still achieving a global and determinate equilibrium. This effect becomes more pronounced as ϕ_y^G increases, highlighting the trade-off between output gap targeting and inflation sensitivity for a central bank.

Looking at the third element in the first row, it becomes apparent that with $\phi_y^G = 1$, there is almost complete substitutability of inflation targeting with output gap targeting. This observation aligns with the findings from the previous section, demonstrating that a central bank can reduce its inflation target below 1 once it starts targeting the output gap. This relationship is represented by the diagonal line that separates the determinacy from the indeterminacy regions in Figure 1.

Another observation is the impact of increased output gap sensitivity on determinacy with respect to inflation sensitivity in the other two economies. It is noticeable that the determinacy region for the other two economies neither significantly increases nor decreases optically. Therefore, in this scenario where the central banks in the other two economies focus solely on inflation targeting, an increase in the output gap target in country G does not influence the ability of the other two central banks to relax their inflation targets. This finding suggests that changes in output gap targeting in one economy have a limited effect on the inflation targeting strategies of other economies under similar conditions.

The second row of Figure 3 presents a similar determinacy region analysis but with a lower level of home bias, reflecting a more open economy. In the earlier section exploring the same Taylor rule calibration, I highlighted that as economies become more open, it becomes increasingly difficult for central banks to replace inflation measures with output gap measures. This challenge arises due to a larger proportion of consumption and final goods originating from abroad, which was illustrated by a steeper diagonal line separating regions of determinacy from indeterminacy.

Comparing the first elements of both rows in Figure 3, it can be observed that in a scenario where all central banks exclusively target inflation, the level of openness does not affect determinacy, as indicated by the identical sizes of both cuboids. This finding aligns with the conclusion from the previous section. In Figure 1, the point where the separating line intersects the x-axis remains consistent.

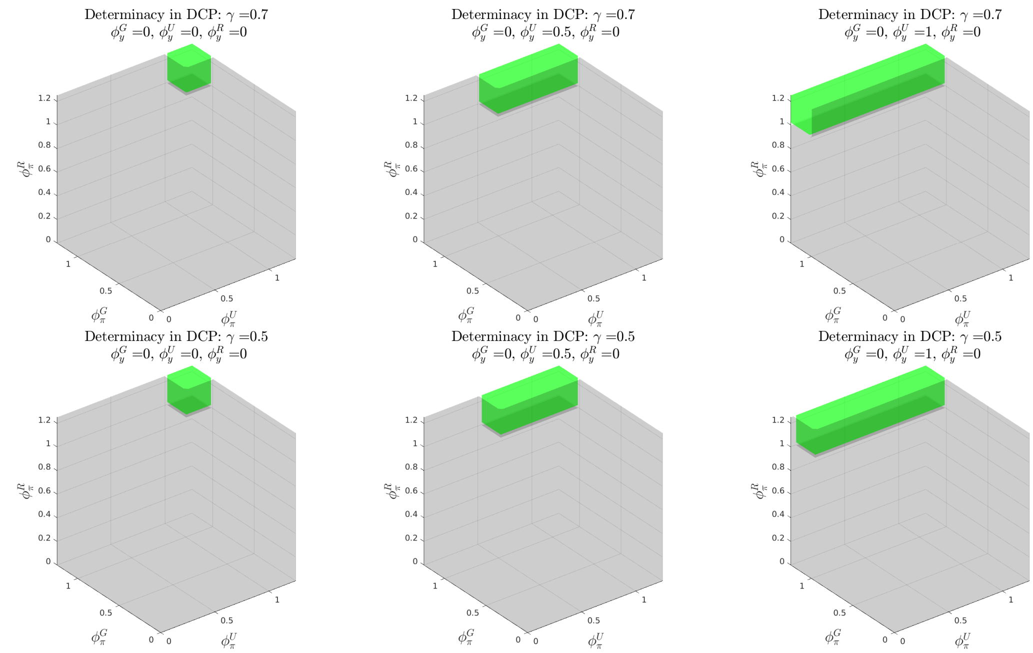
Moving to the second and third columns of the second row in Figure 3, a similar trend is observed where it becomes more challenging for the central bank in economy G to substitute inflation targeting with output targeting as the economy becomes more open. Graphically, the sizes of the cuboids decrease relative to their counterparts in the first row. This signifies that in a more open economy, the ability to shift from inflation to output targeting diminishes progressively from left to right in the second row. The economic intuition remains unchanged: with a greater proportion of final goods sourced from abroad, the effectiveness of central banks' actions targeting domestic output gaps diminishes in achieving a determinant equilibrium.

Interestingly, in this setup, the level of home bias does not appear to impact the determinacy regions of the other two central banks. But again, this is only a visual interpretation and is analyzed in more depth in a later section where the other two central banks also change their level of output gap targeting, creating the whole set of interdependencies.

Figure 3 showed the effects on determinacy of the home central bank, in G, choosing various levels of output gap sensitivity, while the two other ones did not react to the output gap. As right now, I analyze those changes within the DCP framework, it is important to see how and if things change when the central bank in the origin country of the dominant currency changes its output sensitivity and what this means for worldwide equilibrium determinacy. Figure 4 does exactly that and has the same style of investigation as before, but now ϕ_y^U is varied.

Here, it becomes clear that the pricing paradigm has an effect on the determinacy of the whole model. I start by comparing the first rows of Figures 3 and 4, which again is for the case of a relatively closed economy. The first element is identical as it depicts the exact same parametrization. In the second element of the first row the differences begin to emerge. The extension of the cuboid is still towards the respective $\phi_y^U = 0$. But the one in Figure 4 does extend further, meaning that the central bank

Figure 4: Determinacy in DCP - Varying ϕ_y^U

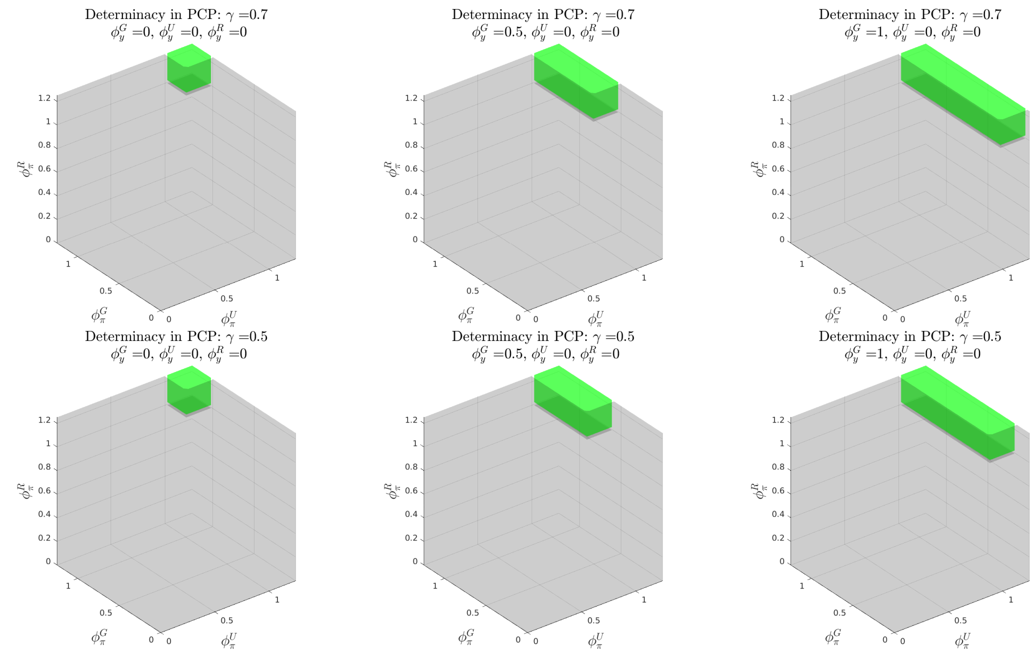


Determinacy regions when all three economies have different Taylor rule parametrization in DCP. The green area shows the Taylor rule parameters consistent with determinacy. The parameter that is adjusted between each subplot is ϕ_y^U, while $\phi_y^G = \phi_y^R = 0$. The first row indicates a more closed economy, $\gamma = 0.7$, whereas the second row shows a more open one, $\gamma = 0.5$. For each of them, the openness parameter is identical in all three regions.

in the dominant currency issuing economy has it easier substituting away from inflation targeting towards output gap targeting while still yielding determinacy. Put differently, it has a larger set of policy options available that yield worldwide equilibrium. This continues when looking at the third element of the first row. There, the central bank in U is able to fully substitute away from inflation targeting by choosing for example $\phi_y^U = 1$ and $\phi_\pi^U = 0$ and still having a determinate solution. In Figure 3, this has not been possible for economy G. Therefore, in a world where all trade is invoiced in a dominant currency, the central bank of this country has more policy options available as the other central banks have to take their respective exchange rates more into account and help stabilizing the dominant currency issuing economy towards determiancy. Figure A1 in the Appendix underscores this finding as the effect on the R economy's determinacy is the same as for G.

In summary, worldwide equilibrium determinacy has to be achieved simultaneously in all three economies by their respective central bank. Thus, if either one country fails to reach a determinate domestic equilibrium, an indeterminate worldwide equilibrium is implied. In the case of DCP, the central bank of the dominant currency issuing country has more parameter options available to ensure determinacy. This insight emphasizes the pivotal role of the dominant currency issuing central bank in maintaining global economic stability, as its policy choices can significantly influence the determinacy outcomes for the entire global economy.

Figure 5: Determinacy in PCP - Varying ϕ_y^G



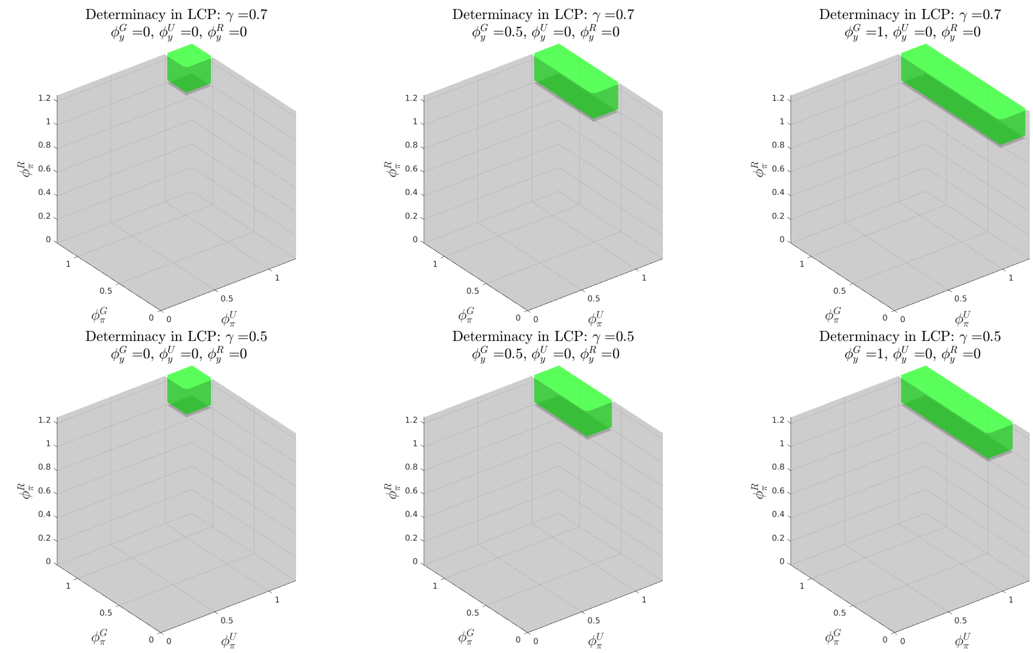
Determinacy regions when all three economies have different Taylor rule parametrization in PCP. The green area shows the Taylor rule parameters consistent with determinacy. The parameter that is adjusted between each subplot is ϕ_y^G, while $\phi_y^U = \phi_y^R = 0$. The first row indicates a more closed economy, $\gamma = 0.7$, whereas the second row shows a more open one, $\gamma = 0.5$. For each of them, the openness parameter is identical in all three regions.

3.2.2 The Cases of PCP and LCP

After having analyzed the more realistic case of dominant currency pricing in international trade, it is crucial to also understand what determinacy effects the much more common paradigms of producer currency pricing (PCP) and local currency pricing (LCP) have. While analyzing these paradigms, it is no longer necessary to separately look at the determinacy effect of the three countries as it was for DCP. The reason for this is that now, all economies - either in PCP or LCP - face the same symmetric pricing in the sense that international trade is invoiced in the producer or local currency independent of the country observed. Thus, for this section I focus on the home economy G exclusively.

Continuing in the same style as before, Figures 5 and 6 show the same determinacy results for the cases of PCP and LCP. As mentioned above, the only parameter changing in a row, without loss of generality, is the output gap target of the central bank in G. While the other two such parameters are fixed at zero, $\phi_y^U = \phi_y^R = 0$, such that those central banks only target inflation. This is again done to disentangle the effects and is adapted later. For relatively closed economies, i.e. the first row, determinacy for PCP and LCP does look very similar. PCP does, however, allow slightly more parameter choices for higher values of ϕ_y^G such that the cuboid extends longer towards $\phi_\pi^G = 0$. This

Figure 6: Determinacy in LCP - Varying ϕ_y^G



Determinacy regions when all three economies have different Taylor rule parametrization in LCP. The green area shows the Taylor rule parameters consistent with determinacy. The parameter that is adjusted between each subplot is ϕ_y^G, while $\phi_y^U = \phi_y^R = 0$. The first row indicates a more closed economy, $\gamma = 0.7$, whereas the second row shows a more open one, $\gamma = 0.5$. For each of them, the openness parameter is identical in all three regions.

indicates that under PCP, the central bank in the home economy has a bit more flexibility in its policy choices compared to LCP.

In both cases, the determinacy region is larger compared to the first row in Figure 3 under the DCP regime. This suggests that when international trade is invoiced in the producer or local currency, the economies have a broader range of policy options to achieve determinacy. This applies only to the economies not issuing the dominant currency in the case of DCP, of course. The symmetry in pricing paradigms in PCP and LCP reduces the complexities faced in DCP, allowing for more straightforward policy coordination.

Making the economy more open in trade again reduces the amount of available parameters across all options. As seen in the second row of Figures 5 and 6, the sizes of the green cuboids decrease as γ decreases, indicating a higher degree of openness. This trend reflects the increasing difficulty for central banks to maintain determinacy with a broader range of policy parameters as their economies become more integrated into the global market.

For both PCP and LCP, the ability to target the output gap diminishes as the economy becomes more open, similar to the findings under DCP. However, the effect is less pronounced compared to DCP. This highlights that while openness imposes constraints on policy flexibility, the symmetric nature of PCP and LCP provides a more stable environment for central banks to operate in compared to the asymmetric DCP.

In summary, the analysis of PCP and LCP reveals that these pricing paradigms offer more policy flexibility and a larger determinacy region compared to DCP. This is due to the symmetric nature of pricing, which simplifies the policy coordination among central banks. However, as economies become more open, the challenge of maintaining determinacy increases across all paradigms, underscoring the importance of careful policy calibration in an interconnected world.

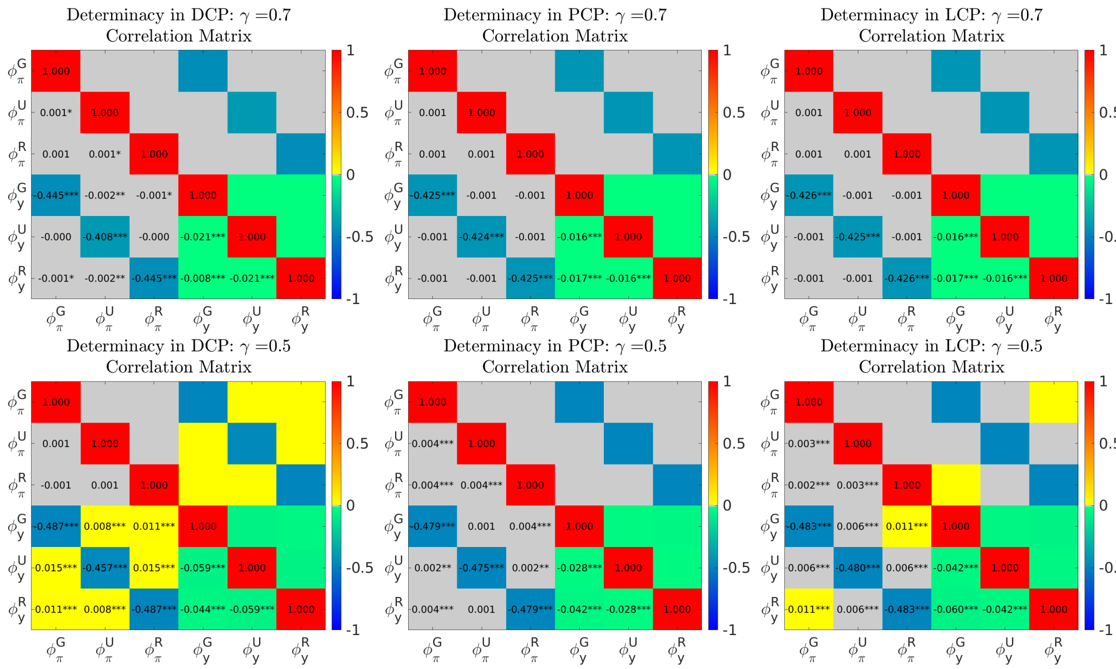
3.3 Parameter Correlations

Before presenting the determinacy region plots for the scenario where all central bank policy parameters vary, correlation plots are introduced to etter understand the interactions among the six specific parameters analyzed across simulations. Previous sections highlighted a notable negative correlation between a country's inflation and output gap targets, a relationship that may vary with the degree of economic openness. Figure 7 presents these correlations derived from multiple simulations, with the two rows representing different levels of home bias (and thus openness). Each row depicts all three pricing paradigms for easier comparison.

In the first row, representing a relatively closed economy similar to Gopinath et al. (2020), clear patterns emerge regarding how different paradigms influence parameter choices that yield determinacy. Across all paradigms, a consistent negative correlation between a central bank's inflation and output gap target parameters is evident. This observation aligns with previous figures, indicating that central banks can shift from pure inflation targeting by increasing their output gap targets. The most prominent negative correlations, shown in blue, are consistent across all three economies under PCP and LCP. However, in DCP, the central bank issuing the dominant currency exhibits a significantly weaker negative correlation, suggesting a less pronounced trade-off between inflation and output gap targeting.

For the Dominant Currency Paradigm (DCP), the correlations show a distinct pattern where the central bank issuing the dominant currency (U) has relatively weaker negative correlations with the output gap parameters in G and R. This suggests that in a DCP setting, the dominant currency's central bank has greater flexibility to influence global economic conditions through output gap stabilization, while other central banks must adjust their inflation targets more significantly. The slight negative correlations (light green) between U's output gap parameter and those of G and R suggest some level of substitutability, though limited. In the open economy scenario, positive correlations (yellow) between ϕ_y^U and ϕ_π^G and ϕ_π^R indicate that G and R need to increase their

Figure 7: Correlation Plot



*Correlation plots between the six central bank parameters governing determinacy. The first row shows correlation plots for all pricing paradigms with relatively closed economies, $\gamma = 0.7$, while the second one does the same for more open economies. For each of them, the openness parameter is identical in all three regions. The numbers in the lower triangular part show the correlation strength. The stars indicate significance levels: * for p-value < 0.1, ** for p-values < 0.05, ** for p-values < 0.01.*

inflation targets when U focuses more on output gap stabilization due to higher import prices invoiced in the dominant currency.

In the Producer Currency Paradigm (PCP), correlations are more uniformly distributed across economies, indicating a more symmetric response to policy changes. The negative correlations between inflation and output gap targets are similar in magnitude for all central banks, reflecting a balanced trade-off in policy adjustments. This suggests that in a PCP setting, central banks play a more equal role in achieving determinacy, with more interdependent policy adjustments. The stronger negative correlations (dark blue) between ϕ_y^G, ϕ_y^U, and ϕ_y^R in both closed and open economies underscore the necessity for central banks to carefully coordinate their output gap and inflation targets to maintain global stability.

In the Local Currency Paradigm (LCP), patterns are similar to PCP but with slight variations. The negative correlations between inflation and output gap targets remain consistent, but there is a slightly higher degree of independence for each central bank's policy choices. These negative correlations are less pronounced than in DCP, indicating that central banks in an LCP setting can achieve determinacy with less stringent coordination compared to DCP. The correlations (light blue) reveal that while interdependence exists, the impact of each central bank's policy on others is marginally reduced, offering more autonomy in policy decisions.

When prices are invoiced in the producer or local currency, the slight negative correlations (light green squares) change but do not significantly influence determinacy outcomes. These consistent sizes across paradigms indicate a more symmetric outcome. Thus, for a relatively closed economy, the strong negative correlation between a country's inflation and output gap targets remains the primary result. This correlation, observed previously, shows that the determinacy cuboid expands towards $\phi_\pi = 0$ as ϕ_y increases, with minimal effects on other central banks' policy options. Extremely large output gap targets may have an effect, which is analyzed further below.

As economies become more open (second row of Figure 7), the correlation patterns shift slightly, but the signs of correlations remain similar. The strong negative correlation between a country's inflation and output stabilizing parameters increases marginally across all pricing paradigms. In DCP, the negative correlation between U's output targeting parameter and those of G and R intensifies to nearly -0.06, providing U's central bank more flexibility to stabilize through output gap adjustments, reducing the need for G and R to do the same. Additionally, new positive correlations (yellow) appear between ϕ_y^U and ϕ_π^G and ϕ_π^R, indicating that when U shifts from inflation targeting to output gap targeting, G and R must increase their inflation targets due to higher import prices invoiced in the dominant currency. Other positive correlations also emerge among various countries' output and inflation targets.

Additionally, it is important to note the overall reduction in the magnitude of correlations in the open economy scenario compared to the closed economy. This trend highlights that higher economic openness leads to more complex interactions among central bank policies, diluting the direct trade-offs observed in less open economies. Therefore, central banks in more open economies need to consider these intricate dynamics when formulating their policy frameworks to achieve determinacy.

The magnitude of the correlations is critical, as it indicates the strength of the trade-offs and interdependencies between different policy targets. In the closed economy scenario, the correlations are stronger, implying more direct and significant trade-offs. In contrast, the open economy scenario shows more subdued correlations, reflecting the complexity and dilution of direct policy impacts due to greater economic openness and interdependence.

Overall, the plots reveal that while the fundamental negative correlation between inflation and output gap targeting remains robust, the degree of openness and the chosen pricing paradigm significantly influence the strength and complexity of these interactions. This underscores the necessity for central banks to adapt their policy strategies in response to the broader economic environment and the specific dynamics of international trade and pricing. The interaction effects highlighted by the correlations show that central bank policies do not operate in isolation but are interlinked, necessitating a coordinated and holistic approach to achieve global economic stability.

3.4 All Parameters Vary

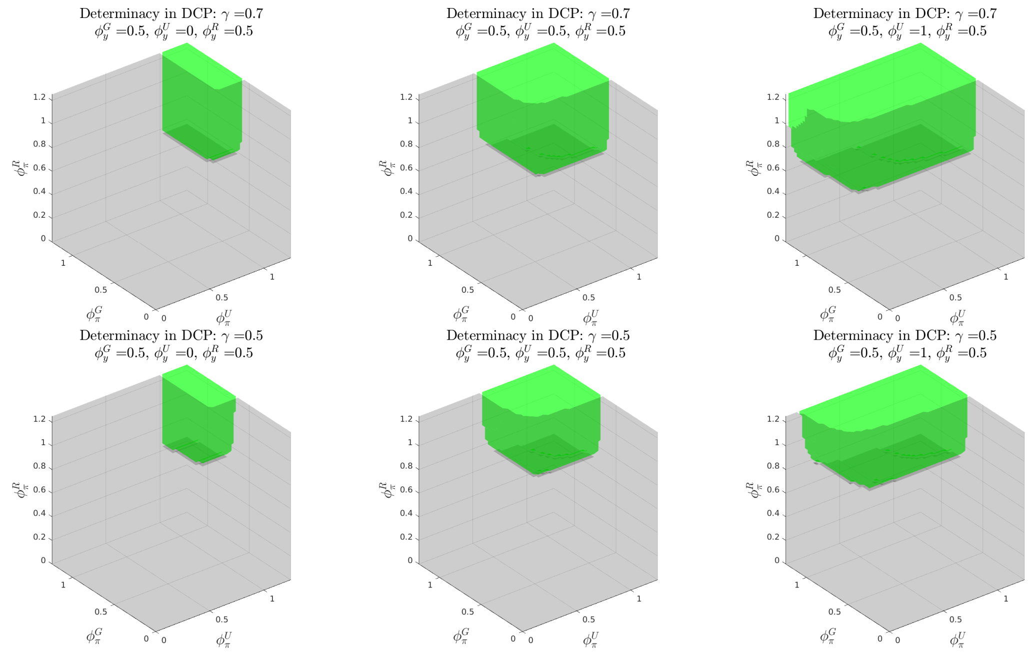
In previous sections, it was established that two crucial aspects determine determinacy in multi-country open economy models: the level of openness and the pricing paradigm used. However, the analysis thus far only explored scenarios where one central bank targeted both inflation and the output gap, while the other two focused solely on inflation. This was important for establishing foundational results. In this section, I extend the analysis to include all three central banks implementing targets for both inflation and the output gap. Insights from the previous section on parameter correlations suggest intriguing trade-offs to explore.

Figure 8 presents a similar analysis as before with DCP, but now all central banks implement targets for both inflation and the output gap. For clarity, I vary the output gap target of country U while keeping the others at $\phi_y^G = \phi_y^R = 0.5$. The spillovers and trade-offs described in the previous section on correlations are visibly apparent here. If there were no interactions between countries' inflation and output gap targets, the cuboids would be perfect squares. However, the rounded edges observed indicate these effects. It is crucial to compare Figure 8 to Figure 4, which depicted a similar determinacy analysis but only with the central bank in U targeting the output gap. In that plot, the sharp edges and cuboid expansion solely represented the inflation and output gap trade-off within a single country, with minimal international spillovers or interactions.

Comparing the second element in the first row of the two mentioned Figures helps to show these results. In Figure 4, the green cuboid extends towards $\phi_\pi^U = 0$ and is very close to $\phi_\pi^U = 0.5$ with almost no effect in other directions. In Figure 8, all three central banks have the same output gap parameter but the green cuboid is smaller in the direction mentioned before and is even more similar to the one in the second row of Figure 4. This effect stems from the positive correlation between ϕ_π^U, ϕ_y^G, and ϕ_y^R depicted in Figure 7. As the only thing that is different between the two plots is the level of output gap targeting in G and R, this makes the central bank in U have to increase its inflation target and thus reduces the size of the cuboid.

As demonstrated in the previous section, the magnitude of parameter correlations increases with the level of openness. This effect is evident in the second row of Figure 8, where the cuboid contracts in all dimensions. A clear example is the positive correlation described earlier, whereby the central bank in U increases its inflation target when central banks abroad target the output gap, including imported goods. The surface also exhibits more irregularities, reflecting additional correlation effects.

Figure 8: Determinacy in DCP



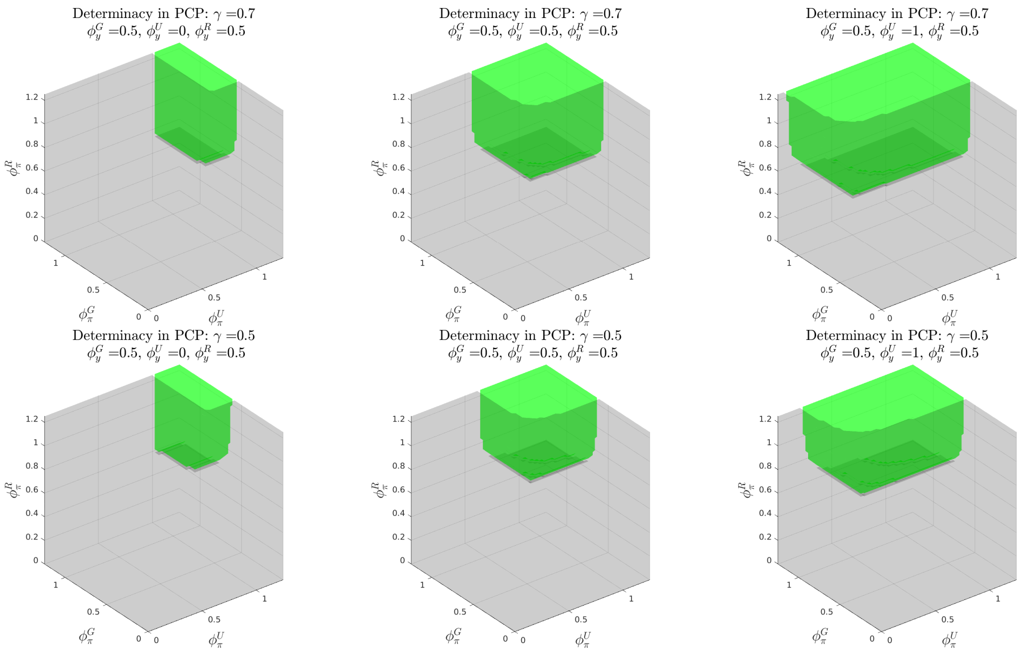
Determinacy regions when all three economies have different Taylor rule parametrization in DCP. The green area shows the Taylor rule parameters consistent with determinacy. The parameter that is adjusted between each subplot is ϕ_y^U, while $\phi_y^G = \phi_y^R = 0.5$. The first row indicates a more closed economy, $\gamma = 0.7$, whereas the second row shows a more open one, $\gamma = 0.5$. For each of them, the openness parameter is identical in all three regions.

Increasing the output gap parameter in U, as shown in the third column, amplifies these effects further. Now, the correlations involving ϕ_y^U, which is increasing, and all other parameters work together to reduce the overall size of the cuboid, constraining the available policy options for all central banks.

These figures elucidate the interplay between various policy rule parameters under different economic openness levels and pricing paradigms. In the first row of Figures 9 and 10, the determinacy regions for a relatively closed economy ($\gamma = 0.7$) are compared across different output gap targets for country U. As ϕ_y^U increases, the green determinacy region contracts, indicating that higher sensitivity to the output gap in country U reduces the flexibility of the other central banks to achieve determinacy. This effect is more pronounced in the DCP scenario, where the invoicing currency plays a dominant role, amplifying the international spillovers and interactions.

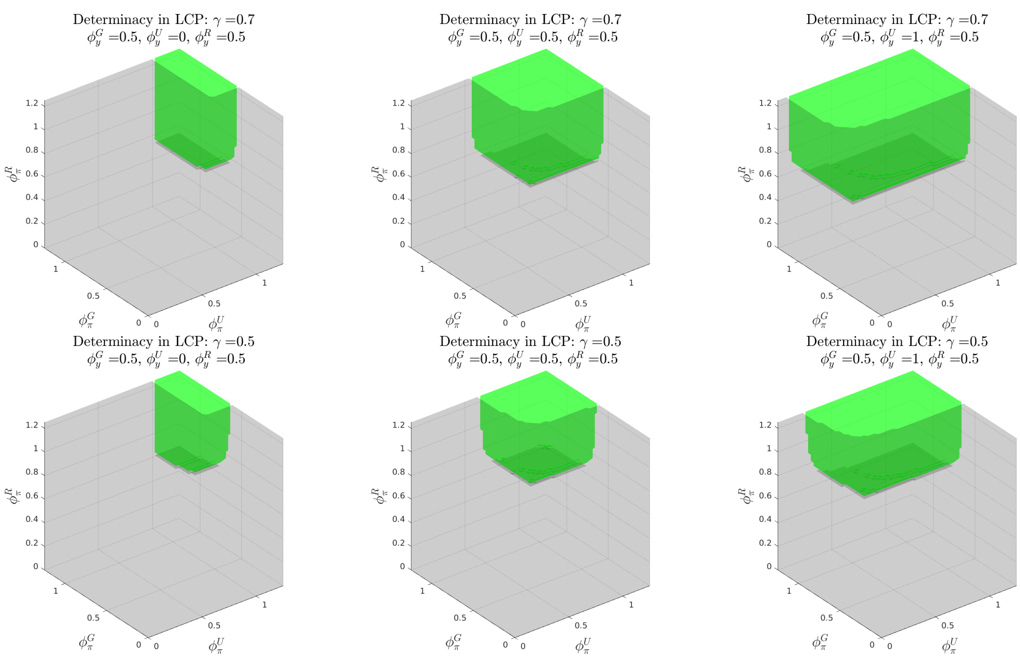
In the second row of Figures 9 and 10, depicting a more open economy ($\gamma = 0.5$), the determinacy regions show even greater contraction as ϕ_y^U increases. This further emphasizes the importance of economic openness in determining the policy flexibility available to central banks. The reduced size of the green area in these figures highlights that in open economies, the trade-offs between targeting inflation and output become more stringent, necessitating careful calibration of the Taylor rule parameters.

Figure 9: Determinacy in PCP



Determinacy regions when all three economies have different Taylor rule parametrization in PCP. The green area shows the Taylor rule parameters consistent with determinacy. The parameter that is adjusted between each subplot is ϕ_y^U, while $\phi_y^G = \phi_y^R = 0.5$. The first row indicates a more closed economy, $\gamma = 0.7$, whereas the second row shows a more open one, $\gamma = 0.5$. For each of them, the openness parameter is identical in all three regions.

Figure 10: Determinacy in LCP



Determinacy regions when all three economies have different Taylor rule parametrization in LCP. The green area shows the Taylor rule parameters consistent with determinacy. The parameter that is adjusted between each subplot is ϕ_y^U, while $\phi_y^G = \phi_y^R = 0.5$. The first row indicates a more closed economy, $\gamma = 0.7$, whereas the second row shows a more open one, $\gamma = 0.5$. For each of them, the openness parameter is identical in all three regions.

Comparing the three pricing paradigms, it is evident that DCP leads to the smallest determinacy regions, particularly in open economies. This is due to the dominant role of the currency in international trade, which intensifies the impact of foreign economic conditions on domestic policy effectiveness. In contrast, PCP and LCP provide relatively larger determinacy regions, suggesting that when prices are set in either the producer's or local currency, the central banks retain more control over their domestic economic conditions, even in a more open setting.

The findings from these figures underscore the complexity of achieving determinacy in a multi-country open economy model. Central banks must account for the interactions between their policy parameters and those of other economies, as well as the prevailing pricing paradigm and level of openness. The nuanced trade-offs illustrated by the varying shapes and sizes of the determinacy regions provide valuable insights for policymakers aiming to design robust monetary frameworks in an interconnected global economy.

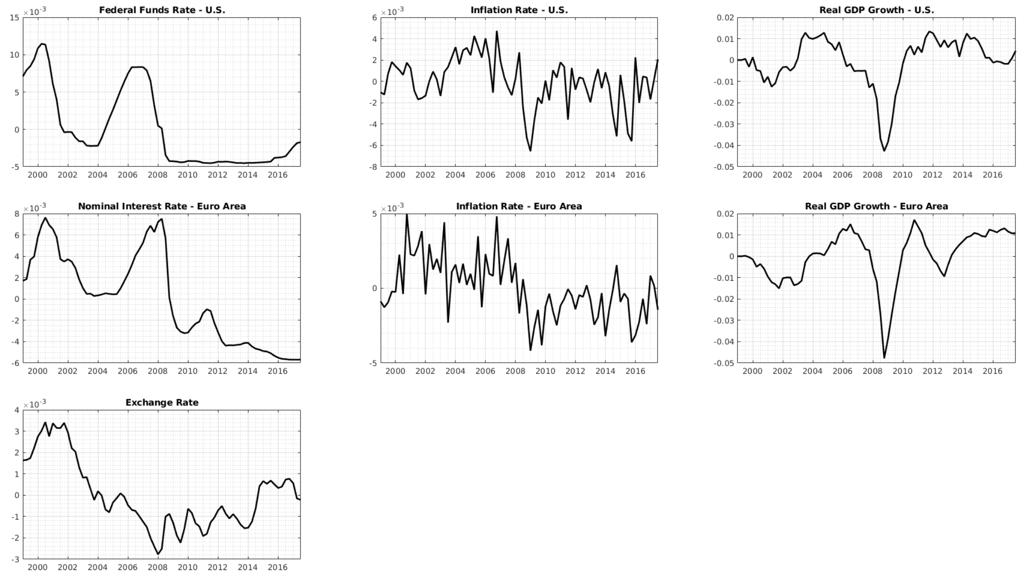
Overall, these analyses reveal that achieving a unique and stable rational expectations equilibrium requires a careful balance between inflation targeting and output gap responsiveness, especially in open economies with significant international trade. The choice of invoicing currency further complicates this balance, highlighting the need for coordinated and well-calibrated policy responses among central banks to ensure global economic stability.

4 Estimation

To analyze whether some of the more theoretical outcomes presented in the sections before arise in reality, I estimate the model on U.S. and Euro Area data. For this exercise, I take a small set of data and estimate key parameters for the three pricing paradigms by applying Bayesian techniques à la An and Schorfheide (2007). Overall, I use 7 macroeconomic data series' that include the nominal interest rate, inflation rate, and real GDP for the U.S. and the Euro Area. The last data series used is the real exchange rate between the Dollar and the Euro. Figure 11 shows all data used. The nominal interest rate for the U.S. is given by the Federal Funds Rate, while the one for the Euro Area is the Euribor 3-month series. Both inflation rates are given by the GDP deflator. These are also used to create real GDP, which is made stationary by taking out the trend using the one-sided HP-filter.

The prior distribution for the standard deviations of the structural shocks follows an Inverse Gamma with prior mean 0.01 and prior standard deviation of 0.05. This means that I stay relatively agnostic on which shocks are driving the business cycle in the model. Similarly, for the autoregressive com-

Figure 11: Estimation Data



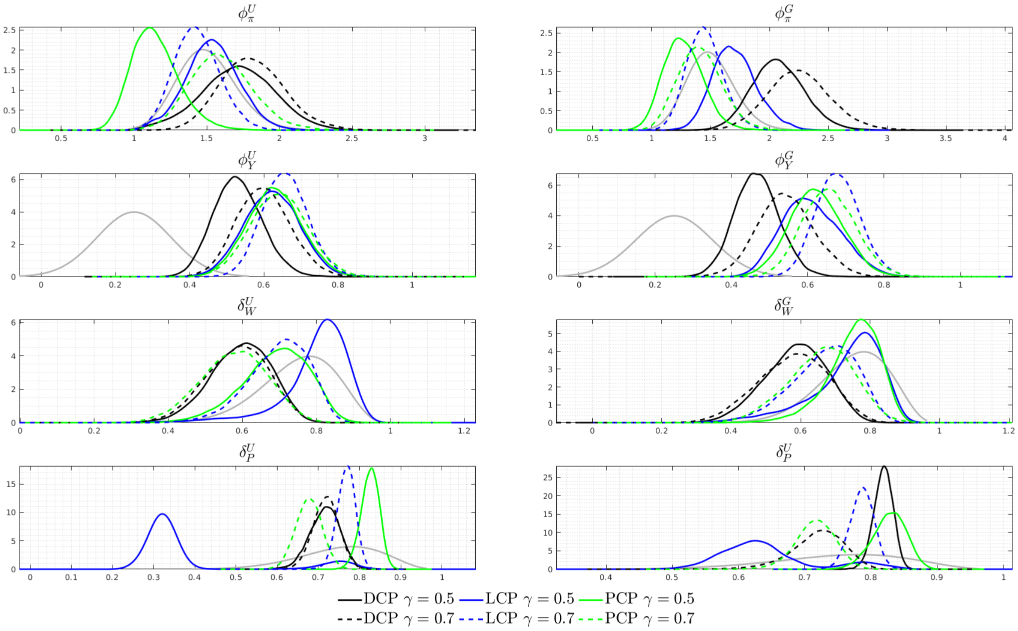
Data used in Bayesian estimation.

ponents of the AR(1) processes, I use a Beta distribution with prior mean 0.75 and prior standard deviation of 0.10. The same prior distribution is used for the wage and price rigidity parameters. The coefficients of the Taylor rules with respect to inflation are estimated with a Gamma distribution with prior mean of 1.5 and standard deviation of 0.20. The coefficients on output gap are estimated on a Normal distribution with prior mean 0.25 and prior standard deviation 0.10.

All other parameters, as is standard in the Bayesian estimation literature, are calibrated as in Table 1. During estimation, I focus only on parameters of two out of the three economies. These are the U.S. issuing the dominant currency, in case of DCP, and the Euro Area. Thus, I do not estimate parameters of the economy R as I do not include any data for it and thus would not add crucial information during the estimation step. These parameters are also calibrated as in Table 1 and thus follow the values used in Gopinath et al. (2020). Furthermore, parameter identification is checked with Dynare Adjemian et al. (2024), which implements various identification checks following for example Iskrev (2010) and Qu and Tkachenko (2012). According to these checks, I find that all parameters within all versions of the models analyzed, given the data explained above, are fully identified. Thus, the results can be interpreted and do not come from unidentified structural parameters.

In this exercise, I estimate in total 6 versions of the model. Each pricing paradigm (DCP, PCP,

Figure 12: Posterior Distributions



Prior and posterior distributions after Bayesian estimation. Solid lines depict relatively open models, with $\gamma = 0.5$, while dashed lines are relatively closed models with $\gamma = 0.7$. Black lines correspond to DCP, blue lines to LCP, and green lines to PCP.

LCP) is estimated for two separate levels of home bias ($\gamma = 0.5$, $\gamma = 0.7$) to include all necessary information from the sections above. For each, I calculate a chain with $500,000$ draws and calculate distributional statistics after discarding the first 50% of the draws. I mainly focus on the parameters analyzed before that include the central banks' decision parameters on inflation and output gap as well as the ones for wage and price rigidities.

All results for the prior and posterior distributions of the parameters and standard deviations of shocks can be seen in Tables 2 and 3. Reported are posterior modes as well as 90% HPD intervals as well as the prior distribution. As can be seen, the various pricing paradigms yield quite different posterior distributions for the estimated parameters.

Figure 12 presents the posterior distributions of the policy parameters ϕ_π^U, ϕ_π^G, ϕ_y^U, ϕ_y^G, δ_W^U, δ_W^G, δ_P^U and δ_P^G under three distinct pricing paradigms: Dominant Currency Pricing (DCP), Local Currency Pricing (LCP), and Producer Currency Pricing (PCP), and two levels of home bias ($\gamma = 0.5$ and $\gamma = 0.7$). These distributions provide insights into the probabilistic characteristics and uncertainty surrounding the policy parameters, influenced by the chosen pricing paradigm and the degree of home bias.

The top row of Figure 12 displays the posterior distributions for the inflation targeting parameters ϕ_π^U and ϕ_π^G. Under DCP (black lines), I find that the distribution of ϕ_π^U and ϕ_π^G are more spread out and moved to the right, indicating greater uncertainty and stronger focus in the inflation response parameter. This is consistent with the characteristics of DCP, where the central bank of the dominant currency country faces significant external spillovers, complicating the calibration of its policy response. For LCP (blue lines) and PCP (green lines), the distributions are more concentrated, reflecting more stable domestic inflation dynamics as these paradigms reduce the direct impact of exchange rate fluctuations on domestic prices. Generally, for both inflation targeting parameters, DCP yields distributions centered around larger values, with LCP in the middle, and PCP centered around the smallest values that has a sizable section of the distribution lower than one.

The difference in distributions under different home biases further highlights the complexity introduced by international trade structures. With a higher home bias ($\gamma = 0.7$, dashed lines), the distributions generally shift rightwards and become more peaked. This indicates that higher home bias allows for a stronger and more predictable inflation targeting response due to reduced exposure to foreign economic conditions. Only the LCP shows opposite behavior such that higher home bias yields distributions shifted to the left.

The second row of the figure shows the posterior distributions for the output gap targeting parameters ϕ_y^U and ϕ_y^G. The spread of ϕ_y^U under DCP is broader, reflecting the heightened uncertainty and the necessity for more flexible policy responses to stabilize the output gap in an economy heavily influenced by the dominant currency. The more concentrated distributions under LCP and PCP suggest that these paradigms provide more stable output gap dynamics, as prices are set in the local or producer's currency, mitigating the volatility induced by exchange rate fluctuations. For both parameters it holds that under DCP, the distributions are more shifted to the left, indicating that central banks under this paradigm put more emphasis on inflation rather than output targeting.

A similar trend is seen when considering the home bias parameter. Higher home bias ($\gamma = 0.7$) results in narrower distributions for ϕ_y^U and ϕ_y^G, indicating that domestic economic policies are more effective and predictable when the economy is less reliant on foreign goods and services.

The last two rows of Figure 12 illustrate the distributions for the wage stickiness parameter δ_W and the price stickiness parameter δ_P in both economies. Under DCP, δ_W^U and δ_W^G show broader - but centered around smaller values - distributions compared to LCP and PCP, indicating greater uncertainty and variability in wage adjustments such that they are adjusted less frequently. This variability reflects the complexity and influence of the dominant currency in trade, which introduces more volatility in wage setting. In contrast, under LCP and PCP, the distributions for δ_W^U and δ_W^G are more concentrated and peaked, suggesting more stable and predictable wage adjustments when prices are set in local or producer currencies. They are, furthermore, adjusted more frequently.

The effect of home bias on wage stickiness parameters is also evident. Under DCP, both countries wages are set equally frequently and does are not critically affected by openness. Under PCP and LCP, however, lower home bias pushes the distributions to the right. This shows the necessity of adjusting domestic behavior to international shocks and thus changing wages more frequently.

The price stickiness parameter δ_P exhibit a slightly different behavior. Under DCP, the distributions for δ_P^U do not seem to change with the level of home bias and are lower compared to δ_P^G, which does indicate different distributions for various levels of home bias. Under PCP, the behavior of the price stickiness parameters is very similar to the previous wage stickiness ones. Under LCP, on the other hand, prices get adjusted much less frequently when the economy becomes more open as all trade is than denominated in the local currency.

These posterior distributions provide valuable insights into the interaction between monetary policy parameters, pricing paradigms, and home bias. Under DCP, the central bank faces greater uncertainty and variability in setting policy parameters due to the dominant currency's pervasive influence on international trade and financial markets. This necessitates a more flexible and adaptive policy approach to achieve macroeconomic stability. In contrast, LCP and PCP offer more predictable and stable economic environments, as prices set in local or producer currencies are less susceptible to exchange rate fluctuations.

Higher home bias further stabilizes the economic environment by reducing dependency on foreign goods and services. This insulates the economy from external shocks, allowing for more precise and effective monetary policy implementation. Consequently, policymakers can achieve better control over inflation and output gap targets, resulting in more predictable wage and price dynamics.

Overall, these findings underscore the importance of considering both the pricing paradigm and the degree of home bias when formulating monetary policy in an open economy. Policymakers must account for these factors to design robust strategies that enhance economic stability and resilience against international shocks.

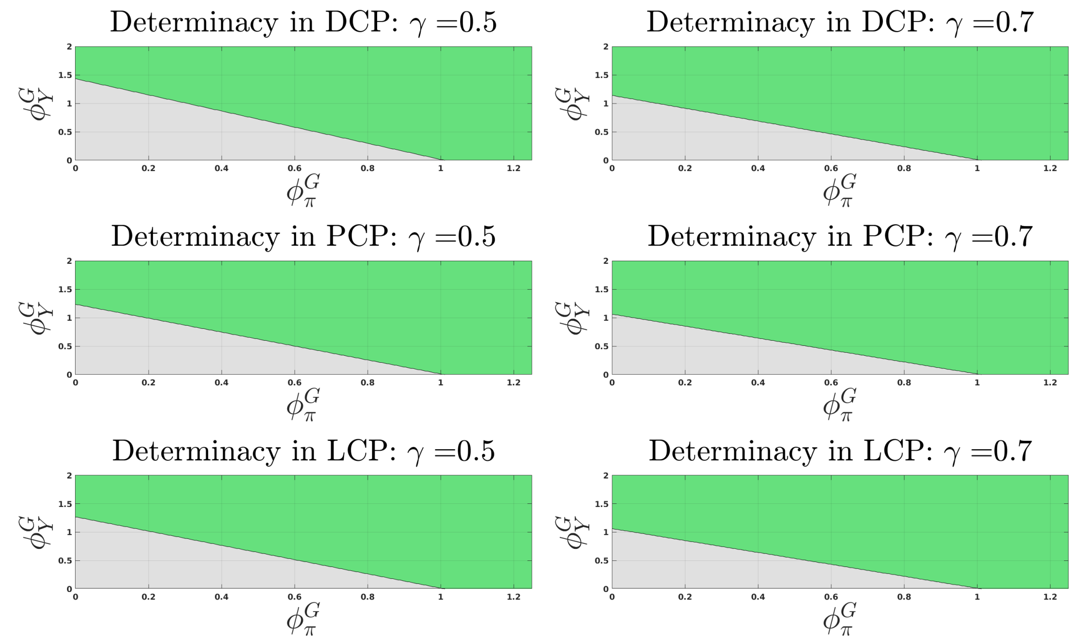
Table 2: Prior and Posterior Distribution - Parameter

			$\gamma = 0.5$						$\gamma = 0.7$					
			DCP		PCP		LCP		DCP		PCP		LCP	
Par.	Description	Prior	Mode	HPD	Mode	HPD	Mode	HPD	Mode	HPD	Mode	HPD	Mode	HPD
ϕ_π^U	U: Infl. Target	$\Gamma\,[1.5, 0.1]$	1.71	$[1.30, 2.13]$	1.15	$[0.88, 1.41]$	1.54	$[1.24, 1.85]$	1.81	$[1.45, 2.17]$	1.60	$[1.24, 1.94]$	1.43	$[1.18, 1.69]$
ϕ_π^G	G: Infl. Target	$\Gamma\,[1.5, 0.1]$	2.07	$[1.70, 2.43]$	1.27	$[1.00, 1.54]$	1.70	$[1.40, 2.01]$	2.25	$[1.82, 2.67]$	1.41	$[1.10, 1.71]$	1.45	$[1.20, 1.69]$
ϕ_Y^U	U: Output Target	$\mathcal{N}\,[0.25, 0.1]$	0.53	$[0.42, 0.63]$	0.63	$[0.51, 0.75]$	0.62	$[0.50, 0.74]$	0.60	$[0.48, 0.72]$	0.64	$[0.51, 0.76]$	0.66	$[0.55, 0.76]$
ϕ_Y^G	G: Output Target	$\mathcal{N}\,[0.25, 0.1]$	0.47	$[0.37, 0.56]$	0.62	$[0.51, 0.74]$	0.61	$[0.48, 0.73]$	0.54	$[0.42, 0.67]$	0.65	$[0.54, 0.77]$	0.68	$[0.58, 0.77]$
δ_P^U	U: Price Rigidity	$B\,[0.75, 0.1]$	0.71	$[0.65, 0.77]$	0.82	$[0.79, 0.86]$	0.38	$[0.26, 0.75]$	0.72	$[0.67, 0.77]$	0.68	$[0.62, 0.73]$	0.77	$[0.73, 0.80]$
δ_W^U	U: Wage Rigidity	$B\,[0.75, 0.1]$	0.59	$[0.46, 0.73]$	0.69	$[0.55, 0.83]$	0.80	$[0.68, 0.93]$	0.58	$[0.45, 0.72]$	0.58	$[0.62, 0.73]$	0.70	$[0.57, 0.83]$
δ_P^G	G: Price Rigidity	$B\,[0.75, 0.1]$	0.82	$[0.80, 0.84]$	0.83	$[0.78, 0.87]$	0.64	$[0.54, 0.80]$	0.72	$[0.66, 0.79]$	0.72	$[0.67, 0.76]$	0.79	$[0.76, 0.82]$
δ_W^G	G: Wage Rigidity	$B\,[0.75, 0.1]$	0.58	$[0.44, 0.73]$	0.75	$[0.64, 0.87]$	0.73	$[0.57, 0.89]$	0.58	$[0.41, 0.74]$	0.66	$[0.50, 0.81]$	0.67	$[0.52, 0.82]$
ρ_m^U	U: Inertia	$B\,[0.75, 0.1]$	0.51	$[0.42, 0.59]$	0.54	$[0.44, 0.64]$	0.57	$[0.47, 0.67]$	0.52	$[0.44, 0.61]$	0.54	$[0.44, 0.63]$	0.49	$[0.41, 0.57]$
ρ_m^G	G: Inertia	$B\,[0.75, 0.1]$	0.49	$[0.41, 0.58]$	0.54	$[0.43, 0.64]$	0.54	$[0.45, 0.62]$	0.59	$[0.51, 0.66]$	0.56	$[0.47, 0.66]$	0.50	$[0.41, 0.58]$
ρ_ε^U	U: MP-AR(1)	$B\,[0.75, 0.1]$	0.94	$[0.92, 0.96]$	0.87	$[0.80, 0.95]$	0.87	$[0.82, 0.92]$	0.93	$[0.90, 0.96]$	0.88	$[0.82, 0.93]$	0.93	$[0.91, 0.95]$
ρ_ε^G	G: MP-AR(1)	$B\,[0.75, 0.1]$	0.96	$[0.95, 0.98]$	0.86	$[0.78, 0.93]$	0.89	$[0.86, 0.93]$	0.93	$[0.90, 0.97]$	0.86	$[0.79, 0.93]$	0.93	$[0.90, 0.95]$
ρ_{uip}^G	G: UIP-AR(1)	$B\,[0.75, 0.1]$	0.98	$[0.97, 0.98]$	0.99	$[0.98, 0.99]$	0.80	$[0.72, 0.99]$	0.95	$[0.94, 0.97]$	0.94	$[0.92, 0.95]$	0.97	$[0.96, 0.98]$
ρ_a^U	U: TFP-AR(1)	$B\,[0.75, 0.1]$	0.82	$[0.77, 0.89]$	0.71	$[0.64, 0.78]$	0.72	$[0.60, 0.84]$	0.85	$[0.80, 0.91]$	0.82	$[0.75, 0.89]$	0.84	$[0.79, 0.88]$
ρ_a^G	G: TFP-AR(1)	$B\,[0.75, 0.1]$	0.91	$[0.87, 0.94]$	0.75	$[0.69, 0.82]$	0.84	$[0.78, 0.90]$	0.86	$[0.81, 0.92]$	0.86	$[0.80, 0.91]$	0.86	$[0.81, 0.90]$

Table 3: Prior and Posterior Distribution - Standard Deviations

			$\gamma = 0.5$						$\gamma = 0.7$					
			DCP		PCP		LCP		DCP		PCP		LCP	
Par.	Description	Prior	Mode	HPD	Mode	HPD	Mode	HPD	Mode	HPD	Mode	HPD	Mode	HPD
σ_m^U	U: MP-Shock	$\Gamma^{-1}\,[0.01, 0.05]$	0.003	$[0.003, 0.004]$	0.003	$[0.002, 0.003]$	0.003	$[0.002, 0.003]$	0.003	$[0.003, 0.004]$	0.003	$[0.002, 0.003]$	0.003	$[0.003, 0.004]$
σ_m^G	G: MP-Shock	$\Gamma^{-1}\,[0.01, 0.05]$	0.003	$[0.002, 0.004]$	0.003	$[0.002, 0.003]$	0.003	$[0.002, 0.003]$	0.003	$[0.002, 0.003]$	0.003	$[0.002, 0.003]$	0.003	$[0.002, 0.003]$
$\sigma_G^\$$	G: Debt-Shock	$\Gamma^{-1}\,[0.01, 0.05]$	0.014	$[0.011, 0.016]$	0.012	$[0.010, 0.014]$	0.007	$[0.004, 0.016]$	0.018	$[0.015, 0.021]$	0.013	$[0.010, 0.015]$	0.027	$[0.023, 0.032]$
σ_a^U	U: TFP-Shock	$\Gamma^{-1}\,[0.01, 0.05]$	0.007	$[0.005, 0.009]$	0.025	$[0.020, 0.029]$	0.006	$[0.005, 0.007]$	0.005	$[0.004, 0.006]$	0.005	$[0.004, 0.006]$	0.006	$[0.004, 0.007]$
σ_a^G	G: TFP-Shock	$\Gamma^{-1}\,[0.01, 0.05]$	0.006	$[0.005, 0.008]$	0.027	$[0.021, 0.032]$	0.005	$[0.003, 0.006]$	0.005	$[0.003, 0.006]$	0.006	$[0.004, 0.007]$	0.005	$[0.004, 0.007]$
σ_{uip}^G	G: UIP-Shock	$\Gamma^{-1}\,[0.01, 0.05]$	0.003	$[0.003, 0.004]$	0.003	$[0.003, 0.004]$	0.003	$[0.003, 0.004]$	0.003	$[0.002, 0.003]$	0.003	$[0.002, 0.003]$	0.003	$[0.003, 0.004]$

Figure 13: Determinacy Analysis after Estimation



Determinacy regions for inflation and output targeting parameters in economy G after key parameter have been estimated under DCP, PCP, and LCP. The green area shows the Taylor rule parameters consistent with determinacy. The first column indicates a more open economy, $\gamma = 0.5$, whereas the second column shows a more closed economy, $\gamma = 0.7$. For each of them, the openness parameter is identical in all three regions.

4.1 Determinacy After Estimation

In this last section, I combine all the work from above to conduct a policy relevant analysis. The purpose is to perform a determinacy analysis for a single economy after important parameters have been estimated. As such, I close the gap between the first part of this paper in which determinacy regions have been analyzed in a calibrated model and a more realistic case where the model fits real world economies better.

For this, I use the model estimated on U.S. and Euro Area data and check what implications this has on determinacy of the Euro Area (denoted as the home economy G). This exercise can be seen as a real-world application of the findings from this paper so far. Put differently, the central bank of the Euro Area - the ECB - uses data to estimate key macroeconomic parameters of the model and then checks what parameter space is still available to it that yields determinate worldwide equilibria.

Figure 13 shows this analysis with a special focus on the Taylor rule parameter in the home economy G (the Euro Area in this example). All other parameters of the other two economies for the various pricing paradigms are set as in Table 2. Hence, it can be assumed that the economies react quite differently compared to the more theoretical calibration used throughout Section 3, which makes this analysis important.

Compared to all investigations before, this one has to be analyzed with a bit more care. The crucial aspect is that now it is no longer possible to compare individual sub-plots of Figure 13 with each other or even different parts of this paper. This comes from the fact that each sub-plot is based on its own estimation and thus has completely different parameters except the Taylor rule ones shown. This makes it impossible to deduct inference between them as strong assumptions on pricing paradigm and level of economic openness were made prior to estimating the models.

What it can be used for is to show how different results a model can produce based on these two mentioned assumptions. As in this example, the ECB tries to navigate its policy parameter space to find valid combinations, it is important to take into account the full set of possibilities and how results depend on them.

The determinacy regions depicted in Figure 13 highlight the sensitivity of the economic system to the chosen policy parameters under different pricing paradigms and levels of openness. Analyzing the DCP scenario first, we observe that for more open economies ($\gamma = 0.5$), the green regions indicating determinacy are moderately restrictive. For more closed economies ($\gamma = 0.7$), DCP becomes the most restrictive among the three paradigms, showing limited flexibility in choosing inflation and output gap targeting parameters. This suggests that under DCP, achieving determinacy requires more precise calibration of the Taylor rule parameters, which can be attributed to the dominant currency's pervasive influence and the associated external spillovers.

Moving to PCP, the determinacy regions are the least restrictive for open economies ($\gamma = 0.5$), offering the greatest flexibility in policy choices. This reflects the stability provided by producer currency pricing, where prices are less affected by exchange rate volatility. However, for more closed economies ($\gamma = 0.7$), PCP is less restrictive than DCP but more restrictive than LCP, indicating a moderate level of policy flexibility. The broader determinacy regions under PCP suggest that the ECB could achieve a stable equilibrium with a wider range of policy combinations, reducing the risk of indeterminacy.

LCP presents the most restrictive determinacy regions for open economies ($\gamma = 0.5$), indicating that achieving determinacy under local currency pricing is challenging when the economy is highly open. However, for more closed economies ($\gamma = 0.7$), LCP becomes the least restrictive, offering the most flexibility in policy parameter choices. This indicates that local currency pricing provides significant insulation from exchange rate fluctuations in less open economies, leading to more stable economic conditions.

An interesting observation is that these findings contrast with the results from previous sections, where the determinacy region generally became smaller as economies became more open. Here, this trend only holds for LCP. For DCP and PCP, the determinacy regions do not uniformly shrink with increased openness, indicating a more complex relationship between openness and policy flexibility. This divergence underscores the necessity for a nuanced understanding of how pricing paradigms and openness interact to influence economic stability.

It is important to note that these comparisons are not apples-to-apples, as each subplot is based on its individually estimated model. Therefore, any inference drawn from these comparisons can only be superficial and should be interpreted with caution. Nevertheless, the exercise illustrates how sensitive the determinacy regions are to the underlying assumptions about pricing paradigms and economic openness.

In conclusion, this extended analysis integrates the theoretical insights and empirical estimations to present a comprehensive view of the challenges faced by central banks in achieving determinacy. By considering the specific characteristics of different pricing paradigms and levels of economic openness, policymakers can better navigate the complex landscape of monetary policy to ensure stable and predictable economic outcomes. This study emphasizes the importance of a nuanced approach to policy formulation, taking into account the interplay between international trade structures, pricing behaviors, and macroeconomic stability.

5 Conclusion

This paper has explored the conditions under which determinacy is achieved in multi-country DSGE models, focusing on the impact of different pricing paradigms — Dominant Currency Paradigm (DCP), Producer Currency Paradigm (PCP), and Local Currency Paradigm (LCP) — and the degree of economic openness. My findings indicate that the choice of pricing paradigm significantly influences the effectiveness of monetary policy and the attainment of a unique and stable equilibrium.

In particular, the Dominant Currency Paradigm introduces complexities due to its amplification of exchange rate effects on international trade, necessitating more aggressive policy responses to stabilize the economy. Conversely, the Producer Currency Paradigm and Local Currency Paradigm offer relatively larger determinacy regions, suggesting that these pricing regimes provide central banks with greater control over domestic economic conditions, even in highly open economies.

My analysis underscores the importance of considering the interplay between monetary policy rules, pricing paradigms, and economic openness when designing robust and adaptive policy frameworks. Policymakers must navigate the trade-offs between targeting inflation and output, especially in an interconnected global economy where international spillovers can significantly impact domestic stability.

The insights gained from this study contribute to the broader understanding of monetary policy determinacy and its implications for global economic stability. By highlighting the critical role of pricing paradigms and economic openness, I provide valuable guidance for central banks aiming to enhance the effectiveness and resilience of their monetary policy frameworks in a complex and dynamic international landscape.

References

Adjemian, Stéphane, Michel Juillard, Fréderic Karamé, Willi Mutschler, Johannes Pfeifer, Marco Ratto, Normann Rion, and Sébastien Villemot, "Dynare: Reference Manual, Version 6," Dynare Working Papers 80, CEPREMAP 2024.

An, Sungbae and Frank Schorfheide, "Bayesian Analysis of DSGE Models," *Econometric Reviews*, April 2007, *26*, 113–172.

Araújo, Eurilton, "Determinacy and learnability of equilibrium in a small-open economy with sticky wages and prices," *International Review of Economics & Finance*, September 2016, *45*, 16–32.

Barnett, William A. and Unal Eryilmaz, "Monetary Policy and Determinacy: An Inquiry into Open Economy New Keynesian Macrodynamics," *Open Economies Review*, April 2023, *34* (2), 217–253.

Blanchard, Olivier Jean and Charles M. Kahn, "The Solution of Linear Difference Models under Rational Expectations," *Econometrica*, 1980, *48* (5).

Bullard, James and Aarti Singh, "Worldwide macroeconomic stability and monetary policy rules," *Journal of Monetary Economics*, October 2008, *55*, S34–S47.

_ **and Kaushik Mitra**, "Learning about monetary policy rules," *Journal of Monetary Economics*, September 2002, *49* (6), 1105–1129.

Calvo, Guillermo A., "Staggered Prices in a Utility-Maximizing Framework," *Journal of Monetary Economics*, September 1983, *12* (3), 383–398.

Evans, George W. and Seppo Honkapohja, "Learning and Expectations in Macroeconomics," in "Learning and Expectations in Macroeconomics," Princeton University Press, 2001.

Farmer, Roger E. A., *The Macroeconomics of Self-fulfilling Prophecies*, MIT Press, 1999.

Galí, Jordi, *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework*, Princeton University Press, 2008.

_ **and Tommaso Monacelli**, "Monetary Policy and Exchange Rate Volatility in a Small Open Economy," *The Review of Economic Studies*, July 2005, *72* (3), 707–734.

Gopinath, Gita, Emine Boz, Camila Casas, Federico J. Díez, Pierre-Olivier Gourinchas, and Mikkel Plagborg-Møller, "Dominant Currency Paradigm," *American Economic Review*, March 2020, *110* (3), 677–719.

Ida, Daisuke, "Cost channel, determinacy, and monetary policy in a two-country new Keynesian model," *Economic Modelling*, February 2023, *119*, 106112.

Iskrev, Nikolay, "Local identification in DSGE models," *Journal of Monetary Economics*, March 2010, *57* (2), 189–202.

Karagiannides, Gabriel and Christos Liambas, "Determinacy and Taylor's Rule with Different Degrees of Trade Openness in a Semi-New Keynesian Model," *Applied Economics and Finance*, September 2019, *6* (6), 43.

Kimball, Miles S., "The Quantitative Analytics of the Basic Neomonetarist Model," 1995.

Klenow, Peter J. and Jonathan L. Willis, "Real Rigidities and Nominal Price Changes," *Economica*, 2016, *83* (331).

Llosa, Luis-Gonzalo and Vicente Tuesta, "Determinacy and Learnability of Monetary Policy Rules in Small Open Economies," *Journal of Money, Credit and Banking*, 2008, *40* (5).

Loisel, Olivier, "New Principles For Stabilization Policy," Technical Report, Center for Research in Economics and Statistics 2022.

Pesaran, Hashem, "The limits to rational expectations," *Oxford, Basil Blackwell*, 1987.

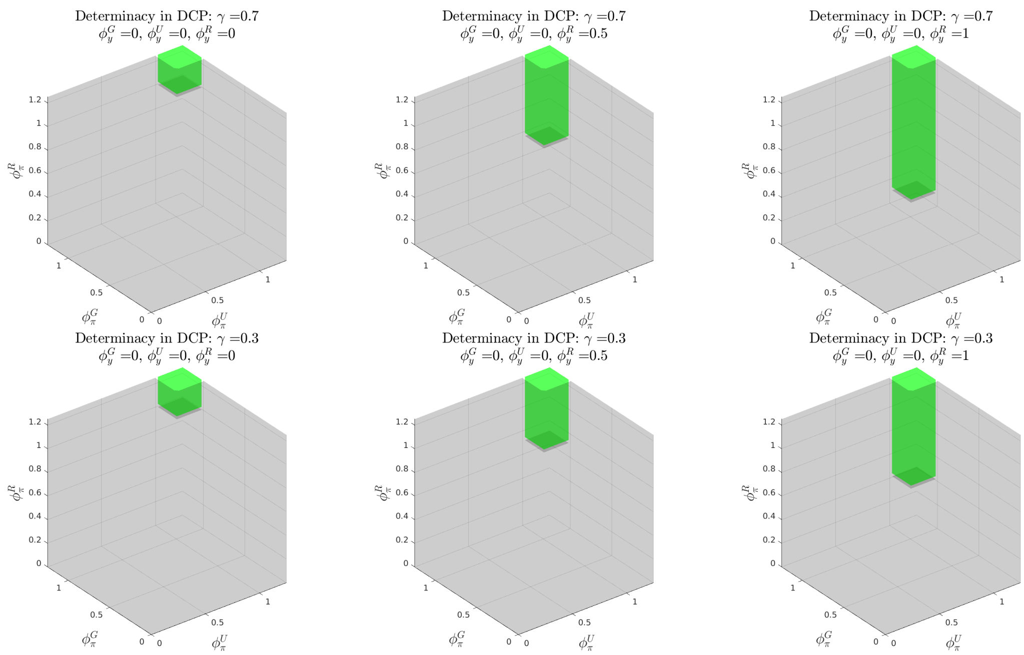
Qu, Zhongjun and Denis Tkachenko, "Identification and frequency domain quasi-maximum likelihood estimation of linearized dynamic stochastic general equilibrium models," *Quantitative Economics*, 2012, *3* (1).

Schmitt-Grohé, Stephanie and Martın Uribe, "Closing small open economy models," *Journal of International Economics*, October 2003, *61* (1), 163–185.

Zhao, Hong, "On the impacts of trend inflation in an open economy," *Journal of International Economics*, September 2022, *138*, 103648.

A Tables and Figures

Appendix Figure A1: Determinacy in DCP - Varying ϕ_y^R



Determinacy regions when all three economies have different same Taylor rule parametrization in DCP. The green area shows the Taylor rule parameters consistent with determinacy. The parameter that is adjusted between each subplot is ϕ_y^R, while $\phi_y^G = \phi_y^U = 0$. The first row indicates a more closed economy, $\gamma = 0.7$, whereas the second row shows a more open one, $\gamma = 0.7$. For each of them, the openness parameter is identical in all three regions.

B Derivation and details of the Kimball (1995) aggregator

All versions of Kimball (1995) used in the paper follow this set-up, in which time subscripts are left out for easier readability and derived using the cost minimization problem of the domestic household. The one for firms' intermediate inputs is derived in identical fashion.

Cost minimization:

$$PC = \min_{C_{iH}(\omega)} \sum_i \int_{\omega \in \Omega_i} P_{iH}(\omega) C_{iH}(\omega) d\omega, \text{with i } \in \{H, US, R\} \tag{22}$$

s.t.

$$\sum_i \frac{1}{|\Omega_i|} \int_{\omega \in \Omega_i} \gamma_i \Upsilon \left(\frac{|\Omega_i| C_{iH}(\omega)}{\gamma_{ij} C} \right) = 1 \tag{23}$$

The first order condition with respect to $C_{iH}(\omega)$ yields (where λ is the Lagrange multiplier):

$$P_{iH}(\omega) = \lambda \Upsilon'(\cdot)\frac{1}{C} \tag{24}$$

Now, the envelope theorem is used w.r.t. to C

$$P = \lambda \sum_i \int_{\Omega_i} \Upsilon'(\cdot)\frac{C_{iH}(\omega)}{C^2}d\omega \tag{25}$$

Combine the two equations

$$\frac{|\Omega_i|C_{iH}(\omega)}{\gamma_{ij}C} = \psi\left(\frac{P_{iH}(\omega)C}{\lambda}\right) \tag{26}$$

where we define $\psi = \Upsilon^{'-1}(\cdot)$.
Further

$$\lambda = \frac{PC}{\widetilde{C}}, \quad \widetilde{C} = \sum_i \int_{\Omega_i} \Upsilon'\left(\frac{|\Omega_i|C_{iH}(\omega)}{\gamma_{ij}C}\right)\frac{C_{iH}(\omega)}{C}d\omega \tag{27}$$

Plugging λ back yields the solution.

The functional form of of the function $\psi(\cdot)$ in Equation 4 is $\psi(\cdot) := \Upsilon^{'-1}(\cdot) > 0$, s.t. $\psi^{'}(\cdot) < 0$. The elasticity of demand is defined, following Gopinath et al. (2020), $\sigma_{ij,t}(\omega) := -\partial logC_{ij,t}(\omega)/\partial logZ_{ij,t}(\omega)$, where $Z_{ij,t}(\omega) := D_{j,t}(P_{ij,t}(\omega)/P_{j,t})$. This makes the log of the optimal flexible price markup time-varying and defined by: $\mu_{ij,t}(\omega) := log(\sigma_{ij,t}/(\sigma_{ij,t}-1))$. This makes $\Gamma_{ij,t}(\omega) := \partial\mu_{ij,t}/\partial logZ_{ij,t}(\omega)$ the elasticity of that markup.

The preference aggregators' functional form for the deamnd function $\Upsilon^{(}\cdot)$ follows the work of Klenow and Willis (2016) and as such introduces the following demand for individual variety ω:

$$Y_{ij,t}(\omega) = C_{ij,t}(\omega) + X_{ij,t}(\omega) = \gamma_i(1 + \epsilon ln\frac{\sigma-1}{\sigma} - \epsilon lnZ_{ij,t}(\omega))^{\sigma/\epsilon}(C_{j,t} + X_{j,t}), \tag{28}$$

where σ and ϵ are the parameters that govern the elasticity of demand and its variability:

$$\sigma_{ij,t}(\omega) = \frac{\sigma}{(1 + \epsilon ln\frac{\sigma-1}{\sigma} - \epsilon ln Z_{ij,t}(\omega))} \tag{29}$$

$$\Gamma_{ij,t}(\omega) = \frac{\epsilon}{(\sigma - 1 - \epsilon ln\frac{\sigma-1}{\sigma} + \epsilon ln Z_{ij,t}(\omega))} \tag{30}$$

As in Gopinath et al. (2020), this makes that in a symmetric steady state $Z_{ij,t}(\omega) = (\sigma-1)/\sigma$, where the elasticity of demand is σ and the elasticity of the markup is $\Gamma = \epsilon/(\sigma-1)$. Strategic complementarities and variable markups arise in all cases with $\epsilon > 0$, while $\epsilon = 0$ yields the constant elasticity case.

Appendix Table A1: Calibrated Parameters of Demand Aggregator

Parameter	Description	Value
σ	Elasticity of Demand	2
ϵ	Super-Elasticity of Demand	1

C Same Taylor Rule Calibration

To further analyze that the findings from Section 3.1 do not solely come from the fact that all three economies are calibrated to the same parameter values, I present here a brief analysis where there are slight differences. Namely, the price rigidity parameters δ_P are varied across the three economies with $\delta_P^U = 0.85$, $\delta_P^G = 0.75$, and $\delta_P^R = 0.65$. Thus, (sunspot) shocks are propagated differently through the economies and so, even if the central banks react with the same strength, there can be different adjustment paths back to steady state.

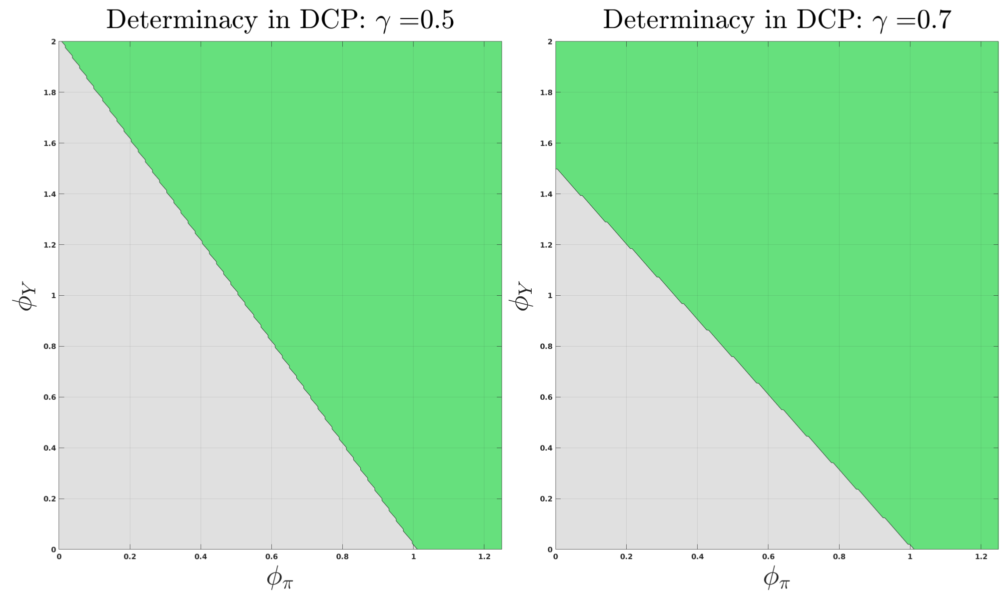
Figures A2 and A3 present the same plots as in Section 3.1 but now with this small adjustment to analyze whether the pricing paradigms effect on determinacy still hold.

It is still true that now, even with different propagation mechanisms, the determinacy areas look very similar when the economies are relatively closed as presented in the right columns.

Opening the economies up, the left columns, the same result as before still applies in that the pricing paradigms begin the make a difference. Again, the cases of DCP and LCP exhibit closer relationships compared to the more conventional PCP scenario.

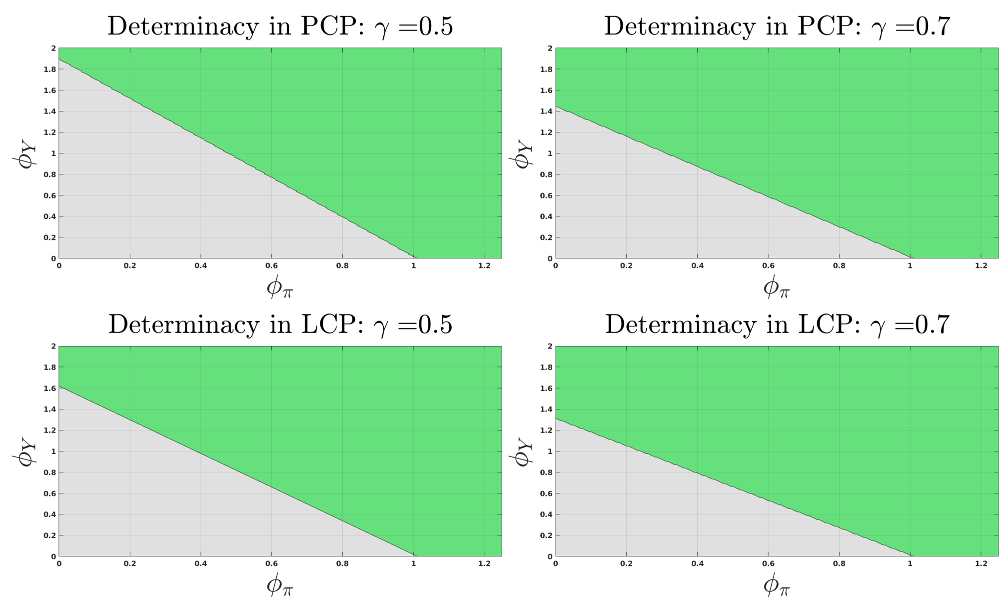
The point where the diagonal intersects the y-axis is shifted in all versions compared to Section 3.1. Thus, the results hold true qualitatively but, of course, change quantitatively as now the trade-offs and spillovers are adabted.

Appendix Figure A2: Determinacy in DCP - Same Calibration v2



Determinacy regions when all three economies follow the same Taylor rule under DCP. The green area shows the Taylor rule parameters consistent with determinacy. The left panel indicates a more open economy, $\gamma = 0.5$, whereas the right panel shows a more close economy, $\gamma = 0.7$. For each panel, the openness parameter is identical in all three regions.

Appendix Figure A3: Determinacy in PCP & LCP - Same Calibration v2



Determinacy regions when all three economies follow the same Taylor rule under PCP and LCP. The green area shows the Taylor rule parameters consistent with determinacy. The first column indicates a more open economy, $\gamma = 0.5$, whereas the second column shows a more close economy, $\gamma = 0.7$. For each of them, the openness parameter is identical in all three regions.